Inverse Scattering filed theory

Kenjiro Kimura
Center for Mathematical and Data Sciences, Kobe University
Noriaki Kimura
Integral Geometry Science Inc.

I Introduction

The most important issue in the theoretical analysis for inverse wave scattering problem is that it should be based on the realistic observation processes and methods. In the theory described in this paper, both the wave transmitting point and the wave receiving point are arbitrarily set on the curved surface to receive the wave scattering data, and the inside of the object is visualized using the acquired data. Transmitting antennas and receiving ones are installed on a curve in a two-dimensional cross section that moves in the x-axis direction. Assuming that the number of transmitting antennas and the number of receiving antennas are both N, the time waveforms of N squared are recorded, and the image of measurement target should be reconstructed using the data. In the overview of this theory, the scattering field function required for visualization should satisfy the partial differential equation defined in 6-dimensional space $(t, x, y_1, y_2, z_1, z_2)$ or 7-dimensional space $(t, x_1, x_2, y_1, y_2, z_1, z_2)$ which corresponds to the coordinates of transmitting points and receiving points extendedly-defined not only on the surface but also the inside. The three-dimensional target image can be obtained by solving this equation using transmitted data and received data on the curved surface as the boundary condition and setting $t \to 0$、$x_1 \to x_2 (= x)$, $z_2 \to z_1 (= z)$, $y_1 \to y_2 (= y)$. This theoretical method is most suitable for computer analysis, and it is considered that it can be effectively used for the research about non-destructive subsurface imaging.

II Inverse Scattering filed theory

II-1 Inverse scattering problem

Under the condition that the wave frequency ω is constant, the wave transmitting point $\mathbf{r}_1$ and the wave receiving point $\mathbf{r}_2$ freely move inside the cross-section D perpendicular to the x direction with a certain constraint condition. when the data obtained at the above situation is expressed for $\mathrm{G}(\mathbf{r}_1, \mathbf{r}_2, \omega)$, this function should be related to the distribution of scattering points in the region. FIG.1 shows a situation in which the wave transmitted from $\mathbf{r}_1$ is reflected at the point ξ and returns to the receiving point $\mathbf{r}_2$. Since there are many scattering points in the region, $\mathrm{G}(\mathbf{r}_1, \mathbf{r}_2, \omega)$ can be expressed as follows.

$$G(\mathbf{r}_1, \mathbf{r}_2, \omega) = \iiint_D \varphi(\mathbf{r}_1 \to \xi \to \mathbf{r}_2, \omega)\, d\xi \quad \text{........... (1-1)}$$

Here, $\varphi(\mathbf{r}_1 \to \xi \to \mathbf{r}_2, \omega)$ indicates the signal of the wave at the receiving point $\mathbf{r}_2$ when the wave is transmitted from the $\mathbf{r}_1$ and scattered at point ξ. The constraint conditions imposed on the wave transmitting point $\mathbf{r}_1$ and the wave receiving point $\mathbf{r}_2$ are as follows. The constraint is that the *x*-coordinates of $\mathbf{r}_1$ and $\mathbf{r}_2$ are always equal. This function $G(\mathbf{r}_1, \mathbf{r}_2, \omega)$ is used to explain the theoretical structure of the inverse scattering problem. Let D be a sub-region in the three-dimensional space, and let ∂D be its boundary. At this situation, the function $G(\mathbf{r}_1, \mathbf{r}_2, \omega)$ is the solution of the following differential equation inside the domain D.

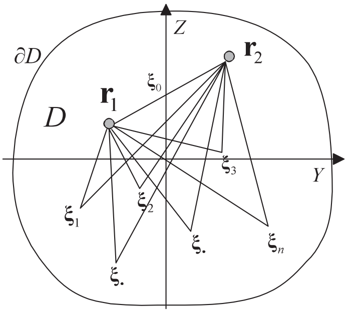

Figure 1: $\mathbf{r}_1$ and $\mathbf{r}_2$ move freely inside region D, respectively. G is the sum of the scattering signals from all points ξ.

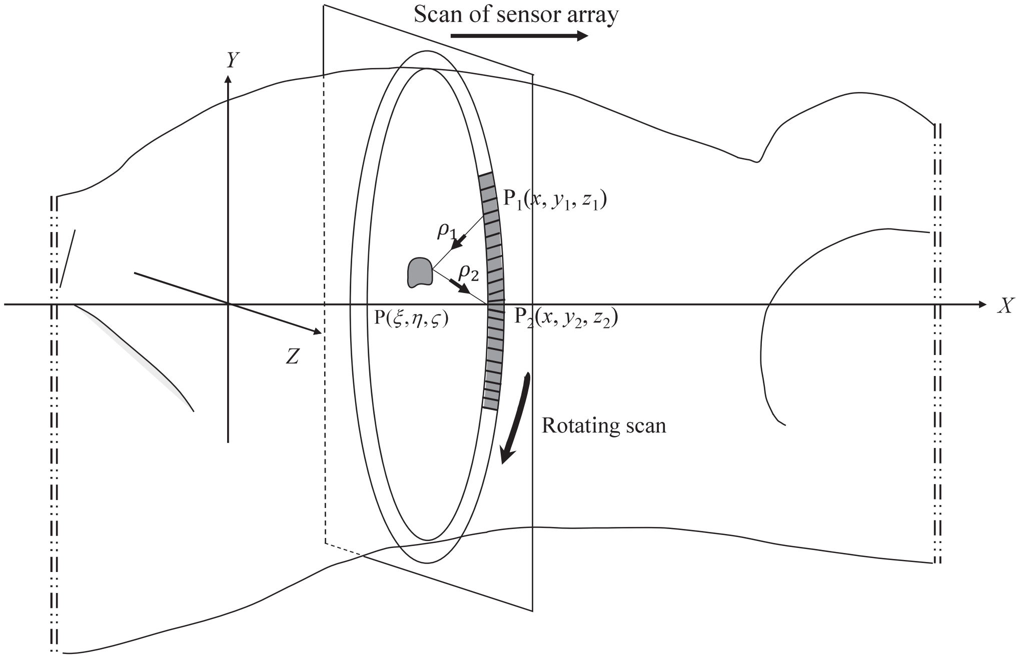

Figure 2: Sensor array

$$L\left(\frac{\partial}{\partial t},\frac{\partial}{\partial \mathbf{r}_1},\frac{\partial}{\partial \mathbf{r}_2}\right)\overline{G}(\mathbf{r}_1,\mathbf{r}_2,t)=0 \qquad \text{........... (1-2)}$$

$\overline{G}(\mathbf{r}_1,\mathbf{r}_2,t)$ is a Fourier transform function of $G(\mathbf{r}_1,\mathbf{r}_2,\omega)$ with respect to ω. Further, the value of $G(\mathbf{r}_1,\mathbf{r}_2,\omega)$ at the boundary ∂D is a signal received by the antennas. The above equation is solved under this boundary condition, and $\rho(\mathbf{r})$ is defined as follows from the result.

$$\rho(\mathbf{r})=\lim_{t\to 0}\left[\mathrm{Tr}\left[\overline{G}(\mathbf{r}_1,\mathbf{r}_2,t)\right]\right]=\overline{G}(\mathbf{r},\mathbf{r},0) \qquad \text{........... (1-3)}$$

This $\rho(\mathbf{r})$ is a function corresponding to the spatial distribution of the gradient of the permittivity inside the region D using microwaves. In this scheme, deriving the differential operator $L(\partial/\partial t,\partial/\partial \mathbf{r}_1,\partial/\partial \mathbf{r}_2)$ that appears here is the most esoteric problem.

II-2 Derivation of $L(\partial/\partial t,\partial/\partial \mathbf{r}_1,\partial/\partial \mathbf{r}_2)$

The method for deriving this differential operator is be described as follows. A case where the sensor array shown in FIG. 2 moves in the x-axis direction along a curved surface while performing a rotational scan in the direction perpendicular to the x-axis is considered. On an arbitrary curve, the y and z coordinates of $\mathbf{r}_1$ and $\mathbf{r}_2$ are not always equal. Specifically, $\mathbf{r}_1=(x, y_1, z_1)$, $\mathbf{r}_2=(x, y_2, z_2)$. The function G is defined as follows.

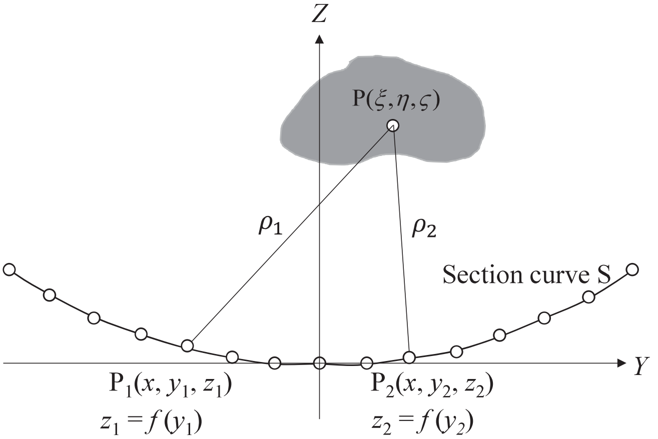

Figure 3: Waves transmitted from P_1 are reflected at point P and return to P_2. z_1 and z_2 are optional. The measurement point moves on the cross-section curve S.

$$G(\mathbf{r}_1,\mathbf{r}_2,\omega)=\iiint_D \varphi(\mathbf{r}_1\to\boldsymbol{\xi}\to\mathbf{r}_2,\omega)\,d\boldsymbol{\xi} \qquad \text{...........(2-1)}$$

The equation that the function G satisfies is derived. Here, $\omega = ck$. c is the propagation velocity and k is the wave number, and the wavelength is λ, there is a relationship of $k = 2\pi/\lambda$. The following function ϕ is introduced.

$$\phi(x, y_1, y_2, z_1, z_2, \omega) = \iint_D \frac{e^{ik\rho_1}}{\rho_1} \frac{e^{ik\rho_2}}{\rho_2} \varepsilon(\xi, \eta, \zeta) d\xi d\eta d\zeta$$
$$\rho_1 = \sqrt{(x-\xi)^2 + (y_1-\eta)^2 + (z_1-\zeta)^2} \qquad \text{...........(2-2)}$$
$$\rho_2 = \sqrt{(x-\xi)^2 + (y_2-\eta)^2 + (z_2-\zeta)^2}$$

Here, the wave number is written as *k*, assuming that the time factor is proportional to exp$(-i\omega t)$. The function in the integrand in the above equation is φ in equation (2-1).

$$\varphi = \frac{e^{ik\rho_1}}{\rho_1} \frac{e^{ik\rho_2}}{\rho_2} \varepsilon(\xi, \eta, \varsigma) \qquad \text{...........(2-3)}$$

A partial differential equation whose an asymptotic solution is equation (2-2) is derived. Here, the higher-order terms of $1/\rho$ resulting from differentiations should be ignored. The abbreviations for differentiations are defined as follows.

$$\frac{\partial}{\partial t} \to \partial_t, \frac{\partial}{\partial x} \to \partial_x, \frac{\partial}{\partial y_1} \to \partial_{y1}, \frac{\partial}{\partial y_2} \to \partial_{y2}, \frac{\partial}{\partial z_1} \to \partial_{z1}, \frac{\partial}{\partial z_2} \to \partial_{z2} \qquad \text{...........(2-4)}$$

As a result of some calculations, it is derived that ϕ satisfies the following equation.

$$\left[\frac{1}{4}\Delta_5^{\ 2} - (ik)^2 \partial_x^{\ 2} - (\partial_{y1}^{\ 2} + \partial_{z1}^{\ 2})(\partial_{y2}^{\ 2} + \partial_{z2}^{\ 2})\right]\phi = 0 \qquad \text{...........(2-5)}$$

This equation is derived assuming a steady state, but it is not difficult to extend it to the unsteady state. Therefore, the variables are replaced as follows.

$$-ik \to \frac{1}{c}\partial_t \qquad \text{...........(2-6)}$$

Finally, the following equation is derived.

$$\left[\frac{1}{4}\Delta_5^{\,2}-\frac{1}{c^2}\partial_t^{\,2}\partial_x^{\,2}-(\partial_{y1}^{\,2}+\partial_{z1}^{\,2})(\partial_{y2}^{\,2}+\partial_{z2}^{\,2})\right]\phi=0 \qquad \text{..........(2-7)}$$

Assuming that the time factor of ϕ is proportional to exp $(-i\omega t)$, the solution of equation (2-7) is derived. Firstly, multiple Fourier transforms are performed on ϕ for t, x, y_1, y_2.

$$\tilde{\phi}(k_x,k_{y_1},k_{y_2},z_1,z_2,\omega)=\int_{-\infty}^{\infty}e^{i\omega t}dt\int_{-\infty}^{\infty}e^{ik_{y_1}y_1}dy_1\int_{-\infty}^{\infty}e^{ik_{y_2}y_2}dy_2\int_{-\infty}^{\infty}e^{ik_x x}\phi(x,y_1,y_2,z_1,z_2,t)dx$$

..........(2-8)

The following equations are obtained by describing the partial derivatives with respect to z_1 and z_2 as D_{z1} and D_{z2}, respectively.

$$\{(D_{z1}^{\,2}+D_{z2}^{\,2}-k_x^{\,2}-k_{y1}^{\,2}-k_{y2}^{\,2})^2-4k^2k_x^{\,2}-4(D_{z1}^{\,2}-k_{y1}^{\,2})(D_{z2}^{\,2}-k_{y2}^{\,2})\}\tilde{\phi}=0$$

..........(2-9)

When this equation is solved, there are two variables, z_1 and z_2. Therefore, the boundary value problem does not hold unless the boundary condition is given in the region with one-dimensional degrees of freedom in the (z_1, z_2) space with respect to the fixed coordinates (x, y_1, y_2), that is the coordinates (k_x, k_{y1}, k_{y2}) in the wave number space. The boundary condition acquired by the actual measurement is given only at one point $(f(y_1), f(y_2))$ in (z_1, z_2) space. To resolve this important issue, the following fact should be taken into account. The theory in this study, where z_1 is independent on z_2, should include a specific case where $z_1 = z$, $z_2 = z$. The solution of equation (2-9) can be expressed as follows.

$$E(k_x,k_{y_1},k_{y_2},z_1,z_2)=\exp(is_1z_1)\exp(is_2z_2) \qquad \text{..........(2-10)}$$

This formula becomes as follows when $z_1 = z_2 = z$.

$$E(k_x,k_{y_1},k_{y_2},z_1,z_2)=\exp\{i(s_1+s_2)z\} \qquad \text{..........(2-11)}$$

Substituting equation (2-10) into equation (2-9) gives the following equation.

$$({s_1}^2+{s_2}^2+{k_x}^2+{k_{y1}}^2+{k_{y2}}^2)^2-4k^2{k_x}^2-4({s_1}^2+{k_{y1}}^2)({s_2}^2+{k_{y2}}^2)=0 \qquad \text{..........(2-12)}$$

Another equation is needed to derive each of s_1 and s_2. Here, from the above-mentioned requirement for the consistency about the fact where this theory should include the specific case $z_1 = z_2 = z$, s_1 and s_2 should satisfy the following relational expression.

$$s_1+s_2=\sqrt{(\sqrt{k^2-{k_{y_1}}^2}+\sqrt{k^2-{k_{y_2}}^2})^2-{k_x}^2} \qquad \text{..........(2-13)}$$

From equations (2-12) and (2-13), s_1 (k_x, k_{y1}, k_{y2}) and s_2 (k_x, k_{y1}, k_{y2}) are derived as follows.

$$s_1(k_x,k_{y_1},k_{y_2})=\frac{\sqrt{k^2-{k_{y_1}}^2}\sqrt{\left(\sqrt{k^2-{k_{y_1}}^2}+\sqrt{k^2-{k_{y_2}}^2}\right)^2-{k_x}^2}}{\sqrt{k^2-{k_{y_1}}^2}+\sqrt{k^2-{k_{y_2}}^2}}$$

$$s_2(k_x,k_{y_1},k_{y_2})=\frac{\sqrt{k^2-{k_{y_2}}^2}\sqrt{\left(\sqrt{k^2-{k_{y_1}}^2}+\sqrt{k^2-{k_{y_2}}^2}\right)^2-{k_x}^2}}{\sqrt{k^2-{k_{y_1}}^2}+\sqrt{k^2-{k_{y_2}}^2}} \qquad \text{..........(2-14)}$$

Specific calculations are performed later, and the solution of equation (2-7) can be described as follows using these s_1 (k_x, k_{y1}, k_{y2}) and s_2 (k_x, k_{y1}, k_{y2}).

$$\phi(x,y_1,y_2,z_1,z_2,k)$$
$$=\frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{-i(k_x x+k_{y_1}y_1+k_{y_2}y_2)}a(k_x,k_{y_1},k_{y_2})e^{is_1(k_x,k_{y_1},k_{y_2})z_1}e^{is_2(k_x,k_{y_1},k_{y_2})z_2}dk_x dk_{y_1} dk_{y_2}$$

..........(2-15)

The equation of the cross-section curve S with the x-coordinate fixed is assumed as follows. Although not an essential assumption, for example, a paraboloid is assumed.

$$f(y)=\alpha y^2 \qquad \text{..........(2-16)}$$

The boundary conditions given on the cross-section curve S are as follows.

$$\phi(x, y_1, y_2, \alpha y_1^2, \alpha y_2^2, k)$$
$$= \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-i(k_x x + k_{y_1} y_1 + k_{y_2} y_2)} a(k_x, k_{y_1}, k_{y_2}) e^{i\alpha\left\{s_1(k_x, k_{y_1}, k_{y_2}) y_1^2 + s_2(k_x, k_{y_1}, k_{y_2}) y_2^2\right\}} dk_x dk_{y_1} dk_{y_2}$$

...........(2-17)

This equation is used to determine $a(k_x, k_{y1}, k_{y2})$. The following abbreviated notation is adopted.

$$a(\mathbf{k}) = a(k_x, k_{y_1}, k_{y_2})$$
$$s_1(\mathbf{k}) = s_1(k_x, k_{y_1}, k_{y_2})$$
$$s_2(\mathbf{k}) = s_2(k_x, k_{y_1}, k_{y_2})$$

...........(2-18)

Equation (2-17) is the following integral equation for $a(k_x, k_{y1}, k_{y2})$.

$$\phi(x, y_1, y_2, \alpha y_1^2, \alpha y_2^2, k)$$
$$= \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-i(k_x x + k_{y_1} y_1 + k_{y_2} y_2)} a(\mathbf{k}) e^{i\alpha\left\{s_1(\mathbf{k}) y_1^2 + s_2(\mathbf{k}) y_2^2\right\}} d\mathbf{k}$$

...........(2-19)

When $a(\mathbf{k})$ can be obtained from this equation, visualization process become accomplished from equation (2-15).

$$\phi(x, y_1, y_2, z_1, z_2, k)$$
$$= \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-i(k_x x + k_{y_1} y_1 + k_{y_2} y_2)} a(\mathbf{k}) e^{is_1(\mathbf{k}) z_1} e^{is_2(\mathbf{k}) z_2} d\mathbf{k}$$

...........(2-20)

In the above equation, $z_1 = z_2 = z$ is set, and then Fourier transform with respect to k is proceeded to obtain the following visualization function $\rho(\mathbf{r})$.

$$\rho(\mathbf{r}) = \lim_{t \to 0}\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} \phi(x, y, y, z, z, k) e^{-ickt} dk\right]$$

...........(2-21)

II-3 Solution of integral equation

$\Phi(k_x, y_I, y_J, k)$, which is Fourier transform of the data $\phi(x, y_I, y_J, z_I, z_J, t)$ measured at points P_I and P_J on the curved surface, is written as follows.

$$\Phi_{I,J}(k_x, y_I, y_J, k) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-ikt-ik_x x}\phi(x, y_I, y_J, z_I, z_J, t)dtdx \quad \text{...........(3-1)}$$

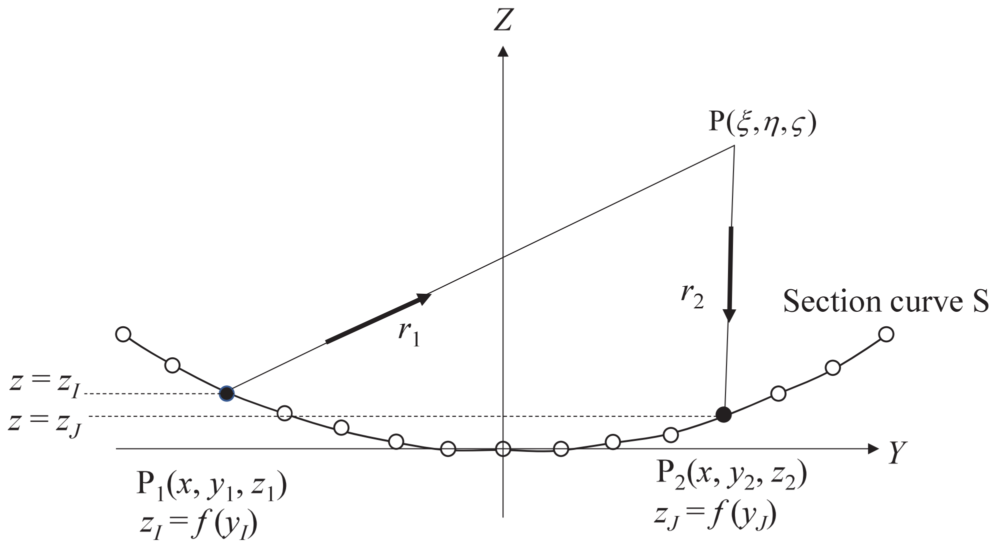

Figure 4: The wave transmitted from P_1 is reflected at point P and returns to P_2. The boundary surface on the *Y* axis from the data on the curved surface is derived.

Here, since z_I and z_J are on the cross-sectional curve, the following equation holds.

$$z_I = f(y_I)$$
$$z_J = f(z_J) \quad \text{...........(3-2)}$$

The following equation is obtained from equation (2-15).

$$\Phi(k_x, y_I, y_J, k) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-i(k_{y_1}y_I + k_{y_2}y_J)} a_{I,J}(k_x, k_{y_1}, k_{y_2}, k) e^{is_1(k,k_x,k_{y_1},k_{y_2})z_I} e^{is_2(k,k_x,k_{y_1},k_{y_2})z_J} dk_{y_1}dk_{y_2}$$

...........(3-3)

This formula can be interpreted and written as follows.

$$\Phi(k_x, y_I, y_J, k)\delta(y_1 - y_I)\delta(y_2 - y_J) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-i(k_{y_1}y_1 + k_{y_2}y_2)} a_{I,J}(k_x, k_{y_1}, k_{y_2}, k) e^{is_1(k,k_x,k_{y_1},k_{y_2})z_I} e^{is_2(k,k_x,k_{y_1},k_{y_2})z_J} dk_{y_1} dk_{y_2}$$

...........(3-4)

The following equation is obtained by taking the Fourier transform of both sides.

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{i(k'_{y_1}y_1 + k'_{y_2}y_2)} \Phi(k_x, y_I, y_J, k)\delta(y_1 - y_I)\delta(y_2 - y_J)\, dy_1 dy_2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{i(k'_{y_1}y_1 + k'_{y_2}y_2)} \left\{\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-i(k_{y_1}y_1 + k_{y_2}y_2)} a_{I,J}(k_x, k_{y_1}, k_{y_2}, k) e^{is_1(k,k_x,k_{y_1},k_{y_2})z_I} e^{is_2(k,k_x,k_{y_1},k_{y_2})z_J} dk_{y_1} dk_{y_2}\right\} dy_1 dy_2$$

...........(3-5)

When integrated,

$$e^{i(k'_{y_1}y_I + k'_{y_2}y_J)} \Phi(k_x, y_I, y_J, k) = \left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \delta(k_{y_1} - k'_{y_1})\delta(k_{y_2} - k'_{y_2}) a(k_x, k_{y_1}, k_{y_2}, k) e^{is_1(k,k_x,k_{y_1},k_{y_2})z_I} e^{is_2(k,k_x,k_{y_1},k_{y_2})z_J} dk_{y_1} dk_{y_2}\right\} = a_{I,J}(k_x, k'_{y_1}, k'_{y_2}, k) e^{is_1(k,k_x,k'_{y_1},k'_{y_2})z_I} e^{is_2(k,k_x,k'_{y_1},k'_{y_2})z_J}$$

...........(3-6)

In this way, $a_{I,J}$ are obtained as follows.

$$a_{I,J}(k_x, k_{y_1}, k_{y_2}, k) = e^{i(k_{y_1}y_I + k_{y_2}y_J)} e^{-is_1(k,k_x,k_{y_1},k_{y_2})z_I} e^{-is_2(k,k_x,k_{y_1},k_{y_2})z_J} \Phi(k_x, y_I, y_J, k)$$

...........(3-7)

The following equation is obtained by calculating the sum for all *I* and *J*.

$$a(k_x, k_{y_1}, k_{y_2}, k) = \sum_{I,J} a_{I,J}(k_x, k_{y_1}, k_{y_2}, k)$$
$$= \sum_{I,J} e^{i(k_{y_1} y_I + k_{y_2} y_J)} e^{-is_1(k,k_x,k_{y_1},k_{y_2})z_I} e^{-is_2(k,k_x,k_{y_1},k_{y_2})z_J} \Phi(k_x, y_I, y_J, k)$$

..........(3-8)

In this way, it becomes possible to convert to the boundary condition at the plane $z = 0$. Deriving the solution of the partial differential equation (2-7) with the boundary condition at $z = 0$ gives the following from Eq. (2-15).

$$\phi(x, y_1, y_2, z_1, z_2, k)$$
$$= \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-i(k_x x + k_{y_1} y_1 + k_{y_2} y_2)} a(k_x, k_{y_1}, k_{y_2}, k) e^{is_1(k_x,k_{y_1},k_{y_2})z_1} e^{is_2(k_x,k_{y_1},k_{y_2})z_2} dk_x dk_{y_1} dk_{y_2}$$

..........(3-9)

At this time, the reconstructed image can be obtained by integrating the following equation with respect to k.

$$\phi(x, y, y, z, k) = \lim_{y_1 \to y} [\phi(x, y_1, y, z, k)]$$
$$= \lim_{y_1 \to y} [\frac{1}{(2\pi)^3} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-i(k_x x + k_{y_1} y_1 + k_y y)} a(k_x, k_{y_1}, k_y, k) e^{i\{\sqrt{(\sqrt{k^2 - k_{y_1}^2} + \sqrt{k^2 - k_y^2})^2 - k_x^2}\} z} dk_x dk_{y_1} dk_y]$$

..........(3-10)

A variable k_z is introduced as follows. The formula for expressing k by k_z and the function obtained by differentiating it are also shown.

$$k_z = \sqrt{\left(\sqrt{k^2 - k_{y_1}^2} + \sqrt{k^2 - k_{y_1}^2}\right)^2 - k_x^2}$$
$$k = \frac{1}{2}\sqrt{k_x^2 + k_z^2 + \frac{(k_{y_1}^2 - k_{y_2}^2)^2}{k_x^2 + k_z^2} + 2(k_{y_1}^2 + k_{y_2}^2)}$$
$$\frac{dk}{dk_z} = \frac{k_z \sqrt{k^2 - k_{y_1}^2}\sqrt{k^2 - k_{y_1}^2}}{k(k_x^2 + k_z^2)}$$

..........(3-11)

Finally, the image function $\rho(x, y, z)$ of the reconstruction result is described as follows.

$$
\begin{aligned}
\rho(x,y,z) &= \int_{-\infty}^{\infty} \phi(x,y,y,z,k)\,dk \\
&= \int_{-\infty}^{\infty} \lim_{y_1 \to y} [\phi(x,y_1,y,z,k)]\,dk \\
&= \int_{-\infty}^{\infty} \lim_{y_1 \to y} [\frac{1}{(2\pi)^3} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-i(k_x x + k_{y_1} y_1 + k_y y)} a(k_x, k_{y_1}, k_y, k) e^{i\{\sqrt{(\sqrt{k^2 - k_{y_1}{}^2} + \sqrt{k^2 - k_y{}^2})^2 - k_x{}^2}\}z} dk_x dk_{y_1} dk_y]\,dk \\
&= \int_{-\infty}^{\infty} \lim_{y_1 \to y} [\frac{1}{(2\pi)^3} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-i(k_x x + k_{y_1} y_1 + k_y y)} e^{ik_z z} a(k_x, k_{y_1}, k_y, k) \left(\frac{dk}{dk_z}\right) dk_x dk_{y_1} dk_y dk_z]
\end{aligned}
$$

...........(3-12)

The theory for the inverse problem of wave scattering in this paper has a wide range of practical implications such as medical imaging and various non-destructive subsurface imaging. Although not mentioned in this paper about the details of experimental setups, the theory that clearly incorporates the physical characteristics of the medium and the structure of the antenna actually used is equipped into the device developed by our research group. As the future work, important improvements should be made according to various application targets based on this theory.

This work was supported by the Research Institute for Mathematical Sciences, an International Joint Usage/Research Center located in Kyoto University, and Medical Research and Development Programs Focused on Technology Transfers: Development of Advanced Measurement and Analysis Systems (AMED-SENTAN) in Japan Agency for Medical Research and Development, and Research on Development of New Medical Devices in Japan Agency for Medical Research and Development, Research on Development of New Medical Devices in Japan Agency for Medical Research and Development, and Grant-in-Aid for Transformative Research Areas in Ministry of Education, Culture, Sports, Science and Technology – Japan.

References

[1] Kenjiro Kimura, “Discovering a theory to visualize the world”, Nature Vol. 588, pp.S124-125 (2020).

[2]Kenjiro Kimura, Noriaki Kimura, “Multi-static Inverse Wave Scattering Theory and Microwave Mammography”, The Institute of Systems, Control and Information Engineers Vol. 64, pp.87-91(2020).

[3]K. Kimura and N. Kimura: Research and development of real-time microwave mammography; ICT Innovation Forum 2016, Strategic Information and Communications R&D Promotion Programe (SCOPE), Makuhari Messe International Conference Center, pp. 26-27 (2016).

Center for Mathematical and Data Sciences
Kobe University
1-1 Rokkodai-cho, Nada-ku, Kobe, Hyogo 657-8501
JAPAN
E-mail address: kimura@gold.kobe-u.ac.jp