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Towards Fairness-Aware Time-Sensitive Asynchronous Federated Learning for Critical Energy Infrastructure

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***Abstract*—Critical energy infrastructure (CEI) systems are vital to underpin the national economy and social development, but vulnerable to cyber attack and data privacy leakage when distributed machine learning technologies are deployed on them. Although federated learning (FL) has promoted distributed collaborative learning while keeping natural compliance with the privacy protection, it is tremendously difficult to schedule edge nodes of CEI collaboratively when asynchronous FL tasks are applied in CEI system, since the CEI system must make an irrevocable immediate decision on whether to hire a participant who arrives and departs dynamically without knowing future information. We tackle this issue by designing fairness-aware and time-sensitive task allocation mechanisms in asynchronous FL for CEI. First, we design an optimal multi-dimensional contract to guarantee the reliability, honesty and fairness, and maximize the learning accuracy for the fixed deadline scenario. Second, we design a multi-metric participant recruitment mechanism to control time consumption for the limited budget scenario, prove that the problem of optimizing this mechanism is NP-Hard, and propose an e-approximation algorithm accordingly. Finally, extensive experiments using both real-world data and simulated data further demonstrate the effectiveness and efficiency of our proposed mechanisms compared to the state-of-the-art approaches.**

***Index Terms*—Critical Energy Infrastructure, Federated Learning, Task Allocation, Fairness-Aware, Time-Sensitive**

I. Introduction

RECENTLY, the prosperities in technologies such as 5G, artificial intelligence, and mobile edge compute have put forward higher demands on Critical Energy Infrastructures (CEI) which are some physical facilities to underpin the national economy and social development. All of them are huge in terms of scale and spatially distributed. Moreover, a large number of edge nodes of CEI makes it difficult to organize cooperative work and vulnerable to cyber-attack and data privacy leakage which can cause huge cascading damage to CEI. Therefore, it is necessary to deploy distributed machine learning technologies on CEI to regulate them collaboratively, and there exist many related research works to combine distributed machine learning technologies with CEI [1]. Federated learning (FL), as a promising technique, naturally solves the problem of privacy protection and has an advantage in distributed machine learning where data are collected and processed locally at distributed devices, and then the updated model parameters are uploaded to a central server for model aggregation [2]. With the popularity of FL, there is an increasing number of FL-related studies devoted to this area [3, 4]. However, when FL is applied in the CEI applications, there are many challenges that lead to low efficiency and worse performance [5]. It is tremendously difficult to schedule edge nodes of CEI effectively to execute collaborative model training and maintain system sustainability in a long term, since a mass of heterogeneous distributed edge nodes are mutually independent and lack of an efficient method to regulate them collaboratively. To this end, how to effectively allocate tasks to improve learning accuracy has become a critical issue when applying FL in CEI systems [6, 7].

There have emerged a lot of works focussing on synchronous FL applied in CEI. On the one hand, it is helpful for the model owner to select a suitable set of participants according to some criteria for different objectives in synchronous FL, *e.g.*, time limitation, communication efficiency, and energy consumption [8, 9]. On the other hand, it incurs higher communication cost, while also leads to higher idle durations waiting for slower participants, as the aggregation must wait for the completion of all local updates [10]. In contrast, asynchronous FL (AFL) allows the model owner to recruit arriving data owners online to continuously train the model, which makes FL more effective. However, time-sensitivity has become a thorny issue in AFL. This is because a large number of heterogeneous participants arrive and depart dynamically in a random manner, meanwhile without knowing future information, the model owner must make an irrevocable immediate decision on whether to hire a participant, and pursue the maximization of individual benefits under some constraints such as time sensitivity and budget feasibility.

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This prevents the task allocation mechanisms applied to synchronous FL from being directly applied to AFL.

Despite the practical application of AFL has received widespread attention [11, 12], how to effectively allocate tasks in AFL, although very urgent, is still largely ignored. To the best of our knowledge, only [13] attempts to efficiently perform distributed learning tasks in an asynchronous manner. However, one-sided pursuit of minimizing the gradient staleness on wireless edge nodes with heterogeneous computing and communication capacities may present a rather unbalanced allocation in shared and limited participant pool, which would be perceived as highly unfair and unacceptable to the worse-off participants [14, 15]. Although a variety of fairness criteria such as Max-Min, Kalai-Smorodinsky, and Proportional Fairness already exist in the literature [16], there are concerns regarding collective fairness rather than individual fairness. Due to natural rationality and selfishness, once a participant feels unfair, she will leave the current system to obtain higher benefit, thereby affecting the sustainability of the system. Consequently, without considering individual fairness, it is not feasible to assume that all participants will unconditionally join the model training task in the practical application of AFL in CEI [17].

In summary, there exist three main challenges that hinder the design of efficient task allocation mechanisms for AFL: **C1:** *Fairness-aware*, *i.e.*, it is critical to address the issue of fairness-aware task allocation mechanisms in AFL in a principled manner as individual fairness will inevitably affect her participation, which will further affect the sustainability of the system. **C2:** *Time-sensitive*, *i.e.*, the model owner in AFL must make an irrevocable immediate decision on whether to hire a participant who arrives and departs dynamically without knowing future information, because time sensitivity is directly related to the effect of AFL. **C3:** *Asymmetric information*, *i.e.*, strategic participants will inevitably take advantage of the information asymmetry and misreport their private information (*e.g.*, arriving time, computation capacity) to seek higher returns.

To tackle these challenges, this paper designs fairness-aware time-sensitive task allocation mechanisms in AFL for CEI. In particular, we make use of the ρ-Lipschitz condition to formalize the individual fairness rather than collective fairness, and define a new form of fairness named acceptance-aware fairness (addressing **C1**), design a multi-metric participants recruitment mechanism, which can iteratively adjust task allocation through forceful interventions to control time consumption of task training (addressing **C2**), and introduce multi-dimensional contract theory, use its self-revealing property to hire heterogeneous participants and incentivize them to complete the training task honestly, thereby maximizing the learning accuracy of the model under the premise of asymmetric information (addressing **C3**). Combining with the above challenges and realistic demands in a CEI system, two typical task allocation scenarios in AFL are taken into account: the fixed deadline scenario where the model training process needs to be completed before a fixed deadline with incomplete information, and the limited budget scenario where the total payment of a model training task to the selected participants cannot exceed a budget. Our main contributions are summarized as follows:

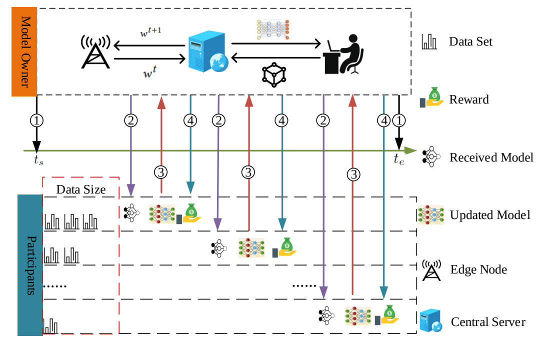

Figure 1. The typical process in AFL in CEI.

- A fairness-aware time-sensitive model for task allocation in AFL is built to formulate task allocation problem by working on two typical scenarios where learning accuracy, time consumption and individual fairness are taken into consideration concurrently.
- For the fixed deadline scenario, we use entropy theory and fuzzy comprehensive evaluation to design an optimal multi-dimensional contract where the optimal contract value is calculated to guarantee the reliability, honesty and fairness, so that the learning accuracy is maximized.
- For the limited budget scenario, we design a multi-metric participant recruitment mechanism to control time consumption by dynamically adjusting task allocation, prove that the problem of optimizing this mechanism is NP-hard, and accordingly propose an e-approximation algorithm based on relaxed iterative optimization.
- Performance evaluations based on real-world data and simulated data further demonstrate the effectiveness and efficiency of our proposed mechanisms in balancing the learning accuracy, fairness and timeliness compared with the state-of-the-art mechanisms.

In the rest of this article, Section II introduces the system model and problem formulation. A multi-dimensional contract is designed for the fixed deadline scenario in Section III. And a multi-metric participant recruitment mechanism is designed for the limited budget scenario in Section IV. Evaluations are performed in Section V. Finally, conclusions are drawn in Section VI .

II. System Model And Problem Formulation

A. Basic Setting

As illustrated in Figure 1, a typical AFL in CEI system consists of a set $\mathcal{P} = \{p_1, \cdots, p_n\}$ of heterogeneous participants (*i.e.*, edge nodes), where each one has different data size and computation capacity. The model owner resides in a cloud-based platform, and aims at building an effective model for some CEI applications, such as traffic prediction and electric vehicle management, and generates a training task by announcing the time constraint $t = (t_s, t_e)$, where t_s and t_e denote the start and end time points respectively. Each participant $p_i \in \mathcal{P}$ arrives in the platform at her

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own time point t_i^a and departs at time point t_i^l. The local computation duration t_i of participant p_i mainly depends on the computation capacity a_i and the training data samples q_i which can be denoted as $t_i = q_i/a_i$. When the local training is completed at $t_i^u = t_i^a + t_i$, each participant should immediately upload the updated weight parameters to the model owner in exchange for rewards.

The process of the model training in AFL can be described as follows: First, the model owner describes and publishes a task consisting the context of training and time constraints (step ①). Next, participants arrive at a random manner. After entering the platform, each participant p_i needs to report to the model owner and download the model parameters from the platform if she is hired (step ②). Then, the hired participants use local resources to train task model and submit the model parameters (step ③). Finally, the model owner pays the participants accordingly after receiving the updated model parameters (step ④). The above process will be repeated until the training task is completed.

From the point of rationality and selfishness, both the model owner and participants are concerned about how to choose her optimal strategy to maximize her own utility. As for each participant p_i, she will determine how much data q_i it will contribute to the model training, and chooses to participate in a task when the reward payment $r_i(t_i^u, q_i)$ can compensate for her cost $c_i q_i$, where c_i is the unit data cost. As participants arrive and depart in a dynamic manner, the reward payment $r_i(t_i^u, q_i)$ is also determined by the uploading time t_i^u. Consequently, the utility of a participant can be defined as follows.

Definition 1. *(Participant's Utility) The utility of participant p_i is defined as:*

$$u_i \triangleq \begin{cases} r_i(t_i^u, q_i) - c_i q_i, & \text{if } \zeta_i = 1, \\ 0, & \text{otherwise}, \end{cases} \tag{1}$$

where the parameter $\zeta_i \in \{0, 1\}$ is a indicator variable referring whether a participant takes part in the model training.

At the model owner side, her strategy is to determine the optimal reward payments in $\mathcal{R}$. The model owner will get more utility by incentivizing participants to contribute more data. This is because the more data participants contribute to training the learning model, the higher the model performance (*i.e.*, prediction accuracy) will be. Following [18], we denote the model performance as

$$\Lambda(\mathcal{Q}) = 1 - e^{-\mu(\mathcal{Q})^v}, \tag{2}$$

where $\mathcal{Q} = \sum_{p_i \in \mathcal{P}} q_i$ refers to the total training data contributed by all participants, μ and v are weight factors. We assume that the global model performance gain has diminishing returns with respect to data quality and quantity. Intuitively, there exists a limit to model performance where $\lim_{\mathcal{Q} \to \infty} 1 - e^{-\mu(\mathcal{Q})^v} = 1$. Formally, the utility of the model owner can be defined as follows.

Definition 2. *(Model Owner's Utility) The utility of the model owner is defined as*

$$u_s(\mathcal{Q}, \mathcal{R}) \triangleq \lambda(1 - e^{-\mu(\mathcal{Q})^v}) - \sum_{r_i \in \mathcal{R}} r_i, \tag{3}$$

Table I
SUMMARY OF NOTATIONS IN THIS PAPER

Variable	Description
$p_i, \mathcal{P}$	ith participant, $\mathcal{P} = \{p_1, \cdots, p_n\}$.
t_i^a, t_i^l, t_i^u	arriving time, leaving time and completing time of ith participant.
$q_i, \tilde{q_i}, \mathcal{Q}, \tilde{\mathcal{Q}}$	contributed, own data of p_i, $\mathcal{Q} = \sum_{p_i \in \mathcal{P}} q_i$, $\tilde{\mathcal{Q}} = \sum_{p_i \in \mathcal{P}} q_i$.
a_i, c_i	computation capacity and unit data cost of ith participant.
ζ_i	indicator variable $\zeta_i \in \{0, 1\}$.
λ, Λ	conversion parameter, model performance function.
u_i, u_s	utility of ith participant, utility of the model owner.
$\xi_m, \hat{\xi_m}$	predicted value and real value by the training model.
θ_i	ith type of participant.
s_i	the task contribution score of ith participant.
$r_i, \mathcal{R}$	the optimal reward of ith participant, $\mathcal{R} = \{r_1, \cdots, r_n\}$.
d, $\mathcal{D}$	difference function of contribution and utility between two participants.

where conversion parameter λ is used to turn the model performance into the model utility.

B. Acceptance-Aware Fairness

How to effectively allocate tasks in AFL is essentially an allocation optimization problem with limited resources. Blindly pursuing the maximization of the model owner's benefit may present a rather unbalanced allocation in shared and limited participant pool. In addition to maximizing the benefit of the model owner, we consider fair allocation which focus on the personal utility obtained by each participant. To this end, we reflect fairness from two aspects: task contribution and payment reward. Therefore, we capture fairness by the principle that participants who make similar contributions to the model training task should receive similar rewards. Hence, we make use of the ρ-Lipschitz condition to formalize the individual fairness rather than collective fairness. First, we modify the ρ-Lipschitz condition to adapt to the payment rule based on [19],

$$\mathcal{D}(\Upsilon(p_i), \Upsilon(p_j)) \leq d(p_i, p_j). \tag{4}$$

The payment rule $\Upsilon : \mathcal{P} \to R^+$ is the strategy for the reward of participants, $\mathcal{D} : R^+ \times R^+ \to R^+$ means the gap of utility and $d : \mathcal{P} \times \mathcal{P} \to R^+$ is a metric on participants which can calculate the similarity between two participants. In order to satisfy the requirement of individual fairness and make participants willing to participate in a task, we design a new form of individual fairness named Acceptance-Aware Fairness (AAF). It guarantees individual fairness by ensuring that any two participants who make similar contributions get similar rewards.

Definition 3. *(AAF) A task allocation mechanism satisfies AAF if*

$$d(p_i, p_j) - \log(\max\{\frac{\Upsilon(p_i)}{\Upsilon(p_j)}, \frac{\Upsilon(p_j)}{\Upsilon(p_i)}\}) \geq 0. \tag{5}$$

C. Problem Formulation

1) Multi-Dimensional Contract: For the fixed deadline scenario, all participants can be classified into different types based on their arriving time and computation capacity. However, these are all private information of the participants. To

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leverage on the self-revealing properties of participants, we design a multi-dimensional contract for this scenario, which establishes some rules that the model owner uses to regulate the behavior of participants.

Definition 4. *(MDC) A Multi-Dimensional Contract is represented as a 3-tuple (σ, π, γ), i.e., a participant classification rule σ, a data quantity requirement rule ϱ, and a payment determination rule γ.*

- $\sigma : \mathcal{P} \to N^*$ *classifies participants into θ-type based on the time constraint $t = (t_s, t_e)$ of the task, the arrival time t_i^a, departure time t_i^l and computation capacity a_i of participant $p_i \in \mathcal{P}$,*
$$\theta = \sigma(t, t_i^a, t_i^l, a_i). \tag{6}$$
- $\pi : N^* \to R^+$ *specifies the amount of data samples q_θ that the θ-type participant should contribute to model training,*
$$q_\theta = \pi(\theta). \tag{7}$$
- $\gamma : N^* \times R^+ \to R^+$ *defines the rules that rewards a participant for its contribution according to her θ-type and the amount of data samples q_θ,*
$$r_\theta = \gamma(\theta, q_\theta). \tag{8}$$

The self-revealing function of contract theory needs to satisfy two fundamental properties, namely individual rationality (IR) and incentive compatibility (IC), otherwise, the honesty of participants cannot be guaranteed. The formal definitions of IR and IC are given below.

Definition 5. *(IR) A participant performs a training task only when her utility is non-negative, i.e.,*
$$u_i = \theta_i r_i - c_i q_i \geq 0, \forall i \in [1, n]. \tag{9}$$

Definition 6. *(IC) A participant can maximize her utility only by honestly choosing the contract item designed for her type. i.e.,*
$$\theta_i r_i - c_i q_i \geq \theta_i r_j - c_i q_j, \forall i \neq j \in [1, n]. \tag{10}$$

For the model owner, the purpose of designing the contract is to maximize her own utility, and it also needs to satisfy AAF, IR and IC constraints, as shown below.

Definition 7. *The optimal design of multi-dimensional contract problem is formalized as follows:*
$$\begin{cases} \max\limits_{(r,q)} u_s = \lambda(1 - e^{-\mu(\sum_{i=1}^N \theta_i q_i)^v}) - \sum\limits_{i=1}^N r_i, \\ \textit{s.t. } \text{Eq.}(5), (9), (10). \end{cases} \tag{11}$$

2) Multi-Metric Participant Recruitment Mechanism: For the limited budget scenario, how to minimize the model training time is a critical issue under the constraints of prediction accuracy and budget feasibility. Since participants can strategically choose tasks, it is difficult to control the model training time, and hence the contract theory cannot be applied to this scenario. To this end, we design a multi-metric participant recruitment mechanism, to achieve optimal control of time sensitivity by designing recruitment rules for participants under the premise of limited budget.

Definition 8. *(MMPR) A Multi-Metric Participant Recruitment mechanism is represented as a 3-tuple (ϕ, φ, ψ), i.e., a participant selection rule ϕ, a data amount contribution rule φ, and a payment determination rule ψ.*

- $\phi : \mathcal{P} \to \{0, 1\}$ *specifies the selection strategy whether participant $p_i \in \mathcal{P}$ with arriving time t_i^a and computation capacity a_i should be selected under the time constraint $t = (t_s, t_e)$ and the budget $\mathcal{B}$,*
$$\zeta_i = \phi(t_i^a, a_i, t, \mathcal{B}) \in 0, 1. \tag{12}$$
- $\varphi : \mathcal{P} \to R^+$ *refers to the amount of data contributed by selected participant $p_i \in \mathcal{P}$ to perform the model training task, which is determined by $\zeta_i \in \{0, 1\}$, t_i^a and a_i,*
$$q_i = \varphi(\zeta_i, t_i^a, a_i). \tag{13}$$
- $\psi : \{0, 1\} \times \{R^+ \cup \{0\}\} \to R^+ \cup \{0\}$ *defines the rules that rewards participant $p_i \in \mathcal{P}$ for its contribution according to ζ_i and q_i,*
$$r_i = \psi(\zeta_i, q_i). \tag{14}$$

We use the Root Mean Square Error (RMSE) to measure the prediction accuracy of the model, and use $\mathcal{M}$ data samples to test the model performance and designing function $\mathcal{G}(\mathcal{M})$ to denote RMSE, which can be calculated by
$$\mathcal{G}(\mathcal{M}) = \sqrt{\frac{1}{\mathcal{M}} \sum_{m=1}^{\mathcal{M}} (\xi_m - \hat{\xi_m})^2}, \tag{15}$$
where ξ_m is predicted by the training model and $\hat{\xi_m}$ is the value of label. On top of that, we also consider a more complex situation, namely budget feasibility. To this end, the optimal design of multi-metric participant recruitment is equivalent to how to minimize the model training time under the premise of satisfying the above constraints.

Definition 9. *The optimal design of multi-metric participant recruitment mechanism can be formalized as follows:*
$$\begin{cases} \min \max\limits_{\forall i \in [1,n]} t_i^u = t_i^a + \dfrac{q_i}{a_i}, \\ \textit{s.t.} \begin{cases} \mathcal{G}(\mathcal{M}) \leq \varepsilon, \\ \sum_{i=1}^n r_i \leq \mathcal{B}, \\ \text{Eq.}(5), (9). \end{cases} \end{cases} \tag{16}$$

The first constraint in Eq. (16) is to ensure the model prediction accuracy achieve a certain performance. The second one is to ensure the whole cost can not exceed the whole budget $\mathcal{B}$. The last one is used to satisfy both AAF and IR.

III. Multi-Dimensional Contract

For the fixed deadline, we first introduce entropy theory to comprehensively measure the contributions of participants from multi-dimensional properties. Based on an auxiliary variable which reflects the participant's type, we next sort all participants and convert the multi-dimensional contract into a single-dimensional one. Then, we relax the constraints for contract feasibility in order to optimize the design.

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A. Conversion Into A Single-Dimensional Contract

Without loss of generality, we assume that participants are indexed in a two-dimensional non-decreasing order: $t_1 \leq \cdots \leq t_n$ and $a_1 \geq \cdots \geq a_n$. First, we use entropy method to estimate the weight of each dimension property. The entropy coefficient ω_j of the jth dimensional property can be calculated as

$$\omega_i = \frac{1-\mathcal{W}_i}{n-\sum_{j=1}^{n}\mathcal{W}_j}, \tag{17}$$

where $\mathcal{W}_i = -\frac{1}{\ln n}\sum_{i=1}^{n}\frac{v_{ji}}{\sum_{k=1}^{n}v_{ki}} \times \log\frac{v_{ji}}{\sum_{k=1}^{n}v_{ki}}$ is the entropy of ith dimensional property, and v_{ij} denotes the value of ith participant's jth dimensional property. From the above analysis, the entropy coefficient indicates the importance of each dimensional property. According to the fuzzy comprehensive evaluation in [20], membership function is introduced to represent the difference between one participant with the others, which can be described as

$$f(v_{ij}) = \frac{v_{ij}-\hat{k_j}}{\tilde{k_j}-\hat{k_j}}, \tag{18}$$

where $\tilde{k_j}$ and $\hat{k_j}$ are the maximum value and the minimum value on the jth dimensional property, respectively. Based on the entropy coefficient and the membership function, the task contribution score s_i of participant p_i can be derived as

$$s_i = \sum_{i=1}^{m}\omega_j \times f(v_{ij}). \tag{19}$$

In order to account for a participant's undertake ability to the model training, we combine these two dimensions into an auxiliary variable θ. We can now sort the n participants by the calculated task contribution score in a non-decreasing order as $\theta_1 \leq \cdots \leq \theta_n$.

B. Feasibility and Optimality of MDC

The necessary and sufficient conditions to guarantee contract feasibility based on IR and IC constraints can be derived as follows:

Property 1. *A feasible contract must meet the following necessary and sufficient conditions:*

$$\begin{cases} q_1 \leq \cdots \leq q_N, \\ r_1 \leq \cdots \leq r_N, \\ \theta_1 r_1 - c_1 q_1 \geq 0, \\ \frac{c_{i-1}}{\theta_{i-1}}(q_i - q_{i-1}) \geq r_i - r_{i-1} \geq \frac{c_i}{\theta_i}(q_i - q_{i-1}), \\ \log\frac{r_i}{r_{i-1}} \leq d(s_i - s_{i-1}). \end{cases} \tag{20}$$

Proof: See Appendix A. ■

To find the optimal contract reward, we first establish the dependence of optimal contract reward $r_i^*, \forall i \in [1,n]$ on the quantity of data $q_i, \forall i \in [1,n]$. Then, the problem we need to address only contains parameter $q_i, \forall i \in [1,n]$.

Theorem 1. *Given a set $\{q_1, \cdots, q_n\}$ that satisfies $0 < q_1 < \cdots < q_n$, the optimal reward r_i^* of a feasible contract is*

$$r_i^* = \frac{1}{\theta_1}c_1 q_1 + \sum_{t=1}^{i}\triangle_t, \forall i \in [1,n], \tag{21}$$

where $\triangle_1 = 0$, and $\triangle_t = \frac{1}{\theta_t}c_t q_t - \frac{1}{\theta_t}c_{t-1}q_{t-1}$, $\forall t \in [2,n]$.

Proof: See Appendix B ■

Given the time end point, the optimal design of a feasible contract is equivalent to the optimal design of $\{q_i\}_{i=1}^{n}$. Hence, Eq. (11) can be rewrote as follows:

$$\begin{cases} \max\limits_{q_i,\forall i\in[1,n]} u_s = \lambda(1 - e^{-\mu(\sum_{i=1}^{n}\theta_i q_i)^v}) \\ \qquad\qquad - \sum\limits_{i=1}^{n}(\frac{1}{\theta_1}c_1 q_1 + \sum\limits_{t=1}^{i}\triangle_t), \\ \text{s.t.} \begin{cases} 0 < q_1 < \cdots < q_n, \\ \log\frac{r_i}{r_{i-1}} \leq d(s_i - s_{i-1}). \end{cases} \end{cases} \tag{22}$$

As the objective function $u_s(q_i)$ is structurally separate from different data quantities $q_i, \forall i \in [1,n]$, *i.e.*, $u_s(q_i)$ is independent of $u_s(q_j)$, and thereby $u_s(q_i)$ is only associated with q_i. As such, the variable of each data quantity q_i can be derived by separately optimizing each $u_s(q_i), \forall i \in [1,n]$. When $i = 1$, we just consider the constraint $q_1 \geq 0$, and compute q_1^* by using the convex optimization tools such as *cvxpy*. When $i \in [2,n]$, we have

$$\begin{cases} q_i^* = \arg\max\limits_{q_i,\forall i\in[2,n]} \lambda(1 - e^{-\mu(\theta_i q_i)^v}) \\ \qquad - (\frac{1}{\theta_1}c_1 q_1 + \sum\limits_{t=1}^{i}\triangle_t), \\ \text{s.t. } q_{i-1} \leq q_i \leq \varpi_i, \forall i \in [2,n], \end{cases} \tag{23}$$

where $\varpi_i = \frac{1}{c_i}(\theta_i r_{i-1}e^{d(s_i - s_{i-1})} + c_{i-1}q_{i-1} - \theta_i r_{i-1})$ can be computed by the AAF constraint. This problem can be transformed into unconstraint optimization problem using Lagrange factors:

$$\begin{aligned} \jmath(q_i,\alpha,\beta) =& \lambda(1 - e^{-\mu(\theta_i q_i)^v}) - (\frac{1}{\theta_1}c_1 q_1 + \sum_{t=1}^{i}\triangle_t) \\ &+ \alpha(q_i - q_{i-1}) + \beta(\varpi_i - q_i), \end{aligned} \tag{24}$$

where $\alpha \geq 0$ and $\beta \geq 0$ are dual variables. The optimization problem in Eq. (23) is a convex problem whose optmal primal and dual variables can be characterized using the Karush-Khun-Tucker (KKT) conditions:

$$\begin{cases} \frac{\partial \jmath}{\partial q_i} = \lambda\phi v\theta_i e^{-\mu(\theta_i q_i)^v})(\theta_i q_i)^v, \\ -\frac{1}{\theta_i}c_i q_i + \alpha - \beta = 0, \\ \alpha(q_i - q_{i-1}) = 0, \\ \beta(\varpi_i - q_i) = 0. \end{cases} \tag{25}$$

Corollary 1. *The optimal value of data quantity q_i^* is*

$$q_i^* = \frac{1}{c_i}(\theta_i r_{i-1}e^{d(s_i - s_{i-1})} + c_{i-1}q_{i-1} - \theta_i r_{i-1}). \tag{26}$$

Proof: See Appendix C. ■

IV. Multi-Metric Participant Recruitment Mechanism

For the limited budget scenario, by turning the time optimization problem into the min-max problem of task assignment which is NP-Hard, we design a multi-metric participants

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recruitment mechanism and propose an e-approximation algorithm based on relaxed iterative optimization for it.

A. Conversion of MMPR Mechanism Design

Since it is intractable to accurately express and quantify the performance of the training model, we simplify the time minimization model training time problem by converting the model performance constraint to the data amount constraint.

Lemma 1. *$\mathcal{G}(\mathcal{M}) \leq \varepsilon$ can be reduced to $\sum_{i=1}^{n} q_i \geq \mathcal{Q}$.*

Proof: See Appendix D. ■

Next, we will design a Payment Determination Rule (PDR) to satisfy AAF and IR.

Definition 10. *(PDR)The payment determination rule is defined as*

$$r_i = \tau s_i q_i, \tag{27}$$

where $\tau = \frac{\max(c_i)}{s_i}$ is the coefficient of proportionality.

Lemma 2. *PDR satisfies IR and AAF.*

Proof: See Appendix E. ■

Then, Eq. (16) can be rewrote as follows:

$$\begin{cases} \min \max\limits_{\forall i \in [1,n]} t_i^u = t_i^a + \dfrac{q_i}{a_i} \\ s.t. \begin{cases} \sum_{i=1}^{n} q_i \leq \tilde{\mathcal{Q}}, \\ \sum_{i=1}^{n} k_i q_i \leq \mathcal{B}, \\ 0 \leq q_i \leq \tilde{q}_i, \forall q_i \in Z^+. \end{cases} \end{cases} \tag{28}$$

B. Design and Analysis of Approximate Algorithm

Theorem 2. Eq. (28) *is NP-Hard.*

Proof: See Appendix F. ■

The problem of training time minimization as shown in Eq. (28) is NP-Hard, as proved in Theorem 2, which means that it is intractable to find the optimal solution for it. Consequently, according to the idea of relaxed iterative optimization, we design an approximation algorithm named Relaxation Neighbor Search (RNS) algorithm to calculate an approximate optimal solution. The pseudo code of RNS is shown in Algorithm 1. First, we divide the task time length $t_e - t_s$ into m time slices of equal lengh. The task completion time can be found by making iteration from the present slice. From line 6 to line 9, we calculate the maximum submission time of each participant and sort participants in descending order based on the maximum submission time. From line 10 to line 12, we then relax the original problem by reducing the budget constraints and use greedy strategy to get a solution. The feasible solution can be obtained to satisfy the budget constraint by dynamic adjustment from line 14 to 24. The computational complexity of RNS mainly depends on two factors: the ordering of participants and the secondary adjustment after greedy search. The computational complexity of sorting participants is $O(mn \log n)$ and the worst time for the adjustment is $O(mn^2 \log n)$. Therefore, the computational complexity of RNS is $O(mn^2 \log n)$.

Lemma 3. *RNS is an e-approximation for Eq. (28).*

Proof: See Appendix G. ■

Algorithm 1: Relaxation Neighbor Search Algorithm

Input: $\mathcal{P}_k \subseteq \mathcal{P}, \forall k \in [1, m]$, $\{t_1^a, \cdots, t_n^a\}$, $\mathcal{C}$, $\mathcal{B}$, $\mathcal{Q}$, $\tilde{\mathcal{Q}}$
Output: t_c
1 **for** $k = 1; k \leq m; k++$ **do**
2   **for** $p_i \in \mathcal{P}_k \subseteq \mathcal{P}$ **do**
3     $s_i = \sum_{i=1}^{m} \omega_j \times f(x_{i,j})$;
4     $t_i^u = t_i^a + \frac{\tilde{q}_i}{c_i}$;
5   resort elements in $\{t_i^u\}_1^{|\mathcal{P}_k|}$, so that $t_1^u > \cdots > t_{|\mathcal{P}_k|}^u$;
6   **for** $t_i^u \in \{t_i^u\}_1^n$ **do**
7   **if** $\sum_{j=1}^{i} \tilde{q}_j \leq \mathcal{Q}$ **then**
8       $\mathcal{M} = \mathcal{M} \cup \{p_j\}$;
9       $r_j = \tau s_j \tilde{q}_j$;
10   **if** $\sum_{i \in \mathcal{M}} r_i \leq \mathcal{B}$ **then**
11     $t_c = \arg\max_{t_i^u}(t_i^a + \frac{q_i}{a_i}), \forall p_i \in \mathcal{M}$;
12   break;
13   **else**
14   resort elements in $\{r_i\}_1^{|\mathcal{M}|}$, so that $r_1 < \cdots < r_{|\mathcal{M}|}$;
15     $\mathcal{B}_d = \sum_{i \in \mathcal{M}} r_i - \mathcal{B}$;
16   **for** $r_i \in \{r_i\}_1^{|\mathcal{M}|}$ **do**
17   **if** $\sum_{j=1}^{i} r_j \leq \mathcal{B}_d$ **then**
18         $\mathcal{W}_h = \mathcal{W}_h \cup \{p_j\}$;
19   **for** $p_i \in \mathcal{P}_k / \mathcal{M}$ **do**
20   **if** $\sum \tau s_i q_i \leq \mathcal{B}_d$ **then**
21         $\mathcal{W}_r = \mathcal{W}_r \cup \{p_i\}$;
22     $\mathcal{M} = (\mathcal{M}/\mathcal{W}_h) \cup \mathcal{W}_r$;
23   update $\mathcal{M}$ using brand and bound method;
24   go to line 10;

V. Performance Evaluation

In this section, we evaluate the performance of our proposed two mechanisms (*i.e.*, multi-dimensional contract and multi-metric participant recruitment mechanism) by using real-world data and simulated data.

A. Experiment Setup

The code of our mechanisms are written in python with pytorch. All the experiments have been carried out on a standard desktop PC with an Intel Core i5 running at 2.3 GHz, and with LPDDR3 8 GB 2133 MHz, running MacOS Mojave 10.14.6 Editions.

1) Datasets: we use both a simulated dataset and a standard real-world dataset to perform comparative experiments.

Simulated Dataset. Without loss of generality, we first divide the participants into 10 different types according to their computation capacity, and assume that the computation capacities of the participants obey a uniform distribution $\theta \sim U(1, 10)$. Next, we assume that 200 participants will arrive at the platform randomly, and their arrival times obey a uniform distribution $t_a \sim U(0, 20)$.

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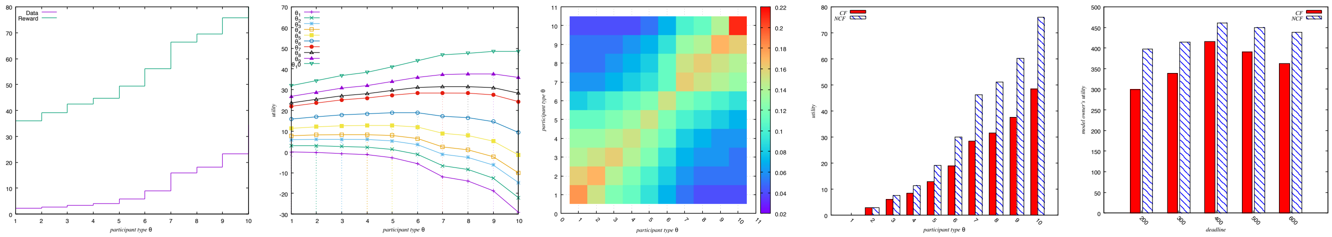
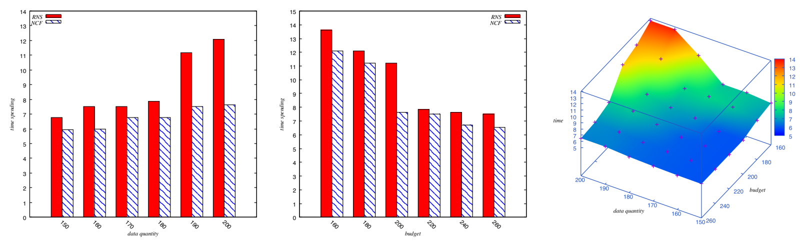


(a) (b) (c) (d) (e)

Figure 2. The performance of multi-dimensional contract from monotonicity, incentive compatibility, fairness and utility of participants with 10 different type participants: (a) Monotonicity; (b) IC; (c) Fairness; (d) Utility of participants; (e) Utility of the model owner.



(a) (b) (c)

Figure 3. Comparison of time consumption between multi-metric participant recruitment mechanism and the unfair one: (a) Fixed budget; (b) Fixed data quantity; (c) Dynamic change.

Real-world Dataset. We utilize MNIST which is a standard real-world dataset widely used in many related research work [21], [22]. It is a good database of 10-class of handwritten digits images which includes a training set of 60000 samples and a testing set of 10000 samples. To show efficiency of our proposed mechanisms, we utilize two most common deeplearning models: Convolutional Neural Networks (CNN) and MultiLayer Perceptron (MLP). The net structure of CNN consists of three convolutional layers, two fully connected layers and a softmax output layer. The net structure of MLP consists of one input layer, four hidden layers and one output layer. The optimizer of them are both stochastic gradient descent.

2) Comparison Mechanisms: In order to make sufficient comparisons with the state-of-the-art mechanisms, we select the following two typical and relevant mechanisms as baselines to compare with our proposed mechanism:

- **NCF**, which only focuses on maximizing utility without taking any fairness into account.
- **EA**, which assigns the same number of tasks to all participants, ignoring both fairness and the heterogeneity of participants [23].

3) Metrics: In the simulated experiment, we use the utility of both the participants and the model owner to evaluate the compared mechanisms. In the real-world dataset-based experiment, we mainly focus on prediction accuracy and time consumption to compare our proposed mechanisms with the others.

B. Results on Simulated Dataset

1) Multi-Dimensional Contract: In this part, we want to validate the monotonicity, IR, IC and AAF of our designed contract. We take 10 different types of participants into consideration from the lowest 1-type to the highest 10-type. From Figure 2(a), both the data sample size and the reward of a participants monotonically increase with her type value. This is because a participant with a higher type value should make higher contribution to model training and receive more reward. Thus, the numerical result verifies that MDC satisfies the monotonicity. From Figure 2(b), a participant always achieves the highest utility when it chooses the right contract designed for her own type. On the contrary, the participant will reduce her utility when choosing an inappropriate contract for her type. This phenomenon is consistent with incentive compatibility. In Figure 2(c), we use the palette to quantify the difference in all type participants. The value of the color with small number indicates low difference and strong fairness. Obviously, our designed contract can guarantee that most participants get fair rewards. According to the role of fairness, the system of AFL will be much stable and sustainable. As for participant's utility in Figure 2(d), the participant's utility of all possible types has non-negative returns which satisfies the IR constraint, and the utility of our designed contract with fairness (CF) is less than NCF with minor difference. Then, we demonstrate the change of the model owner's utility with different task deadline in Figure 2(e). The utility first increases with the passage of time. This is because the participants who stay in the system for a long time can use more data samples to train the task to obtain higher model performance. As time progresses, the prediction accuracy improves slowly, while the rewards paid to participants increase linearly. Revenues and payments are just offset from each other, and thus the mode owner's utility begins to decrease monotonically.

2) Multi-Metric Participant Recruitment Mechanism: In order to show the efficiency of MMPR on time consumption, we make a comparison between NCF and RNS. In Figure 3(a), we fix the budget with 200. It can be seen that the time

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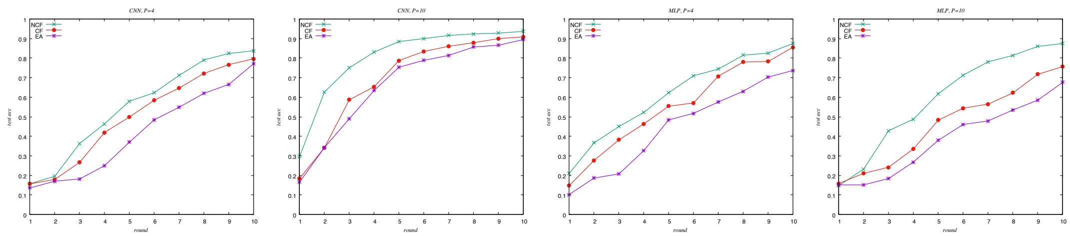
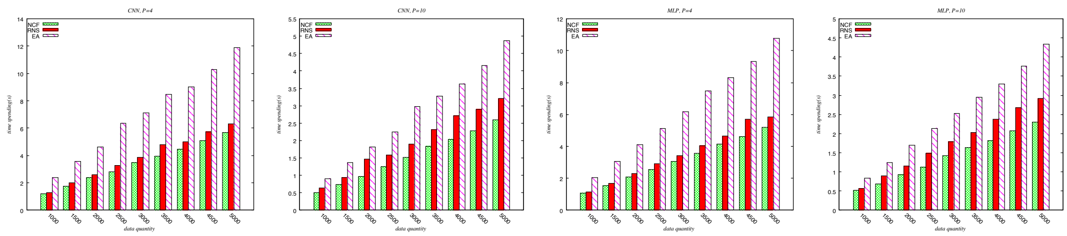


(a) (b) (c) (d)

Figure 4. The prediction accuracy of multi-dimensional contract using MNIST: (a) CNN & P=4; (b) CNN & P=10; (c) MLP & P=4; (d) MLP & P=10.



(a) (b) (c) (d)

Figure 5. The time consumption of multi-metric participant recruitment mechanism using MNIST: (a) CNN & P=4; (b) CNN & P=10; (c) MLP & P=4; (d) MLP & P=10.

consumption of RNS monotonically increases with the data quantity, while the time consumption of NCF is less than that of RNS. This is because RNS takes more time to guarantee the fair constraints. In Figure 3(b), we fix the data quantity with 200. It can be seen from the graph that as the budget increases, the time consumption will decrease. Obviously, the model owner with a higher budget can hire more expensive participants to perform tasks. Figure 3(c) draws a 3D graph of time consumption changes under different budgets and data quantities. It can be seen that the time consumption monotonically increases with the data quantity, monotonically decreases with the budget value, and the reason is the same as the previous two figures.

C. Results on Real-world Dataset

1) Prediction Accuracy: Figure 4(a) and Figure 4(b) show the prediction accuracy of the CNN model with different numbers of participants. As the number of training rounds increases, the prediction accuracy of CNN for each mechanism increases monotonically. The prediction accuracy of CF is generally lower than NCF but higher than EA. This is because compared with NCF, CF takes into account the fairness factor, which means that the difference in the amount of data that participants should contribute is compressed, and the total amount will be reduced accordingly. However, compared with EA, EA ignores the heterogeneity of participants, resulting in some unfinished tasks and low resource utilization. Figure 4(c) and 4(d) show the prediction accuracy of the MLP model with different numbers of participants. The performance curve trends of the three mechanisms and the differences between them are basically the same as those in Figure 4(a) and Figure 4(b), and the reasons are also the same. The main difference is that we can find that the prediction accuracy has fluctuated growth, and there are some overlaps. The main reason for these phenomena is that the training model from the training data set is unstable, and cannot be applied to the test data set after a few rounds. As the number of training rounds increases, the fluctuations gradually disappear.

2) Time Consumption: Figure 5 show the time consumption of the different models (*i.e.*, CNN, MLP) with different numbers of participants. The curve trends of the four figures are very similar. No matter which model is used, and how many participants there are, the time consumption monotonically increases with the data quantity. Compared Figure 5(a) with Figure 5(b), the larger the number of participants, the less time for model training. This is the same phenomenon in Figure 5(c) with Figure 5(d). Compared the difference between the three mechanisms in each figure, we can see that EA is the most time-consuming of the three mechanisms, and the difference between EA and other mechanisms will change as the amount of data increases. This is because it ignores the heterogeneity between devices, resulting in low resource utilization. In contrast, RNS is far superior to EA and very close to NCF. As the number of participants increases, the gap between EA and others further increases, and the mechanism of NCF and RNS will become more stable.

VI. Conclusions

In this paper, we have studied the fairness-aware time-sensitive task allocation in AFL of CEI. For the fixed deadline scenario, we have designed a multi-dimensional contract based incentive mechanism to select heterogeneous participants to finish task training honestly such that the social utility can be maximized. For the limited budget scenario, we have designed a multi-metric participant recruitment mechanism which can iterative adjust task allocation to control time consumption, proved the time control problem is NP-Hard and proposed

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an e-approximation algorithm based on relaxed iterative optimization for it. We have also used real-world and simulated data to conduct sufficient performance evaluations to further validate the effectiveness and efficiency of our proposed two mechanisms in balancing the learning accuracy, fairness and timeliness

Appendix

A. Proof of Property 1

The first two inequalities are about the data size and reward monotonicity. The monotonicity of feasible contract can be proofed by using IC constraint. We first prove the sufficiency, *e.g.*, if $q_x \geq q_y$, it follows $r_x \geq r_y$. From IC constraint, we have

$$\begin{cases} \theta_x r_x - c_x q_x \geq \theta_x r_y - c_x q_y, \\ \theta_y r_y - c_y q_y \geq \theta_y r_x - c_y q_x. \end{cases} \tag{29}$$

Based on the first inequality in Eq. (29), we can further get

$$\theta_x(r_x - r_y) \geq c_x(q_x - q_y) \geq 0. \tag{30}$$

Since $\theta_x \geq 0$ and $c_x \geq 0$, it follows that $r_x \geq r_y$. Next, we prove the necessity, *e.g.*, if $r_x \geq r_y$, it follows that $q_x \geq q_y$. Similarly, according to Eq. (30), we have

$$c_y(q_x - q_y) \geq \theta_y(r_x - r_y) \geq 0. \tag{31}$$

It is obvious that $q_x \geq q_y$. Therefore, data quantity strictly monotonically increases with the payment reward.

The third inequality is the sufficient and necessary conditions of IR. Following IC and the condition $\theta_1 \leq \cdots \leq \theta_n$, we can obtain that

$$\theta_i r_i - c_i q_i \geq \theta_1 r_1 - c_1 q_1 \geq 0. \tag{32}$$

As such, if the IR of type-θ_1 participant is satisfied, the IR of other participants with different types are naturally satisfied.

Next, we prove the fourth inequality. Based on the monotonically of contract, IC can be reduced into incentive compatibility between the adjacent participants. Thus, we only need to prove the Local Downward Incentive Compatibility (LDIC) and the local upward one. Let's first take LDIC into account. Consider three types of workers that $\theta_{x-1} \leq \theta_x \leq \theta_{x+1}$. Following IC constraint, we have

$$\begin{cases} \theta_{x+1} r_{x+1} - c_{x+1} q_{x+1} \geq \theta_{x+1} r_x - c_{x+1} q_x, \\ \theta_x r_x - c_x q_x \geq \theta_x r_{x-1} - c_x q_{x-1}. \end{cases} \tag{33}$$

Based on the monotonicity, we know that $r_x \geq r_y$ when $\theta_x \geq \theta_y$. As such, we can get inequalities as follows:

$$\theta_{x+1}(r_x - r_{x-1}) \geq \theta_x(r_x - r_{x-1}) \geq c_{x+1}(q_x - q_{x-1}) \tag{34}$$

As such, we have

$$\theta_{x+1} r_{x+1} - c_{x+1} q_{x+1} \geq \theta_{x+1} r_{x-1} - c_{x+1} q_{x-1}. \tag{35}$$

According to the property of monotonicity, we obtain

$$\theta_{x+1} r_{x+1} - c_{x+1} q_{x+1} \geq \theta_{x+1} r_1 - c_{x+1} q_1. \tag{36}$$

Hence, if the IC constraint can be guaranteed by type-θ_{x+1} participants, it will also be guaranteed by type-θ_x participants. This process can be extended downward from type-θ_{x+1} to type-θ_1, *e.g.*, all LDICs are held and it is same as the upward one.

At last, we give the proof of the sufficient and necessary condition of AAF constraints: based on the monotonicity of contract, AAF constraints can be reduced to guarantee individual fairness between the adjacent participants. Consider three types of workers that $\theta_{x-1} \leq \theta_x \leq \theta_{x+1}$, we can get inequalities as follows:

$$\begin{cases} \log \frac{r_{x+1}}{r_x} \leq d(s_{x+1} - s_x), \\ \log \frac{r_x}{r_{x-1}} \leq d(s_x - s_{x-1}). \end{cases} \tag{37}$$

By adding the two inequalities in Eq.(37) to obtain

$$\log \frac{r_{x+1}}{r_x} + \log \frac{r_x}{r_{x-1}} = \log \frac{r_{x+1}}{r_{x-1}} \leq d(s_{x+1} - s_{x-1}). \tag{38}$$

As such, the sufficient and necessary conditions of monotonicity, IR, IC and AAF constraints are proved.

B. Proof of Theorem 1

First of all, we prove that the following equation holds.

$$r_i^* = \begin{cases} \frac{1}{\theta_i} c_i q_i, & \text{if } i = 1, \\ r_{i-1}^* - \frac{1}{\theta_i} c_{i-1} q_{i-1} + \frac{1}{\theta_i} c_i q_i, & \text{otherwise.} \end{cases} \tag{39}$$

For the sake of contradiction, there exist some r^+ in a feasible contract that yields greater profit for the model owner, *i.e.*, $\pi(r^+) > \pi(r^*)$. For simplicity, we only need to consider the reward of the model owner's utility function in this proof, *e.g.*, $\sum_{i=1}^n r_i^+ < \sum_{i=1}^n r_i^*$. This implies that there exists at least a $t \in [1, n]$ that satisfies inequality $r_t^+ < r_t^*$. According to the LDIC conditions in the proof of Property 1, we have

$$\begin{cases} r_t^+ > r_{t-1}^+ - \frac{1}{\theta_t} c_{t-1} q_{t-1} + \frac{1}{\theta_t} c_t q_t, \\ r_t^* = r_{t-1}^* - \frac{1}{\theta_t} c_{t-1} q_{t-1} + \frac{1}{\theta_t} c_t q_t. \end{cases} \tag{40}$$

We thus can deduce that $r_{(t-1)}^+ < r_{(t-1)}^*$. Continuing this process, we eventually obtain $r_1^+ < r_1^* = \frac{1}{\theta_1} c_1 q_1$. However, this violates IR constraint. Therefore, this theorem is proved.

C. Proof of Corollary 1

There are four cases: (i) $\alpha = 0$ and $\beta = 0$: it's impossible for the first equation in Eq. (25) to be true because the utility of participant is greater than zero. (ii) $\alpha > 0$ and $\beta = 0$: it doesn't work for the same reason as the first case. $(iii) \alpha > 0$ and $\beta > 0$: it cannot make both the second equation and the third one in Eq. (25) to be true at the same time. (iv) $\alpha = 0$ and $\beta > 0$: there is a solution that makes all three equations true, and hence we can obtain q_i^* as shown in Eq. (26).

D. Proof of Lemma 1

We consider n participants with the local data sets $\{q_1, q_2, \ldots, q_n\}$ and denote w^* as the optimal solution of model parameters. In the learning problem, the task is to find the objective parameters w by optimizing the loss function. For each local data set q_j of participant p_j, the loss function is

$$F_j(w) = \frac{1}{q_j} \sum_{j=1}^{n} f_j(w). \tag{41}$$

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Then, the global loss function can be defined as

$$F(w) = \frac{\sum_{j=1}^{n} q_j F_j(w)}{\mathcal{Q}}. \quad (42)$$

Where $\mathcal{Q} = \sum_{j=1}^{n} q_j$ denotes the total training data amount. In order to achieve the model performance to a certain extent, we need to control the error between the global loss function and the optimal solution of the model within a certain range. Without loss of generality, we assume that $F(w)$ is convex and L-smooth and after T global update on the server, we have

$$E[F(w^T) - F(w^*)] \leq (1 - 2\mu\gamma\eta_k)^T [F(w^0) - F(w^*)]. \quad (43)$$

Each update consume data amount Ψ and the total data amount which needs to converges to a global optimum w^* is $Q = T * \Psi$. Therefore, we can find that the model can achieve certain performance by controlling the data volume of model training, and thus $\sqrt{\frac{1}{M}\sum_{m=1}^{M}(\xi_m - \hat{\xi_m})^2} \leq \varepsilon$ can be reduced to $\sum_{i=1}^{N} q_i \geq Q$.

E. Proof of Lemma 2

IR is naturally satisfied for each p_i, its utility is larger than zero because $\tau s_i - c_i \geq 0$ where $\tau = \frac{\max(c_i)}{s_i}$.

As for AAF, we assume that participant p_i is superior to participant p_j which means that $r_i \geq r_j$. Thus, we need to hold

$$\log \frac{r_i}{r_j} \leq d(p_i - p_j). \quad (44)$$

According to the definitions of AAF and the payment determination rules γ and ψ, we can get

$$\log \frac{r_i}{r_j} = \log \frac{s_i q_i}{s_j q_j} \leq d((s_i, q_i), (s_j, q_j)). \quad (45)$$

Then, we only need to prove that

$$s_i q_i - \log s_i q_i \geq s_j q_j - \log s_j q_j. \quad (46)$$

Therefore, we need to proof that $f(x) = x - \log x$ monotonically increases with the specified interval. It is obvious that $\frac{df(x)}{dx} = 1 - \frac{1}{x} > 0$ as for $s_i q_i$ and $s_j q_j$. Thus, the proposed PDR satisfies AAF.

F. Proof of Theorem 2

We show that Eq. (28) is NP-Hard by reducing the NP-hard instance of bounded knapsack problem to its special case. In the instance of bounded knapsack problem, for a set of items numbered from 1 to n, $\mathcal{U} = \{u_1, \cdots, u_n\}$ where item u_i is specified by its positive profit v_i, its positive weight w_i, a bound $n_i \in N$ on the number of available copies and a knapsack of capacity $\mathcal{C}$. It can be described as a decision problem: Is there a subset of items with total weight at most b, such that the corresponding value is at least a. Given a bounded knapsack problem instance $\mathcal{X}(\mathcal{U}, \mathcal{C})$ and the special case of time sensitivity control problem of task allocation instance $\mathcal{Y}(\mathcal{P}, \mathcal{B})$, the $\mathcal{Y}(\mathcal{P}, \mathcal{B})$ can be reduced from the $\mathcal{X}(\mathcal{U}, \mathcal{C})$ by following steps: (i) Let a set $\mathcal{P}$ of the participants is the universal set $\mathcal{U}$, and the knapsack's capacity $\mathcal{C}$ is equal to the whole budget $\mathcal{B}$. (ii) Add the data amount γ_i of each participant p_i for each bound $n_i \in N$. (iii) As for the payment $k_i q_i$ of each participant, let the positive weight w_i be same as it if it is selected. Otherwise the payment is zero. It is obvious that the solution of $\mathcal{Y}(\mathcal{P}, \mathcal{B})$ which participants' payment should not exceed the budget and the final task yields must reach a certain threshold is equal to the solution of $\mathcal{X}(\mathcal{U}, \mathcal{C})$ in polynomial time. Hence, this theorem is proved.

G. Proof of Lemma 3

We first relax the original problem by reducing the budget constraints. Then we use greedy strategy to get the solution to the relaxation problem and use $\mathcal{M}$ to denote the solution to the relaxation problem without budget. Next, we define the optimal solution of the original problem as opt and the same of RNS as apx. It is obvious that the solution apt is better than opx and inferior to $\mathcal{M}$. We define the amount of task data that needs secondary adjustment as q which needs to transform from high payment participant r_h to low payment participant r_l. Based on the proposed determination rule and the fairness constraint, we have

$$\frac{s_h}{s_l} = \frac{r_h}{r_l} \leq e^{d(p_i, p_j)} \leq e, \quad (47)$$

where $d(p_i, p_j) \leq 1$. Based on the entropy theory, we have

$$\frac{s_h}{s_l} = \frac{w_1 f(c_h) + w_2 f(t_h)}{w_1 f(c_l) + w_2 f(t_l)} > \frac{w_1 f(c_h)}{w_1 f(c_l)}. \quad (48)$$

Based on the definition of membership function in Eq. (18), we can further calculate as follows:

$$\frac{f(c_h)}{f(c_l)} = \frac{c_h - c_{min}}{c_l - c_{min}} > \frac{c_h}{c_l}. \quad (49)$$

Since $t = \frac{q}{c}$, we have

$$\frac{t_l}{t_h} = \frac{c_h}{c_l} < \frac{s_h}{s_l} < e. \quad (50)$$

We then can obtain that the time consumption of the solution is less than the one of e*$\mathcal{M}$, and hence this is an e-approximation.

References

[1] M. Elhoseny, et al., "Secure Automated Forensic Investigation for Sustainable Critical Infrastructures Compliant with Green Computing Requirements," IEEE Trans. Sustain. Comput., vol. 5, no. 2, pp. 174-191, Apr.-Jun. 2020.

[2] O. A. Wahab, A. Mourad, H. Otrok, and T. Taleb, "Federated Machine Learning: Survey, Multi-Level Classification, Desirable Criteria and Future Directions in Communication and Networking Systems," IEEE Commun. Surv. Tutorials, vol. 23, no. 2, pp. 1342-1397, Feb. 2021.

[3] Y. Qu, et al., "A Blockchained Federated Learning Framework for Cognitive Computing in Industry 4.0 Networks," IEEE Trans. Ind. Informatics, vol. 17, no. 4, pp. 2964-2973, Apr. 2021.

[4] S. Pokhrel, and S. Singh, "Compound TCP Performance for Industry 4.0 WiFi: A Cognitive Federated Learning Approach," IEEE Trans. Ind. Informatics, vol. 17, no. 3, pp. 2143-2151, Mar. 2021.

[5] S. Otoum, I. A. Ridhawi and H. Mouftah, "Securing Critical IoT Infrastructures with Blockchain-Supported Federated Learning," IEEE Internet Things J., doi: 10.1109/JIOT.2021.3088056.

[6] C. T. Dinh, et al., "Federated Learning Over Wireless Networks: Convergence Analysis and Resource Allocation," IEEE/ACM Trans. Netw. vol. 29, no. 1, pp. 398-409, Feb. 2021.

[7] H. H. Yang, Z. Liu, T. Q. S. Quek and H. V. Poor, "Scheduling Policies for Federated Learning in Wireless Networks," IEEE Transactions on Communications, vol. 68, no. 1, pp. 317-333, Jan. 2020.

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[8] M. Chen, et al., “A Joint Learning and Communications Framework for Federated Learning Over Wireless Networks,” IEEE Transactions on Wireless Communications, vol. 20, no. 1, pp. 269-283, Jan. 2021.

[9] S. Lee, and D. H. Choi, “Federated Reinforcement Learning for Energy Management of Multiple Smart Homes with Distributed Energy Resources,” IEEE Trans. Ind. Informatics, doi: 10.1109/TII.2020.3035451.

[10] S. Wang, et al., “When Edge Meets Learning: Adaptive Control for Resource-Constrained Distributed Machine Learning,” in Proc. 37th Int. Conf. Computer Communications, Honolulu, HI, USA, 16-19 Apr., 2018, pp. 63-71.

[11] Y. Lu, et al., “Differentially Private Asynchronous Federated Learning for Mobile Edge Computing in Urban Informatics,” IEEE Trans. Ind. Informatics, vol. 16, no. 3, pp. 2134-2143, Mar. 2020.

[12] Y. Chen, X. Sun, and Y. Jin, “Communication-Efficient Federated Deep Learning With Layerwise Asynchronous Model Update and Temporally Weighted Aggregation,” IEEE Trans. Neural Networks Learn. Syst., vol. 31, no. 10, pp. 4229-4238, Oct. 2020.

[13] U. Mohammad, and S. Sorour, “Adaptive task allocation for asynchronous federated mobile edge learning,” arXiv:1905.01656, 2019, [online] Available: http://arxiv.org/abs/1905.01656.

[14] X. Wang, et al., “Location-Aware Crowdsensing: Dynamic Task Assignment Assignment and Truth Inference,” IEEE Trans. Mob. Comput., vol. 19, no. 2, pp. 362-375, Feb. 2020.

[15] G. Sun, Y. Wang, X. Ding, and R. Hu, “Cost-Fair Task Allocation in Mobile Crowd Sensing With Probabilistic Users,” IEEE Trans. Mob. Comput., vol. 20, no. 2, pp. 403-415, Feb. 2021.

[16] A. Agnetis, B. Chen, G. Nicosia, and A. Pacifici, “Price of fairness in two-agent single-machine scheduling problems,” Eur. J. Oper. Res., vol. 276, no. 1, pp.79-87, Jun. 2019.

[17] W. Yang, et al., “Hierarchical Incentive Mechanism Design for Federated Machine Learning in Mobile Networks,” IEEE Internet Things J., vol. 7, no. 10, pp. 9575-9588, Oct. 2020.

[18] G. Cong, et al., “Improving data quality: Consistency and accuracy,” in Proc. 33rd Int. Conf. Very large data bases, University of Vienna, Austria, 23-27 Sep., 2007, pp.315-326.

[19] C. Dwork, et al., “Fairness Through Awareness,” in Proc. 3rd Innovations in Theoretical Computer Science Conference, New York, NY, USA, Jan., 2012, pp. 214-226.

[20] Y. Jiang, and C. Jiang, “Application of Fuzzy Theory and Entropy for the Undertake Ability of Service Outsourcing Evaluation,” in Proc. 4th International Symposium on Computational Intelligence and Design, Hangzhou, China, 28-30 Oct., 2011.

[21] H. T. Nguyen, et al., “Fast-Convergent Federated Learning,” IEEE Journal on Selected Areas in Communications, vol. 39, no. 1, pp. 201-218, Jan., 2021.

[22] Y. Huang, et al., “Personalized Cross-Silo Federated Learning on Non-IID Data,” in proc. of 35th AAAI Conference on Artificial Intelligence, Virtual Event, 2-9 Feb. 2021, pp. 7865-7873.

[23] B. McMahan, et al., “Communication-efficient learning of deep networks from decentralized data,” in Proc. 20th Int. Conf. Artif. Intell. Statist., 20-22 Apr., 2017, pp. 1273-1282.

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