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# Extended Aluthge Transforms and Applications (Research on structure of operators by order and related topics)

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CITATION:

Benhida, Chafiq. Extended Aluthge Transforms and Applications (Research on structure of operators by order and related topics). 数理解析研究所講究録 2021, 2202: 35-35

ISSUE DATE:

2021-10

URL:

<http://hdl.handle.net/2433/266246>

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# Extended Aluthge Transforms and Applications

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## Abstract

Given a bounded linear operator  $T$  with canonical polar decomposition  $T = V|T|$ , the Aluthge transform of  $T$  is the operator  $\Delta(T) := \sqrt{|T|}V\sqrt{|T|}$ .

For  $P$  an arbitrary positive operator such that  $VP = T$ , we define the *extended* Aluthge transform of  $T$  associated with  $P$  by  $\Delta_P(T) := \sqrt{P}V\sqrt{P}$ .

First, we establish some basic properties of  $\Delta_P$ ; second, we study the fixed points of the extended Aluthge transform; third, we consider the case when  $T$  is an idempotent; next, we discuss whether  $\Delta_P$  leaves invariant the class of complex symmetric operators.

We also study how  $\Delta_P$  transforms the numerical radius and numerical range.

As a key application, we prove that the spherical Aluthge transform of a commuting pair of operators corresponds to the extended Aluthge transform of a  $2 \times 2$  operator matrix built from the pair; thus, the theory of extended Aluthge transforms yields results for spherical Aluthge transforms.

(This is a joint work with Raul E. Curto)