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Laís Martins Fracasso

Empirical Asset Pricing

The relationship between three categories of risk measures and the cross-section of expected stock returns

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Empirical Asset Pricing

The relationship between three categories of risk measures and the cross-section of expected stock returns

Dissertação apresentada à Escola de Administração da Universidade Federal do Rio Grande do Sul como requisito parcial para obtenção do grau de mestre em Administração.

Fernanda Maria Müller, Dra.
Advisor

Guilherme Kirch, Dr.
Committee Member

Henrique Pinto Ramos, Dr.
Committee Member

Marcelo Brutti Righi, Dr.
Committee Member

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Resumo

Este estudo examinou as evidências da relação entre treze medidas de risco e os retornos esperados de ações. Consideramos medidas que avaliam a perda, o desvio e ambos os conceitos de risco simultaneamente. Usamos os retornos das ações do mercado dos Estados Unidos para o período de Janeiro 1982 a Janeiro 2021. Utilizando os métodos de análise univariada de portfólio e de regressões *cross-sectional*, encontramos uma relação positiva entre os retornos esperados mensais e as medidas de risco analisadas, exceto pela *Expected Loss* (EL), que é negativa. Analisando a diferença dos retornos dos portfolios de maior risco e dos de menor risco, vemos que há diferença significativa para todas as medidas de risco testadas, exceto pela *Shortfall Deviation*. Também realizamos regressões de séries temporais usando os retornos dos três fatores de Fama-French e o fator Momentum como variáveis independentes. Os *Alphas* dessas regressões indicam que apenas a EL, o Desvio Padrão e o Semi-Desvio Negativo são estatisticamente significativos nas carteiras igualmente ponderadas. Nas carteiras ponderadas por valor, apenas a EL é significativa. Encontramos resultados semelhantes usando o método de regressões *cross-sectional*, em que apenas a *Shortfall Deviation* não apresentou coeficiente estatisticamente diferente de zero. No entanto, quando fatores de controle são incluídos no modelo, como os três fatores de Fama-French, o fator Momentum, o fator de Reversão de Curto Prazo e o fator de Volatilidade Idiossincrática, vemos que eles já explicam essa relação de que ações com maior risco atingem retornos mais altos e nenhuma medida de risco se mostra estatisticamente significativa para explicar os retornos esperados.

Palavras-chave: Retornos de ações. Precificação. Investimento em fatores. Medidas de risco. Medidas de Perda-Desvio.

Abstract

This study examined the evidence of the relation between thirteen risk measures and expected stock returns. We consider measures that assess the loss, deviation and both risk concepts simultaneously. We use United States market stock returns for the period January 1982 to January 2021. Using the single-sorting method and cross-sectional regressions, we find a positive relationship between the cross-section of expected returns and the risk measures analyzed, except for Expected Loss, which is negative. Analyzing the difference between the returns of the higher risk and lower risk portfolios, we see that there is a significant difference for all risk measures tested, except for Shortfall Deviation. We also perform time-series regressions using the returns of Fama-French three-factors and Momentum factor as independent variables. The Alphas of these regressions indicate that only the EL, the Standard Deviation and the Negative Semi-Deviation are statistically significant in the equally weighted portfolios. In the value-weighted portfolios, just the EL is significant. We find similar results using the method of cross-sectional regressions, in which only Shortfall Deviation did not present a coefficient statistically different from zero. However, when controlling factors are included in the model, such as the Fama-French three-factors, the Momentum factor, the Short-Term Reversal factor, and the Idiosyncratic Volatility factor, we see they already explain the finding that equities with higher risk achieve higher one-month-ahead returns and no risk measure proves to be statistically significant to explain the expected returns.

Keywords: Stock returns. Pricing. Factor investing. Risk measures. Loss-deviation measures.

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1 Introduction

The study of Markowitz (1952) on Portfolio Theory discusses how investors should use the principle of diversification to optimize their portfolios, considering that decisions on investment selection must be made according to the risk-return analysis. Based on the fundamentals of this theory, such as the behavior of investors concerning risk and distributions of risk and return, the Capital Assets Price Model (CAPM) has been independently developed by Sharpe (1964) and Treynor (1962), and extended by Lintner (1965a), Lintner (1965b), and Mossin (1966). This model establishes a theoretical relationship between asset returns that can be tested empirically by a simple formula. The single-period CAPM formula assumes a simple linear relationship in which the cost of capital equals the rate of return that investors demand as compensation for the market risk to which they are exposed.

Due to its easy applicability, empirical tests did not take long to appear with the works of Black et al. (1972), Fama and MacBeth (1973), Blume and Friend (1975), Basu (1977) and Banz (1981). These studies, however, do not empirically confirm the total validity of the CAPM formula. Thus, for some companies, the β factor (the slope in the regression of a security's return on the market's return) does not explain the cross-section of expected returns. These studies suggested the existence of additional relevant factors for asset pricing. Banz (1981), for example, contributed by adding the size effect to explain the cross-section of average returns. According to the author, smaller companies, measured by their market equity, ME (a stock's price times shares outstanding), would achieve higher returns, given their β estimates. But the average returns of larger companies would be lower. In this context, Merton (1973) proposed the Intertemporal Capital Asset Pricing Model (ICAPM), and Ross (1976) developed the Arbitrage Pricing Theory (APT). The latter suggests that the premium for the risk of the securities can be better explained, instead of only the market return used in CAPM, by several underlying factors, e.g., product, interest rate, inflation, and credit risk. The APT, however, did not clearly identify what these other explanatory factors would be.

Afterward, Fama and French (1992) found no significant cross-sectional relationship between the historical betas and the historical returns of over 2,000 shares between 1963 and 1990, revealing that the magnitude of the historical beta of a share is not related to the level of its historical return. They have also shown that even in a greater period with 50 years (1941-1990), a simple relation between β and average return is also weak. Therefore, they tested several factors identified in previous works with a strong role in explaining the cross-section of average stock returns, like the size effect of Banz (1981). Other previous works that could identify variables related to returns are Stattman (1980), Rosenberg et al. (1985) and Chan et al. (1991), who encountered a positive cross-sectional relation between expected returns and the book-to-market ratio, BE/ME (a firm's book value of common equity to its market value). Basu

(1983) discovered this relation in the earnings-price ratio (E/P) in tests that also include size and β, and Bhandari (1988) found an increase in explanation cross-section of average returns when leverage is included as an explanatory variable with size (ME) and β. According to the CAPM, the leverage should also be captured by the β. Fama and French (1992) evaluated the individual and joint roles of market β, size, E/P, leverage, and BE/ME in the cross-section of average returns on US-listed stocks. They concluded that the univariate relations between average return and these factors (except market β) are strong. The multivariate tests showed that the combination of size and book-to-market equity absorbed the roles of leverage and E/P.

Posteriorly, Fama and French (1993) expanded the previous study and used time-series regressions to test this three-factor model, including the market factor, size, and book-to-market and achieved good results in explaining the average returns on stocks. Since then, new studies have tested other variables, like the profitability factor by Novy-Marx (2013) and the expected investment by Aharoni et al. (2013), and detected that the three-factor model was incomplete. Both used cross-sectional regressions and sorting analysis. This evidence led Fama and French (2015) to expand their three-factor model with these two new elements (profitability and investment), creating the five-factor model. There are other similar papers, such as Hou et al. (2015) and Carhart (1997), which add a momentum factor. It is common for most of the explanatory variables chosen by researchers to be a set of firm-specific attributes, such as data extracted from financial statements or balance sheets.

In recent years, researchers are also using risk measures as independent variables to explain expected stock returns. We note these studies use as methodology cross-sectional regressions and sorting analysis. Some examples are Kelly and Jiang (2014), who proposed a new measure of time-varying tail risk that positively forecasts excess market returns. Zaremba (2019) included as measure the price range (the difference between previous maximum and minimum prices) and concluded that stocks with the highest price range outperform those with the lowest price range, i.e., higher volatility is compensated by higher returns. But Mohanty (2019) found that lower market risk results in higher excess return in 19 out of the 22 developed markets analyzed. Long et al. (2019) focused on the tail risk and verified a negative relationship between risk and expected stock returns. Atilgan et al. (2019) applied downside risk with six distinct metrics and found a negative relationship between systematic downside risk and the cross-section of equity returns. Bi and Zhu (2020) tested as independent variable the 1%, 5% and 10% Expected Shortfall (ES) and Value at Risk (VaR), but their relationship with expected returns varies according to the investor's sentiment levels. We can see that most studies focus on VaR and ES, and there are almost no studies that compare the results with a large number of risk measures. For studies that consider measures other than VaR and ES, we note these studies analyze separately the two main concepts of risk, which are the possibility of a negative outcome (a loss), and the variability in terms of an expected result (a deviation) (RIGHI; CERETTA, 2016; RIGHI, 2019).

To shed some new light on pricing literature, we reexamine the evidence of the relation between cross-section expected returns and risk measures. We use 13 risk measures that assess the loss, deviation, and both risk concepts simultaneously by using loss-deviation measures proposed by Righi (2019). The loss measures analyzed are Expected Loss, Value at Risk, Expected Shortfall, Expectile Value at Risk, Entropic, and Maximum Loss. The deviation measures are Standard Deviation, Negative Semi-Deviation, and Shortfall Deviation. Finally, as loss-deviation measures, we use Expected Loss Deviation, Shortfall Deviation Risk, Deviation Expectile Value at Risk, and Deviation Entropic. We also include controlling factors in the model, such as the Fama-French three-factors, the Momentum factor, the Short-Term Reversal factor, and the Idiosyncratic Volatility of stocks. We consider the Historical Simulation method and the significance levels (α) equal to 1% and 5% for the risk estimation. We perform the tests according to a non-parametric and a parametric technique, using data from 1982 to 2020 of listed companies from the US exchanges NYSE (New York Stock Exchange) and NASDAQ (National Association of Securities Dealers Automated Quotations). In the non-parametric method, we form portfolios based on the risk level of the companies and we compare their average returns. To compose the portfolios, stocks are sorted by their level of risk, according to the 13 previously presented measures and the controlling variables. The list of companies is divided into 5 portfolios. The first quintile contains stocks with the lowest risk and the fifth quintile, the highest risk. After that, we analyze whether there is a difference between the returns of each portfolio to determine its relationship with the risk measures. The parametric method consists of monthly cross-sectional regressions of Fama and MacBeth (1973) where the dependent variable is the one-month-ahead excess returns of each stock and the independent variables are the lagged 13 risk measures, analyzed independently and with the control variables. We calculate if the coefficients of the factors statistically differ from zero and evaluate the relationship between these risk measures and the expected stock returns.

Our study aims to contribute to the literature in three main ways. First, we are broadening the variety of risk measures already tested, including some that have never been tried as a pricing variable before, such as the Expectile Value at Risk and Entropic. We could not find papers that used these measures, even though new risk measures have been increasingly considered recently, such as the papers and variables cited in the previous paragraphs. Previous studies have focused their investigation on the use mainly of VaR and ES as predictors. See Atilgan et al. (2019), Atilgan et al. (2020), and Bi and Zhu (2020) for some investigations carried out. Second, we explore measures belonging to three categories: loss, deviation, and loss-deviation measures. Losses-based measures, like Value at Risk and Expected Shortfall, achieved more popularity due to major financial crises of recent decades, such as the Subprime crisis in 2008, for capturing the least likely losses, known as tail risks. However, historically, measures based on the dispersion of financial returns are the basis of portfolio theories since Markowitz (1952). To consider both dimensions of risk simultaneously, we include the loss-deviation measures, combining the characteristics of both groups. To the best of our knowledge, we are the first to apply in pricing this new approach of loss-deviation risk measures. According to Righi and Borenstein (2018), loss-deviation measures achieve higher performance than loss and deviation measures individually. It occurs because the deviation adds information to the loss measure, guaranteeing more solid protection. With many options to calculate risk and inconsistencies in empirical results of previous studies, it is of particular interest to identify, for the selected sample, which particular measure has greater predictive power and which category of risk measures has it. In addition, considering that previous studies diverged on the nature of the relationship between risk measures and cross-section expected returns – whether positive or negative –, this large number of risk measures can offer a better understanding of this relation by kind of risk measure. Third, we analyze the relationship between each risk measure and cross-section expected returns in different moments of the business cycles. This way, we can verify whether this relationship is maintained in recession periods and in periods of strong economy, or whether the relationship is inverted at each moment of the cycle. Bi and Zhu (2020) compared the relationship of Value at Risk and cross-section expected returns in different levels of investor sentiment and discovered that they are negatively related in high sentiment periods, but the relation is mixed during low sentiment periods. They use the BW investor sentiment index of Baker and Wurgler (2007) to evaluate the sentiment. Atilgan et al. (2020) also tested something similar, but splitting their sample based on the level of the Chicago Fed National Activity Index (CFNAI) and applying the macroeconomic uncertainty index (JNL) developed by Jurado et al. (2015). In this study, we prefer to use the recession periods classified by the Business Cycle Dating Committee of the National Bureau of Economic Research (NBER) and other periods of financial crises added by Danielsson et al. (2016), because they consider a wide range of monthly measures of aggregate real economic activity.

Besides these academic contributions, this study contributes to the nonacademic financial community. Several asset management companies worldwide are specializing in factor investing, an area being increasingly studied for the formation of investment fund portfolios based on specific factors. As this study was empirically performed and can be applied in practice, it aims to add the risk measures analyzed as options for possible factor-based investment portfolios.

The remainder of this study is structured as follows: Chapter 1 describes the concepts and historic review of the Capital Asset Pricing Model and factor models. Chapter 2 demonstrates some risk measures, properties and applications. Chapter 3 introduces the research data and methodology of the prediction model. Chapters 4 and 5 present the results and conclusions.

2 The CAPM and Factor Models

In this section, previous pricing models will be briefly reviewed. We demonstrate the chronology and main contributions of the CAPM and factor models analyzed here.

Markowitz (1952) was the first to introduce the concept of decision-making in finance from a risk-return relationship and diversification for finance, starting from some assumptions about investor behavior (RUBINSTEIN, 2011). First, all investors would want high and stable returns, not subject to variation. Investors would prefer more return to less return and less risk to more risk. These preferences would lead them to select the portfolio with lower risk for a given expected return or the portfolio with higher expected return for a level of risk. (MARKOWITZ, 1959).

Rubinstein (2002) also summarized Markowitz (1952) paper indicating that an investor should maximize expected portfolio return ($E[r_p]$) and minimizing portfolio variance of return (σ_p^2) at the same time. The most important is to analyze the contribution each security makes to the variance of the entire portfolio, which is primarily a question of its covariance with all the other securities in this portfolio. This follows from the relation between the variance of the return of a portfolio and the variance of return of its constituent assets.

Based on the work developed by Markowitz (1952), the CAPM was introduced by Sharpe (1964), Lintner (1965a), Mossin (1966), among other authors who improved it later. The authors added premises to those imposed by Markowitz to formulate the model:

- Investors have homogeneous expectations regarding return, risk and correlation between assets, and the distribution of asset returns follows a normal distribution;
- Individual assets are infinitely divisible, meaning that an investor can buy the fraction of stock he wants;
- Investors select diversified portfolios based on the expected return, the standard deviation and the correlation between the return on assets;
- Investors would be averse to risk, so they prefer less risk to more risk;
- Between two portfolios with the same risk, investors always prefer the one with the highest expected return;
- All investors decide at the same time;
- The market is in balance and there is no pressure for changes in the equilibrium price;
- There is a risk-free rate in the economy, the same for all investors, with which they can lend and borrow in the amount they wish;

- The same information is available to all investors at no additional cost;
- There are no transaction costs, laws, taxes, restrictions on short selling, or any other market imperfection.

CAPM occupies a fundamental place among the models for calculating the expected return on investment in risky conditions. Shortly, CAPM provides a linear relationship between the expected return of an asset or portfolio, in a balanced market, and the market portfolio. Capiński and Kopp (2014) highlighted that the two pieces of information are connected by a parameter commonly named beta (β), which affords a measure of undiversifiable risk of an asset or portfolio. This parameter (β) indicates the expected changes in the return of a portfolio or individual asset in response to the behaviour of the market. It is represented by the equation (2.1):

$$\beta_i = \frac{\sigma_{im}}{\sigma_m^2}, \tag{2.1}$$

where β_i is the beta of an asset or portfolio i, σ_{im} is the covariance of the returns of asset or portfolio i with the market portfolio and σ_m^2 is the variance of market portfolio returns.

Given the hypotheses of homogeneous expectations and the possibility of applying and capturing unlimited amounts at the risk-free rate, all investors will have the market portfolio. Thus, the investor will have a very diversified portfolio. As the investor is supposed to be concerned only with expected return and risk, the only dimensions of a security that should be of interest are expected return and beta.

The expected return of an individual asset or a portfolio is a linear function of the beta coefficient β_i, being represented by

$$E[r_i] = R_{rf} + \beta_i\,(E[R_m] - R_{rf}), \tag{2.2}$$

in which, $E[R_m]$ refers to the expected market return and R_{rf} is the risk-free asset. For a CAPM derivation we suggest Sharpe (1964) and Capiński and Kopp (2014).

This model is logical and intuitive based on a solid theoretical foundation and maintains a central place in academic studies and finance professionals such as portfolio managers, investment advisors and risk analysts. However, the hypotheses necessary for its construction are restrictive and have caused controversy over the years. Many studies have tested the CAPM empirically in different markets and periods and failed to find the relationship expected by the model. Because of these CAPM restrictions, Ross (1976) proposed a new and different approach to explain the formation of asset prices. The alternative view of the relationship between risk and return, presented by this author, is embodied in the arbitration arguments. The Arbitrage Pricing Theory is based on the single price law: two identical assets cannot be sold at different prices. Strong assumptions about preferences made in building the CAPM are unnecessary. The description of the balance by the APT is more general than that provided by the CAPM, in the sense that the formation of prices can be affected by other factors, besides the return of the market (BROWN et al., 2004). Empirical evidence (Chen et al. (1986), Burmeister and Wall (1986), Khoury (2015), for instance) suggests that factors that measure systematic responses to macroeconomic variables (inflation, interest rates, gross domestic product, foreign exchange, etc.) and related to the characteristics of the companies (size, book value/market value (BE/ME), price/earnings (P/E), etc.) are very relevant to explain expected returns.

Fama and French (1992) investigated the explanatory power of the returns of some factors associated with the characteristics of companies, such as size, book value/market value (BE/ME), leverage, earnings/price (E/P) ratio. The authors started from the premise that much of what happens in relation to the prices of assets traded in the market originates not only with market behavior, which is already measured by CAPM, but also with other variables. They found that such variables captured a relevant portion of the portfolio return not explained by the CAPM beta. Based on these results, the authors proposed the use of a three-factor model to explain the expected return on the asset or portfolio i:

$$E[r_i] = R_{rf} + \beta_m (R_m - R_{rf}) + \beta_{smb} SMB + \beta_{hml} HML, \tag{2.3}$$

in which β_m is the market loading factor (exposure to market risk, different from CAPM beta), β_{smb} is the size factor (the level of exposure to size risk), SMB is the small-minus-big (the size premium), β_{hml} is the value factor (the level of exposure to value risk) and HML is the high-minus-low (the value premium).

Fama and French (1993) formed the factors HML and SMB by ranking NYSE stocks in June each year between 1963 and 1991 according to their market value, segregating them into two groups called small (S) and big (B). Likewise, they ranked the same stocks according to the BE/ME ratio, segregating them into three groups: low (L), medium (M) and high (H). Group L included 30% of the stocks, classified as low BE/ME (growth stocks); group M, 40% of the stocks, with a medium BE/ME; finally, group H, with the stocks with the highest BE/ME (value stocks), representing 30% of the total. They then built six portfolios at the intersection of the two size groups with the three BE/ME groups. These portfolios were denominated S/L, S/M, S/H, B/L, B/M and B/H, with the S/L portfolio representing stocks classified simultaneously as small and low, and so on.

Thus, SMB is the average return on the three small stock portfolios minus the average return on the three big stock portfolios,

$$SMB = 1/3\,(S/H + S/M + S/L) - 1/3\,(B/H + B/M + B/L). \tag{2.4}$$

HML is the average return on the two value (high BE/ME) stock portfolios minus the average return on the two growth (low BE/ME) stock portfolios,

$$HML = 1/2\,(S/H + B/H) - 1/2\,(S/L + B/L). \tag{2.5}$$

Further studies (Novy-Marx (2013), and Titman et al. (2004), for instance) showed that many anomalies cause problems to the three-factor model, so the choice of profitability and investment was anchored by Gordon's dividend discount model and Miller and Modigliani (1961). Fama and French (2015) stated that there is much evidence that the average assets return is related to the BE/ME ratio, and evidence that profitability and investments increase the descriptive power of the BE/ME ratio. According to the authors, the dividend discount model can help explain the relationship between the profitability, investment and BE/ME variables with the average return, since the model presents the market value of an asset as the discount value of the expected dividends per share.

Therefore, Fama-French five-factor model is given by equation (2.6)

$$E[r_i] = R_{rf} + \beta_m (R_m - R_{rf}) + \beta_{smb} SMB + \beta_{hml} HML + \beta_{rmw} RMW + \beta_{cma} CMA, \tag{2.6}$$

in which β_{rmw} is the profitability factor, RMW is the robust-minus-weak (the profitability premium), B_{cma} is the investment factor and CMA is the conservative minus aggressive (the conservative investment premium).

Explaining thoroughly, RMW is the average return on the two robust operating profitability stock portfolios minus the average return on the two weak operating profitability stock portfolios. Similar to the three-factor model, these portfolios were denominated S/R, S/W, B/R, and B/W, with the S/R portfolio representing stocks classified simultaneously as small and robust, and so on.

$$RMW = 1/2\,(S/R + B/R) - 1/2\,(S/W + B/W). \tag{2.7}$$

CMA is the average return on the two conservative investment stock portfolios minus the average return on the two aggressive investment stock portfolios. Here, the portfolios were denominated S/C, S/A, B/C, and B/A, with the S/C portfolio representing stocks classified simultaneously as small and conservative, and so on.

$$CMA = 1/2\,(S/C + B/C) - 1/2\,(S/A + B/A). \tag{2.8}$$

Fama and French (2015) used data from 1963 to 2013 and tried some alternative portfolio formations. The natural construction could result in 25 portfolios given the possibilities of combinations of the factors (5 x 5). Another idea was to form two Size groups (small and big), using the median market cap for NYSE stocks as the breakpoint, and NYSE quartiles to form four groups for each of the other two sort variables, resulting in 2 x 4 x 4 = 32 portfolios. They could also form 81 portfolios (3 x 3 x 3 x 3) resulting from the division by three of each factor, but it would produce poorly diversified portfolios.

3 Risk and Deviation Measures

In this chapter, risk, deviation and loss-deviation measures will be further addressed, with definitions, properties and examples.

Consider the random result X of any asset where $X \geq 0$ is a gain, $X < 0$ is a loss and $E[X]$ is the expected value of X. Respectively, $\operatorname{ess\,inf} X$ and $\operatorname{ess\,sup} X$ refers to the essential infimum and essential supremum of X. F_X is the probability function of X and its inverse (left) F_X^{-1} is defined as $F_X^{-1}(\alpha) = \inf\{x : F_X(x) \geq \alpha\}$. Let $L^p := L^p(\Omega; \mathcal{F}; \mathbb{P})$, with $1 \leq p \leq \infty$, be the space of equivalence classes of random variables defined by the norm $\|X\|_p = (E[|X|^p])^{\frac{1}{p}}$ with finite p and $\|X\|_\infty = \inf\{k : |X| \leq k\}$. $X \in L^p$ indicates that $\|X\|_p < \infty$. We define $X^- = \max(-X, 0)$.

A risk measure is defined as $\rho : L^p \to \mathbb{R} \cup \{\infty\}$ and may comply with these properties:

Monotonicity: if $X \leq Y$, then $\rho(X) \geq \rho(Y), \forall X, Y \in L^p$, i.e., if one investment yields worse results than another one, the risk of the former should be greater;

Translation Invariance: $\rho(X + C) = \rho(X) - C, \forall X \in L^p, C \in \mathbb{R}$, i.e., if a certain gain is achieved in an investment, its risk decreases by the same amount;

Sub-additivity: $\rho(X + Y) \leq \rho(X) + \rho(Y), \forall X, Y \in L^p$, i.e., the risk of a combined position is smaller than the sum of the individual risks of the assets in the portfolio;

Positive Homogeneity: $\rho(\lambda X) = \lambda\rho(X), \forall X \in L^p, \lambda \geq 0$, i.e., the risk increases according to the increase in the size of an investment;

Convexity: $\rho(\lambda X + (1 - \lambda)Y) \leq \lambda\rho(X) + (1 - \lambda)\rho(Y), \forall X, Y \in L^p, 0 \leq \lambda \leq 1$, i.e., the risk of a diversified investment is less or equal to the weighted average of the individual risks of the assets in the portfolio. This is a well-known property of functions that can be understood as a relaxed version of sub-linearity;

Law Invariance: if $F_X = F_Y$, then $\rho(X) = \rho(Y), \forall X, Y \in L^p$, i.e, two investments with the same probability function have the same risk;

Limitedness: $\rho(X) \leq -\operatorname{ess\,inf} X = \operatorname{ess\,sup} -X, \forall X \in L^p$, i.e., the risk of an investment is never greater than the maximum loss.

A risk measure that fulfills the properties of Monotonicity, Translation Invariance, Sub-additivity and Positive Homogeneity is known as coherent, in the sense proposed by Artzner et al. (1999). Sub-additivity and Positive Homogeneity imply that the risk measure also respects Convexity. Given that the measure respects Monotonicity, Translation Invariance and Convexity, it is known as convex, in the sense formalized by Frittelli and Gianin (2002) and Föllmer and

Schied (2002). Coherent and convex risk measures comply Limitedness. See Righi (2019). For more details regarding the properties above, we refer Delbaen (2012).

Some examples of risk measures can be mentioned, as follows, for $0 \leq \alpha \leq 1$, where α is the significance level, and for $\forall\ X \in L^p$.

- Expected Loss (EL): $EL(X) = E[-X]$;
- Value at Risk (VaR): $VaR^{\alpha} = -F_X^{-1}(\alpha)$;
- Expected Shortfall (ES): $ES^{\alpha}(X) = E[-X|X \leq F_X^{-1}(\alpha)]$;
- Expectile Value at Risk (EVaR): $EVaR^{\alpha} = -\arg\min_{\gamma} E[|\alpha - 1_{X\leq\gamma}|(X-\gamma)^2]$;
- Entropic (ENT): $ENT^{\theta}(X) = \frac{1}{\theta}\log(E[e^{-\theta X}]), \theta \geq 0$;
- Maximum Loss (ML): $ML(X) = -\text{ess}\inf X = \text{ess}\sup -X$.

The first measure, EL, refers to the expected value of position. The next measure, VaR, is a canonical tail risk measure and represents the α-quantile of X. It is the maximum expected loss given a certain significance level and period. This is the only one among the illustrated measures that does not satisfy Convexity or Sub-additivity. VaR fulfills Monotonicity, Translation Invariance, Positive Homogeneity and Law Invariance, among the properties mentioned. Expected Shortfall (called Tail-VaR by some authors) represents the expected value of a loss, given it exceeds the quantile of interest, – the VaR. Expectile Value at Risk is the only coherent risk measure beyond the mean loss that possesses the property of elicitability[1]. It is more sensitive to extreme losses compared to VaR, since the tail probability is determined by the underlying distribution. Entropic is a convex, but not coherent, risk measure. ENT depends on the θ, which represents the parameter of the investor's risk aversion through the exponential utility function. Finally, the Maximum Loss is the more conservative risk measure, since it represents the most extreme loss of a financial position.

Now we define a deviation measure and some of its properties.

A deviation measure is a functional $\mathcal{D} : L^p \rightarrow \mathbb{R}^+ \cup \{\infty\}$ that may fulfill these properties:

Non-negativity: For all $X \in L^p, \mathcal{D}(X) = 0$ for constant X, and $\mathcal{D}(X) > 0$ for non-constant X, i.e., there is dispersion only for non-constant investments;

Translation Insensitivity: $\mathcal{D}(X + C) = \mathcal{D}(X), \forall\ X \in L^p, C \in \mathbb{R}$, i.e., the deviation does not change if a constant value is included;

Sub-additivity: $\mathcal{D}(X + Y) \leq \mathcal{D}(X) + \mathcal{D}(Y), \forall\ X, Y \in L^p$, i.e., the risk of a combined position is less than the sum of individual risks;

[1] A functional is named elicitable when it is the minimizer of expectation of some score function (ZIEGEL, 2016).

Positive Homogeneity: $\mathcal{D}(\lambda X) = \lambda\mathcal{D}(X), \forall X \in L^p, \lambda \geq 0$, i.e., the risk increases according to the increase in the size of an investment;

Lower Range Dominance: $\mathcal{D}(X) \leq E[X] - \text{ess inf } X, \forall X \in L^p$, i.e., it restricts the measure to an interval below the interval between expectation and the minimum value;

Law Invariance: if $F_X = F_Y$, then $\mathcal{D}(X) = \mathcal{D}(Y), \forall X, Y \in L^p$, i.e., two investments with the same probability function have the same risk.

If the deviation measure fulfills Non-negativity, Translation Insensitivity, Positive Homogeneity and Sub-additivity it is known as a generalized deviation measure, in the way proposed by Rockafellar et al. (2006).

Some examples of deviation measures are:

- Standard Deviation (StD): $StD(X) = \|(X - E[X])\|_p$;
- Semi-Deviation (SD^-): $SD^-(X) = \|(X - E[X])^-\|_p$;
- Shortfall Deviation (SD^α): $SD^\alpha(X) = \|(X + ES^\alpha(X))^-\|_p$.

Standard Deviation quantifies the dispersion of returns around the expectation, by averaging the distances between potential returns and their mean value. Negative Semi-Deviation is an alternative deviation measure in relation to StD and VaR but restrains only to the lower part of expected value. Shortfall Deviation is a dispersion truncated by ES. This measure refers to the standard deviation of results that represent losses larger than the ES. The three measures are generalized deviation measures.

A combination of these risk and deviation measures was proposed by Righi (2019) with the formulation $\rho + \mathcal{D}$, maintaining the theoretical properties that are essential to risk measures theory. The author could relate Limitedness with Monotonicity and Lower Range Dominance. He highlights that normally it is obtained $\mathcal{D}(X) \leq -\rho(X) - \text{ess inf } X$, i.e., the dispersion term considers "financial information" from the range between the loss represented by ρ and the maximum loss $-\text{ess inf } X = \text{ess sup} -X$. With that, the author can state this combination is again a coherent risk measure. Under Translation Invariance, one can think in $\rho(X) + \mathcal{D}(X)$ as $\rho(X')$, where $X' = X - \mathcal{D}(X)$, i.e., a real valued penalization on the initial investment X.

Let $\rho : L^p \rightarrow \mathbb{R} \cup \{\infty\}$ and $\mathcal{D} : L^p \rightarrow \mathbb{R}^+ \cup \{\infty\}$. Then:

- if ρ fulfills Sub-additivity (Convexity) and Limitedness, then it possesses Monotonicity;
- if ρ fulfills Translation Invariance and Monotonicity, then it possesses Limitedness;
- if $\rho + \mathcal{D}$ is a coherent (convex) risk measure, then $\mathcal{D}$ possesses Lower Range Dominance;

- If ρ is a coherent risk measure and $\mathcal{D}$ a generalized deviation measure, then $\rho + \mathcal{D}$ is a coherent risk measure if and only if it fulfills Limitedness;
- If ρ is a convex risk measure and $\mathcal{D}$ a generalized deviation measure, then $\rho + \mathcal{D}$ is a convex risk measure if and only if it fulfills Limitedness.

Therefore, we present below examples of loss-deviation measures that respect these premises. The role of β is similar to an aversion term. It chooses the proportion of dispersion that should be included and can range between 0 and 1 ($0 \leq \beta \leq 1$).

- Expected Loss Deviation (ELD): $ELD(X) = E[-X] + \beta\|(X - E[X])^{-}\|_p$;
- Shortfall Deviation Risk (SDR): $SDR^{\alpha}(X) = ES^{\alpha}(X) + \beta\|(X + ES^{\alpha}(X))^{-}\|_p$;
- Deviation Expectile Value at Risk (DEVaR): $DEVaR^{\alpha}(X) = EVaR^{\alpha}(X)$ $+\beta\|(X + EVaR^{\alpha}(X))^{-}\|_p$;
- Deviation Entropic (DENT): $DENT^{\theta}(X) = ENT^{\theta}(X) + \beta\|(X + ENT^{\theta}(X))^{-}\|_p$.

Expected Loss Deviation combines the Expected Loss with the Negative Semi-Deviation. According to Righi and Ceretta (2016), Shortfall Deviation Risk is the expected loss that occurs with a probability aggravated by the dispersion of losses worse than this expectation. It associates the Expected Shortfall with the Shortfall Deviation. Last, the Deviation Expectile Value at Risk and the Deviation Entropic also penalizes their counterpart's risk measures EVaR and ENT by adding a deviation penalty. The mathematical proofs for the Loss-Deviation measures are omitted and can be consulted at Righi (2019).

4 Method

This section presents the method used in the empirical study, detailing the data analyzed, models and evaluation criteria.

4.1 Data Description

The source of daily and monthly stock prices for returns and shares outstanding is Thomson Reuters Datastream. The quarterly accounting data, such as shareholders' equity, are also from Thomson Reuters Datastream. The monthly returns of portfolios formed by the Fama-French three-factors – excess returns on the market, size, and value – and Momentum factor by Carhart (1997) are from Kenneth R. French's data library (FRENCH, 2020). They are used as independent variables in the regressions to calculate the so-called Jensen's alpha in the sorting analysis, which are exposed in Section 4.4. Excess returns are the return difference between the stock's raw return and the one-month US T-bill rate at time $t + 1$ available in the Federal Reserve database. This reference as the risk-free rate is widely used in literature, as seen in the works by Fama and MacBeth (1973), Burmeister and Wall (1986), and Bali and Cakici (2004). French (2020) also considers this rate in his online database.

We include listed companies from the major United States (US) stock exchanges: New York Stock Exchange (NYSE) and Nasdaq Stock Market (NASDAQ). This study focuses mainly on the US market motivated by data availability and close correlation with global indicators. The United States represents by far the most important and largest equity market worldwide, and its data is the most used in previous studies that seek to verify the relationship between risk and expected return, such as Bi and Zhu (2020) and Atilgan et al. (2020). Thus, since we are evaluating risk measures not previously considered, we can compare the results found here with those identified by them.

The research period runs from January 1, 1982, to January 31, 2021. We consider as dependent variable the one-month-ahead stock returns, so the period of analysis for this variable runs from February 1982 to January 2021, while the independent variables run from January 1982 to December 2020. This period contains turbulent markets and less volatile periods. Some examples of global financial crisis within these dates are the 1987 Black Monday, the 2008 Subprime mortgage crisis, and the 2020 COVID-19 outbreak. In contrast, the periods from 1983 to 1986 and from 1991 to 1997, among others, did not present any recessions, according to the National Bureau of Economic Research ((NBER), 2020). According to Zarnowitz (1992), cycles are attributes of open economies with highly developed commerce and industry, division of labor, and use of credit. The business cycle theory was developed in the early twentieth century because the empirical study of fluctuations across periods of prosperity and decline showed a regularity in the characteristics of these cycles (COOLEY; PRESCOTT, 2021). The National Bureau of Economic Research (NBER) (2020) maintains a chronology of US business cycles, identifying the dates of peaks and troughs that frame economic recessions and expansions. Zarnowitz (1992) says that the NBER reference chronologies began in 1834 for the United States. Given that the stock market reflects the economy's movements, the cycles are also highlighted in the historical analysis of stock returns. According to NBER, in the period from January 1982 to January 2021, there are five recession periods (period between a peak of economic activity and its subsequent trough, or lowest point), which are (i) July 1981 to November 1982, (ii) July 1990 to March 1991, (iii) March 2001 to November 2001, (iv) December 2007 to June 2009, and (v) February 2020 to April 2020. Danielsson et al. (2016) added the stock market crash of October 1987 to January 1988 and the LTCM/Russian crisis from August 1998 to November 1998. In this work, we compare all the results obtained in the months in the five recession periods of NBER and the two more of Danielsson et al. (2016) – called crisis periods – with the results of the remaining months, which are considered the normal or non-crisis periods.

At every two years, considering a rolling scheme, we form sub-samples with daily stock prices. We use this scheme because we consider a rolling estimation window of 250 observations to predict risk measures. So, to have a forecast year (approximately 250 observations), we need approximately a previous year of data, 250 observations. Thus, we change the list of stocks every two years to consider all companies that remained listed during these two years of sampling, even if they were delisted later, avoiding survivorship bias (ATILGAN et al., 2019). To ensure that small and illiquid stocks do not drive results, we clean the data following the previous literature: (i) We include only the common stocks, not considering depository receipts, real estate investment trusts, preferred shares, investment funds, and other stocks with special characteristics (LONG et al., 2019). (ii) We exclude stocks with an average price in the period analyzed of less than \$5 (ATILGAN et al., 2019). (iii) We exclude stocks for which the ratio of nonzero return days to the number of trading days is less than 80% (LONG et al., 2019). (iv) To avoid outliers and unrealistically extreme daily returns, instead of a winsorization applied in other studies, we exclude stocks with a standard deviation of daily returns higher than 1000% in each 2-year period or daily returns higher than 100%. Long et al. (2019) removed daily returns greater than 200%, but we were more restrictive as we realized that a few stocks affected the averages a lot. The final sample contains an average of 2,605 stocks for every 2 years. After sample selection, we compute daily returns for each stock. The stocks kept in the daily database were the same ones selected in the monthly database.

4.2 Variables

This study is performed with a monthly focus, although daily data were used to calculate some variables. The objective is to analyze the relationship of the risk measures, described in more detail in this section, with one-month-ahead stock returns[1] ($t + 1$).

To investigate the relationship between risk and expected equity returns, we use 13 distinct measures to quantify the risk, including (i) six risk measures (Expected Loss, Value at Risk, Expected Shortfall, Expectile Value at Risk, Entropic, and Maximum Loss), (ii) three deviation measures (Standard Deviation, Negative Semi-Deviation, and Shortfall Deviation), and (iii) four loss-deviation measures (Expected Loss Deviation, Shortfall Deviation Risk, Deviation Expectile Value at Risk, and Deviation Entropic). These measures are defined in Chapter 3. This selection of measures is chosen because it represents different classes and levels of complexity and contains very common measures widely used in the literature and their loss-deviation counterparts.

Risk measures are estimated using a non-parametric method, known as historical simulation (HS). According to Pérignon and Smith (2010), HS does not create assumptions about the data distribution and is the most used method by banks and literature. This approach, in pricing problem, is also considered by Bali et al. (2009) and Atilgan et al. (2020). We consider this approach because it is common in predicting risk measures (see Kuester et al. (2006)).

As pointed out in the previous section, we consider a rolling estimation window of 250 daily observations to compute the 13 risk measures. Thus, for each day in the out-sample period, we use the last 250 observations to estimate the risk measures. The choice for this rolling window is based on Basel Committee on Banking Supervision (2013), which recommends considering at least 250 observations to estimate risk measures. We conduct out-of-sample risk estimation using R programming language (R Core Team, 2020). We use the *riskR* (RIGHI, 2015) to estimate the risk measures by non-parametric approach. As α, we use 1% and 5%, because it is usually considered in risk forecasting (RIGHI; CERETTA, 2015; MÜLLER; RIGHI, 2018) and pricing studies (ATILGAN et al., 2019). After estimating the risk measures, we consider the arithmetic mean of the daily data for each month in the sample. Bi and Zhu (2020) and Atilgan et al. (2020) also use daily observations to get a monthly risk estimate.

To examine the possibility that a set of firm-specific attributes impacts the relationship between the risk measures and cross-sectional expected returns, we include them as controlling variables. The first attributes considered are the Fama-French three-factors (market beta, size, value). Bi and Zhu (2020) and Long et al. (2019) also used the three-factors model in their studies. Besides these, the Momentum factor, the Short-Term Reversal factor, and Idiosyncratic Volatility factor are also tested. These variables are also considered because they are widely used in literature, highlighting the works of Zaremba (2019), Atilgan et al. (2019) and Kelly and Jiang (2014).

We calculate the market beta (beta factor), the size, and the value factors similarly to Bi

[1] The expression one-month-ahead stock returns, so as in asset pricing literature, is used as synonym of expected returns.

and Zhu (2020). The market beta (BETA) is calculated as:

$$(R_{i,d} - R_{rf,d}) = \alpha_i + \beta_{i,d}(R_{m,d} - R_{rf,d}) + \epsilon_{i,d}, \qquad d = 1, ..., D_y, \tag{4.1}$$

where $(R_{i,d} - R_{rf,d})$ is the excess return of stock i in day d, $(R_{m,d} - R_{rf,d})$ is the market excess return on day d, and D_y is the number of trading days in year y. To estimate the market beta, it is considered a rolling estimation window of 250 observations. Therefore, the last 250 observations (approximately 1 business year) are used to obtain the first estimate. After estimation, for each month in the sample, the arithmetic mean of the daily market betas is computed.

The Size factor (SIZE) is the natural logarithm of the market value of equity in millions USD at the end of each month t. The market value of a stock is calculated as the number of shares outstanding times the stock's price at the end of the month. According to Bali et al. (2016), the natural logarithm is preferred because the distribution of market value is highly skewed. The Value factor (VAL) is the natural logarithm of equity's ratio book value/market value (BE/ME) at the end of the month t. The book value is calculated using the shareholders' equity of common equity for the firm's fiscal year ending in the prior calendar year. Like Fama and French (1992), to guarantee that the accounting variables are known before the returns they are used to explain, we correspond the BE/ME calculated for all fiscal year-ends in calendar year $t - 1$ with the returns for July of year t to June of $t + 1$. In this way, we may consider that book value is available 6 months after the reporting date. This gap between fiscal year-end and the return tests is conservative, as firms are required to file their financial reports within 90 days of their fiscal year-ends.

The Momentum factor (MOM) is measured as Jegadeesh and Titman (1993), using the 11-month cumulative return of the stock during the period beginning 12 months before and ending one month before the measurement date, covering months $t - 11$ through $t - 1$. This lagged momentum return, skipping the month before the holding month $t + 1$, is standard in momentum tests, according to Fama and French (2012). This convention of excluding the stock return during the month t from the calculation of momentum aims to separate the medium-term momentum effect from the short-term reversal effect (BALI et al., 2016).

The Short-Term Reversal factor (STR) uses the one-month-lagged return for each stock, similarly to Jegadeesh (1990). According to Bali et al. (2016), this factor is important because stock's last month returns tend to have a negative cross-sectional relation with returns over the next period. Thus, a strategy that could generate positive returns is to buy recent losers and sell recent winners.

The last factor measured is the Idiosyncratic Volatility (IVOL), the standard deviation of idiosyncratic daily returns within month t. According to Bi and Zhu (2020), the return residuals are calculated as in Equation 4.1, where $\epsilon_{i,d}$ is the idiosyncratic return on day d for stock i and D_t is the number of trading days for month t. Then, the IVOL of stock i in month t is defined by,

$$IVOL_{i,t} = \sqrt{var(\epsilon_{i,d})}, \qquad d = 1, ..., D_t. \tag{4.2}$$

4.3 Descriptive Statistics

This section presents the basic summary statistics for the variables described in Section 4.2 to provide to the reader a strong understanding of the sample used. The procedures adopted in this section follow Bali et al. (2016) and they were used in the works of Bali et al. (2009), Bollerslev et al. (2015), and Atilgan et al. (2020). The variables, such as the 13 risk measures (e.g., Expected Loss and Value at Risk) and the controlling factors, are X. We define as t the period and as i stocks in the sample. Thus, the variable X for stock i during period t will be referenced using $X_{i,t}$. Data are organized in panels containing as entries a combination of the basic statistics and period (months, in this study) for each variable analyzed. The data corresponding to any period is called a cross-section. For each month (cross-section data), we calculate the mean ($Mean$), standard deviation (SD), skewness ($Skew$), kurtosis ($Kurt$), minimum (Min), median (Med), and maximum (Max) of the stock returns. Then, for each statistic, we compute the time-series averages of the cross-sectional values. For simplicity, we present in this study only the averages of each statistic.

Explaining in more detail, the calculation procedure is executed in two steps. In the first step, called "periodic cross-sectional summary statistics", we assemble one panel for each variable described in section 4.2 and also for the monthly stock excess returns. For better understanding, we include as a variable in the statistics the excess return of stock i in a month t, calculated as the return of stock i in a month t minus the return of the risk-free security in a month t. Each panel combines the statistics (columns) and the months t in the sample (rows). Thus, considering that the statistics are displayed in the same order, in the panel of the variable Size factor, for example, the element of the first column ($Mean$) and first row (January 1982) displays the average of the Size factors in January 1982 of all stocks in the sample for this year.

The second step is called "average cross-sectional summary statistics" and consists of the time-series averages of the periodic cross-sectional values calculated in the first step. It is simply the average of all data in each column of each panel of X. In the previous example, we would compute the average of all Size factor values of the first column $Mean$ and then following to the other columns (SD, Med, ...). So, the last row in the panel would be the averages over all periods t of each statistic.

To fully understand how the variables analyzed relate to the expected results, we also calculate the correlations between each variable and the stock returns in $t + 1$. The correlation procedure consists of two steps, very similar to the steps of the summary statistics procedure. In the first step, called "periodic cross-sectional correlations", we calculate the correlation between each variable described in section 4.2 in each period t with the stock returns in period $t + 1$, forming a panel of correlations with the variables in the columns and the periods t in the rows. For example, the first column could be the correlation between the variable BETA in t and stock returns in $t + 1$ and each row of this column would be this correlation calculated month by month, starting with January 1982 in the first row. These correlations are calculated using the non-parametric method Spearman rank correlation, which is most applicable when the relation between the variables is thought to be monotonic, but not necessarily linear (BALI et al., 2016).

Likewise, the summary statistics procedure, the second step calculates the time-series averages of the periodic cross-sectional correlations between each variable and the returns. It is called "average cross-sectional correlations" and averages all the values in each column.

4.4 Portfolio Analysis

This section is dedicated to analyzing the relationship between the expected returns and the variables of this study using portfolios or sorting analysis, as it is also named. We refer to the independent variables (or sort variables) as X and the dependent variable (or outcome) as Y. In this study, the sort variables X to form the portfolios are the 13 risk measures describe in section 4.2, calculated with both α of 1% and 5%. The outcome Y is the expected stock excess returns (one-month-ahead stock returns minus the risk-free rate). We use the univariate portfolio analysis, likewise Long et al. (2019) and Bi and Zhu (2020). Following the method elucidated by Bali et al. (2016), the univariate procedure has four steps: (i) calculate the breakpoints to divide the stocks used into portfolios, (ii) form the portfolios, (iii) compute the average expected returns within each portfolio for each period t, and (iv) examine the average values of outcome Y across the portfolios.

In the first step, we create the breakpoints. They are used to divide the sample into portfolios. We rank all stocks by each sort variable X (risk measures), from lowest to highest, and then define cut-off points along that ranking, forming different portfolios. The breakpoints are calculated for each period t and they are determined by increasing percentiles. For example, if the X analyzed is VaR and we want to divide the sample into five portfolios (quintiles). Stocks should be classified from the lowest risk to the highest and then divided in 5 equal groups. Here, the breakpoints are 20%, 40%, 60%, and 80%. Stocks with a risk lower than the first breakpoint (20%) are placed in the first portfolio, and so on. Therefore, we have one breakpoint less than the number of portfolios formed since, in the example given, the last portfolio will be formed by the stocks with greater risk than the last breakpoint. Also, if a stock has a value of $X_{i,t}$ exactly equal to a breakpoint, it must be included in both portfolios before and after this breakpoint.

According to Bali et al. (2016), the most common is to use between 3 and 20 portfolios, with the majority preferring 5 or 10 (quintiles or deciles). Portfolios also can be formed with a different number of assets by portfolio. It is frequent in financial empirical studies to form smaller portfolios with extreme values, maintaining the middle breakpoints more distant between them. Following Long et al. (2019) and Bollerslev et al. (2015), our portfolios are divided into quintiles.

The second step is to form the portfolios. To compose them, stocks are sorted by their level of risk, according to the 13 previously presented measures. The first quintile contains stocks with the lowest risk and the fifth quintile, the highest risk.

After forming the portfolios, the third step is to compute the average value of the expected return Y for each portfolio in each period t. While Bollerslev et al. (2015) focused on a simple average, Bi and Zhu (2020), Atilgan et al. (2020), and Long et al. (2019) considered both equal-weighted and value-weighted returns. In the latter case, they used market capitalization as weight variable. We use as Y the one-month-ahead excess stock return (r_{t+1}) of each portfolio, which allows us to identify whether there is a significant relationship between riskier portfolios and excess returns. It is expected that the portfolios with high-risk equities also achieve higher returns and vice-versa. To verify this, we calculate for each t the difference between the average value of Y of the last and the first portfolios:

$$\bar{Y}_{Diff,t} = \bar{Y}_{n_p,t} - \bar{Y}_{1,t} \tag{4.3}$$

where $\bar{Y}_{n_p,t}$ is the average value of Y of the last portfolio (with the highest values) and $\bar{Y}_{1,t}$ is the average value of Y of the first portfolio (with the lowest values). We call the $\bar{Y}_{Diff,t}$ as the average value of the difference portfolio. Using our previous example, we calculate the VaR of each stock at the end of each month t and form 5 portfolios (quintiles). We consider then that we have bought each portfolio by the stock prices of the last trading day of month t (portfolio formation). The portfolios are held without further trading for the entire month $t + 1$ (holding period) and sold at the closing price of its last trading day. We compute the average excess returns for each of the 5 equal-weighted portfolios and for the difference portfolio. We then repeat it with the other months. The procedure for value-weighted portfolios is similar, but instead of a simple average, a weighted-average of excess returns is used. The weights in each portfolio are determined by the market capitalization measured as of the end of the month t.

The fourth and last step is to proceed with the calculations of the time-series means of the period average values of Y for each portfolio, and the difference portfolio. Next, we use the Wilcoxon Test to assess whether the median of the distribution of the difference portfolio differs statistically from zero. The advantage of the Wilcoxon Test is that it does not assume that the data is normally distributed. If the average return of the difference portfolio is statistically nonzero, it is evidence that a cross-sectional relationship exists between X and Y.

We also want to test if the results remain after controlling for a set of firm-specific factors. For that, we use time-series regression with the Fama-French three-factors and the Momentum factor of Carhart (1997) as independent variables to adjust the portfolio returns previously calculated:

$$r_{p,t} = \alpha + \beta_m (R_{m,t} - R_{rf,t}) + \beta_{smb} SMB_t + \beta_{hml} HML_t + \beta_{mom} MOM_t + \epsilon_t \tag{4.4}$$

where α is the Jensen's alpha, explained further. This model is known as the Fama, French, and Carhart (FFC) four-factor model, and Bali et al. (2016) refers to this model as one of the three most commonly used risk model. Previous works, such as Atilgan et al. (2020), also used this four-factor model to check whether it could explain differences in returns across the five portfolios (quintiles).

For the estimation of Equation (4.4), we use the Newey and West (1986) adjustment because, often, the values used during the time-series regressions may exhibit autocorrelation and/or heteroscedasticity. The important result of the regressions is the so-called Jensen's alpha (the intercept coefficient) (JENSEN, 1968), which indicates the abnormal results not due to sensitivity to the factors included in the model. We can investigate if the portfolio generates average abnormal returns by analyzing whether the α statistically differs from zero, using the t-statistics and p-value associated with the intercept coefficient. If not, the risk measure used to form the portfolio is not explaining the returns (BALI et al., 2016).

4.5 Regression Analysis

The advantage of the portfolio analysis, presented in the previous section, is that it is a non-parametric technique, which requires no assumptions about the relationship of the analyzed variables. The disadvantage is that it allows fewer control variables (BALI et al., 2016). Another solution to examine the relationship between the risk measures and return, with the appropriate control of other variables that could influence in the results, is the regression analysis proposed by Fama and MacBeth (1973). However, in this case, we have to assume the nature of the relationship between variables, which usually is linear.

This procedure follows Bali et al. (2016) and it is implemented in two steps: (i) compute regressions using data from each period t (cross-sectional regression), and (ii) analyze the time series of each of the regression coefficients created in the first step to determine whether the average coefficient differs from zero and analyze the coefficient of determination.

In the first step, we run a regression of the dependent variable Y, which is the one-month-ahead excess return of the given stock ($r_{i,t+1}$), on the independent variables X for each period t. The independent variables X are the previously presented 13 risk measures (calculated with α of 1% and 5%) and the controlling variables. The risk measures are included independently (one by one) and we regress them just with the market beta and also with all the controlling variables, following the regression specifications below:

$$\begin{aligned} r_{i,t+1} = \delta_{0,t} + \delta_{1,t}\,\mathcal{D}_{i,t} + \delta_{2,t}\,BETA_{i,t} + \delta_{3,t}\,SIZE_{i,t} + \delta_{4,t}\,VAL_{i,t} \\ +\delta_{5,t}\,MOM_{i,t} + \delta_{6,t}\,STR_{i,t} + \delta_{7,t}\,IVOL_{i,t} + \epsilon_{i,t+1}, \end{aligned} \tag{4.5}$$

$$\begin{aligned} r_{i,t+1} = \delta_{0,t} + \delta_{1,t}\,\rho_{i,t} + \delta_{2,t}\,BETA_{i,t} + \delta_{3,t}\,SIZE_{i,t} + \delta_{4,t}\,VAL_{i,t} \\ +\delta_{5,t}\,MOM_{i,t} + \delta_{6,t}\,STR_{i,t} + \delta_{7,t}\,IVOL_{i,t} + \epsilon_{i,t+1}, \end{aligned} \tag{4.6}$$

$$r_{i,t+1} = \delta_{0,t} + \delta_{1,t}\rho *_{i,t} + \delta_{2,t} BETA_{i,t} + \delta_{3,t} SIZE_{i,t} + \delta_{4,t} VAL_{i,t} + \delta_{5,t} MOM_{i,t} + \delta_{6,t} STR_{i,t} + \delta_{7,t} IVOL_{i,t} + \epsilon_{i,t+1}, \quad (4.7)$$

where $\delta_{0,t}$ is the intercept, $\delta_{1,t}, \delta_{2,t}$, etc. are the slope coefficients, $\mathcal{D}$ represents one deviation measure (StD, SD^-, or SD^α) ρ represents one loss measure (EL, VaR, ES, EVaR, ENT, or ML) and $\rho*$ represents one loss-deviation measure (ELD, SDR, DEVaR, or DENT). For simplicity, the equations are summarized, but they represent one equation by risk measure. We also calculate the R-squared (R_t^2), and adjusted R-squared ($AdjR_t^2$) of each period t. Likewise, Atilgan et al. (2019), the cross-sectional regressions for each t are estimated using the ordinary least squares (OLS) method, using the R function *lm* from *stats* package.

These monthly cross-sectional regressions produce time-series averages of slope coefficients, R_t^2, and $AdjR_t^2$. The average slope coefficient is the time-series average of the monthly regression slopes. This leads us to the second step: to verify if the average coefficient is statistically different from zero. With that in mind, we calculate the standard errors and the associated t-statistics and p-values to test the null hypothesis (average coefficient equal to zero), all adjusted by Newey and West (1986). If the results indicate a statistically significant average slope coefficient, there is a relationship between the cross-section expected returns and the independent variables. We considered 1%, 5% and 10% as significance levels. If the coefficient differs from zero (significant) using one risk measure as an independent variable but it is not significant when we add the controlling factors; then, the relationship between X and Y seems explained by some linear combination of the added control variables.

Once found if there is some relationship between X and Y, it is also important to determine the level of this explanation by analyzing the averages R_t^2 and $AdjR_t^2$. These indicators directly provide the percentage of explanation for the variation in expected stock returns explained by the independent variables used in the regression. For example, if the R_t^2 of a regression using VaR as X is 0,024, it means VaR explains only 2,4% of the variation in expected stock returns.

Regardless of the method used (Portfolio or Regression Analysis), we expect that there is a significant relationship of each risk measures analyzed with the one-month-ahead stock returns. We assume this relationship to be positive, as riskier assets are likely to achieve higher returns to compensate investors for the risk incurred. Focusing on loss measures, Atilgan et al. (2020) translates this presumption into the left-tail risk framework saying that, in the presence of under-diversification pertaining to higher-order moments of the return distribution, stocks with higher left-tail risk would be anticipated to have lower prices in compensation for the higher probability and magnitude of large losses associated with them. Hence, it is expected to find higher returns from stocks with higher left-tail risk. Bali and Cakici (2004) found a strong positive relationship between average returns and VaR, being robust for different investment horizons and loss probability levels. Chen et al. (2014) also tested VaR and could find a positive relationship with returns, but in a less developed stock market – Taiwan's stock market. As for the deviation measures, Ghysels et al. (2016) says that one version of the ICAPM of Merton (1973) states that the expected excess equity market return is positively related to its conditional variance. Guo and Whitelaw (2006) estimated a variant of Merton's ICAPM and found a positive and statistically significant estimated coefficient of relative risk aversion.

5 Results

This section presents the descriptive statistics of the data in the analysis and the results of the two empirical procedures adopted in this dissertation, parametric and non-parametric.

5.1 Descriptive Statistics

The variables analyzed in this study included the 13 risk measures calculated with significance levels (α) equal to 1% and 5%, and, as controlling factors, the Fama-French three-factors (BETA, SIZE, and VAL), the Momentum factor (MOM), the Short-Term Reversal factor (STR) and the Idiosyncratic Volatility factor (IVOL) for the period from January 1982 to December 2020. All these variables were used as independent variables to explain the one-month-ahead stock returns (r_{t+1}). As the dependent variable is considered one-month-ahead, the analysis period of this variable runs from February 1982 to January 2021. Each month is calculated the mean ($Mean$), standard deviation (SD), skewness ($Skew$), kurtosis ($Kurt$), minimum (Min), median (Med), and maximum (Max) values of the cross-sectional distribution of each of these variables. We present in Table 1 the time-series means for each cross-sectional value.

The monthly results of the stocks in our sample have a mean of 1.3%. However, a high standard deviation of 12.3%, indicating there are times when returns are positive, and at other times they are negative, offsetting each other, following business cycles. This also may be seen in the minimum of -60% and a maximum of 131%. The cross-sectional distribution of r_{t+1} is usually highly skewed and leptokurtic, as stated by Bali et al. (2016) because the minimum possible return is -100%, but there is no upper bound. Figure 1 shows the 12-month moving average of sample stock returns. The shaded areas mark these crisis periods of NBER and Danielsson et al. (2016), detailed in Section 4.1. We prefer to use moving averages of monthly returns to smooth the chart line and make recession and expansion periods more evident, highlighting the cycles. It is important to emphasize that the crisis periods do not match exactly their beginnings with the peak of the graph line and their endings with the trough because the NBER's Business Cycle Dating Committee determines the months of peaks and troughs based on a range of monthly measures of aggregate real economic activity published by the federal statistical agencies.

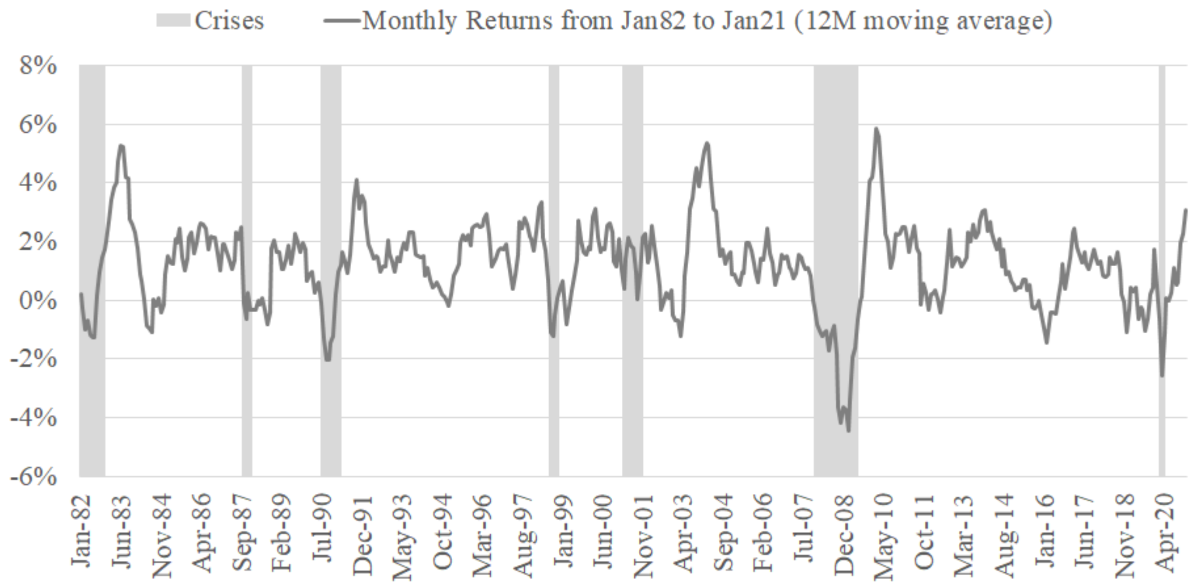
The cross-sectional distribution of the risk measures has positive means, except for Expected Loss, which is -0.1%. As expected, the deviation measures have values lower than the other categories of measures, as they do not assess the loss but quantify the variability of the data about a central value. We verify that lower significance levels imply higher average risk values. Lower levels are associated with higher losses, which justifies this result. We see that, for the same α, the VaR means are lower than the ES means, which are lower than the SDR means. This is likely to happen because VaR is the maximum expected loss given a significance

Table 1 – Descriptive Statistics of monthly expected returns and the independent variables. The sample period extends between January 1982 and January 2021.

	Mean	*SD*	*Skew*	*Kurt*	*Min*	*Med*	*Max*
Dependent Variable *(Values in monthly percentage)*							
r_{t+1}	1.3	12.3	155.1	2182.4	-60.7	0.5	131.5
Loss Measures *(Values in monthly percentage)*							
EL	-0.1	0.2	-0.5	7.2	-1.1	-0.1	0.7
VaR 1%	6.5	3.2	1.6	8.3	0.7	5.8	28.0
VaR 5%	3.9	1.9	1.4	6.6	0.2	3.5	14.5
ES 1%	8.7	4.8	1.9	9.8	1.0	7.5	43.3
ES 5%	5.7	2.8	1.5	7.7	0.6	5.1	22.5
EVaR 1%	5.2	2.7	1.7	7.9	0.6	4.5	19.7
EVaR 5%	2.9	1.4	1.4	6.3	0.3	2.6	10.9
ENT	6.5	7.0	3.3	22.3	0.0	4.3	70.4
ML	11.1	7.5	2.8	17.8	1.2	9.1	75.9
Deviation Measures *(Values in monthly percentage)*							
StD	2.7	1.4	1.5	7.6	0.3	2.4	10.7
SD^{-}	1.8	0.9	1.4	6.2	0.2	1.6	6.7
SD^{α} 1%	0.2	0.2	4.6	39.1	0.0	0.1	2.6
SD^{α} 5%	0.4	0.4	3.7	26.8	0.0	0.3	4.0
Loss-Deviation Measures *(Values in monthly percentage)*							
ELD	1.8	0.9	1.4	6.3	0.2	1.6	6.8
SDR 1%	8.8	4.9	2.0	10.1	1.0	7.6	44.6
SDR 5%	6.1	3.0	1.5	7.6	0.7	5.4	24.6
DEVaR 1%	5.7	3.0	1.8	8.7	0.7	4.9	23.4
DEVaR 5%	3.8	1.8	1.5	6.7	0.4	3.3	14.6
DENT	6.9	7.0	3.3	22.6	0.2	4.7	70.7
Controlling Factors							
BETA	0.9	0.5	0.5	3.9	-0.9	0.8	3.2
SIZE ($ln(MktCap)$)	6.2	1.8	0.0	4.0	-2.1	6.2	12.4
VAL ($ln(BE/ME)$)	-0.7	0.8	-1.0	11.9	-7.4	-0.7	5.7
MOM (*% 11 Months*)	13.8	52.0	391.0	5413.1	-88.8	6.4	806.0
STR (*% 1 Month*)	1.2	12.3	151.2	2120.3	-60.4	0.5	129.7
IVOL	2.5	1.4	163.5	775.3	0.2	2.2	10.4

Note: Values presented here are the time-series means of the cross-sectional mean (*Mean*), standard deviation (*SD*), skewness (*Skew*), kurtosis (*Kurt*), minimum (*Min*), median (*Med*), and maximum.

Figure 1 – 12-month moving average of the monthly returns of stocks in the sample. Shaded areas are crisis periods. The sample period extends between January 1982 and January 2021.



level over time, and ES equals the average of all losses beyond the VaR threshold. Thus, for the same significance level, ES must be greater than VaR. Besides that, the mean values of SDR are higher because this measure is obtained from the composition between ES and SD^{α}. Thus, as for the SDR, we find that the values of the loss deviation measures are greater than their risk counterparty. EVaR is lower than VaR and ES because, according to Bellini and Bernardino (2017), for a normally distributed X, the EVaR is closely comparable to the VaR 1% and ES 2.5% when we consider a significance level equal to 0.145% (EVaR 0.145%). Means equal to 6.5% for VaR 1% and 3.9% for VaR 5% are consistent with other studies, such as Atilgan et al. (2020), which found 6% and 4%, respectively, for the US-based common stocks trading on NYSE and NASDAQ in the period 1962 to 2014. ES means equal to 8.7% and 5.7% are also consistent with Atilgan et al. (2020), which found 8% and 5%, respectively, for α of 1% and 5% for the same data. The highest mean value among the risk measures refers to the ML, as expected since this measure is the most aggressive of the tail measures used in the study.

About the Controlling factors statistics, it is expected that the BETA mean is approximately 1. The CAPM theory states that the BETA shows how much the price of a particular stock varies compared with the entire stock market. If a share price moves exactly in line with the market, then the stock's beta is 1. As the sample of stocks used in this study is wide, it contains stocks more and others less correlated to the market return, leading the average to 1.

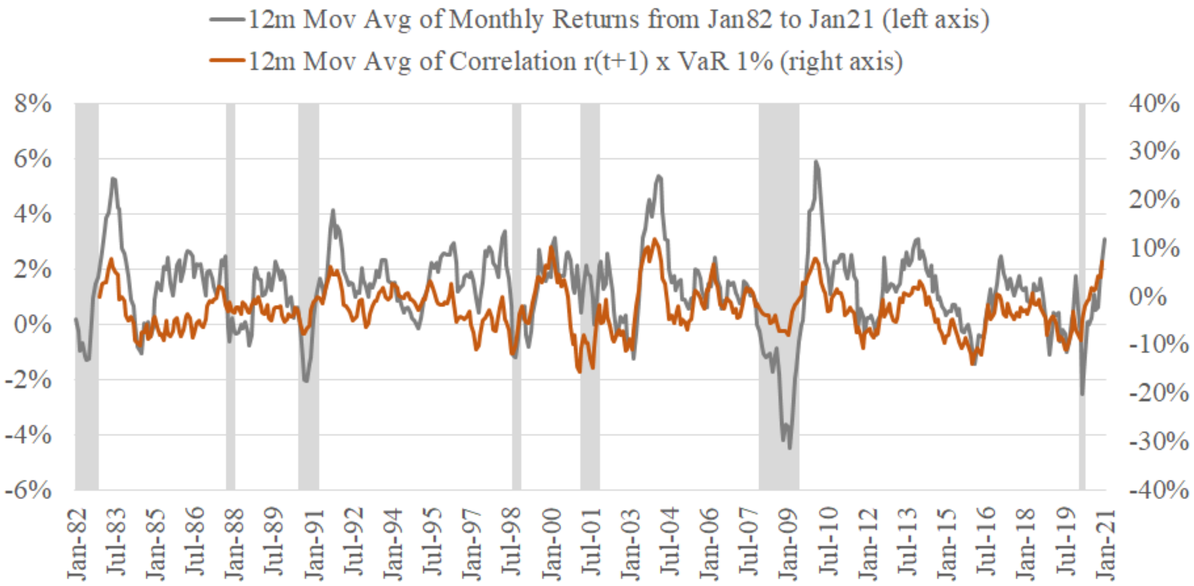
Table 2 shows the time-series averages of the cross-sectional correlations between the r_{t+1} and each independent variable. All correlation means of risk measures are slightly negative, but close to zero. The minimum and maximum values demonstrate that the correlations change in from positive to negative over the years analyzed. It is expected that the behavior of the correlation

Table 2 – Correlations between monthly expected returns and the independent variables. The sample period extends between January 1982 and January 2021.

	Total Period			Crisis Periods			Normal Periods		
	Mean	Min	Max	Mean	Min	Max	Mean	Min	Max
Loss Measures									
EL	-2.7	-37.8	42.8	-1.9	-31.1	42.8	-2.8	-37.8	40.6
VaR 1%	-2.7	-44.4	40.7	0.2	-41.8	37.9	-3.1	-44.4	40.7
VaR 5%	-2.7	-46.3	44.4	-0.1	-44.2	38.8	-3.1	-46.3	44.4
ES 1%	-2.5	-42.4	39.9	0.5	-40.6	37.4	-3.0	-42.4	39.9
ES 5%	-2.7	-45.5	43.2	0.2	-42.9	38.8	-3.1	-45.5	43.2
EVaR 1%	-2.7	-43.9	41.3	0.3	-41.9	38.2	-3.1	-43.9	41.3
EVaR 5%	-2.8	-45.9	44.5	0.0	-44.0	39.2	-3.2	-45.9	44.5
ENT	-2.4	-39.3	37.5	0.4	-39.2	35.4	-2.8	-39.3	37.5
ML	-2.3	-38.8	37.1	0.5	-38.5	35.4	-2.8	-38.8	37.1
Deviation Measures									
StD	-2.6	-48.5	39.9	0.1	-43.8	36.3	-3.0	-48.5	39.9
SD^{-}	-2.6	-46.6	42.1	0.3	-43.8	38.6	-3.0	-46.6	42.1
SD^{α} 1%	-1.4	-24.8	24.5	0.6	-23.0	24.5	-1.7	-24.8	22.9
SD^{α} 5%	-1.9	-31.4	31.2	0.9	-31.1	29.3	-2.3	-31.4	31.2
Loss-Deviation Measures									
ELD	-3.0	-45.6	44.8	-0.1	-45.1	40.8	-3.4	-45.6	44.8
SDR 1%	-2.5	-42.1	39.8	0.5	-40.5	37.3	-2.9	-42.1	39.8
SDR 5%	-2.7	-45.1	42.4	0.3	-42.6	38.9	-3.1	-45.1	42.4
DEVaR 1%	-2.6	-43.5	41.0	0.3	-41.7	38.0	-3.1	-43.5	41.0
DEVaR 5%	-2.8	-45.4	43.0	0.1	-43.4	39.2	-3.2	-45.4	43.0
DENT	-2.4	-39.3	37.5	0.4	-39.2	35.4	-2.8	-39.3	37.5
Controlling Factors									
BETA	-0.2	-50.5	49.8	2.3	-45.4	39.5	-0.5	-50.5	49.8
SIZE	1.8	-36.8	28.1	0.2	-21.5	18.9	2.0	-36.8	28.1
VAL	0.7	-22.4	35.9	-1.5	-22.4	18.3	1.0	-21.1	35.9
MOM	3.4	-48.2	36.9	1.9	-47.7	31.3	3.7	-48.2	36.9
STR	-3.0	-41.1	26.6	-4.1	-38.2	23.7	-2.9	-41.1	26.6
IVOL	-2.7	-49.2	39.0	-0.1	-41.1	33.4	-3.1	-49.2	39.0

Note: Correlations presented here are the time-series means of the cross-sectional monthly correlations between the one-month-ahead returns and each independent variable analyzed. Correlations are multiplied by 100 so they are represented as a percentage. Min refers to minimum, and Max is the maximum correlation.

Figure 2 – 12-month moving average of the monthly returns of stocks in the sample and the correlation between VaR and Returns ($t + 1$). Shaded areas are crisis periods. The sample period extends between January 1982 and January 2021.



change in crisis periods in relation to non-crisis periods, which could help to justify this behavior. However, considering just the crisis periods of NBER and Danielsson et al. (2016), the numbers do not change much, but the distance between the minimum and maximum correlation decreases, taking the mean to the positive side in most risk measures. Correlations in normal periods have a greater variation between the minimum and maximum and negative means for most measures. A possible explanation for this result is that, according to Ghysels et al. (2016), crises are characterized by flights to safety, so investors look for investment options with less risk. Therefore, the dependence between assets increases in periods of crisis, and thus, the behavior of assets becomes more homogeneous.

Figure 2 adds to the previous graph 1 the 12-month moving average of cross-sectional correlations between VaR 1% and r_{t+1} (orange line). In this graph, we can see that indeed, when the returns are higher, the correlation increases, and vice-versa. We randomly highlighted the correlation of r_{t+1} with VaR 1% rather than the other risk measures, as we realized that all correlations of risk measures have a similar behavior over the years. The EL was the only one that did not seem to follow the movements of the correlations of the other risk measures. The different behavior of the EL is since it evaluates losses in terms of the central trend and ignores the loss variability and less likely losses, known as tail risk. We also do not present the other illustrations for brevity, but they are available on request.

5.2 Portfolio Analysis

In this section, we interpret the results of the univariate portfolio-level analysis, where quintiles are formed every month from January 1982 to December 2020 by sorting the stocks in the sample from lower to higher values of each risk measure considered in this study. The one-month-ahead excess returns are calculated for each quintile to test whether a zero-cost portfolio that takes a long position in stocks with the highest risk and a short position in stocks with the lowest risk generates a significant return. Besides comparing the returns of each portfolio, we also perform time-series regressions using, as dependent variable, the portfolios' expected excess returns for the full period and, as independent variables, the returns of portfolios formed by the market beta, size, value, and momentum factors of Fama and French (1993) and Carhart (1997). We use the intercept coefficient of these regressions, called Alphas, to examine whether the excess return of each portfolio and also the difference in returns between the extreme risk quintiles can be explained by standard asset pricing models. These Alphas are also known as abnormal returns or Jensen's alpha. We verify if the Alphas are statistically different from zero using the Newey and West (1986) adjusted *t*-statistics. The portfolios are formed both equal-weighting and value-weighting the stocks in the sample. The market capitalization of each stock is used to compose the value-weighted portfolios.

Starting the explanation by the equally weighted portfolios, Tables 3, 4 and 5 show the time-series averages of cross-sectional one-month-ahead excess returns of equal-weighted quintiles and of the difference portfolio, which is the fifth portfolio return minus the first portfolio return. We also call the difference portfolio Portfolio 5-1. These tables report, for each risk measure analyzed, the one-month-ahead excess returns (first row), the Alphas for each quintile (second row) and the significance level of the Alphas (third row). The column *Port 5-1* shows the differences of monthly excess returns and Alphas between quintiles 5 and 1. The last column *W Text* refers to the bilateral Wilcoxon signed-rank test and it shows the symbols ***, **, and * indicating statistical significance at the 10%, 5%, and 1% levels, respectively, of the Portfolio 5-1 expected returns. The significance level of the Alphas of each quintile and of the Portfolio 5-1 are shown in the third row of each risk measure. These symbols demonstrate if the Alphas are statistically different from zero. This result means that the factors used as independent variables in the regressions do not explain all the expected returns of the portfolios formed by risk measures.

Regarding the Loss Measures portfolios (Table 3), we observe that only the Expected Loss has a negative excess return in the Portfolio 5-1. It means that the 5th portfolio, which has the stocks with the highest EL level, delivered a lower return than the 1st quintile, with the stocks with the lowest level of EL. As we can see in the last column of the Wilcoxon Test, this excess return of the Portfolio 5-1 for EL is significant at 1%. The Alpha of the Portfolio 5-1 of EL is also negative. Even after controlling by the Fama-French three-factors and the Momentum Factor, the Alpha remained robust in the Portfolio 5-1. All the other Loss Measures had a positive expected

Table 3 – Excess returns of equal-weighted quintiles and difference portfolio (5th minus 1st quintile) of Loss Measures. The sample period extends between January 1982 and January 2021.

	Equal-weighted portfolios						
	Port 1	Port 2	Port 3	Port 4	Port 5	Port 5-1	W Test
EL	1.56	0.96	0.70	0.64	0.94	-0.62	***
Alpha	0.0152	0.0089	0.0062	0.0058	0.0089	-0.0093	
	***	***	***	**	**	***	
VaR 1%	0.54	0.78	0.91	1.10	1.46	0.92	**
Alpha	0.0050	0.0074	0.0087	0.0104	0.0135	0.0055	
	***	***	***	***	***	>10%	
VaR 5%	0.53	0.78	0.90	1.10	1.49	0.96	**
Alpha	0.0049	0.0075	0.0085	0.0106	0.0135	0.0057	
	***	***	***	***	***	>10%	
ES 1%	0.56	0.78	0.94	1.10	1.43	0.87	**
Alpha	0.0052	0.0073	0.0088	0.0103	0.0132	0.0050	
	***	***	***	***	***	>10%	
ES 5%	0.54	0.75	0.93	1.11	1.48	0.94	**
Alpha	0.0050	0.0071	0.0088	0.0106	0.0136	0.0056	
	***	***	***	***	***	>10%	
EVaR 1%	0.56	0.76	0.94	1.10	1.44	0.88	**
Alpha	0.0052	0.0072	0.0089	0.0105	0.0133	0.0051	
	***	***	***	***	***	>10%	
EVaR 5%	0.55	0.76	0.94	1.08	1.48	0.93	**
Alpha	0.0051	0.0071	0.0089	0.0103	0.0135	0.0055	
	***	***	***	***	***	>10%	
ENT	0.58	0.80	0.97	1.07	1.39	0.81	**
Alpha	0.0055	0.0075	0.0092	0.0099	0.0129	0.0045	
	***	***	***	***	***	>10%	
ML	0.59	0.78	0.98	1.06	1.39	0.80	**
Alpha	0.0055	0.0073	0.0093	0.0099	0.0130	0.0045	
	***	***	***	***	***	>10%	

Note: Values presented here are the time-series means of the cross-sectional one-month-ahead excess returns of the equal-weighted portfolios formed by sorting equities by each analyzed loss measure. The last column W Test shows the results of the Wilcoxon Test by the symbols ***, **, and * indicating statistical significance at the 10%, 5%, and 1% levels, respectively, of the Portfolio 5-1 expected returns. The Alpha rows exhibit the intercept coefficient of the regressions with, as dependent variables, the time-series excess returns of each portfolio formed by each risk measure, and, as independent variables, the returns of the Fama-French three-factors and the Momentum factor. The rows below the Alpha rows show if the Alphas are significantly different from zero (Newey and West (1986) adjusted). Excess returns are multiplied by 100 so they are represented as a percentage.

return in the Portfolio 5-1, indicating that equities with higher loss risk have significantly higher expected excess returns, as demonstrated by the Wilcoxon Test. However, when we adjust by other known pricing models, the Alphas of the Portfolios 5-1 are positive but not statistically different from zero. It means that the Fama-French three-factors and the Momentum Factor already explain the finding that equities with higher loss measures earn higher one-month-ahead returns.

Atilgan et al. (2020) also tested VaR at 1% using portfolio analysis between 1962 and 2014 and, unlike this study, they have found that VaR 1% has a negative relationship with expected results. It is counterintuitive, as investors should not be willing to take more risk to receive less return. Atilgan et al. (2020) themselves, in their conclusions, say the findings are an anomaly. Long et al. (2019) tested the expected return relationship with three measures of tail risk. These measures were calculated differently from the measures we used in this study, but it is interesting to analyze their results. Their sample spans 1980 to 2015 and includes stocks in 21 developed markets and 18 emerging markets. Analyzing only the findings for the United States, two of the three measures presented significant negative average monthly excess returns of the Portfolio 5-1. In the full sample, with all countries, Long et al. (2019) also found a negative relationship for these two measures. The other one did not present a significant relationship with expected results either in the US or the full sample. Long et al. (2019) comment this contradicts the findings of Kelly and Jiang (2014), which tested the same measure and found a positive relationship in a sample of US market returns from 1963 to 2010. Likewise, Atilgan et al. (2020), Long et al. (2019) highlight that the negative relationship they found contradicts what is normally expected of a risk-return tradeoff. They initially attributed this contradiction to the "idiosyncratic volatility puzzle" found in the US market (ANG et al., 2006) and the international markets (ANG et al., 2009). However, after other robustness checks for the US market, using momentum effect, skewness, heterogeneous beliefs, liquidity, leverage, and profitability, they conclude that the negative relationship between tail risk and stock returns cannot be explained. Thus, this anomaly remains an unsolved puzzle.

Table 4 summarizes the findings of Deviation Measures. All the three Deviation Measures tested show an increase in expected returns as the portfolio's risk level increases. However, only the expected returns of the Portfolios 5-1 of the Standard Deviation and of the Semi-Deviation are statistically robust at 1%. Even after adjusting the commonly used factors, the Alpha of the Portfolio 5-1 is statistically different from zero at 10%, meaning they cannot explain the higher one-month-ahead returns that equities with higher StD and SD^- earn.

Table 5 shows the results for Loss-Deviation Measures. We see that all four tested measures presented positive excess returns for Portfolio 5-1, indicating a positive relationship between the risk measure and expected returns. These results are significant according to the Wilcoxon Test, but the Alphas of the Portfolios 5-1 suggest that all the higher results can be explained by the Fama-French three-factors and the Momentum factor.

Table 4 – Excess returns of equal-weighted quintiles and difference portfolio (5th minus 1st quintile) of Deviation Measures. The sample period extends between January 1982 and January 2021.

	Equal-weighted portfolios						
	Port 1	Port 2	Port 3	Port 4	Port 5	Port 5-1	W Test
StD	0.51	0.75	0.90	1.10	1.54	1.03	***
Alpha	0.0047	0.0071	0.0085	0.0105	0.0142	0.0065	
	***	***	***	***	***	*	
SD^-	0.53	0.74	0.91	1.09	1.53	1.00	***
Alpha	0.0048	0.0070	0.0086	0.0105	0.0141	0.0063	
	***	***	***	***	***	*	
SD^α 1%	0.81	0.89	0.96	1.01	1.14	0.33	>10%
Alpha	0.0075	0.0083	0.0089	0.0095	0.0108	0.0003	
	***	***	***	***	***	>10%	
SD^α 5%	0.68	0.85	0.94	1.11	1.23	0.55	>10%
Alpha	0.0064	0.0079	0.0088	0.0102	0.0117	0.0024	
	***	***	***	***	***	>10%	

Note: Values presented here are the time-series means of the cross-sectional one-month-ahead excess returns of the equal-weighted portfolios formed by sorting equities by each analyzed deviation measure. The last column W Test shows the results of the Wilcoxon Test by the symbols ***, **, and * indicating statistical significance at the 10%, 5%, and 1% levels, respectively, of the Portfolio 5-1 expected returns. The Alpha rows exhibit the intercept coefficient of the regressions with, as dependent variables, the time-series excess returns of each portfolio formed by each risk measure, and, as independent variables, the returns of the Fama-French three-factors and the Momentum factor. The rows below the Alpha rows show if the Alphas are significantly different from zero (Newey and West (1986) adjusted). Excess returns are multiplied by 100 so they are represented as a percentage.

Comparing the three risk categories (loss, deviation and loss-deviation) for equally weighted portfolios, we see that only the loss and the deviation categories contain measures with Alphas statistically different from zero. After adjusting for factors already established in the literature of pricing, the measures continue to show explanatory potential for expected returns. Among the loss measures, EL is the only measure presenting a significant relationship when adjusted for these factors, with a negative relationship. As for the deviation measures, the StD and the SD^- show a significant relationship at 10% (weaker than the EL), but with a positive relationship. In the loss-deviation measures, there is no measure with a significant relationship after adjusting for the Fama-French three factors and Carhart's momentum factor. That means they already explain the relationship between risk and expected returns. If we analyze only the 5 quintiles, without considering the Portfolio 5-1, we see that all risk measures, from all categories, have Alphas statistically different from zero. This indicates that each portfolio produces positive excess returns in the average month. According to Bali et al. (2016), this is expected because stocks are known to generate higher average returns than the risk-free security.

We now repeat the analysis using value-weighted portfolios. Tables 6, 7 and 8 present the results for the portfolios formed, respectively, by loss, deviation, and loss-deviation measures weighting stocks in sample by their market capitalization. Stocks with a higher market value gained more weight in the formation of portfolios. These tables are structured similarly to the previous tables with equal-weighted portfolios. As we could see in the equal-weighted portfolios, all the analyzed risk measures, except for the EL, showed a positive relationship between risk

Table 5 – Excess returns of equal-weighted quintiles and difference portfolio (5th minus 1st quintile) of Loss-Deviation Measures. The sample period extends between January 1982 and January 2021.

Equal-weighted portfolios							
	Port 1	Port 2	Port 3	Port 4	Port 5	Port 5-1	W Test
ELD	0.56	0.79	0.96	1.04	1.45	0.89	**
Alpha	0.0052	0.0075	0.0091	0.0100	0.0132	0.0050	
	***	***	***	***	***	>10%	
SDR 1%	0.57	0.78	0.94	1.09	1.43	0.86	**
Alpha	0.0053	0.0073	0.0088	0.0103	0.0133	0.0050	
	***	***	***	***	***	>10%	
SDR 5%	0.54	0.76	0.92	1.11	1.46	0.92	**
Alpha	0.0050	0.0072	0.0087	0.0107	0.0135	0.0055	
	***	***	***	***	***	>10%	
DEVaR 1%	0.56	0.77	0.94	1.11	1.43	0.87	**
Alpha	0.0052	0.0072	0.0088	0.0105	0.0132	0.0051	
	***	***	***	***	***	>10%	
DEVaR 5%	0.55	0.77	0.93	1.10	1.46	0.91	**
Alpha	0.0051	0.0072	0.0087	0.0106	0.0134	0.0053	
	***	***	***	***	***	>10%	
DENT	0.58	0.79	0.98	1.06	1.39	0.81	**
Alpha	0.0055	0.0074	0.0093	0.0099	0.0130	0.0046	
	***	***	***	***	***	>10%	

Note: Values presented here are the time-series means of the cross-sectional one-month-ahead excess returns of the equal-weighted portfolios formed by sorting equities by each analyzed loss-deviation measure. The last column W Test shows the results of the Wilcoxon Test by the symbols ***, **, and * indicating statistical significance at the 10%, 5%, and 1% levels, respectively, of the Portfolio 5-1 expected returns. The Alpha rows exhibit the intercept coefficient of the regressions with, as dependent variables, the time-series excess returns of each portfolio formed by each risk measure, and, as independent variables, the returns of the Fama-French three-factors and the Momentum factor. The rows below the Alpha rows show if the Alphas are significantly different from zero (Newey and West (1986) adjusted). Excess returns are multiplied by 100 so they are represented as a percentage.

and expected returns. However, with value-weighted portfolios, only EL had a significantly excess return for the Portfolio 5-1. All the other measures did not present significantly difference in the expected return of the portfolios formed by the stocks with the highest level of each risk measure and with the lowest ones. One possible explanation for this divergence between the results found in equally weighted portfolios and in value-weighted portfolios is that the results could be driven by small stocks. As indicated by the size factor of Fama and French (1993), portfolios formed by companies with higher market value tend to achieve lower returns, so the value weighting harmed portfolio returns. Also, with value-weighted portfolios, the Alphas are all negative, indicating that the return is lower than the expected return.

Table 6 – Excess returns of value-weighted quintiles and difference portfolio (5th minus 1st quintile) of Loss Measures. The sample period extends between January 1982 and January 2021.

	Value-weighted portfolios						
	Port 1	Port 2	Port 3	Port 4	Port 5	Port 5-1	W Test
EL	0.93	0.67	0.42	0.41	0.46	-0.47	**
Alpha	0.0096	0.0065	0.0046	0.0038	0.0042	-0.0083	
	***	***	**	>10%	>10%	***	
VaR 1%	0.5	0.63	0.67	0.74	0.75	0.25	>10%
Alpha	0.0051	0.0063	0.0068	0.0074	0.0073	-0.0008	
	***	***	**	*	>10%	>10%	
VaR 5%	0.48	0.66	0.72	0.71	0.73	0.25	>10%
Alpha	0.0050	0.0068	0.0071	0.0070	0.0072	-0.0008	
	***	***	**	*	>10%	>10%	
ES 1%	0.52	0.55	0.68	0.78	0.66	0.14	>10%
Alpha	0.0054	0.0056	0.0067	0.0077	0.0069	-0.0014	
	***	**	**	**	>10%	>10%	
ES 5%	0.49	0.63	0.70	0.80	0.68	0.19	>10%
Alpha	0.0052	0.0062	0.0071	0.0077	0.0070	-0.0011	
	***	**	**	*	>10%	>10%	
EVaR 1%	0.52	0.56	0.73	0.71	0.68	0.16	>10%
Alpha	0.0054	0.0056	0.0071	0.0071	0.0072	-0.0012	
	***	**	**	*	>10%	>10%	
EVaR 5%	0.48	0.64	0.75	0.74	0.58	0.10	>10%
Alpha	0.0051	0.0065	0.0076	0.0073	0.0059	-0.0022	
	***	***	***	*	>10%	>10%	
ENT	0.53	0.57	0.74	0.58	0.70	0.17	>10%
Alpha	0.0055	0.0058	0.0076	0.0053	0.0074	-0.0010	
	***	**	***	>10%	*	>10%	
ML	0.53	0.54	0.75	0.58	0.71	0.18	>10%
Alpha	0.0055	0.0055	0.0076	0.0054	0.0076	-0.0009	
	***	**	***	>10%	*	>10%	

Note: Values presented here are the time-series means of the cross-sectional one-month-ahead excess returns of the value-weighted portfolios formed by sorting equities by each analyzed loss measure. The market capitalization of the stocks were used as weights. The last column W Test shows the results of the Wilcoxon Test by the symbols ***, **, and * indicating statistical significance at the 10%, 5%, and 1% levels, respectively, of the Portfolio 5-1 expected returns. The Alpha rows exhibit the intercept coefficient of the regressions with, as dependent variables, the time-series excess returns of each portfolio formed by each risk measure, and, as independent variables, the returns of the Fama-French three-factors and the Momentum factor. The rows below the Alpha rows show if the Alphas are significantly different from zero (Newey and West (1986) adjusted). Excess returns are multiplied by 100 so they are represented as a percentage.

Table 7 – Excess returns of value-weighted quintiles and difference portfolio (5th minus 1st quintile) of Deviation Measures. The sample period extends between January 1982 and January 2021.

Value-weighted portfolios							
	Port 1	Port 2	Port 3	Port 4	Port 5	Port 5-1	W Test
StD	0.48	0.66	0.74	0.78	0.70	0.22	>10%
Alpha	0.0050	0.0066	0.0074	0.0077	0.0069	-0.0011	
	***	***	***	*	>10%	>10%	
SD^{-}	0.48	0.63	0.73	0.82	0.75	0.27	>10%
Alpha	0.0050	0.0064	0.0073	0.0080	0.0078	-0.0002	
	***	***	***	**	>10%	>10%	
SD$^{\alpha}$ 1%	0.57	0.53	0.57	0.55	0.68	0.11	>10%
Alpha	0.0057	0.0055	0.0057	0.0054	0.0072	-0.0015	
	***	**	**	*	**	>10%	
SD$^{\alpha}$ 5%	0.52	0.59	0.60	0.62	0.70	0.18	>10%
Alpha	0.0053	0.0060	0.0062	0.0057	0.0075	-0.0008	
	***	***	**	*	**	>10%	

Note: Values presented here are the time-series means of the cross-sectional one-month-ahead excess returns of the value-weighted portfolios formed by sorting equities by each analyzed deviation measure. The market capitalization of the stocks were used as weights. The last column W Test shows the results of the Wilcoxon Test by the symbols ***, **, and * indicating statistical significance at the 10%, 5%, and 1% levels, respectively, of the Portfolio 5-1 expected returns. The Alpha rows exhibit the intercept coefficient of the regressions with, as dependent variables, the time-series excess returns of each portfolio formed by each risk measure, and, as independent variables, the returns of the Fama-French three-factors and the Momentum factor. The rows below the Alpha rows show if the Alphas are significantly different from zero (Newey and West (1986) adjusted). Excess returns are multiplied by 100 so they are represented as a percentage.

Table 8 – Excess returns of value-weighted quintiles and difference portfolio (5th minus 1st quintile) of Loss-Deviation Measures. The sample period extends between January 1982 and January 2021.

Value-weighted portfolios							
	Port 1	Port 2	Port 3	Port 4	Port 5	Port 5-1	W Test
ELD	0.49	0.66	0.77	0.58	0.60	0.11	>10%
Alpha	0.0051	0.0066	0.0078	0.0058	0.0060	-0.0021	
	***	***	***	>10%	>10%	>10%	
SDR 1%	0.53	0.54	0.68	0.77	0.67	0.14	>10%
Alpha	0.0054	0.0055	0.0067	0.0077	0.0070	-0.0014	
	***	**	**	**	>10%	>10%	
SDR 5%	0.50	0.62	0.72	0.78	0.65	0.15	>10%
Alpha	0.0052	0.0062	0.0072	0.0076	0.0068	-0.0013	
	***	***	**	*	>10%	>10%	
DEVaR 1%	0.53	0.57	0.70	0.71	0.65	0.12	>10%
Alpha	0.0054	0.0058	0.0069	0.0072	0.0069	-0.0015	
	***	**	**	**	>10%	>10%	
DEVaR 5%	0.49	0.62	0.74	0.73	0.64	0.15	>10%
Alpha	0.0051	0.0063	0.0074	0.0073	0.0066	-0.0015	
	***	***	***	*	>10%	>10%	
DENT	0.52	0.59	0.73	0.57	0.70	0.18	>10%
Alpha	0.0055	0.0060	0.0075	0.0052	0.0075	-0.0010	
	***	***	***	>10%	*	>10%	

Note: Values presented here are the time-series means of the cross-sectional one-month-ahead excess returns of the value-weighted portfolios formed by sorting equities by each analyzed loss-deviation measure. The market capitalization of the stocks were used as weights. The last column W Test shows the results of the Wilcoxon Test by the symbols ***, **, and * indicating statistical significance at the 10%, 5%, and 1% levels, respectively, of the Portfolio 5-1 expected returns. The Alpha rows exhibit the intercept coefficient of the regressions with, as dependent variables, the time-series excess returns of each portfolio formed by each risk measure, and, as independent variables, the returns of the Fama-French three-factors and the Momentum factor. The rows below the Alpha rows show if the Alphas are significantly different from zero (Newey and West (1986) adjusted). Excess returns are multiplied by 100 so they are represented as a percentage.

We now investigate if the previous findings are consistent in crisis periods and normal periods. To do so, we separate the months from January 1982 to December 2020 in two groups: Crisis months (which are the shaded areas in Figure 1) and the other months (called here as "Normal"). Tables 9, 10 and 11 show the time-series averages of the Excess Returns of equal-weighted portfolios for (i) the Total Period from 1982 to 2020, (ii) only for the crisis periods and (iii) only for the normal periods. Analyzing the average returns only in crisis periods, we see that the excess returns of the 5th quintiles, formed by the stocks with the highest risk, are consistently higher than the excess results of the 1st quintiles. In other words, the positive relationship between risk and expected returns is even more emphasized in periods of crisis. Even for EL, which we find a negative relationship when we analyze the full period, we now find a positive relationship when considering only the average returns of crisis months.

Table 9 – Excess returns of equal-weighted quintiles and difference portfolio (5th minus 1st quintile) of Loss Measures separated by Crisis and Normal Periods. The sample period extends between January 1982 and January 2021.

	Equal-weighted portfolios							
	Period	Port 1	Port 2	Port 3	Port 4	Port 5	Port 5-1	W Test
EL	Total	1.56	0.96	0.70	0.64	0.94	-0.62	***
	Crises	1.43	0.68	0.59	0.86	2.46	1.03	
	Normal	1.58	1.00	0.71	0.61	0.73	-0.85	
VaR 1%	Total	0.54	0.78	0.91	1.10	1.46	0.92	**
	Crises	0.19	0.55	0.95	1.61	2.71	2.52	
	Normal	0.59	0.82	0.91	1.03	1.28	0.69	
VaR 5%	Total	0.53	0.78	0.90	1.10	1.49	0.96	**
	Crises	0.18	0.59	0.85	1.70	2.68	2.50	
	Normal	0.58	0.81	0.91	1.01	1.32	0.74	
ES 1%	Total	0.56	0.78	0.94	1.10	1.43	0.87	**
	Crises	0.18	0.58	0.93	1.50	2.81	2.63	
	Normal	0.62	0.81	0.94	1.04	1.23	0.61	
ES 5%	Total	0.54	0.75	0.93	1.11	1.48	0.94	**
	Crises	0.21	0.49	0.90	1.58	2.82	2.61	
	Normal	0.59	0.79	0.93	1.04	1.28	0.69	
EVaR 1%	Total	0.56	0.76	0.94	1.10	1.44	0.88	**
	Crises	0.21	0.54	0.92	1.53	2.80	2.59	
	Normal	0.61	0.79	0.95	1.04	1.24	0.63	
EVaR 5%	Total	0.55	0.76	0.94	1.08	1.48	0.93	**
	Crises	0.21	0.48	0.90	1.58	2.84	2.63	
	Normal	0.60	0.80	0.94	1.00	1.28	0.68	
ENT	Total	0.58	0.80	0.97	1.07	1.39	0.81	**
	Crises	0.27	0.58	0.98	1.45	2.73	2.46	
	Normal	0.63	0.83	0.97	1.01	1.19	0.56	
ML	Total	0.59	0.78	0.98	1.06	1.39	0.80	**
	Crises	0.26	0.58	0.96	1.50	2.71	2.45	
	Normal	0.64	0.81	0.98	1.00	1.20	0.56	

Note: Values presented here are the time-series means of the cross-sectional one-month-ahead excess returns of the equal-weighted portfolios formed by sorting equities by each analyzed loss measure. The Total rows refer to the full period from January 1982 to January 2021. The Crises rows consider only the average of the Crisis periods by (NBER) (2020) and Danielsson et al. (2016). The Normal rows consider the average of all the other months. The last column W Test shows the results of the Wilcoxon Test by the symbols ***, **, and * indicating statistical significance at the 10%, 5%, and 1% levels, respectively, of the Portfolio 5-1 expected returns. Excess returns are multiplied by 100 so they are represented as a percentage.

Table 10 – Excess returns of equal-weighted quintiles and difference portfolio (5th minus 1st quintile) of Deviation Measures separated by Crisis and Normal Periods. The sample period extends between January 1982 and January 2021.

	Equal-weighted portfolios							
	Period	Port 1	Port 2	Port 3	Port 4	Port 5	Port 5-1	W Test
StD	Total	0.51	0.75	0.90	1.10	1.54	1.03	***
	Crises	0.18	0.54	0.89	1.67	2.74	2.56	
	Normal	0.56	0.78	0.90	1.02	1.37	0.81	
SD$^-$	Total	0.53	0.74	0.91	1.09	1.53	1.00	***
	Crises	0.16	0.52	0.92	1.58	2.83	2.67	
	Normal	0.58	0.77	0.91	1.02	1.34	0.76	
SD$^\alpha$ 1%	Total	0.81	0.89	0.96	1.01	1.14	0.33	>10%
	Crises	0.62	0.93	1.08	1.29	2.08	1.46	
	Normal	0.83	0.88	0.94	0.97	1.00	0.17	
SD$^\alpha$ 5%	Total	0.68	0.85	0.94	1.11	1.23	0.55	>10%
	Crises	0.49	0.68	0.86	1.53	2.44	1.95	
	Normal	0.70	0.87	0.95	1.04	1.06	0.36	

Note: Values presented here are the time-series means of the cross-sectional one-month-ahead excess returns of the equal-weighted portfolios formed by sorting equities by each analyzed deviation measure. The Total rows refer to the full period from January 1982 to January 2021. The Crises rows consider only the average of the Crisis periods by (NBER) (2020) and Danielsson et al. (2016). The Normal rows consider the average of all the other months. The last column W Test shows the results of the Wilcoxon Test by the symbols ***, **, and * indicating statistical significance at the 10%, 5%, and 1% levels, respectively, of the Portfolio 5-1 expected returns. Excess returns are multiplied by 100 so they are represented as a percentage.

Table 11 – Excess returns of equal-weighted quintiles and difference portfolio (5th minus 1st quintile) of Loss-Deviation Measures separated by Crisis and Normal Periods. The sample period extends between January 1982 and January 2021.

	Equal-weighted portfolios							
	Period	Port 1	Port 2	Port 3	Port 4	Port 5	Port 5-1	W Test
ELD	Total	0.56	0.79	0.96	1.04	1.45	0.89	**
	Crises	0.21	0.58	0.88	1.46	2.88	2.67	
	Normal	0.61	0.82	0.97	0.98	1.25	0.64	
SDR 1%	Total	0.57	0.78	0.94	1.09	1.43	0.86	**
	Crises	0.20	0.58	0.92	1.48	2.82	2.62	
	Normal	0.62	0.81	0.94	1.03	1.23	0.61	
SDR 5%	Total	0.54	0.76	0.92	1.11	1.46	0.92	**
	Crises	0.20	0.51	0.92	1.53	2.86	2.66	
	Normal	0.59	0.80	0.93	1.05	1.26	0.67	
DEVaR 1%	Total	0.56	0.77	0.94	1.11	1.43	0.87	**
	Crises	0.21	0.54	0.94	1.53	2.80	2.59	
	Normal	0.61	0.80	0.94	1.05	1.24	0.63	
DEVaR 5%	Total	0.55	0.77	0.93	1.10	1.46	0.91	**
	Crises	0.21	0.51	0.85	1.57	2.88	2.67	
	Normal	0.59	0.80	0.94	1.03	1.26	0.67	
DENT	Total	0.58	0.79	0.98	1.06	1.39	0.81	**
	Crises	0.28	0.57	1.00	1.42	2.74	2.46	
	Normal	0.63	0.82	0.97	1.01	1.20	0.57	

Note: Values presented here are the time-series means of the cross-sectional one-month-ahead excess returns of the equal-weighted portfolios formed by sorting equities by each analyzed loss-deviation measure. The Total rows refer to the full period from January 1982 to January 2021. The Crises rows consider only the average of the Crisis periods by (NBER) (2020) and Danielsson et al. (2016). The Normal rows consider the average of all the other months. The last column W Test shows the results of the Wilcoxon Test by the symbols ***, **, and * indicating statistical significance at the 10%, 5%, and 1% levels, respectively, of the Portfolio 5-1 expected returns. Excess returns are multiplied by 100 so they are represented as a percentage.

Tables 12, 13 and 14 show the results of value-weighted portfolios separating the averages by crisis and normal periods. Just like in equal-weighted portfolios, there is also a higher positive relationship between risk and expected return in crisis periods than in normal periods, but the returns of the difference portfolios are smaller than the ones found in equal-weighted portfolios. Among the reasons for this are those mentioned earlier.

Table 12 – Excess returns of value-weighted quintiles and difference portfolio (5th minus 1st quintile) of Loss Measures separated by Crisis and Normal Periods. The sample period extends between January 1982 and January 2021.

	Value-weighted portfolios							
	Period	Port 1	Port 2	Port 3	Port 4	Port 5	Port 5-1	W Test
EL	Total	0.93	0.67	0.42	0.41	0.46	-0.47	***
	Crises	0.39	0.23	0.31	-0.03	0.61	0.22	
	Normal	1.01	0.73	0.44	0.48	0.44	-0.57	
VaR 1%	Total	0.50	0.63	0.67	0.74	0.75	0.25	**
	Crises	0.13	0.29	0.29	0.54	0.90	0.77	
	Normal	0.55	0.68	0.72	0.77	0.72	0.17	
VaR 5%	Total	0.48	0.66	0.72	0.71	0.73	0.25	**
	Crises	0.08	0.38	0.37	0.55	0.90	0.82	
	Normal	0.54	0.71	0.77	0.73	0.70	0.16	
ES 1%	Total	0.52	0.55	0.68	0.78	0.66	0.14	**
	Crises	0.15	0.25	0.19	0.62	1.02	0.87	
	Normal	0.58	0.60	0.75	0.80	0.61	0.03	
ES 5%	Total	0.49	0.63	0.70	0.80	0.68	0.19	**
	Crises	0.16	0.26	0.34	0.50	0.98	0.82	
	Normal	0.54	0.68	0.76	0.84	0.64	0.10	
EVaR 1%	Total	0.52	0.56	0.73	0.71	0.68	0.16	**
	Crises	0.22	0.23	0.21	0.41	1.13	0.91	
	Normal	0.56	0.61	0.80	0.76	0.61	0.05	
EVaR 5%	Total	0.48	0.64	0.75	0.74	0.58	0.10	**
	Crises	0.14	0.28	0.49	0.60	0.83	0.69	
	Normal	0.53	0.70	0.79	0.76	0.54	0.01	
ENT	Total	0.53	0.57	0.74	0.58	0.70	0.17	**
	Crises	0.21	0.25	0.38	0.36	0.98	0.77	
	Normal	0.57	0.62	0.79	0.62	0.65	0.08	
ML	Total	0.53	0.54	0.75	0.58	0.71	0.18	**
	Crises	0.23	0.10	0.53	0.45	0.89	0.66	
	Normal	0.58	0.60	0.78	0.60	0.68	0.10	

Note: Values presented here are the time-series means of the cross-sectional one-month-ahead excess returns of the value-weighted portfolios formed by sorting equities by each analyzed loss measure. The market capitalization of the stocks were used as weights. The Total rows refer to the full period from January 1982 to January 2021. The Crises rows consider only the average of the Crisis periods by (NBER) (2020) and Danielsson et al. (2016). The Normal rows consider the average of all the other months. The last column W Test shows the results of the Wilcoxon Test by the symbols ***, **, and * indicating statistical significance at the 10%, 5%, and 1% levels, respectively, of the Portfolio 5-1 expected returns. Excess returns are multiplied by 100 so they are represented as a percentage.

Table 13 – Excess returns of value-weighted quintiles and difference portfolio (5th minus 1st quintile) of Deviation Measures separated by Crisis and Normal Periods. The sample period extends between January 1982 and January 2021.

	Value-weighted portfolios							
	Period	Port 1	Port 2	Port 3	Port 4	Port 5	Port 5-1	W Test
StD	Total	0.48	0.66	0.74	0.78	0.70	0.22	***
	Crises	0.10	0.26	0.44	0.68	0.98	0.88	
	Normal	0.53	0.71	0.78	0.79	0.66	0.13	
SD$^-$	Total	0.48	0.63	0.73	0.82	0.75	0.27	***
	Crises	0.08	0.31	0.39	0.57	1.26	1.18	
	Normal	0.53	0.68	0.78	0.85	0.68	0.15	
SD$^\alpha$ 1%	Total	0.57	0.53	0.57	0.55	0.68	0.11	>10%
	Crises	0.12	0.44	0.39	-0.15	1.10	0.98	
	Normal	0.63	0.54	0.60	0.64	0.62	-0.01	
SD$^\alpha$ 5%	Total	0.52	0.59	0.60	0.62	0.70	0.18	>10%
	Crises	0.15	0.26	0.27	0.46	0.86	0.71	
	Normal	0.58	0.64	0.65	0.65	0.67	0.09	

Note: Values presented here are the time-series means of the cross-sectional one-month-ahead excess returns of the value-weighted portfolios formed by sorting equities by each analyzed deviation measure. The market capitalization of the stocks were used as weights. The Total rows refer to the full period from January 1982 to January 2021. The Crises rows consider only the average of the Crisis periods by (NBER) (2020) and Danielsson et al. (2016). The Normal rows consider the average of all the other months. The last column W Test shows the results of the Wilcoxon Test by the symbols ***, **, and * indicating statistical significance at the 10%, 5%, and 1% levels, respectively, of the Portfolio 5-1 expected returns. Excess returns are multiplied by 100 so they are represented as a percentage.

Table 14 – Excess returns of value-weighted quintiles and difference portfolio (5th minus 1st quintile) of Loss-Deviation Measures separated by Crisis and Normal Periods. The sample period extends between January 1982 and January 2021.

	Value-weighted portfolios							
	Period	Port 1	Port 2	Port 3	Port 4	Port 5	Port 5-1	W Test
ELD	Total	0.49	0.66	0.77	0.58	0.60	0.11	**
	Crises	0.17	0.21	0.57	0.36	1.10	0.93	
	Normal	0.54	0.72	0.80	0.62	0.53	-0.01	
SDR 1%	Total	0.53	0.54	0.68	0.77	0.67	0.14	**
	Crises	0.18	0.22	0.18	0.58	1.08	0.90	
	Normal	0.58	0.59	0.76	0.80	0.61	0.03	
SDR 5%	Total	0.50	0.62	0.72	0.78	0.65	0.15	**
	Crises	0.15	0.30	0.37	0.40	1.12	0.97	
	Normal	0.55	0.66	0.77	0.83	0.59	0.04	
DEVaR 1%	Total	0.53	0.57	0.70	0.71	0.65	0.12	**
	Crises	0.20	0.24	0.22	0.39	1.14	0.94	
	Normal	0.57	0.62	0.77	0.76	0.58	0.01	
DEVaR 5%	Total	0.49	0.62	0.74	0.73	0.64	0.15	**
	Crises	0.14	0.27	0.41	0.52	1.17	1.03	
	Normal	0.54	0.67	0.78	0.76	0.56	0.02	
DENT	Total	0.52	0.59	0.73	0.57	0.70	0.18	**
	Crises	0.20	0.26	0.42	0.33	0.99	0.79	
	Normal	0.57	0.64	0.78	0.60	0.66	0.09	

Note: Values presented here are the time-series means of the cross-sectional one-month-ahead excess returns of the value-weighted portfolios formed by sorting equities by each analyzed loss-deviation measure. The market capitalization of the stocks were used as weights. The Total rows refer to the full period from January 1982 to January 2021. The Crises rows consider only the average of the Crisis periods by (NBER) (2020) and Danielsson et al. (2016). The Normal rows consider the average of all the other months. The last column W Test shows the results of the Wilcoxon Test by the symbols ***, **, and * indicating statistical significance at the 10%, 5%, and 1% levels, respectively, of the Portfolio 5-1 expected returns. Excess returns are multiplied by 100 so they are represented as a percentage.

5.3 Regression Analysis

This section shows all the results achieved from cross-sectional regressions performed for each month on the period from January 1982 to December 2020. In the first stage, we are interested in verifying a fundamental question about the relationship between the risk measures and expected returns in the cross-section. Thus, we apply simple cross-sectional regressions in the style of Fama and MacBeth (1973). For each month, we use as the dependent variable the one-month-ahead excess returns on each stock in the sample and, as independent variables, the market beta and the risk measures analyzed in this study independently. In the second stage, we add, as independent variables, various controlling factors: the Fama-French three-factors, the Momentum factor, the Short-Term Reversal factor, and the Idiosyncratic Volatility factor.

Table 15 exhibits the time-series median of the coefficients, R^2 and adjusted R^2 of the cross-sectional regressions performed with each risk measure and the market beta as independent variables. All values are reported as percentages. We prefer to report medians rather than means as they are not affected by extreme values, but we also analyzed the means, which are available upon request. The first row of the table shows the coefficients, R^2 and adjusted R^2 of the regression only with the market beta as an independent variable, with no risk measure as variable. We can see that in all regressions, the market beta factor has coefficients with a negative sign. This is consistent with other studies that used regressions of Fama and MacBeth (1973) to analyze the relationship of risk measures with expected returns, such as Atilgan et al. (2020), Long et al. (2019) and Zaremba (2019). Bali et al. (2016) states that the market beta measures the sensitivity of the given stock to the return of the market portfolio, so the average coefficient on the market beta should be an estimate of the premium associated with taking one unit of market risk. The average coefficient of -0.12 indicates that one unit of market risk leads to a premium of -0.12% per month, which is certainly contradictory to what we would expect. Therefore, the most important prediction of theoretical asset pricing has failed in most empirical studies. This finding is possibly the most persistent empirical anomaly in all of empirical asset pricing (BALI et al., 2016).

Now, comparing the results of all risk measures, the Shortfall Deviation with a significance level equal to 1% is the only measure that did not present a coefficient significantly different from zero. All the coefficients of the other risk measures are statistically different from zero. It means these risk measures explain a large part of the expected returns. However, the intercepts remain significant, indicating that only the risk measures and the market beta are not enough to explain the expected excess returns. As in Portfolio Analysis, regressions also show that risk measures have a positive relationship with expected returns. Thus, the greater the risk, the greater the expected return, and also, the lower the risk, the lower the expected return. Chen et al. (2014) and Bali and Cakici (2004) also performed cross-sectional regressions for VaR at 1% and at 5% and in both cases they have found significantly different from zero positive coefficients, just like us. Only the Expected Loss has a negative coefficient, indicating a negative relationship. Thus, the smaller the average expected loss, the greater the expected return, and the reverse is also true. This result is consistent with the portfolio analysis.

The last two columns of Table 15 present the R^2 and adjusted R^2 of the cross-sectional regressions performed. We can see that the values are very low, a little more than 2% for most risk measures. R^2 indicate the total variation in expected returns explained by the risk measures and market beta (independent variables) in the average period. This result is certainly disappointing news, but Bali et al. (2016) says that low levels of R^2 and adjusted R^2 are common in research that examines the ability to predict expected stock returns. One possible reason for this is that predicting future stock returns is a very difficult task and realized stock returns are a very noisy proxy for expected stock returns (ELTON, 1999).

In addition, just as we did for Portfolio Analysis, we also separate the months between crisis periods and normal periods and computed the averages of the coefficients separately. This is not presented in the tables, but it is available upon request. One interesting finding is that the coefficient of the risk measure EL is highly positive (135.07%) when we consider only the

Table 15 – Coefficients of the cross-sectional regressions of expected excess returns as dependent variable and the risk measures analyzed and market beta as independent variables. The sample period extends between January 1982 and January 2021.

	Alpha		Risk		BETA		R^2	Adj R^2
Regression without the risk measure variable								
	1.09	***			-0.12	***	1.1	1.0
Loss Measures								
EL	1.01	***	-196.82	**	-0.19	***	2.2	2.1
VaR 1%	0.43	***	5.39	***	-0.33	***	2.6	2.5
VaR 5%	0.38	***	10.37	***	-0.40	***	2.7	2.6
ES 1%	0.50	***	3.62	**	-0.27	***	2.3	2.3
ES 5%	0.33	***	5.93	***	-0.40	***	2.6	2.5
EVaR 1%	0.43	***	7.40	**	-0.27	***	2.4	2.3
EVaR 5%	0.36	***	11.65	***	-0.41	***	2.6	2.6
ENT	0.90	***	1.61	**	-0.21	***	2.1	2.0
ML	0.78	***	1.46	**	-0.23	***	2.1	2.0
Deviation Measures								
StD	0.35	***	15.98	***	-0.41	***	2.7	2.6
SD^-	0.27	***	20.46	***	-0.42	***	2.6	2.5
SD^α 1%	0.97	***	1.02	>10%	-0.10	***	1.4	1.3
SD^α 5%	0.94	***	8.73	*	-0.18	***	1.6	1.5
Loss-Deviation Measures								
ELD	0.45	***	10.77	***	-0.39	***	2.7	2.6
SDR 1%	0.49	***	3.22	**	-0.30	***	2.3	2.2
SDR 5%	0.35	***	5.88	***	-0.37	***	2.6	2.5
DEVaR 1%	0.44	***	6.19	**	-0.28	***	2.4	2.3
DEVaR 5%	0.36	***	8.75	***	-0.39	***	2.6	2.6
DENT	0.89	***	1.43	**	-0.20	***	2.1	2.0

Note: Values presented here are the time-series medians of the coefficients from cross-sectional regressions with the one-month-ahead excess returns as dependent variable and the risk measures and market beta as independent variables. The symbols ***, **, and * indicate if the coefficients are statistically different from zero at the 10%, 5%, and 1% levels, respectively. Last 2 columns exhibit the R^2 and the adjusted R^2. Coefficients, R^2, and adjusted R^2 are multiplied by 100 so they are represented as percentages.

average in months of crises. This result is similar to the findings in the Portfolio Analysis, in Table 9, which shows a positive return in Portfolio 5-1 of EL for periods of crisis, but a negative return for other months. For the other risk measures, the averages of the coefficients are always positive, whether or not we analyze periods of crisis.

As we realized that the intercepts remained significant in Table 15, indicating that other factors could explain the expected return, we add the following well-established forecasting factors as independent variables: the Fama-French three-factors, the Momentum factor, the Short-Term reversal factor, and the Idiosyncratic Volatility factor. The time-series medians of the coefficients, R^2 and adjusted R^2 are presented in Tables 16, 17 and 18. The first row of Table 16 exhibits the coefficients of a regression using only the factors as independent variables, without including any risk measure. In this regression, the coefficient of the market beta is

Table 16 – Coefficients of the cross-sectional regressions of expected excess returns as dependent variable and the loss measures analyzed and controlling factors as independent variables. The sample period extends between January 1982 and January 2021.

	Alpha	Risk	BETA	SIZE	VAL	MOM	STR	IVOL	R^2	Adj R^2
Regression without the risk measure variable										
	0.97		-0.25	-0.05	0.05	0.63	-2.81	14.65	4.8	4.5
	*		***	*	>10%	**	>10%	**		
Loss Measures										
EL	0.96	-300	-0.21	-0.07	0.04	-0.15	-3.23	6.53	5.2	4.9
	*	>10%	***	*	>10%	>10%	>10%	**		
VaR1%	0.75	5.08	-0.25	-0.05	0.05	0.59	-2.79	11.17	5.0	4.7
	*	>10%	***	*	>10%	**	>10%	>10%		
VaR5%	0.68	10.09	-0.25	-0.05	0.03	0.56	-2.64	-8.12	5.0	4.7
	*	>10%	***	*	>10%	*	>10%	>10%		
ES1%	0.93	1.35	-0.19	-0.06	0.03	0.64	-2.89	2.53	5.0	4.8
	*	>10%	***	*	>10%	*	>10%	>10%		
ES5%	0.65	10.58	-0.32	-0.05	0.03	0.57	-2.77	-9.07	5.0	4.6
	*	>10%	***	*	>10%	*	>10%	>10%		
EVaR1%	0.90	6.37	-0.22	-0.05	0.03	0.68	-2.91	-5.72	5.0	4.7
	*	>10%	***	*	>10%	*	>10%	>10%		
EVaR5%	0.60	25.88	-0.28	-0.06	0.04	0.62	-2.60	-11.20	5.0	4.6
	*	>10%	***	*	>10%	*	>10%	>10%		
ENT	0.83	0.44	-0.17	-0.06	0.03	0.59	-2.83	8.25	5.0	4.6
	*	>10%	***	*	>10%	**	>10%	*		
ML	0.90	0.39	-0.17	-0.07	0.03	0.61	-2.88	8.21	5.0	4.6
	*	>10%	***	*	>10%	**	>10%	*		

Note: Values presented here are the time-series medians of the coefficients from cross-sectional regressions with the one-month-ahead excess returns as dependent variable and, as independent variables, the risk measures, the market beta, the Size factor ($ln(MktCap)$), the Value factor ($ln(BE/ME)$), the Momentum factor (lagged 11 months return), the Short-Term Reversal factor (one-month-lagged return), and the Idiosyncratic Volatility factor. The symbols ***, **, and * indicate if the coefficients are statistically different from zero at the 10%, 5%, and 1% levels, respectively. Last 2 columns exhibit the R^2 and the adjusted R^2. Coefficients, R^2, and adjusted R^2 are multiplied by 100 so they are represented as percentages.

significant at 1%, MOM and IVOL are significant at 5%, and the Size factor and the intercept remain significant, but only at 10%. The Value factor and the STR are not good predictors of expected returns, according to these regressions. When we add the risk measures analyzed in this study as independent variables, along with these other factors, we see that none proves to be statistically significant to explain the expected returns. Except for EL, all the others have positive coefficients, but the p-value adjusted by Newey and West (1986) could not indicate these coefficients are statistically different from zero. The different results presented by EL can be explained by the different characteristics of this measure. EL assesses the central trend of losses and is not concerned with less likely losses (such as ES and VaR, for example) nor with variability (such as semi-deviation, for example).

Table 17 – Coefficients of the cross-sectional regressions of expected excess returns as dependent variable and the deviation measures analyzed and controlling factors as independent variables. The sample period extends between January 1982 and January 2021.

	Alpha	Risk	BETA	SIZE	VAL	MOM	STR	IVOL	R^2	Adj R^2
Deviation Measures										
StD	0.81	92.65	-0.41	-0.07	0.05	0.61	-2.80	-80.86	5.1	4.9
	*	>10%	>10%	*	>10%	*	>10%	>10%		
SD^-	0.80	56.24	-0.33	-0.05	0.03	0.64	-2.77	-16.47	4.9	4.6
	*	>10%	**	*	>10%	*	>10%	>10%		
$SD^{\alpha}1\%$	0.88	13.00	-0.21	-0.07	0.04	0.59	-2.80	14.92	5.0	4.7
	*	>10%	***	*	>10%	**	>10%	**		
$SD^{\alpha}5\%$	0.90	1.30	-0.17	-0.06	0.03	0.61	-2.84	7.71	5.0	4.7
	*	>10%	***	*	>10%	**	>10%	**		

Note: Values presented here are the time-series medians of the coefficients from cross-sectional regressions with the one-month-ahead excess returns as dependent variable and, as independent variables, the risk measures, the market beta, the Size factor ($ln(MktCap)$), the Value factor ($ln(BE/ME)$), the Momentum factor (lagged 11 months return), the Short-Term Reversal factor (one-month-lagged return), and the Idiosyncratic Volatility factor. The symbols ***, **, and * indicate if the coefficients are statistically different from zero at the 10%, 5%, and 1% levels, respectively. Last 2 columns exhibit the R^2 and the adjusted R^2. Coefficients, R^2, and adjusted R^2 are multiplied by 100 so they are represented as percentages.

Table 18 – Coefficients of the cross-sectional regressions of expected excess returns as dependent variable and the loss-deviation measures analyzed and controlling factors as independent variables. The sample period extends between January 1982 and January 2021.

	Alpha	Risk	BETA	SIZE	VAL	MOM	STR	IVOL	R^2	Adj R^2
Loss-Deviation Measures										
ELD	0.88	15.33	-0.31	-0.05	0.02	0.62	-2.71	3.50	5.0	4.6
	**	>10%	**	*	>10%	>10%	>10%	>10%		
SDR1%	0.92	1.47	-0.18	-0.06	0.03	0.65	-2.90	2.75	5.0	4.8
	*	>10%	***	*	>10%	**	>10%	>10%		
SDR5%	0.76	9.94	-0.26	-0.05	0.02	0.60	-2.83	-2.87	5.0	4.6
	*	>10%	***	*	>10%	*	>10%	>10%		
DEVaR1%	0.92	4.09	-0.21	-0.06	0.03	0.67	-2.91	2.47	5.0	4.7
	*	>10%	***	*	>10%	*	>10%	>10%		
DEVaR5%	0.76	20.81	-0.29	-0.05	0.02	0.62	-2.77	-7.27	4.9	4.6
	*	>10%	***	*	>10%	*	>10%	>10%		
DENT	0.83	0.48	-0.17	-0.06	0.03	0.59	-2.83	8.18	5.0	4.6
	*	>10%	***	*	>10%	**	>10%	*		

Note: Values presented here are the time-series medians of the coefficients from cross-sectional regressions with the one-month-ahead excess returns as dependent variable and, as independent variables, the risk measures, the market beta, the Size factor ($ln(MktCap)$), the Value factor ($ln(BE/ME)$), the Momentum factor (lagged 11 months return), the Short-Term Reversal factor (one-month-lagged return), and the Idiosyncratic Volatility factor. The symbols ***, **, and * indicate if the coefficients are statistically different from zero at the 10%, 5%, and 1% levels, respectively. Last 2 columns exhibit the R^2 and the adjusted R^2. Coefficients, R^2, and adjusted R^2 are multiplied by 100 so they are represented as percentages.

6 Conclusions

This study examines the potential of a set of risk measures to predict expected returns in the US market from January 1982 to December 2020. The risk measures analyzed are part of three distinct categories: loss, deviation, and loss-deviation. The loss measures are Expected Loss, Value at Risk, Expected Shortfall, Expectile Value at Risk, Entropic, and Maximum Loss. The deviation measures are Standard Deviation, Negative Semi-Deviation, and Shortfall Deviation. Finally, the loss-deviation measures are Expected Loss Deviation, Shortfall Deviation Risk, Deviation Expectile Value at Risk, and Deviation Entropic. By using daily returns from an average of 2,605 stocks each year, we estimated the risk measures and performed both a non-parametric and a parametric technique to investigate the relationship between the risk measures and the one-month-ahead equity returns.

The non-parametric method is the univariate portfolio examination. Each month, stocks were sorted by each risk measure and separated into quintiles. The first quintile contains stocks with the lower estimated risk and the last (fifth) quintile with the higher risk. These portfolios were formed by weighting the stocks equally and also by market value. Analyzing the one-month-ahead excess returns of each portfolio, we can conclude that all risk measures have a positive relationship with expected returns, except for the EL, which presented results in the opposite direction (negative relationship). This positive relationship means that, as expected, the expected return also increases as the risk of a stock increases. For the equally weighted portfolios, the difference between the 5th and 1st portfolios returns is statistically different from zero for all risk measures, except for the Shortfall Deviation. However, for the value-weighted portfolios, only the EL presented returns for the Portfolio 5-1 statistically different from zero. One possibility for these findings is that small stocks drive the results for equally weighted portfolios. Next, we examine whether standard asset pricing models can explain the excess return of the Portfolio 5-1. To do so, we tested if the intercept coefficients (Alphas) of regressions with the portfolios expected excess return as the dependent variable and, as independent variables, the returns of portfolios formed by the market beta, size, value, and momentum factors of Fama and French (1993) and Carhart (1997) differ from zero. In these regressions, only the Expected Loss, the Standard Deviation, and the Semi-Deviation show Alphas statistically different from zero. This means that, for the other risk measures, the Fama-French three-factors and the Momentum Factor already explain the finding that equities with higher risk measures earn higher one-month-ahead returns. We also analyzed the average returns for each portfolio, only considering crisis periods. The findings are a return of the Portfolio 5-1 even higher than of the Total Period.

The parametric method was cross-sectional regressions performed similarly to Fama and MacBeth (1973). The results from the regressions are consistent with the findings in the Portfolio Analysis. Each month, we run regressions considering as dependent variable the one-month-ahead excess returns and, as independent variables, each risk measure and the market beta. Only the EL presented a negative median of cross-sectional coefficients, indicating a negative relationship between the expected returns and the risk measure. All other risk measures exhibit a positive relationship. The time-series medians of the cross-sectional coefficients for all risk measures are statistically different from zero, except for the Shortfall Deviation at 1%. Likewise, in the Portfolio Analysis, for robustness check, we added some well-established factors as independent variables, such as the Fama-French three-factors, the Momentum factor, the Short-Term reversal factor, and the Idiosyncratic Volatility factor. When we add the risk measures analyzed in this study as independent variables, along with these other factors, we see that none proves to be statistically significant to explain the expected returns. It indicates a relationship between the risk measures and the expected returns, but when we add other factors, they lose their explanatory power. According to the common-sense assumptions regarding the risk-return tradeoff and our expectations, we could find a positive relationship between expected returns and the risk measures of all categories: loss, deviation and loss-deviation. EL is the only exception, which only shows a positive relationship in crisis periods. However, we would expect to find a significant relationship even after including the controlling factors in the model, but we see in the results these variables already explain the expected stock returns.

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