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Lau, Buon Kiong; Capek, Miloslav; Hassan, Ahmed M.

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Characteristic Modes – Progress, Overview, and Emerging Topics

Buon Kiong Lau, *Senior Member, IEEE*, Miloslav Capek, *Senior Member, IEEE*,
and Ahmed M. Hassan, *Senior Member, IEEE*

***Abstract*—Over the past decade, characteristic mode analysis (CMA) research has grown from a niche topic to a mainstream topic, warranting a tutorial-style special issue to survey the significant progress that has been made in this field. In this introductory article (PAPER 1), the focus is on providing the big picture. We start with a simple description of characteristic modes. Next, we examine the trends in this field, followed by providing further insights into CMA's historical development. We will also address common myths surrounding the subject. Then, leaving the detailed coverage of major topics to the following papers, we summarize recent applications of CMA in scattering and other emerging topics. Finally, we conclude with some future perspectives on this field.**

***Index Terms*—Characteristic mode analysis, modal resonance, computational electromagnetics, electromagnetic scattering.**

I. INTRODUCTION

AFTER the presentation of a paper on the application of characteristic modes (CMs) for antenna design at the 2015 European Microwave Conference, a professor in microwave theory commented that he was not surprised about the usefulness of some kind of modes for antenna design. After all, the microwave community has benefited tremendously from modal theory in the design/analysis of microwave circuits.

In fact, the antenna community has for many years its own version of modal theory, in the form of spherical wave expansion (SWE) [1]. Unlike waveguide modes [2], confined to fields within guided media, SWE can deal with radiating currents and fields. The radiating properties of canonical structures (*e.g.*, a sphere, a spherical shell) can be quantified in analytical form, which improves the understanding of the radiation mechanisms and how to manipulate them. However, the simplicity and straightforward use of SWE is lost for arbitrarily shaped structures. In this context, CMs provide a versatile tool to analyze the inherent resonance properties of an arbitrarily shaped radiating structure [3]. CMs share with SWE the property of orthogonal far fields and introduce modal ratio between reactive and radiated power, an important physical measure.

As in the modal concepts utilized by the microwave community, the most valuable feature of CMs is that they provide the resonance properties of the object in question, without considering the excitation source [4]. Similar to a guitar string resonating and releasing acoustic waves in different frequencies and harmonics according to string tension and boundary conditions, upon being plucked (excited), an object can resonate to radiate electromagnetic waves in different "harmonics" (modes) in response to an external excitation, be it near-field (antenna feed) or far-field (incident plane wave). The ability to extract the detailed resonance properties independently of the excitation makes CMs particularly suitable for designing and analyzing radiating and scattering objects.

A. Previous CM Overview Literature

After the pioneering works of Garbacz [3] and Harrington and Mautz [4], the basic theory together with some extensions were summarized in a few book chapters [5]–[7]. However, these chapters more or less follow the content of the original papers [4], [8] only.

The practical value of acquiring the modal reactive power, current and fields was the main message of the first overview paper [9] from 2007, which demonstrated that these properties of CMs help to simplify the design of various real-life antennas, including patch antenna, reflect-array, planar monopole, and terminal antenna. A follow-up overview paper in 2015 [10] continues with the same theme, but with the emphasis on how CMA provides a step-by-step deterministic approach to antenna design and correct antenna placement, exemplified with high-frequency aircraft antenna design, aircraft radar cross section reduction, low band compensation of a horn antenna, and conformal LTE antenna design.

Also, in 2015, a book dedicated to CMs was published [11], giving a comprehensive summary of major achievements in the field. Shortly after, a special issue on "Theory and Applications of Characteristic Modes" [12] was published in the IEEE Transactions on Antennas and Propagation (TAP), presenting a cluster of the latest advances in the field.

B. Motivation of CM Special Issue

In the previous review literature [5]–[7], [9]–[12], the focus was to introduce CMs as a viable and efficient tool for analysis and design in antenna and scattering problems. However, the vast majority of journal articles in this field are published after 2016, with Fig. 1 showing the trend for TAP. Moreover, it can be seen in Fig. 2 that more than half of all CM journal papers

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B. K. Lau is with the Department of Electrical and Information Technology, Lund University, 221 00 Lund, Sweden (E-mail: bkl@eit.lth.se).

M. Capek is with the Department of Electromagnetic Field, Faculty of Electrical Engineering, Czech Technical University in Prague, 166 27 Prague, Czech Republic (e-mail: miloslav.capek@fel.cvut.cz).

A. M. Hassan is with the Department of Computer Science and Electrical Engineering, University of Missouri–Kansas City, Kansas City, MO 64110, USA (E-mail: hassanam@umkc.edu).

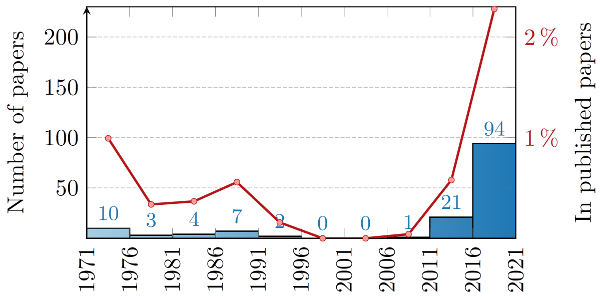

Figure 1. Publication of CM papers in IEEE Transactions on Antennas and Propagation (TAP) over time, in absolute number (blue bars) and as a percentage (red curve) of all the published papers. CM papers in all journals were first extracted from using keywords of characteristic modes using Web of Science and filtered to remove irrelevant papers. These are then supplemented by other CM papers identified by the authors of this special issue.

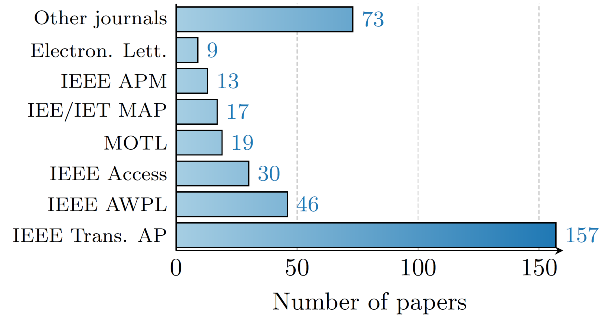

Figure 2. Distribution of CM papers in journals, *e.g.*, IEEE Antennas and Propagation Magazine (APM), IEE/IET Microwave, Antennas and Propagation (MAP), Microwave and Optical Technology Letters (MOTL), IEEE Antennas and Wireless Propagation Letters (AWPL). The data collection method is described in the caption of Fig. 1.

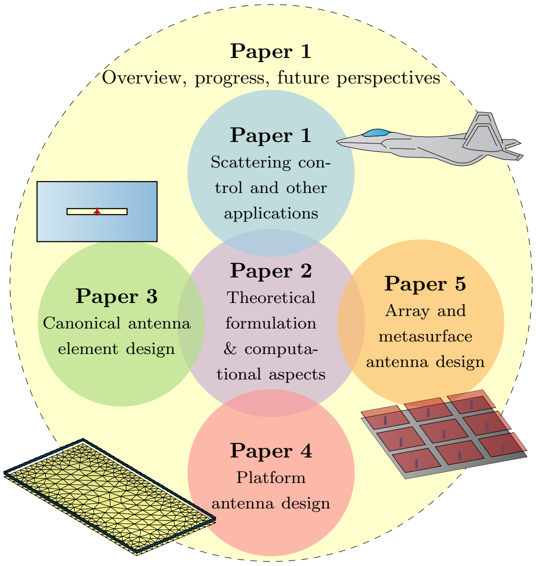

Figure 3. Mapping of CM topics to special issue papers.

are published in the flagship journal (TAP) and letter (AWPL) of our society, giving an indication of quality work being produced in this field. Therefore, the time is ripe for a tutorial-style review of advances in the field, as well as presenting in-depth analyses, experiences and guidance of the subject area to the antenna community at large. Another overarching purpose of this review is to "clean up the topic", given the large number of papers published, to address common myths and help prevent re-discovery of known facts.

Apart from the popularity of CMs in academia, as evidenced by the large number of papers, the industry has also taken an increasing interest in the field. Due to the confidential nature of commercial information, the strongest clue on industry interest is perhaps the progressive adoption and refinements of CM analysis (CMA) as a standard feature ("solver") in all major commercial electromagnetic software vendors today, *i.e.*, Altair (2012), CST (2016), and ANSYS (2017). This is because the features and functionalities in commercial software are mainly driven by industry customers. Today, CMA stands side-by-side with other computational methods like finite-difference time domain method, finite element method, and method of moments. Recent work has also validated existing CMA solvers, including those of Altair FEKO, CEM One, AToM, and WIPL-D [13], [14]. Good performance was generally observed [13].

The increasing availability of software tools for CMA since 2016 has also contributed to the trend in Fig. 1, as many researchers do not have in-house CMA tools. But more fundamentally, the momentum in applying CMA has been growing hand-in-hand with computational power. Garbacz could only calculate the CMs of a few canonical obstacles [3]. Today, more than 50 years later, the theory is largely the same. But advances in computing power, computational electromagnetics, and modern algorithms [15] have enabled even problems involving electrically large structures to be analyzed [16].

C. *Organization of CMA Review*

The special issue presents the topics as schematically depicted in Fig. 3. This paper (PAPER 1) takes the role of justifying the need for a comprehensive overview of CMA, summarizing historical developments, addressing myths, and elaborating on emerging topics and future perspectives. Developments in both theoretical and computational aspects are covered in PAPER 2. Progress on these fronts enables CMA to solve a greater variety of electromagnetic problems and more efficient CMA tool. We then focus on the main applications of CMA, *i.e.*, antenna design and analysis. It seems natural for us to start with canonical antennas in PAPER 3, where CMA offers possibilities of how to enhance performance of existing structures. The utility of CMA is consolidated by its use to design platform antennas in PAPER 4, a field which has traditionally been relying on time-consuming optimization of parameters (*e.g.*, antenna location) over a large design space. Finally, PAPER 5 is dedicated to recent developments on multi-antenna design for array applications and different metasurface designs. We further note that the mapping of topics into PAPERS 1–5 also, to some extent, accounts for the number of journal papers published on these topics.

II. RE-TRACING THE ORIGIN AND EVOLVING TRENDS

It is commonly accepted that the theory of CMs for conducting bodies, suggested for antenna application, is formalized

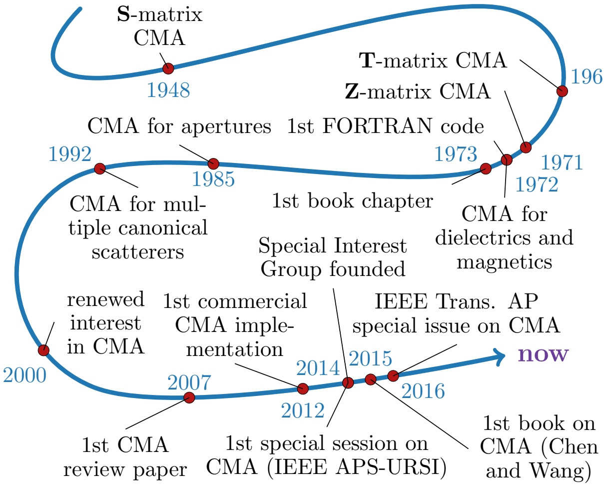

Figure 4. Notable milestones on the development of CM research.

by Mautz and Harrington in their pioneer work [4]. Using boundary condition of the incident and scattered fields[1], and the impedance operator that relates the fields to the currents on the conductor, current modes that are related to the external resonances of the object can be obtained via a generalized eigenvalue decomposition. However, the CM concept was first explored for electromagnetic scattering [3], and it is obtained through diagonalization of the perturbation (transition) operator. In fact, as illustrated in Fig. 4, the inspiration of the theory can be traced further back in history to 1948, when the diagonalization of the scattering operator was mentioned by Montgomery *et al.* [18].

The historical evolution of the CM theory predetermined its applications. They were first mainly from the area of scattering control and cancellation, *e.g.*, [19]–[21]. The antenna applications were addressed afterwards, *e.g.*, [22], [23]. Nowadays – similarly as tens years ago – the application domain of CMA closely follows actual trends. That is why CMs-assisted design is especially popular for electrically small and embedded IoT antennas [24], MIMO antennas [25], with a steep increase of interest in metamaterials [26], metasurfaces [27], 2D materials [28], and nanoantennas [29], [30].

III. COMMON MYTHS

The surge of interest in CMA has brought along with it a fair number of myths, which should be debunked to promote more productive development of this research field. We address some common myths below.

Myth 1: CMA is One-Size-Fits-All

CMA were originally developed to analyze the scattering of conducting objects, by means of their externally resonating modes [3]. Following Harrington's and Mautz' seminal work [4], it has found far more popular application today in the analysis and design of resonant antennas. Moreover, attempts are being made to extend CMA to other fields of application, *e.g.*, time-domain analysis [31] and electrically large structure analysis [16]. However, it is safe to say that CMA is not expected to be well suited for every electromagnetic problem under the sun. For example, since CMA is formulated for far-field scattering and radiation, it is not well-converging for near-field problems, such as design of coil systems for inductive wireless power transfer or implanted antennas. However, there might be an alternative modal decomposition more suitable for these engineering problems, see PAPER 2 for details.

Myth 2: CMs Cannot Be Utilized for Feeding Synthesis

The usefulness of the extracted CMs for a given structure for the purpose of guiding antenna design can be questioned: Can these modes be utilized for feed design in reality? The motivation is that the addition of any feed structure, either resonant or non-resonant coupling element, will fundamentally change the CMs of the structure without the feed, as found in some previous studies [32].

Although it is correct that the CMs of the entire structure may be changed by adding the feed(s), recent work has addressed this issue by limiting the size of the feed element, so that it does not cause any appreciable effect on the modes of interest [33]. However, the smaller the coupling element, the more challenging it is to match the antenna over a given modal bandwidth [34]. To confirm that the desired modes are unchanged, CMA can be repeated with the feed structure included and modal weighting coefficient (MWC) can be calculated to verify the desired modes are indeed excited [35].

Closely related is a problem of selective excitation, *i.e.*, how to excite only a selected mode. This is nearly impossible in practice, especially when there are many resonant modes, in other words, when the structure is not electrically small. Nevertheless, the contribution of other modes can be minimized by increasing the number of properly placed/phased feeders [35] or by reactive loading.

Myth 3: Modal Tracking is Useless

CMs are solved separately at each frequency without considering any excitation. Hence, the utility of modal tracking, *i.e.*, a technique interconnecting eigenvalues and corresponding eigenvectors across small frequency spacing, should be discussed. A benefit of tracking is that the parameters based on continuity of eigentraces, *e.g.*, modal Q-factor [19], [36], can be directly evaluated. The same applies for excitation of multi-band resonances [37]. Another benefit of tracking is based on time-domain characteristics inferred from modal quantities [38], where well-tracked traces embody causal response.

However, the tracking procedure is opened to interpretations induced by von Neumann-Wigner theorem [38], [39] and that the tracking becomes irrelevant when all the modal currents are superposed after excitation is added. Moreover, one should not deny that correct modal tracking can be challenging for certain structures where modal properties may change significantly over a small frequency interval even for the same mode.

Fortunately, recent results have addressed many previous pitfalls in modal tracking, a subject that will be treated in PAPER 2.

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[1] {Typically, the electric field integral equation is employed for the CMA but the combined field integral equation is preferred for perfectly electric conducting (PEC) objects with internal resonances [17].

Myth 4: Orthogonal Far-fields are Unique to CMs

A well-known feature, greatly appreciated in both scattering and antenna applications, is that CMs have orthogonal far-fields. It has to be pointed out that this is not a unique property pertaining to CMs. The far-field orthogonality can be realized via (infinitely many) alternative modal decompositions as well, see PAPER 2 of this review series. This also indicates that not every decomposition granting orthogonal far fields should be called characteristic mode decomposition.

IV. EMERGING TOPICS

Research in CMA has taken off over the past decade (see Fig. 1), especially in applications involving antenna design on conducting bodies, surveyed in PAPERS 3-5. In the following, we review several emerging CMA topics, covering theoretical work for material bodies and scattering-related applications.

A. Formulations for Dielectric and Magnetic Structures

The original forms of the volume integration equation (VIE) and surface integral equation (SIE) for the CMA of material bodies (dielectric and magnetic materials) [40], [41] were presented shortly after that of conducting bodies. However, it was not until recent years that the subject of material bodies has received more interest. The turnaround is due the resurging interest in CMA and the presence/integration of material bodies in many antennas (*e.g.*, dielectric resonator antennas, antennas with radomes, substrate integrated antennas).

As it turned out, while there is consensus that the CMs for lossless dielectrics from VIE [40] are correct, the classic work is problematic in several ways, as hinted in [41]: "Many questions are still left unanswered in the interpretation and application of characteristic modes to material objects." First, a problem that applies not just to material bodies, but also to conducting bodies, is the impact of losses on the far-field orthogonality of CMs, a key property of CMA. This is because the losses add an extra real-value term in the inner product $\langle \boldsymbol{J}_m^*, \mathcal{Z}(\boldsymbol{J}_n)\rangle$ [6], with $\boldsymbol{J}_n$ and $\mathcal{Z}$ being the current density of mode n and impedance operator, respectively. This in general introduces far-field correlation if the operator $\mathcal{Z} = \mathcal{R} + \mathrm{j}\mathcal{X}$ is chosen such that the decomposition of $\mathcal{X}(\boldsymbol{J}_n) = \lambda_n \mathcal{R}(\boldsymbol{J}_n)$ retains real-valued eigenvalue λ_n. The problem was shown for the VIE CM formulation in [40]. The same paper proposes another VIE CM formulation involving complex eigenvalue and characteristic currents that provides orthogonal far-fields based on $\langle \boldsymbol{J}_m, \mathcal{Z}(\boldsymbol{J}_n)\rangle$, but $\langle \boldsymbol{J}_m^*, \mathcal{Z}(\boldsymbol{J}_n)\rangle$ is needed when the characteristic current is complex. Fortunately, in practice, low losses typical in good conductors and PCB substrate materials are found to only marginally disturb the far-fields [42]. In these cases, CMA can be well utilized.

Secondly, it was reported in [11] that the computationally attractive surface formulation [41] has the problem of spurious modes among the CM solution. These modes do not exist in the volume formulation of [40] and they have been attributed to redundancy in the modal decomposition of PMCHWT [5] SIE formulations [43]. Specifically, both the equivalent electric and magnetic currents for the exterior problem have been shown earlier to be related to a single auxiliary equivalent current density, and Schur complement can be applied to eliminate the magnetic currents [44]. Although the redundancy does not matter for scattering problems, it doubles the solution space if the PMCHWT impedance matrix is used to calculate the characteristic electric and magnetic currents, resulting in spurious modes [43]. Inspired by [44], the dependency between the electric and magnetic currents can be eliminated by a new operator, so that only one type of currents should be solved directly from a generalized eigenvalue equation [43]. This idea has been extended to solve the CMs of dielectric-PEC composite problems [45], [46].

In another interpretation, these spurious modes have been attributed to internal resonances induced by forced symmetry of the PMCHWT [5] SIE [42]. Therefore, post-processing techniques have been suggested to remove these spurious modes, based on them having zero radiation power. However, such an approach fails when dealing with lossy structures [40], since even the correctly calculated modes can have very small radiated powers, which due to numerical accuracy be confused with internal resonances [42]. To alleviate this problem, the fundamental lower bound on modal radiated power was proposed as a threshold to remove spurious modes [42].

A final issue that has plagued CMA for material bodies is the interpretation of the CMs when both electric and magnetic currents are needed to represent the integral equation [47], *i.e.*, always the case for SIE and only for VIE when both dielectric and magnetic materials are present. In such a case, it was shown that the real eigenvalue do not represent the ratio of the reactive power to the radiated/scattered power [47]. For VIE, this issue can be traced to the original paper [40], where it was mentioned that "The imaginary part of $\langle f^*, Tf\rangle$ is not simply related to reactive power", where the vector f contains both equivalent electric and magnetic currents, and T is the VIE impedance operator $\mathcal{Z}$. To solve this issue (and avoid spurious modes), recent work [48]–[53] proposes a different weighting matrix, and the resulting eigenvalue is complex, with the real and imaginary parts dealing with reactive power and dissipated power (ohmic loss). The method has been applied to design handset with robust antenna performance [54]. However, the modal far-field orthogonality has not been confirmed.

Another way to handle the presence of material bodies is to utilize a method-of-moments solver with an infinite-ground-plane-based Green's function for multilayered medium, to avoid calculating equivalent surface currents over the substrate faces, *e.g.*, in [26]. Losses in the material bodies can be modeled, but as before, these losses will also disturb modal far-fields orthogonality.

Summarizing, recent literature utilizes different perspectives to shed more light and offer solutions to CMA for material bodies. Both VIE and SIE based CMA have been shown to be feasible for low-loss dielectrics, with the latter being more computationally attractive. However, further developments may be needed in handling mixed dielectric/magnetic materials and higher ohmic losses.

B. Application in Scattering Control

Applying CMA to scattering problems seems natural, since CMs were first proposed in the context of scattering due to plane wave excitation [3]. Therefore, it is not surprising that the potential use of CMA to control the radar scattering pattern of an object was first explored as far back as 1972 [19]–[21]. Referring to the matrix form of the fundamental CM equation in Paper 2, *i.e.*, $\mathbf{XI}_n = \lambda_n \mathbf{RI}_n$, where $\mathbf{I}_n$ is the current density of mode n, $\mathbf{R}$ and $\mathbf{X}$ are the real and imaginary parts of the impedance matrix $\mathbf{Z}$, the eigenvalue λ_n will be nonzero when the frequency of interest is not equal to mode n's resonant frequency. If purely reactive loads are added to the object of interest, the CM equation becomes [19]–[21]

$$\left(\mathbf{X} + \mathbf{X}_{\mathrm{L}}\right) \mathbf{I}_n = \lambda_n \mathbf{RI}_n, \tag{1}$$

where $\mathbf{X}_{\mathrm{L}}$ is a diagonal load matrix that accounts for the added reactive loads [19]–[21]. If a particular mode, say the m-th mode, generates the desired scattering characteristics, it can be made to resonate by tuning the purely reactive elements such that $\mathbf{I}_m^{\mathrm{H}} \left(\mathbf{X} + \mathbf{X}_{\mathrm{L}}\right) \mathbf{I}_m = 0$ and by making sure that it has a non-zero projection onto the incident field. Then, the total current and the scattering characteristics of the object will be dominated by mode m as desired.

Motivated in part by the growing interest in stealth technology, the control of scattering characteristics using CMA has been developed to deal with more complex objects such as aircrafts [55]. Also, multiple studies have used CMA to reduce the in-band radar cross-section (RCS) of antennas [56]–[58]. As illustrated in Fig. 5, the first step in these studies is to calculate the modal significance (MS) and the MWCs (defined in Paper 2) of the scatterer, the latter also depends on specific incident excitation. Although a high value of MS is necessary for a mode to contribute to scattering, like antenna applications discussed in Papers 3–5, the actual contribution to the scattering response also critically depends on MWCs. The modes are then classified into desirable or undesirable modes based on the application. The current distributions of the undesirable modes are then plotted to identify the "hot spots", *i.e.*, the locations where the currents of the undesirable modes are maximal. CMA identifies these locations with no trial and error, with negligible addition to the computation cost. The undesirable modes can then be suppressed using techniques shown in Fig. 5, while making sure that the desirable modes are unaffected. A similar CMA procedure was used in detecting and suppressing unintentional radiators in electronic systems [59], [60] and for the design and optimization of frequency-selective rasorbers [61].

It is important to emphasize that the CMA approach is significantly more advantageous than just looking at the hot spots of the total current of the scatterer. More than one mode can contribute to the total current at a particular frequency and excitation. Therefore, without CMA, it is challenging to identify the hot spots of the undesired modes from the hot spots of the desired modes. Moreover, some undesired modes are only excited at specific angles of incidence and polarization. In traditional scattering control analysis, exhaustive simulations are needed to make sure that no incident waveform excites an undesired mode. On the other hand, the CMA approach easily identifies all possible modes in the excitation-independent MS spectrum with no need to test multiple excitation scenarios.

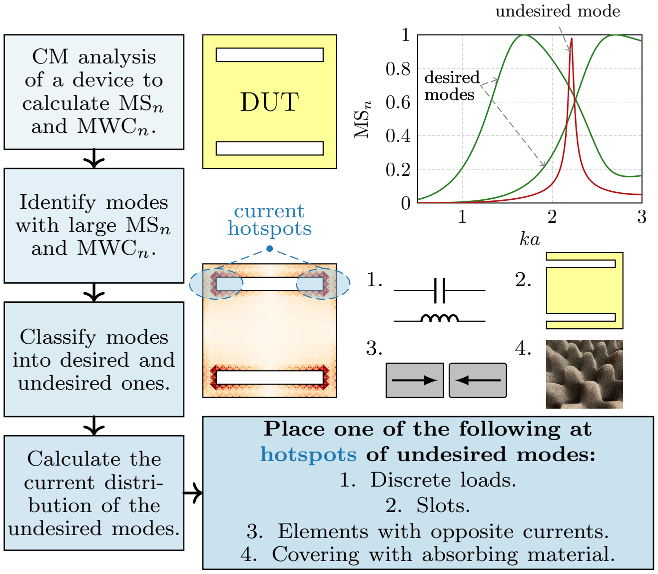

Figure 5. A flow chart showing how the CMA can be used to control the scattering from a Device Under Test (DUT). The inset shows an example of a DUT that is studied using this approach.

In addition to scattering control, CMA has also been employed for the analysis of scattering from microstrip antenna arrays where the modes are used as entire basis functions, which offers significantly higher efficiency than subdomain basis functions, especially in the analysis of large arrays [62]. In the following section, we discuss additional examples of how CMs can be employed for scattering control and analysis.

C. Nanoparticles Scattering Analysis and Optimization

At a much smaller scale than the previous application, in the order of nanometers, CMA has been recently used to analyze and optimize the absorption and scattering characteristics of a wide range of metallic nanoparticles (NPs) as shown in Fig. 6a [28]–[30], [63], [64]. Plasmonic NPs contain losses. As discussed in Section IV-A, the CMA of an impedance matrix with lossy materials included yields modes with far-fields that are not orthogonal, which disagrees with the original definition of CMs [3], [65]. Yet, the approach and tools of CMA, such as the MS spectrum and the current distribution of the modes, can be used to understand and optimize the response of plasmonic NPs. For example, the shape, size, and material of a split-ring nanoantenna were optimized using CMA to yield more than 700% near-field intensity enhancement at the desired frequency [64].

Moreover, the complexity of NP analysis can be mitigated by using CMA. In the frequency range of interest, the modes with high MS represent the potential coupling pathways to the NPs [30]. By identifying the current or distribution of the significant modes, the incident field distribution to excite each mode can be identified [30]. Modes that cannot be excited by any incident field distribution, termed non-radiating modes or dark modes, can also be identified [29], [30].

CMA has also been used to explain the electromagnetic response of complex-shaped NPs, such as wormlike carbon

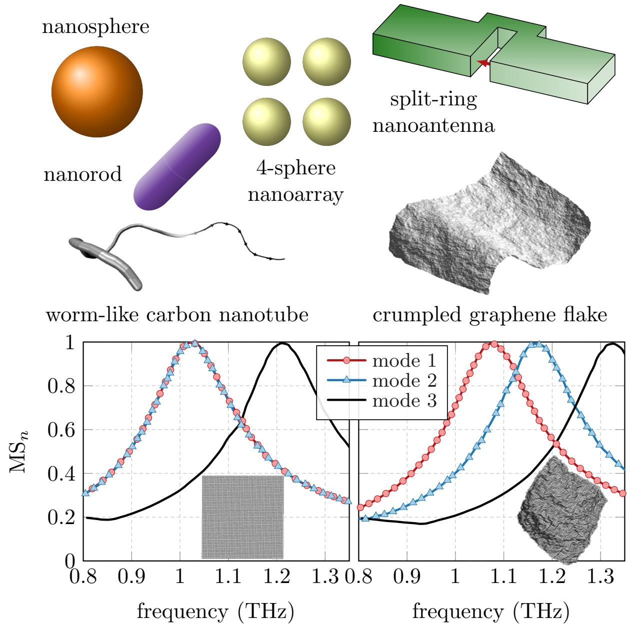

Figure 6. (top) Nanoparticles studied using CMA, (bottom) modal significance of flat vs. crumpled square graphene flakes.

nanotubes and crumpled graphene flakes, where the breakage of geometrical symmetry can lead to the emergence of new resonances, separation of modes (*e.g.*, Fig. 6b), and/or bandwidth broadening [28], [29]. In a related application, the feeding location of a complex-shapes graphene-based plasmonic nanoantennas was varied to excite multiple modes, minimizing the input impedance and increasing the radiation efficiency [63]. Similar CMA approaches has also been used to study spoof localized surface plasmon resonators that mimic the behavior of plasmonic NPs but in the microwave/terahertz range [66].

D. *Other Applications*

CMA has also been employed for electromagnetic compatibility (EMC) applications. In the context of interference, electronics are typically shielded by a metallic enclosure that separates the protected volume from the surrounding media. Practically, the enclosures are typically perforated with apertures where electromagnetic waves can penetrate. Early CM research has provided guidance on the penetration of electromagnetic waves via a slot in a cylinder [67], the penetration of electromagnetic waves through a conducting plane perforated with slots [68], and the penetration of electromagnetic waves through multiple slot-perforated conducting planes [69]. The true significant modes that dominate the penetration of electromagnetic waves are not always easy to identify. However, recently, new criteria were developed to facilitate the detection of the dominant modes for studying the shielding effectiveness of apertures [70].

V. CONCLUSIONS AND FUTURE PERSPECTIVES

This paper gives a general overview on the maturing field of CMA, outlining the reasons for a tutorial-style special issue, the progression of the covered topics, and common myths. Several emerging CM topics are also discussed. Despite strong indications that CMA has become a mainstream topic, there are still many opportunities for significant contributions to be made in this field. For example, further theoretical developments and unifications are needed in dealing with other (extended) orthogonal relations, periodic structures, large structures and even lossy dielectric/magnetic materials.

Another important stimulus for even more widespread application of CMA is the continuing development of commercial software tools, which at the moment is trailing behind the latest research findings. For example, to our knowledge the theory of eigenvalue crossing (detailed in PAPERS 2) has not been applied to modal tracking.

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