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Functional central limit theorems for locally compact groups: the use of infinite dimensional Fourier analysis

by Herbert Heyer

In the theory of functional central limit theorems one considers scaled sums of infinitesimal arrays of d-dimensional random vectors of the form

$$X_n(t) := \sum_{\ell=1}^{k_n(t)} X_{n\ell}$$

on a probability space $(\Omega, \mathfrak{A}, \mathbf{P})$ and studies the corresponding sequences $\{X_n : n \in \mathbf{N}\}$ of stochastic processes $X_n = \{X_n(t) : t \in \mathbf{R}_+\}$ as functions in the Skorokhod space $D(\mathbf{R}_+, \mathbf{R}^d)$. One of the most profound contributions to the theory was to establish necessary and sufficient conditions for a sequence $\{X_n : n \in \mathbf{N}\}$ of process X_n to converge in distribution on $D(\mathbf{R}_+, \mathbf{R}^d)$ towards an increment process $X := \{X(t) : t \in \mathbf{R}_+\}$. A classical tool used in solving the convergence problem is the Lévy-Khintchine bijection

$$\mathbf{P}_X \leftrightarrow (a, B, \eta) \tag{1}$$

between the set $\mathcal{IP}(\mathbf{R}^d)$ of distributions of increment processes X in $\mathbf{R}^d$ and the set $P(\mathbf{R}_+, \mathbf{R}^d)$ of characteristic triplets (a, B, η) consisting of shift mappings a, diffusion mappings B and Lévy measures η. The solution to the problem given for example in [12] consists in characterizing the convergence

$$X_n \rightarrow X \tag{2}$$

of an increment process in terms of convergence conditions on the scaled sums of moments towards the characteristic objects in the triplet (a, B, η).

Functional central limit theorems of the described type can also be looked at within the framework of general locally compact groups G provided a Lévy-Khintchine bijection similar to (1) is available. For Lie projective groups G this work was carried out in [8] and [13]. On the other hand the Lévy-Khintchine bijection for Moore groups G described in [14] and [6] suggests the search for at least sufficient conditions for the convergence (2) in terms of generalized characteristic functions of G-valued random variables or synonymously, in terms of the Fourier transforms of their distributions on the dual of G. The definition of the Fourier transform of a probability measure on G therefore involves infinite dimensional unitary representations of G. The method of infinite dimensional Fourier transforms has been efficiently applied to commutative arrays and stationary increment processes in [15]. In their papers [9] and [10] G. Pap and the author make use of infinite dimensional Fourier transforms in order to propose sufficient conditions in terms of integrating families related to the given infinitesimal array.

The present article aims at surveying the methodical tools and some of the results achieved on the way to a solution of the problem in (2). In particular the author will elaborate on an axiomatic approach to the Lévy continuity property which plays an important role in arriving at the desired functional central limits. The subsequent discussion can be viewed as a supplement actualizing the very useful survey [13].

1. The case of a Lie projective group

For the general setting we suppose that G is a second countable locally compact group with neutral element e. Given an array $\{X_{n\ell} : n, \ell \in \mathbf{N}\}$ of rowwise independent G-(valued) random variables and a scaling sequence $\{k_n : n \in \mathbf{N}\}$ consisting of increasing càd functions $k_n : \mathbf{R}_+ \to \mathbf{Z}_+$ with $k_n(o) = o$ and $k_n(\mathbf{R}_+) = \mathbf{Z}_+$, such that the family $\{X_{n\ell} : n \in \mathbf{N}, 1 \leq \ell \leq k_n(t)\}$ is infinitesimal in the sense that

$$\lim_{n\to\infty} \max_{1\leq \ell \leq k_n(t)} \mathbf{P}([X_{n\ell} \in V^c]) = o$$

for all Borel neighborhoods V of e and all $t \in \mathbf{R}_+$, we look at the sequence $\{X_n : n \in \mathbf{N}\}$ of functional processes

$$X_n := \prod_{\ell=1}^{k_n(\cdot)} X_{n\ell}$$

(with G as their state space). For any increment process $X = \{X(t) : t \in \mathbf{R}_+\}$ in G (normalized by $X(o) = e$ and càdlàg) the family $\{\mu(s,t) : (s,t) \in \mathbf{S}\}$ of distributions $\mu(s,t) := \mathbf{P}_{X(s)^{-1}X(t)}$ forms a convolution hemigroup in the set $M^1(G)$ of all probability measures on G, i.e. $\mu(s,r) * \mu(r,t) = \mu(s,t)$ for all $s \leq r \leq t, \mu(t,t) = \varepsilon_e$, and the mapping $(s,t) \mapsto \mu(s,t)$ from $\mathbf{S} = \{(u,v) \in \mathbf{R}_+^2 : u \leq v\}$ into $M^1(G)$ (together with the weak topology $\mathcal{T}_w$) is càdlàg in each variable. X is stochastically continuous if and only if $(s,t) \mapsto \mu(s,t)$ is continuous. Returning to the initial array and to the sequence $\{X_n : n \in \mathbf{N}\}$ of functional processes in G we have finite dimensional convergence

$$X_n \to X$$

if and only if

$$\prod_{\ell=k_n(s)+1}^{k_n(t)} {}^{*}\, \mu_{n\ell} \to \mu(s,t)$$

for all $(s,t) \in \mathbf{S}$ in the sense of the topology $\mathcal{T}_w$ on $M^1(G)$.

Applying the fact that to any continuous convolution hemigroup $\{\mu(s,t) : (s,t) \in \mathbf{S}\}$ in $M^1(G)$ there corresponds the family $\{T_{s,t} : (s,t) \in \mathbf{S}\}$ of translation operators $T_{s,t} := T_{\mu(s,t)}$ defined in the space $\mathcal{L}(C^o(G), C^o(G))$ of all linear operators on the space $C^o(G)$ of all continuous functions on G vanishing at infinity, by

$$T_{s,t}f(x) := T_{\mu(s,t)}f(x) := \int_G f(xy)\mu(s,t)(dy)$$

whenever $f \in C^o(G), x \in G$, one obtains a bijection

$$\mathbf{H}(G) \leftrightarrow Evol(C^o(G))$$

between the sets $\mathbf{H}(G)$ of continuous convolution hemigroups in $M^1(G)$ and $Evol(C^o(G))$ of (strongly continuous, positive, left invariant) evolution families of contractions on $C^o(G)$. This bijection extends to a bijection

$$\mathbf{S}(G) \leftrightarrow Contr(C^o(G))$$

between continuous convolution semigroups and semigroups of contraction operators on $C^o(G)$.

For the following we assume to be known what it means that a mapping F from $\mathbf{S}$ or $\mathbf{R}_+$ into a Banach space E is of (continuous) finite (bounded) variation. A convolution hemigroup $\{\mu(s,t) : (s,t) \in \mathbf{S}\}$ is said to be of (continuous) *weak finite variation on a subspace* C of $C^o(G)$ if

$$(s,t) \mapsto (T_{\mu(s,t)} - I)f(e)$$

from $\mathbf{S}$ into $\mathbf{R}$ is of (continuous) bounded variation for every $f \in C$.

From now on let G be a Lie projective group with Lie algebra $L(G)$, projective basis $\{X_i : i \in I\}$ and projective (weak) coordinate system $\{x_i : i \in I\}$ (associated with $\{X_i : i \in I\}$). Examples of Lie projective groups are all locally compact abelian groups, all compact groups, in particular the torus group $\mathbf{T}^{\mathbf{N}}$ and the solenoidal group $\mathbf{Q}_d^\wedge$ (which both are not Lie groups), and all maximally almost periodic groups generated by a compact neighborhood of the identity. For Lie projective groups G the space $D(G)$ of (Bruhat) test functions is contained in the space $C_2(G)$ of twice left differentiable functions on G. The bijection

$$\mathbf{S}(G) \leftrightarrow P(G)$$
$$\{\mu(t) : t \in \mathbf{R}_+\} \leftrightarrow (a, B, \eta)$$

between $\mathbf{S}(G)$ and the set $P(G) := \mathbf{R}^I \times \mathbf{M}_{I,+} \times \mathbf{L}(G)$ of triplets (a, B, η) consisting of vectors a, symmetric positive semidefinite matrices B and Lévy measures η has been established in final form in [2], where also the tools for the general framework have been collected. The corresponding bijection

$$\mathbf{H}_{wfv}(G) \leftrightarrow P_{fv}(\mathbf{R}_+, G)$$
$$\{\mu(s,t) : (s,t) \in \mathbf{S}\} \leftrightarrow (a, B, \eta)$$

between the set $\mathbf{H}_{wfv}(G)$ of continuous hemigroups $\{\mu(s,t) : (s,t) \in \mathbf{S}\}$ of weakly finite variation on G and the set $P_{fv}(\mathbf{R}_+, G)$ of triplets (a, B, η), where a is a continuous mapping $\mathbf{R}_+ \to \mathbf{R}^I$ of finite variation with $a(o) = o, B$ an increasing continuous mapping $\mathbf{R}_+ \to \mathbf{M}_{I,+}$ with $B(o) = o$ and η a measure in $M^1(\mathbf{R}_+ \times G)$ such that $\eta(\mathbf{R}_+ \times \{e\}) = o, \eta([o,t] \times \cdot) \in \mathbf{L}(G)$ for all $t \in \mathbf{R}_+$, and

$$t \mapsto \int f(y)\eta([o,t] \times dy)$$

is continuous for all $f \in D(G)_+$ with $f(e) = o$. The set of all such measures η will be denoted by $\mathbf{L}(\mathbf{R}_+, G)$. While the first cited (Hunt) bijection is produced by a generating function, the letter one requires generating mappings and the notion of a weak backward equation.

The following functional convergence result has been proved in [8].

1.1 Theorem. Let $\{\mu_{n\ell} : n, \ell \in \mathbf{N}\}$ be an array of measures in $M^1(G), \{k_n : n \in \mathbf{N}\}$ a scaling sequence, and let D denote a dense subset of $\mathbf{R}_+$. It is assumed that

(i) there exists a continuous function $t \mapsto a(t) = (a_i(t))_{i \in I}$ on $\mathbf{R}_+$ such that for all $t \in D, i \in I$

$$\sum_{\ell=1}^{k_n(t)} \int x_i d\mu_{n\ell} \to a_i(t) \text{ as } n \to \infty,$$

(ii) there exists a continuous function $t \mapsto B(t) := (b_{ij}(t))_{i,j \in I}$ on $\mathbf{R}_+$ such that for all $t \in D, i, j \in I$

$$\sum_{\ell=1}^{k_n(t)} \int x_i x_j d\mu_{n\ell} \to b_{ij}(t) + \int_G x_i(y)x_j(y)\eta([o,t] \times dy) \text{ as } n \to \infty,$$

(iii) there exists a measure $\eta \in \mathbf{L}(\mathbf{R}_+, G)$ such that for all $t \in D$ and bounded continuous functions f on G vanishing in a neighborhood of e

$$\sum_{\ell=1}^{k_n(t)} \int f d\mu_{n\ell} \to \int_G f(y)\eta([o,t] \times dy),$$

(iv) for all $T > o, i \in I$

$$\limsup_{n \to \infty} \sup_{\substack{o \le s \le t \le T \\ t-s \le \delta}} \sum_{\ell=k_n(s)+1}^{k_n(t)} \left| \int x_i d\mu_{n\ell} \right| \to o \text{ as } \delta \to o.$$

Then $(a, B, \eta) \in P_{fv}(\mathbf{R}_+, G)$, and

$$\prod_{\ell=k_n(s)+1}^{k_n(t)} {}^{\bullet}\ \mu_{n\ell} \rightarrow \mu(s,t)$$

for all $(s,t) \in \mathbf{S}$, where $\{\mu(s,t) : (s,t) \in \mathbf{S}\} \in \mathbf{H}_{wfv}$ and

$$\{\mu(s,t) : (s,t) \in \mathbf{S}\} \leftrightarrow (a, B, \eta).$$

The proof of the theorem is based on the corresponding result for a Lie group G established in [7].

2. Infinite dimensional Fourier transforms

In this section G is assumed to be an arbitrary locally compact group. By a representation of G we always mean a continuous homomorphism U from G into the group $\mathcal{U}(\mathcal{H}(U))$ of unitary operators on the complex representing Hilbert space $\mathcal{H}(U)$. The set of all representations of G will be denoted by $Rep(G)$. Of particular importance is the subset $Irr(G)$ of all irreducible representations U of G which by definition admit no nontrivial closed U-invariant subspace of $\mathcal{H}(U)$. The famous Gelfand-Raikov theorem states that $Irr(G)$ separates the points of G. We also introduce for any cardinal α the α-dimensional Hilbert space $\mathcal{H}(\alpha)$ and the sets $Rep_\alpha(G)$ and $Irr_\alpha(G)$ of all $U \in Rep(G)$ or $U \in Irr(G)$ respectively with $\mathcal{H}(U) = \mathcal{H}(\alpha)$. For the union of the sets $Rep_n(G)$ for $n \in \mathbf{N}$ we write $Rep_f(G)$. The prominent class of *Moore groups* G is defined by the inclusion $Irr(G) \subset Rep_f(G)$. It contains all compact and all abelian locally compact groups and has a well understood structure as is cited in [5].

Now we look at the set $\hat{G} := Irr(G)/\sim$ of unitary equivalence classes of irreducible representations. In the standard references [4] and [18] from which we pick most of the subsequent information, $\hat{G}$ is called the *dual* of G. For any $U \in \hat{G}$ we consider the space $\mathcal{H}_{(1)}(U)$ of all $u \in \mathcal{H}(U)$ with $\|u\| = 1$. We note that the symbol U will be used for the class in $\hat{G}$ as well as for any of its representations. For a given $U \in \hat{G}$ and $u, v \in \mathcal{H}(U)$ the corresponding coefficient of U is defined by $p_{u,v}(U) :=< U(\cdot)u, v >$. In the case that $u = v$ we write $p_u(U)$ instead of $p_{u,v}(U)$. The next definition concerns the *reduced dual* of G introduced as the set $\hat{G}_r$ of all $U \in \hat{G}$ such that there exists a $u \in \mathcal{H}_{(1)}(U)$ admitting the approximation (in the sense of the compact open topology $\mathcal{T}_{co}$)

$$p_u(U) = \lim_{n\to\infty} f_n * f_n^{\sim}$$

for some sequence $(f_n)_{n\geq 1}$ in $C^c(G)$.

Since $\hat{G}$ can be identified with the dual $C^*(G)^\wedge$ of the C^*-algebra $C^*(G)$ of G where $C^*(G)^\wedge$ carries the hull-kernel topology, we obtain the *Fell topology* on $\hat{G}$. A base of the Fell topology at the identity representation $\mathbf{1}$ of G is given by the family of finite intersections of sets of the form

$$V(C,\varepsilon) := \{U \in \hat{G} : \text{There exists } u \in \mathcal{H}_{(1)}(U) : |p_u(U)(x) - 1| < \varepsilon \text{ for all } x \in C\},$$

where C is a compact subset of G and $\varepsilon > o$. Furnished with the Fell topology $\hat{G}$ is a quasi-locally compact (Baire) space which is second countable if G is second countable. $\hat{G}_r$ is a closed subspace of $\hat{G}$. The equality $\hat{G}_r = \hat{G}$ can be characterized by either of the subsequent statements

(i) $\mathbf{1} \in \hat{G}_r$

(ii) Every continuous positive definite functions on G can be approximated (in the sense of $\mathcal{T}_{co}$) by functions of the form $f * f^{\sim}$ with $f \in C^c(G)$.

(iii) The constant function 1 on G can be approximated (in the sense of $\mathcal{T}_{co}$) by function of the form $f * f^{\sim}$ with $f \in C^c(G)$.

For any cardinal α the sets $Rep_\alpha(G)$ and $Rep_\alpha(C^*(G))$ are bijectively related to each other. Consequently the weak topology on $Rep_\alpha(C^*(G))$ induces a topology on $Rep_\alpha(G)$ which supplies an equivalent definition of the topology of $\hat{G}_\alpha$ as the subspace $\hat{G}$ consisting of all $U \in \hat{G}$ of dimension α.

We are now prepared to introduce the main tool of harmonic analysis on a locally compact group G: the *Fourier transform* $\hat{\mu}$ of a measure $\mu \in M^b(G)$ given for any $U \in Rep(G)$ as an element $\hat{\mu}(U)$ of the space $\mathcal{L}(\mathcal{H}(U))$ of all linear operators on $\mathcal{H}(U)$, by

$$< \hat{\mu}(U)u, v > := \int p_{u,v}(U) d\mu$$

whenever $u, v \in \mathcal{H}(U)$. Clearly, $\|\hat{\mu}\| \leq \|\mu\|$. Moreover, the application $\mu \mapsto \hat{\mu}$ from $M^b(G)$ into the set of mappings from $Rep(G)$ into $\bigcup\{\mathcal{L}(\mathcal{H}(U)) : U \in Rep(G)\}$ is linear, multiplicative, injective and bicontinuous in the sense of the following equivalences expressed for a sequence $(\mu_n)_{n \geq 1}$ and a measure μ both in $M^1(G)$:

(i) $\mu_n \to \mu$ (in the weak topology $\mathcal{T}_w$)

(ii) $\hat{\mu}_n(U)u \to \hat{\mu}(U)u$ for all $U \in Irr(G), u \in \mathcal{H}(U)$.

(iii) $< \hat{\mu}_n(U)u, v > \to < \hat{\mu}(U)u, v >$ for all $U \in Irr(G), u, v \in \mathcal{H}(U)$.

The implication (iii)$\Rightarrow$(i) can be considered as a narrow version of the Lévy continuity theorem for probability measures on a locally compact group. For the problem dealt with in [10] it turned out to be helpful to work with a wider version of Lévy's theorem which is axiomatized as follows.

2.1 Definition. G is said to *admit* the *Lévy continuity property* (LCP) *with respect to a subset* Γ of $Rep(G)$ if there exists a topology on Γ with the following property: Given a sequence $\{\mu_n : n \in \mathbf{N}\}$ in $M^1(G)$ and a mapping $h : \Gamma \to \bigcup\{\mathcal{L}(\mathcal{H}(U)) : U \in \Gamma\}$ which is continuous on $\Gamma \cap Rep_\alpha(G)$ for all cardinals α, satisfying

$$\hat{\mu}_n(U) \to h(U)$$

whenever $U \in \Gamma$ then there exists a measure $\mu \in M^1(G)$ such that

$$\mu_n \to \mu$$

and

$$\hat{\mu}(U) = h(U)$$

for all $U \in \Gamma$.

It is shown in [5] that any Moore group G admits (LCP) with respect to $\Gamma := Rep_f(G)$ the topology on Γ being $\mathcal{T}_{co}$ on $\bigcup\{Rep_n(G) : n \in \mathbf{N}\}$.

Following the note [3] we report on a different axiomatization of the Lévy continuity theorem.

Let G be a second countable locally compact group and Γ a subset of $\hat{G}$ such that $\mathbf{1} \in \Gamma$. A mapping $h : \Gamma \to \mathcal{L} := \bigcup\{\mathcal{L}(\mathcal{H})/\sim : \mathcal{H}$ is a Hilbert space$\}$ with $h(\mathbf{1})$ being a scalar (operator) is said to be continuous in $\mathbf{1}$ if for every $\varepsilon > o$ there exists a neighborhood V of $\mathbf{1}$ (with respect to the Fell topology in $\hat{G}$) satisfying the following property: If $U \in V \cap \Gamma$ then there is a representative $h(U)$ of the class $h(U) \in \mathcal{L}(\mathcal{H})/\sim$ for some Hilbert space $\mathcal{H}$, and a vector $u \in \mathcal{H}$ with $\|u\| = 1$ such that

$$| < h(U)u, u > -h(\mathbf{1})| < \varepsilon.$$

Obviously, the Fourier transform $\hat{\mu}$ of any measure $\mu \in M^b(G)$ considered as mapping $\Gamma \to \mathcal{L}$ is continuous at $\mathbf{1}$.

For subsets Γ of $\hat{G}$ (for groups G that are amenable and of type I) such that $\sigma(\Gamma^c) = o$, where σ denotes a representing measure (in the direct integral decomposition) of the left regular representation of G, the following modification of (LCP) holds.

2.2 Definition. Let G be a second countable locally compact group and $\Gamma \subset \hat{G}$ with $\mathbf{1} \in \Gamma$. G is said to *admit the modified Lévy continuity property* (MLCP) *with respect to* Γ if for any given sequence $\{\mu_n : n \in \mathbf{N}\}$ in $M^1(G)$ and any mapping $h : \Gamma \to \mathcal{L}$ which is continuous at $\mathbf{1}$ and satisfies

$$\hat{\mu}_n(U) \to h(U) \in \mathcal{L}$$

for all $U \in \Gamma$ there exists a measure $\mu \in M^1(G)$ such that

$$\mu_n \to \mu$$

and

$$\hat{\mu}(U) = h(U)$$

for all $U \in \Gamma$.

Following the exposition in [3] we note that if G is of type I (f.e. if G is nilpotent or solvable or a Moore group) then there exists a representing measure σ of the left regular representation of G such that $\sigma(\hat{G}_r^c) = o$. If, in addition, G is amenable (f.e. if G is an almost connected nilpotent or a Moore group) then $\mathbf{1} \in supp\sigma$ for every representing measure σ, and hence G admits (MLCP) with respect to any subset Γ of $\hat{G}$ with $\sigma(\Gamma^c) = o$.

On the other hand G admits (MLCP) with respect to $\hat{G}$ provided every neighborhood of $\mathbf{1}$ (in $\hat{G}$) contains a representation U such that for any $u \in \mathcal{H}(U)$ the coefficient $p_u(U)$ vanishes at infinity. Applying this fact it turns out that a noncompact, connected simple Lie group G with finite center admits (MLCP) with respect to $\hat{G}$ if and only if G violates the Kazdhan property which states that $\mathbf{1}$ is isolated in $\hat{G}$.

3. Convergence of scaled arrays of distributions

A (continuous) convolution hemigroup $\{\mu(s,t) : (s,t) \in \mathbf{S}\}$ of probability measures on a locally compact group G is characterized by the fact that the corresponding family $\{\hat{\mu}(s,t)(U) : (s,t) \in \mathbf{S}\}$ of operators in $\mathcal{L}(\mathcal{H}(U))$ is a (continuous) evolution family for each $U \in Irr(G)$. Given a subset Γ of $Rep(G)$ we define a convolution hemigroup $\{\mu(s,t) : (s,t) \in \mathbf{S}\}$ in $M^1(G)$ to be of (continuous) *$\mathcal{F}$-finite variation with respect to* Γ if for each $U \in \Gamma$ the mapping

$$(s,t) \mapsto \hat{\mu}(s,t)(U) - I$$

from $\mathbf{S}$ into $\mathcal{L}(\mathcal{H}(U))$ is of (continuous) finite variation.

3.1 Definition. Let $\{\mu(s,t) : (s,t) \in \mathbf{S}\}$ be a convolution hemigroup in $M^1(G)$ and let $\Gamma \subset Rep(G)$. A family $\{\varphi^U : U \in \Gamma\}$ of mappings $\varphi^U \in FV(\mathbf{R}_+, \mathcal{L}(\mathcal{H}(U)))$ is called an *integrating family related to* $\{\mu(s,t) : (s,t) \in \mathbf{S}\}$ if for all $U \in \Gamma, \varphi^U(o) = o$ and

$$\mu(s,t)^\wedge(U) = I + \int_{]s,t]} \hat{\mu}(s,\tau-)^\wedge(U)\varphi^U(d\tau)$$

whenever $(s,t) \in \mathbf{S}$.

If a convolution hemigroup $\{\mu(s,t) : (s,t) \in \mathbf{S}\}$ admits an integrating family for $\Gamma \subset Rep(G)$ then $\{\mu(s,t) : (s,t) \in \mathbf{S}\}$ is of $\mathcal{F}$-finite variation with respect to Γ. Conversely, if $\{\mu(s,t) : (s,t) \in \mathbf{S}\}$ is a convolution hemigroup of $\mathcal{F}$-finite variation with respect to Γ then it admits an integrating family for Γ. Moreover, let $\{\mu(s,t) : (s,t) \in \mathbf{S}\}$ be a convolution hemigroup of continuous $\mathcal{F}$-finite variation with respect to $\Gamma \subset Rep(G)$. Then the integrating family $\{\varphi^U : U \in \Gamma\}$ related to $\{\mu(s,t) : (s,t) \in \mathbf{S}\}$ is uniquely determined, and $\varphi^U \in C(\mathbf{R}_+, \mathcal{L}(\mathcal{H}(U)))$ for all $U \in \Gamma$.

In the classical situation of $G = \mathbf{R}^d$ (for $d \geq 1$), where $Irr(G) \cong \mathbf{R}^d$, any convolution hemigroup $\{\mu(s,t) : (s,t) \in \mathbf{S}\}$ in $M^1(G)$ can be characterized by a triplet (a, B, η) in $P(\mathbf{R}_+, G)$ such that

$$\mu(s,t)^\wedge(U) = exp\{i < U, a(t) - a(s) > -\frac{1}{2} < U, (B(t) - B(s))U >$$

$$+ \int (e^{i<U,y>} - 1 - i < U, h(y) >)\eta(]s,t] \times dy)\}$$

for all $U \in Irr(G)((s,t) \in \mathbf{S})$, where h denotes a truncation function on G. It turns out that $\{\mu(s,t) : (s,t) \in \mathbf{S}\}$ is of $\mathcal{F}$-finite variation if and only if a is of finite variation, and in this case the integrating family $\{\varphi^U : U \in Irr(G)\}$ related to $\{\mu(s,t) : (s,t) \in \mathbf{S}\}$ consists of functions $\varphi^U \in FV(\mathbf{R}_+, \mathcal{L}(\mathcal{H}(U)))$ given by

$$\varphi^U(\tau) = \log \ \mu(o,\tau)^\wedge(U)$$

whenever $\tau \in \mathbf{R}_+$. In terms of increment processes associated with hemigroups the above stated Lévy-Khintchine correspondence

$$\{\mu(s,t) : (s,t) \in \mathbf{S}\} \leftrightarrow (a,b,\eta)$$

between the sets $\mathbf{H}(G)$ and $P(\mathbf{R}_+, G)$ is proved in [12].

A similar description of the integrating family can be given in the case of Moore groups G which are known to be Lie projective. The necessary argument relies on Section 5 of [5] and the method developed in [14]. In the special case of abelian locally compact groups a comparison of the various versions of convolution hemigroups of finite variation has been carried out in [1].

Results for specified limits

3.2 Theorem. For every $n \in \mathbf{Z}_+$ let $\{\mu_n(s,t) : (s,t) \in \mathbf{S}\}$ be a convolution hemigroup admitting an integrating family $\{\varphi_n^U : U \in Irr(G)\}$. Suppose that for every $U \in Irr(G)$

(i) there exists a dense subset D of $\mathbf{R}_+$ such that for all $t \in D$

$$\varphi_n^U(t) \to \varphi_o^U(t),$$

(ii) for the sequence of moduli of continuity

$$\limsup_{n\to\infty} \omega_T(V_{\varphi_n^U};\delta) \to o \text{ as } \delta \to o$$

whenever $T > o$.

Then

$$\mu_n(s,t) \to \mu_o(s,t)$$

for all $(s,t) \in \mathbf{S}$, and $\{\mu_o(s,t) : (s,t) \in \mathbf{S}\}$ is a convolution hemigroup of continuous $\mathcal{F}$-finite variation with respect to $Irr(G)$.

3.3 Theorem (Convergence). Let $\{\mu_{n\ell} : n,\ell \in \mathbf{N}\}$ be an array in $M^1(G)$ and $\{k_n : n \in \mathbf{N}\}$ a scaling sequence. Moreover, let $\{\mu(s,t) : (s,t) \in \mathbf{S}\}$ be a convolution hemigroup in $M^1(G)$ admitting an integrating family $\{\varphi^U : U \in Irr(G)\}$. Suppose that for every $U \in Irr(G)$

(i) there exists a dense subset D of $\mathbf{R}_+$ such that for all $t \in D$

$$\sum_{\ell=1}^{k_n(t)} (\hat{\mu}_{n\ell}(U) - I) \to \varphi^U(t),$$

(ii)

$$\limsup_{n\to\infty} \sup_{\substack{0\le s\le t\le T\\ t-s\le\delta}} \sum_{\ell=k_n(s)+1}^{k_n(t)} \|\hat{\mu}_{n\ell}(U) - I\| \to o \text{ as } \delta \to o$$

whenever $T > o$.

Then

$$\prod_{\ell=k_n(s)+1}^{k_n(t)} {}^{*} \mu_{n\ell} \to \mu(s,t)$$

for all $(s,t) \in \mathbf{S}$, and $\{\mu(s,t) : (s,t) \in \mathbf{S}\}$ is a convolution hemigroup of continuous $\mathcal{F}$-finite variation with respect to $Irr(G)$.

Results for unspecified limits

Here we assume that G is a locally compact group admitting (LCP) for some fixed $\Gamma \subset Rep(G)$.

3.4 Theorem. For every $n \in \mathbf{N}$ let $\{\mu_n(s,t) : (s,t) \in \mathbf{S}\}$ be a convolution hemigroup in $M^1(G)$ admitting an integrating family $\{\varphi_n^U : U \in \Gamma\}$. Suppose that for every $U \in \Gamma$

(i) there exists a dense subset D of $\mathbf{R}_+$ such that for all $t \in D$ the sequence $\{\varphi_n^U : n \in \mathbf{N}\}$ converges in $\mathcal{L}(\mathcal{H}(U))$,

(ii) $\limsup_{n\to\infty)} \omega_T(V_{\varphi_n^U};\delta) \to o$ as $\delta \to o$ whenever $T > o$.

Then there exists a family $\{\varphi^U \in FV(\mathbf{R}_+, \mathcal{L}(\mathcal{H}(U))) \cap C(\mathbf{R}_+, \mathcal{L}(\mathcal{H}(\mathcal{U}))) : \mathcal{U} \in \Gamma\}$ such that

$$\varphi_n^U \to \varphi^U$$

locally uniformly for all $U \in \Gamma$.

If, in addition,

(iii) the mapping $U \mapsto \varphi^U$ from $\Gamma \cap Rep_\alpha(G)$ into $C(\mathbf{R}_+, \mathcal{L}(\mathcal{H}(\alpha)))$ is continuous for each α,

(iv) the mapping $U \mapsto V_{\varphi^U}$ from $\Gamma \cap Rep_\alpha(G)$ into $C(\mathbf{R}_+, \mathbf{R}_+)$ is locally bounded for each α,

then there exists a convolution hemigroup $\{\mu(s,t) : (s,t) \in \mathbf{S}\}$ of continuous $\mathcal{F}$-finite variation with respect to Γ such that

$$\mu_n(s,t) \to \mu(s,t)$$

for all $(s,t) \in \mathbf{S}$, and $\{\varphi^U : U \in \Gamma\}$ is an integrating family related to $\{\mu(s,t) : (s,t) \in \mathbf{S}\}$.

3.5 Theorem (Convergence). Let $\{\mu_{n\ell} : n, \ell \in \mathbf{N}\}$ be an array in $M^1(G)$ and $\{k_n : n \in \mathbf{N}\}$ a scaling sequence. Suppose that for every $U \in \Gamma$

(i) there exists a dense subset D of $\mathbf{R}_+$ such that for all $t \in D$

$$\left\{ \sum_{\ell=1}^{k_n(t)} (\hat{\mu}_{n\ell}(U) - I) : n \in \mathbf{N} \right\} \text{ converges in } \mathcal{L}(\mathcal{H}(U)),$$

(ii)

$$\limsup_{n\to\infty} \sup_{\substack{o \le s \le t \le T \\ t-s \le \delta}} \sum_{\ell=k_n(s)+1}^{k_n(t)} \|\hat{\mu}_{n\ell}(U) - I\| \to o \text{ as } \delta \to o$$

whenever $T > o$.

Then there exists a family $\{\varphi^U \in FV(\mathbf{R}_+, \mathcal{L}(\mathcal{H}(U))) \cap C(\mathbf{R}_+\mathcal{L}(\mathcal{H}(U))) : U \in \Gamma\}$ such that

$$\sup_{t\in[o,T]} \| \sum_{\ell=1}^{k_n(t)} (\hat{\mu}_{n\ell}(U) - I) - \varphi^U(t)\| \to o$$

for all $U \in \Gamma$ whenever $T > o$.

If, in addition, conditions (iii) and (iv) of Proposition 3.4 hold, then

$$\prod_{\ell=k_n(s)+1}^{k_n(t)\ \bullet} \mu_{n\ell} \to \mu(s,t)$$

for all $(s,t) \in \mathbf{S}$, and $\{\mu(s,t) : (s,t) \in \mathbf{S}\}$ is a convolution hemigroup of continuous $\mathcal{F}$-finite variation admitting $\{\varphi^U : U \in \Gamma\}$ as its related integrating family.

For the technical background and proofs of the results we refer the reader to [10]. The main idea is to reduce the study of convolution hemigroups on G via Fourier transform to the study of evolution families of operators and related operator-valued integrating functions which are chosen to be of finite variation. These integrating functions are applied in order to obtain integral representations of the given evolution families the integral involved being a (Bogdanowicz) generalization of the (bilinear) Lebesgue-Bochner-Stieltjes integral for operator-valued integrands and integrators.

4. Convergence of scaled arrays of random variables

In this section we wish to reformulate the previous results in terms of increment processes and scaled products of random variables taking their values in a second countable locally compact group G which is also a complete separable metric group. Let $X := \{X(t) : t \in \mathbf{R}_+\}$ be an increment process in second countable G and let $\{\mu(s,t) : (s,t) \in \mathbf{S}\}$ denote the associated convolution hemigroup of distributions $\mu(s,t)$ of increments $X(s)^{-1}X(t)$ of X. The process X is said to be of (continuous) finite $\mathcal{F}$-variation with respect to $\Gamma \subset Rep(G)$ if the convolution hemigroup $\{\mu(s,t) : (s,t) \in \mathbf{S}\}$ is of $\mathcal{F}$-finite variation with respect to Γ in the sense of Section 3, and to admit an integrating family for $\Gamma \subset Rep(G)$ if $\{\mu(s,t) : (s,t) \in \mathbf{S}\}$ does.

Results for specified limits

4.1 Theorem. For every $n \in \mathbf{N}$ let $X_n = \{X_n(t) : t \in \mathbf{R}_+\}$ be a càdlàg increment process in G which is of $\mathcal{F}$-finite variation with respect to $Irr(G)$ and admits an integrating family $\{\varphi_n^U : U \in Irr(G)\}$. Moreover, let $\{\mu(s,t) : (s,t) \in \mathbf{S}\}$ denote any convolution hemigroup of $\mathcal{F}$-finite variation with respect to $Irr(G)$ and let $\{\varphi^U : U \in Irr(G)\}$ be some integrating family related to $\{\mu(s,t) : (s,t) \in \mathbf{S}\}$. We assume the conditions (i) and (ii) of Theorem 3.2 to be satisfied.

Then there exists a G-valued stochastically continuous càdlàg increment process $X = \{X(t) : t \in \mathbf{R}_+\}$ of continuous $\mathcal{F}$-finite variation with respect to $Irr(G)$ such that

$$X_n \to X$$

in distribution on $D(\mathbf{R}_+, G)$, and $\mathbf{P}_{X(s)^{-1}X(t)} = \mu(s,t)$ whenever $(s,t) \in \mathbf{S}$.

4.2 Theorem. Let $\{X_{n\ell} : n, \ell \in \mathbf{N}\}$ be an array of rowwise independent random variables with values in G, and let $\{k_n : n \geq 1\}$ be a scaling sequence. Moreover, let $\{\mu(s,t) : (s,t) \in \mathbf{S}\}$ denote any convolution hemigroup in $M^1(G)$ admitting an integrating family $\{\varphi^U : U \in Irr(G)\}$. We assume that for every $U \in Irr(G)$

(i) there exists a dense subset D of $\mathbf{R}_+$ such that for all $t \in D$

$$\sum_{\ell=1}^{k_n(t)} (\mathbf{E}(U \circ X_{n\ell}) - I) \to \varphi^U(t),$$

(ii)

$$\limsup_{n\to\infty} \sup_{\substack{o \leq s \leq t \leq T \\ t-s \leq \delta}} \sum_{\ell=k_n(s)+1}^{k_n(t)} \|\mathbf{E}(U \circ X_{n\ell}) - I\| \to o \text{ as } \delta \to o$$

whenever $T > o$.

Then there exists a G-valued stochastically continuous càdlàg increment process $X = \{X(t) : t \in \mathbf{R}_+\}$ of $\mathcal{F}$-finite variation with respect to $Irr(G)$ such that

$$\prod_{\ell=1}^{k_n(\cdot)} X_{n\ell} \to X$$

in distribution on $D(\mathbf{R}_+, G)$, and $\mathbf{P}_{X(s)^{-1}X(T)} = \mu(s,t)$ whenever $(s,t) \in \mathbf{S}$.

Results for unspecified limits

Similar to Section 3 we need also here the additional hypothesis that G admits (LCP) for some fixed $\Gamma \subset Rep(G)$.

4.3 Theorem. For every $n \in \mathbf{N}$ let $X_n := \{X_n(t) : t \in \mathbf{R}_+\}$ be a càdlàg increment process in G which is of $\mathcal{F}$-finite variation with respect to Γ and admits an integrating family $\{\varphi_n^U : U \in \Gamma\}$. Suppose that for every $U \in \Gamma$ conditions (i) and (ii) of Theorem 3.4 are satisfied.

Then there exists a family $\{\varphi^U : U \in \Gamma\}$ of mappings $\varphi^U \in FV(\mathbf{R}_+, \mathcal{L}(\mathcal{H}(U))) \cap C(\mathbf{R}_+, \mathcal{L}(\mathcal{H}(U)))$ such that

$$\varphi_n^U \to \varphi^U$$

locally uniformly for all $U \in \Gamma$.

If, in addition, conditions (iii) and (iv) of Theorem 3.4 are fulfilled, then there exists a stochastically continuous càdlàg increment process $X = \{X(t) : t \in \mathbf{R}_+\}$ of continuous $\mathcal{F}$-finite variation with respect to Γ such that

$$X_n \to X$$

in distribution on $D(\mathbf{R}_+, G)$, and $\{\varphi^U : U \in \Gamma\}$ is an integrating family related to the convolution hemigroup of distributions of increments $X(s)^{-1}X(t)$ of X.

4.4 Theorem. Let $\{X_{n\ell} : n, \ell \in \mathbf{N}\}$ be an array of rowwise independent random variables with values in G, and let $\{k_n : n \geq 1\}$ be a scaling sequence. Suppose that for every $U \in \Gamma$

(i) there is a dense subset D of $\mathbf{R}_+$ such that for all $t \in D$ the sequence

$$\left\{ \sum_{\ell=1}^{k_n(t)} (\mathbf{E}(U \circ X_{n\ell}) - I) : n \in \mathbf{N} \right\}$$

converges in $\mathcal{L}(\mathcal{H}(U))$,

(ii)

$$\limsup_{n\to\infty} \sup_{\substack{o \leq s \leq t \leq T \\ t-s \leq \delta}} \sum_{\ell=k_n(s)+1}^{k_n(t)} \|\mathbf{E}(U \circ X_{n\ell}) - I\| \to o \text{ as } \delta \to o$$

whenever $T > o$.

Then there exists a family $\{\varphi^U : U \in \Gamma\}$ of mappings $\varphi^U \in FV(\mathbf{R}_+, \mathcal{L}(\mathcal{H}(U))) \cap C(\mathbf{R}_+, \mathcal{L}(\mathcal{H}(U)))$ such that

$$\varphi_n^U \to \varphi^U$$

locally uniformly for all $U \in \Gamma$.

If, in addition, conditions (i) and (ii) of Theorem 3.4 are fulfilled, then there exists a stochastically continuous càdlàg increment process $X = \{X(t) : t \in \mathbf{R}_+\}$ of continuous $\mathcal{F}$-finite variation with respect to Γ such that

$$\prod_{\ell=1}^{k_n(\cdot)} X_{n\ell} \to X$$

in distribution on $D(\mathbf{R}_+, G)$, and $\{\varphi^U : U \in \Gamma\}$ is an integrating family related to the convolution hemigroup of distributions of increments $X(s)^{-1}X(t)$ of X.

5. Suggestions for further research on the subject

An open problem in functional limit theory for locally compact groups is the specification of sufficient conditions enforcing the limiting process to be a diffusion. For Lie projective groups diffusion hemigroups and their corresponding increment processes have been characterized in [8] and [1]. We recall the following

5.1 Definition. A convolution hemigroup $\{\mu(s,t) : (s,t) \in \mathbf{S}\}$ on a locally comact group G is said to be a *diffusion hemigroup* if for all $T > o$ and for every neighborhood V of e

$$\lim_{\substack{t-s\to o \\ o\le s<t\le T}} \frac{1}{t-s}\mu(s,t)(V^c) = o.$$

Under Lipschitz conditions one shows that a convolution hemigroup on G is a diffusion hemigroup if and only if the corresponding increment process is a *diffusion process* in the sense that it has continuous paths.

For convolution semigroups $\{\mu(t) : t \in \mathbf{R}_+\}$ on G and their corresponding stationary increment processes the analoguous diffusion property

$$\lim_{t\to o} \frac{1}{t}\mu(t)(V^c) = o$$

valid for every neighborhood V of e defines *Gaussian semigroups* and *Gaussian processes* respectively.

In the sequel we shall sketch theorems on the convergence towards a Gaussian semigroup and on the martingale characterization of Gaussian semigroups, two results whose possible extensions to diffusion hemigroups by means of infinite dimensional Fourier transforms would be of great value for the development of functional central limit theory.

Let $\{\mu(t) : t \in \mathbf{R}_+\}$ be a convolution semigroup on G and $\{\mu(t)^\wedge(U) : t \in \mathbf{R}_+\}$ the associated semigroup of operators $\mu(t)^\wedge(U)$ in $\mathcal{L}(\mathcal{H}(U))$ whenever $U \in Rep(G)$. For any $U \in Rep(G)$ one introduces the infinitesimal generator$(N(U), \mathcal{N}(U))$ of the *representing semigroup* $\{\mu(t)^\wedge(U) : t \in \mathbf{R}_+\}$. It turns out that the domain $\mathcal{N}(U)$ of $N(U)$ contains the space $\mathcal{H}_o(U)$ of U-differentiable vectors of $\mathcal{H}(U)$, and $\mathcal{H}_o(U)$ contains the Gårding space $\mathcal{H}_1(U)$. If $U \in Rep_f(G)$ then $\mathcal{H}_1(U) = \mathcal{H}_o(U) = \mathcal{H}(U)$. For arbitrary $U \in Rep(G)$ the operator $N(U)$ admits a Lévy-Khintchine representation on $\mathcal{H}_o(U)$, and $\{\mu(t) : t \in \mathbf{R}_+\}$

is uniquely determined by the family $\{Res_{\mathcal{H}_1(U)}N(U) : U \in Irr(G)\}$. The author of [15] studies the convergence of sequences of convolution semigroups towards a limiting convolution semigroup on G. In particular he achieves the following central limit result.

5.2 Theorem. Let G be a Lie projective group, and let $\{\mu_{n\ell} : n, \ell \in \mathbf{N}\}$ be a commutative infinitesimal array in $M^1(G)$ satisfying the condition that

$$\lim_{n\to\infty} \sum_{\ell=1}^{k_n} \mu_{n,\ell}(V^c) = o$$

whenever V is a neighborhood of e. Suppose, moreover, that

$$\limsup_{n\to\infty} \sum_{\ell=1}^{k_n} | < \hat{\mu}_{n\ell}(U)u - u, u > | < \infty$$

for all $U \in Irr(G)$ and $u \in \mathcal{H}_o(U)$.

Then the sequence $\{\mu_n : n \in \mathbf{N}\}$ of row products

$$\mu_n := \prod_{\ell=1}^{k_n}{}^{*} \mu_{n\ell}$$

is uniformly tight, and for any of its nondegenerate limit points μ there exists a Gaussian semigroup $\{\mu(t) : t \in \mathbf{R}_+\}$ on G such that $\mu(1) = \mu$.

Next we describe a martingale characterization of a Gaussian semigroup or process in terms of its representing semigroup as it is shown in [16].

For any Hilbert space $\mathcal{H}$ we consider *$\mathcal{L}(\mathcal{H})$-martingales* $\{Z(t) : t \in \mathbf{R}_+\}$ (with respect to a filtration $\{\mathcal{F}(t) : t \in \mathbf{R}_+\}$) defined by the property that for all $u, v \in \mathcal{H}$ the $\mathbf{C}$-valued process $\{< Z(t)u, v >: t \in \mathbf{R}_+\}$ is a martingale with respect to $\{\mathcal{F}(t) : t \in \mathbf{R}_+\}$). Now, let $\{\mu(t) : t \in \mathbf{R}_+\}$ be a convolution semigroup with representing semigroup $\{\mu(t)^\wedge(U) : t \in \mathbf{R}_+\}$ for $U \in Rep(G)$. Let Γ be a subset of $Rep(G)$ such that for all $U \in \Gamma$ and all $t \in \mathbf{R}_+$ the operator $\mu(t)^\wedge(U)$ is invertible in $\mathcal{L}(\mathcal{H}(U))$, and that the Fourier mapping $\mu \mapsto \hat{\mu}$ from $M^b(G)$ into the set of mappings from Γ into $\bigcup\{\mathcal{L}(\mathcal{H}(U)) : U \in \Gamma\}$ is injective. Then a stochastic process $X = \{X(t) : t \in \mathbf{R}_+\}$ in G is a (stationary) increment process corresponding to $\{\mu(t) : t \in \mathbf{R}_+\}$ if and only if for each $U \in \Gamma$ the process $\{\mu(t)^\wedge(U)^{-1}U \circ X(t) : t \in \mathbf{R}_+\}$ is an $\mathcal{L}(\mathcal{H}(U))$-valued martingale with respect to the canonical filtration of X. One notes that this equivalence holds provided G is almost periodic in the sense that $Rep_f(G)$ separates the points of G, and $\Gamma := Irr(G) \cap Rep_f(G)$. If, moreover, G is a Moore group, it clearly holds for $S := Irr(G)$.

5.3 Theorem. Let G be a compact group for which a faithful representation $F \in Rep_f(G)$ exists. Given a convolution semigroup $\{\mu(t) : t \in \mathbf{R}_+\}$ on G and a stochastic process $X = \{X(t) : t \in \mathbf{R}_+\}$ in G with filtration $\{\mathcal{F}(t) : t \in \mathbf{R}_+\}$ which has continuous paths, the following statements are equivalent:

(i) X is a Gaussian process corresponding to $\{\mu(t) : t \in \mathbf{R}_+\}$.

(ii) For each $U \in \{F, F \otimes F\}$ the process $\{\mu(t)^\wedge(U)^{-1} U \circ X(t) : t \in \mathbf{R}_+\}$ is an $\mathcal{L}(\mathcal{H}(U))$-valued martingale with respect to the filtration of X.

As for the hypothesis on G in the theorem it should be noted that a compact group G admits a faithful finite dimensional representation if and only if G is isomorphic as a topological group to a (compact) group of orthogonal (or unitary) matrices, or equivalently to G being a Lie group. Further equivalences can be found in [11].

In the proof of the implication (ii)⇒(i) of the theorem the author of [16] applies the fact that for any convolution semigroup $\{\mu(t) : t \in \mathbf{R}_+\}$ on a locally compact group G and any càdlàg process $\{X(t) : t \in \mathbf{R}_+\}$ in G the process $\{\mu(t)^\wedge(U)^{-1} U \circ X(t) : t \in \mathbf{R}_+\}$ is an $\mathcal{L}(\mathcal{H}(U))$-valued local L^2-martingale (for $U \in Rep(G)$) if and only if the process $\{U \circ X(t) - N(U) \int_o^t U \circ X(s) ds : t \in \mathbf{R}_+\}$ has that property.

In the case of an arbitrary locally compact group G admitting a faithful real representation in $Rep_f(G)$ a result similar to Theorem 5.3 can be found in [17].

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