Automata with One-way Jumping Mode

Kaito Hoshi, Akihiro Yamamura, Szilard Zsolt Fazekas,

Akita University, Graduate School of Engineering Science
Department of Mathematical Science and Electrical-Electronic-Computer Engineering
1-1 Tegata Gakuen-machi, Akita 010-8502, JAPAN

Abstract

Recently, new types of non-sequential machine models have been introduced and studied, such as jumping automata and one-way jumping automata. We study the abilities and limitations of automata with these two jumping modes of tape heads with respect to how they affect the class of accepted languages. We give several methods to determine whether a language is accepted by a machine with jumping mode. We also consider relationships among the classes of languages defined by the new machines and their classical counterparts.
keywords: Jumping mode, One-way jumping finite automata, Pushdown automata, Pumping lemma, Context free language

1 Introduction

We study the ability of the jumping mode of tape heads to strengthen accepting power of automata. Recently a mode of tape head movement has been introduced and examined with respect to how the class of languages accepted is affected ([1-5,7-9,12]). We study the abilities and limitations of the new mode of tape head move by comparing several machine models.

In 2012, Jumping finite automaton(JFA) were introduced by A. Meduna and P. Zemek in [8]. The way of reading strings for JFA is different from usual finite automata. Meduna et al. proved that JFA and finite automata differed in their computing power.

One-way jumping (deterministic) finite automata (OWJFA), a variant of jumping finite automata, were introduced and analyzed in [4]. They have another mode of tape head; the head moves in one direction only and starts at the beginning of the input word. It moves from left to right (and jumps over parts of the input it cannot read) and when the tape head reaches the end of the input, it is returned to the beginning of the input and continues the computation until all the letters are read or the automaton is stuck in the sense that it can no longer read any letter of the remaining input. Several properties and characterization results were provided in [2].

In this paper we consider deterministic or nondeterministic finite automata and pushdown automata in a uniform manner. However, we restrict our study to automata that do not rewrite the input letters and so we exclude linear bounded automata and Turing machines from consideration.

We recall notations of automata (see [6,10]). We denote $\Sigma_\epsilon = \Sigma \cup \{\epsilon\}$, where ϵ is the empty word and $P(Q)$ stands for the power set of Q.

A *nondeterministic finite automaton* (denoted by NFA) M is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where Q is set of states, Σ is finite set, $\delta : Q \times \Sigma \to P(Q)$ is a transition relation, q_0 is the initial state, F is set of accept state. In Section 3 we deviate from this definition by allowing the NFA to have multiple initial states, a change that is known not to affect the class of accepted languages in the classical case, but makes a difference in the alternative tape head modes. If δ is a mapping $Q \times \Sigma_\epsilon \to P(Q)$, then M is called $\epsilon-$NFA. M is called *deterministic* (denoted by a DFA) if (1) it is an ε-free NFA and (2) for $\forall p \in Q$ and $\forall a \in \Sigma$, there is no more than one $q \in Q$ such that $\delta(p, a) = q$.

A *nondeterministic pushdown automaton* (denoted by NPDA) M is a 6-tuple $(Q, \Sigma, \Gamma, \delta, q_0, F)$, where Q is finite set of states, Σ is finite set, Γ is a finite stack alphabet, $\delta : Q \times \Sigma_\epsilon \times \Gamma_\epsilon \to P(Q \times \Gamma_\epsilon)$ is the transition function, q_0 is the initial state, F is set of accept states, and \$ is a bottom marker of stack. M is called *deterministic* (denoted by DPDA) if it satisfies (1) For $\forall q \in Q, \forall a \in \Sigma \cup \{\epsilon\}, \forall b \in \Gamma$ we have $|\delta(q, a, b)| \leq 1$ and (2) For $\forall q \in Q, \forall a \in \Sigma, \forall b \in \Gamma$, we have $\delta(q, a, b) = \emptyset$ if $\delta(q, \epsilon, b) \neq \emptyset$. The language accepted by DPDA is defined as the set of inputs on which the automaton ends up in a final state (not the empty stack acceptance condition) after reading the whole input.

2 Modes of Tape Head Move

First we define modes of tape head movement to capture characteristics of (R)OWJFA. Transitions between configurations of automata are considered to be rewriting of strings on state and input alphabets. We study two ways of rewriting configurations of automata; the standard mode and the one-way jumping mode. The first one is the traditional way to rewrite configurations of automata as defined in [6]. The second one are introduced by [4]. The two modes can be applied to any automata with deterministic/nondeterministic transition functions, with/without stacks, and with/without rewriting and erasing a letter in an input. Our objective with this paper is to extend the examination of how the two modes relate to each other with respect to the accepting power of automata, in particular, nondeterministic finite and pushdown automata.

2.1 Tape Head Modes

Suppose M is a (deterministic or nondeterministic) finite automaton.

standard mode
A configuration of M is a string in $Q \times \Sigma^*$. A transition from configuration $q_1 aw$ to configuration $q_2 w$, written as $q_1 aw \to q_2 w$, is possible when $q_2 = \delta(q_1, a)$. In the standard manner, we extend $\to$ to $\to^m$, where $m \geq 0$. Let $\to^+$ and $\to^*$ denote the transitive and the transitive-reflexive closure of $\to$, respectively.

A FA with *standard mode* is a rewriting system $(M, \to)$ based on $\to^*$. The language accepted by $(M, \to)$ is $L(M, \to) = \{w \mid w \in \Sigma^*, sw \to^* f, f \in F\}$.

One-way jumping mode
The *right one-way jumping relation* (denoted by $\circlearrowright$ here) between configurations from $Q\Sigma^*$, was defined in [4]. Let $x, y \in \Sigma^*$, $a \in \Sigma$ and $p, q \in Q$ such that $q = \delta(p, a)$. Then the right one-way jumping automaton M makes a jump from the configuration $pxay$ to the configuration qyx, symbolically written as $pxay \circlearrowright qyx$ if x belongs to $\{\Sigma \setminus \Sigma_p\}^*$ where $\Sigma_p = \{b \in \Sigma \mid \exists q \in Q \ s.t. \ q \in \delta(p, b)\}$. In the standard manner, we extend $\circlearrowright$ to $\circlearrowright^m$, where $m \geq 0$. We denote by $\circlearrowright^*$ the transitive-reflexive closure of $\circlearrowright$. Intuitively, a machine in right one-way jumping mode will look for the closest letter to the right of its current position, for which it has a defined transition. This means that when the automaton is completely defined, then one-way jumping mode works the same way as the standard reading mode. While incomplete FA have the same accepting power as complete ones in the classical case, in this new mode of tape head movement non-regular and even non-context-free languages can be accepted by incomplete finite state machines.

We define a FA with *one-way jumping mode of tape head* to be a rewriting system $(M, \circlearrowright)$ based on $\circlearrowright^*$. The language accepted by $(M, \circlearrowright)$ is defined to be $L(M, \circlearrowright) = \{w \mid w \in \Sigma^*, sw \circlearrowright^* f, f \in F\}$.

In a similar manner we define a (deterministic or nondeterministic) pushdown automaton with *standard mode*, *one-way jumping mode*, respectively, to be the rewriting systems $(M, \to)$ and $(M, \circlearrowright)$, respectively, where M is a (deterministic or nondeterministic) pushdown automaton.

2.2 Language Classes

We consider deterministic and nondeterministic finite automata and deterministic and nondeterministic pushdown automata, denoted by DFA, NFA, DPDA, NPDA, respectively. Then we classify these automata with three modes of tape head from the standpoint of languages accepted. We denote the language classes accepted by DFA, NFA, DPDA, NPDA with two modes $\rightarrow, \circlearrowright$ by $(\rightarrow, \circlearrowright)$-**DFA**, $(\rightarrow, \circlearrowright)$-**NFA**, $(\rightarrow, \circlearrowright)$-**DPDA**, $(\rightarrow, \circlearrowright)$-**NPDA**, respectively, in this paper. For example, $\rightarrow$**DFA** coincides with $\rightarrow$**NFA** and they comprise the class of regular languages, and $\rightarrow$**NPDA** is the class of context-free languages. Versions of the statement regarding the acceptance of inputs by M in the different modes have been shown in [4] for $\circlearrowright$.

2.3 Differences of Modes of Tape Head Move

Chigahara et al. got following important Theorem and Corollary.

Theorem. 1 (see [4], Theorem 10)**.** *Let M be the DFA. If $w \in L(M, \circlearrowright)$, then there exists a permutation ϕ such that $\phi(w) \in L(M, \rightarrow)$. Moreover, $L(M, \rightarrow) \subseteq L(M, \circlearrowright)$.*

Corollary. 1 (see [4], Corollary 11)**.** *For any language L accepted by a $\circlearrowright$DFA there exists a constant n, such that for every string $w \in L$ with $|w| \geq k$, there exists a permutation $\phi(w)$ of w, which can be written as $\phi(w) = xyz$, satisfying the following conditions:*

1. *$y \neq \epsilon$.*
2. *$|xy| \leq n$.*
3. *$xy^iz \in L$, for all $i \geq 0$.*

Corollary 1 is a pumping lemma for $\circlearrowright$DFA. We obtain extensions of Theorem 1 and Corollary 1.

Theorem. 2. *Let M be an NFA, DFA, DPDA or NPDA. If $w \in L(M, \circlearrowright)$, then there exists a permutation ϕ such that $\phi(w) \in L(M, \rightarrow)$. Moreover, $L(M, \rightarrow) \subseteq L(M, \circlearrowright)$.*

Corollary. 2. *Let M be an NFA, DFA, DPDA or NPDA, and $N_M > 0$ a constant which depends on M. If a pumping lemma holds for all words $w \in L(M, \rightarrow)$ with $|w| \geq N_M$, then for all words $w \in L(M, \circlearrowright)$ with $|w| \geq N_M$, there exists a permutation ϕ such that the lemma holds for $\phi(w)$.*

Proof is similar to Theorem 1 and Corollary 1. $\square$

3 One-Way Jumping Nondeterministic Finite Automata ($\circlearrowright$NFA)

First, we describe about $\circlearrowright$NFA.

Definition. 1. *($\circlearrowright$NFA) A (right) one-way jumping nondeterministic finite automaton is an ϵ-free NFA with multiple initial states in $\circlearrowright$ execution mode.*

Example. 1. *The language $K = \{w \in \{a, b\}^* : |w|_b = 0 \text{ or } |w|_a = |w|_b\}$ is accepted by $\circlearrowright$NFA $M = (\{q_0, q_1, q_2, q_3, q_4, q_5\}, \{a, b\}, \delta, \{q_0\}, \{q_1, q_3\})$, where $\delta(q_0, a) = \{q_1, q_3\}, \delta(q_1, a) = \{q_2\}, \delta(q_2, b) = \{q_1\}, \delta(q_3, a) = \{q_3\}$.*

In [2] it is shown that $K \notin \circlearrowright$**DFA**; this establishes $\circlearrowright$**DFA**$\subsetneq\circlearrowright$**NFA**.

Theorem. 3. *The class $\circlearrowright$**NFA** is closed under union.*

Proof. Let $M_1 = (Q_1, \Sigma_1, \delta_1, S_1, F_1)$ and $M_2 = (Q_2, \Sigma_2, \delta_2, S_2, F_2)$ be two NFA such that $Q_1 \cap Q_2 = \emptyset$ (if this does not hold, we can simply rename the states). It is straightforward to see that $M_3 = (Q_1 \cup Q_2, \Sigma_1 \cup \Sigma_2, \delta_1 \cup \delta_2, S_1 \cup S_2, F_1 \cup F_2)$ accepts the union of the languages accepted by M_1 and M_2 in $\circlearrowright$ execution mode, that is, $L(M_3, \circlearrowright) = L(M_1, \circlearrowright) \cup L(M_2, \circlearrowright)$. $\square$

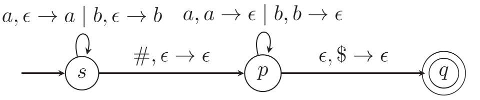

Figure 1: DPDA M with $\circlearrowright$ mode accepting $L_{\mathrm{ppal}} = \{w\#\phi(w) \mid w \in \{a,b\}^*, \phi \in S_{|w|}\}$.

Chigahara et al. got that $\{w \mid w \in \Sigma^*, |w|_a = |w|_b\} \in \circlearrowright$**DFA** (see [5], Example 1), $\{wa \mid w \in \Sigma^*, |w|_a = |w|_b\} \notin \circlearrowright$**DFA** (see [5], Theorem 7) and $\{a^n b^n \mid n \geq 0\} \notin \circlearrowright$**DFA** (see [5], Corollary 12). We can prove following Corollary in the same way as $\circlearrowright$**DFA**.

Corollary. 3. $\{wa \mid w \in \Sigma^*, |w|_a = |w|_b\} \notin \circlearrowright$**NFA** $\square$

Corollary. 4. $\{a^n b^n \mid n \geq 0\} \notin \circlearrowright$**NFA** $\square$

These Corollary mean that $\circlearrowright$**NFA** is not closed under intersection and concatenation.

4 One-Way Jumping Pushdown Automaton: ($\circlearrowright$)NPDA, ($\circlearrowright$)DPDA

In this section we describe PDA with OWJ mode. First, we state two extended pumping lemmas for $\circlearrowright$ mode. The proofs of these lemmas are trivial based on Corollary 2.

Corollary. 5 (Bar-Hillel lemma for $\circlearrowright$NPDA)**.** *For any language L accepted by a $\circlearrowright$NPDA there exists a constant n, such that for every string $w \in L$ with $|w| > n$, there exists a permutation w_σ, which can be written as $w_\sigma = uvxyz$, satisfying (1) $|vy| \geq 1$, (2) $|vxy| \leq n$ and (3) $uv^ixy^iz \in L$ for every $i \geq 0$.*

Corollary. 6 (Pumping Lemma for $\circlearrowright$DPDA, original version in [11])**.** *Suppose L is accepted by a $\circlearrowright$DPDA M. Then there exists a constant n for L such that for any pair of words $w, w' \in L$ if*

(1) $s = xy$ and $s' = xz$, $|x| > n$, and

(2) (first symbol of y) = (first symbol of z),

where s and s' are permutations of w and w', such that $s, s' \in L(M, \to)$, then either (3) or (4) holds:

(3) there is a factorization $x = x_1x_2x_3x_4x_5, |x_2x_4| \geq 1$ and $|x_2x_3x_4| \leq n$, such that for all $i \geq 0, x_1{x_2}^ix_3{x_4}^ix_5y$ and $x_1{x_2}^ix_3{x_4}^ix_5z$ are in L;

(4) there exist factorizations $x = x_1x_2x_3, y = y_1y_2y_3$ and $z = z_1z_2z_3, |x_2| \geq 1$ and $|x_2x_3| \leq n$, such that for all $i \geq 0, x_1{x_2}^ix_3y_1{y_2}^iy_3$ and $x_1{x_2}^ix_3z_1{z_2}^iz_3$ are in L.

Let $L_{\mathrm{ppal}} = \{w\#\phi(w) \mid w \in \{a,b\}^*, \phi \in S_{|w|}\}$.

Example. 2. *Construct DPDA $M = (\{s,p,q\}, \{a,b,\#\}, \delta, s, \{q\})$ where δ satisfies Fig 1 . We get $L(M, \circlearrowright) = L_{\mathrm{ppal}}$.*

Proposition 1. $L_{\mathrm{ppal}} \notin \to$**NPDA**$\cup\circlearrowright$**DFA**.

Proof. Note that $L_{\mathrm{ppal}} \notin \to$**NPDA**, by a simple application of the Bar-Hillel lemma. Suppose that $M = (Q, \Sigma, \delta, q_0, F)$ is a DFA, such that $L(M, \circlearrowright) = L_{\mathrm{ppal}}$. Consider the word $w = a^p\#a^p \in L_{\mathrm{ppal}}$, with $p = |Q|+1$. By Theorem 2 there exists a permutation P such that $P(w) \in L(M, \to) \subseteq L_{\mathrm{ppal}}$ and the pumping lemma for regular languages says that it can be written as $P(w) = xyz$, where $y \neq \epsilon, |xy| \leq |Q|$ and $xy^iz \in L_{\mathrm{ppal}}, \forall i \geq 0$. Since $L_{\mathrm{ppal}} \cap (a + \#)^*$ has only one word of each length, we have $P(w) = a^p\#a^p$. From $|xy| \leq |Q|$, we get a contradiction when $i = 2$ as $xy^iz \notin L_{\mathrm{ppal}}$. $\square$

Proposition 2. $L_{\mathrm{ambig}} = \{a^ib^i | i \geq 0\} \cup \{a^ib^{2i} | i \geq 0\} \notin \circlearrowright$**DPDA**.

Proof. Assume there exists DPDA M=$(Q,\Sigma,\Gamma,\delta,q_0,F)$ such that $L(M,\circlearrowright)=L_{\text{ambig}}$ and let C be the constant for L_{ambig} in Corollary 6.

Choose $w=a^nb^n$ and $w'=a^nb^{2n}$ for some integer $n>C$, then there exist permutations σ and σ' such that $w_\sigma, w'_{\sigma'} \in L(M,\rightarrow)$. By $w_\sigma, w'_{\sigma'} \in L(M,\rightarrow) \subseteq L(M,\circlearrowright) = L_{\text{ambig}}$, we get that $w_\sigma = a^nb^n$ and $w'_{\sigma'} = a^nb^{2n}$. Let $x=a^nb^{n-1}$, $y=b$, and $z=b^{2n-1}$. The choice of $w_\sigma = xy$ and $w'_{\sigma'}=xz$ satisfies (1) and (2) of Corollary 6. According to Corollary 6, either (3) or (4) should hold.

Let us consider (3) first. The only possible factorization $x=x_1x_2x_3x_4x_5$ such that $|x_2x_4|>0$ and for all i, $x_1{x_2}^ix_3{x_4}^ix_5y \in L_{\text{ambig}}$ must satisfy the condition $x_2=a^k$ and $x_4=b^k$ for some $k>0$. But then $x_1{x_2}^0x_3{x_4}^0x_5z = x_1x_3x_5z = a^{n-k}b^{2n-k} \notin L_{\text{ambig}}$. Therefore (3) does not hold.

Now,we consider (4). Any factorization $x=x_1x_2x_3$ such that $|x_2|>0$ and $|x_2x_3| \le C < n$ will result in $x_2 \in b^+$ and $y_2 \in b^*$, so $x_1x_3y_1y_3 = a^nb^{n-|x_2|-|y_2|} \notin L_{\text{ambig}}$. So (4) does not hold either.

This contradicts the $\circlearrowright$-DPDA pumping lemma, so $L_{\text{ambig}} \notin \circlearrowright$**DPDA**. □

By Proposition 2 $\circlearrowright$**DPDA** is not closed under union. The language L_{ambig} is a classic example of nondeterministic context-free language. At the same time, as mentioned in [4],

$$L_{abc} = \{w \in \{a,b,c\}^* \mid |w|_a = |w|_b = |w|_c\} \in \circlearrowright\mathbf{DFA} \setminus \rightarrow\mathbf{NPDA}.$$

It is also straightforward that

$$L_{\text{lin}} = \{a^nb^n \mid n>0\} \in \circlearrowright\mathbf{DPDA} \cap \rightarrow\mathbf{NPDA},$$

Since $\rightarrow$**NPDA** is trivially included in $\circlearrowright$**NPDA**.

The following propositions provide us with the separation results which, added to the previously known relationships, add up to Fig. 2.

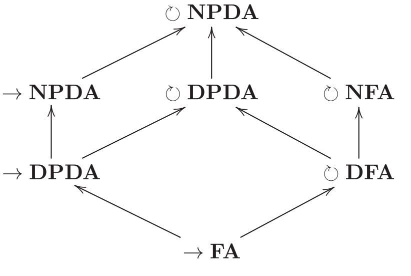

Figure 2: Relationship between REG=→DFA=→NFA, CFL=→NPDA, $A \rightarrow B$ means $A \subsetneq B$, and one-way jumping classes.

Next, we will prove that $L_1 = \{a^nb^nc^n | n \ge 0\} \notin \circlearrowright$**NPDA**.

Theorem. 4. $L_1 = \{a^nb^nc^n | n \ge 0\} \notin \circlearrowright$**NPDA**.

Proof. Suppose that M=$(Q,\{a,b,c\},\Gamma,\delta,q_0,F)$ and $L(M,\circlearrowright)=L_3$. Consider the word $w=a^nb^nc^n \in L_1$. By Corollary 5 there exists a permutation σ such that $w_\sigma \in L_1$ can be written as $w_\sigma = uvxyz$, where $|vy| \ge 1$, $|vxy| \le n$, $uv^ixy^iz \in L_1$, for all $i \ge 0$, where n is the contant from the Bar-Hillel lemma for $\circlearrowright$**NPDA**. Depending on the decomposition $uvxyz$, we have two cases

1. If vxy is generated by one symbol, then uv^2xy^2z does not include the same number of a,b,c. This contradicts $uv^ixy^iz \in L_1$, for $i=2$.
2. If vxy contains two kinds of symbols, then $uv^2xy^2z \notin L_1$, because the number of copies of the third letter (the one not in vxy) does not match the other two. This contradicts $uv^ixy^iz \in L_1$, when $i=2$.

Therefore, no $\circlearrowright$NPDA accepts L_1. □

Since $L_{abc} \in \circlearrowright$**NPDA** and $a^*b^*c^* \in \circlearrowright$**NPDA**, from Theorem 4 we get that the class $\circlearrowright$**NPDA** is not closed under intersection.

5 Summary

We discussed two modes of tape head. But several questions remain open with respect to the one way jumping modes. An unanswered decidability problem, which so far resisted attempts, is whether there exists an algorithm which decides $L(M, \circlearrowright) \in$REG for a given FA M. Here, the technique of completing $\circlearrowright$DFA presented in Section 3 might help, but the problem seems rather difficult, because of the unusual languages which these classes of machines can accept.

6 Future Work

We also studied two-way jumping mode, which extended one-way jumping mode (see [12]). And got as results what some closure properties for DFA with two-way jumping mode. As future work, we would like to study NFA and PDA with two-way jumping mode models.

References

1. Beier, S., Holzer, M.: Decidability of right one-way jumping finite automata. In: Hoshi, M., Seki, S. (eds.) Developments in Language Theory - 22nd International Conference, DLT 2018, Tokyo, Japan, September 10-14, 2018, Proceedings. Lecture Notes in Computer Science, vol. 11088, pp. 109-120 (2018)
2. Beier, S., Holzer, M.: Properties of right one-way jumping finite automata. Theoretical Computer Science 798, 78 - 94 (2019)
3. Bensch, S., Bordihn, H., Holzer, M., Kutrib, M.: On input-revolving deterministic and nondeterministic finite automata. Inf. Comput. 207(11), 1140-1155 (2009)
4. Chigahara, H., Fazekas, S.Z., Yamamura, A.: One-way jumping finite automata. Int. J. Found. Comput. Sci. 27(3), 391-405 (2016)
5. Fazekas, S.Z., Hoshi, K., Yamamura, A.: Enhancement of automata with jumping modes. In: A. Castillo-Ramirez, P.P.B de Oliveira(eds.) Cellular Automata and Discrete Complex Systems, pp. 62-76. Springer International Publishing, Cham (2019)
6. Hopcroft, J.E., Ullman, J.D.: Introduction to Automata Theory, Languages and Computation. Addison-Wesley (1979)
7. Jančar, P., Mráz, F., Plátek, M., Vogel, J.: Restarting automata. In: Reichel, H. (ed.) Fundamentals of Computation Theory. pp. 283-292. Springer Berlin Heidelberg, Berlin, Heidelberg (1995)
8. Meduna, A., Zemek, P.: Jumping finite automata. Int. J. Found. Comput. Sci.23(7), 1555-1578 (2012)
9. Nagy, B., Otto, F.: Finite-state acceptors with translucent letters. In: BILC 2011 - 1st International Workshop on AI Methods for Interdisciplinary Research in Language and Biology, ICAART 2011 - 3rd International Conference on Agents and Articial Intelligence. pp. 3-13 (2011)
10. Sipser, M.: Introduction to the Theory of Computation. Course Technology, second edn. (2006)
11.Yu, S.: A pumping lemma for deterministic context-free languages. Inf. Process. Lett. 31(1), 47-51 (1989)
12. Fazekas, S.Z., Hoshi, K., Yamamura, A.: Two-way Jumping automata. In: FAW2020, to appear