NULL SHELLS IN KINETIC GRAVITY BRAIDING SCALAR-TENSOR THEORIES

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Abstract. We derive the equation of motion of null shells generated by distributional sources with support on the shell in a class of second-order scalar-tensor theories that is both compatible with gravitational wave observations and produce valid cosmological evolution.

The shell equation of motions and thus the junction conditions required for smooth matching along a null hypersurface are obtained by considering a 2+1+1 decomposition given by the null generators of the hypersurface and an auxiliary null vector field.

The equations relate jumps of geometric quantities to the components of the distributional sources and also provide a constraint relation on the source components.

Key words: General Relativity - Gravitation - Junction Conditions - Thin Shells.

1. INTRODUCTION

Despite the success of general relativity (GR) as a classical theory of physics, its modifications are heavily searched for even in the classical regime.

The predictions of GR on galactic and larger scales are only compatible with observation if hitherto unknown forms of matter, dark matter and dark energy are introduced. Present observations imply that only $\sim 4.898\%$ of the total matter in the Universe is observable (baryonic), while the remaining $\sim 26.212\%$ is dark matter and $\sim 68.89\%$ is dark energy (Plank Collaboration, 2018).

It is hoped that the effects that point towards the existence of dark matter and dark energy can be explained by modifying GR on large scales.

Lovelock's theorem (Lovelock, 1971) implies that in four dimensions, the Einstein field equation (extended by the cosmological constant) is the only equation of motion that is i) variational with diffeomorphism-invariant action functional, ii) second order, iii) contains the metric tensor as the sole dynamical variable. Thus, the most sensible modification in four spacetime dimensions is to add additional degrees of freedom, since higher order theories have a tendency to suffer from pathologies (Woodard, 2015).

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The simplest possible way to add further gravitational degrees of freedom is by coupling scalar fields nonminimally to gravity. Such scalar-tensor theories are of interest also as scalar field decay is able to drive cosmological inflation; also higher dimensional theories based on compactified extra dimensions, such as various Kaluza-Klein and string theories contain a phletora of scalar fields in the low-energy limit.

The most general scalar-tensor theory with second order equations of motion in four dimensions was given by Horndeski (Horndeski, 1974; Deffayet *et al.*, 2011). The recent gravitational wave observations, restricting the propagation speed of the tensorial perturbations to the speed of light have severely constrained its form (Baker *et al.*, 2017).

When describing phenomena involving sharp changes through some boundary surface, such as shockwaves, stellar boundaries, phase transitions, it is a useful idealization to allow the change to be instantaneous. Since the dynamical fields are not necessarily differentiable, care must be taken at the boundary surface to ensure that the fields provide a valid (distributional) solution to the equations of motion. In addition to that, a singular transition across the boundary surface can be interpreted as a thin shell consisting of distributional sources supported on the boundary surface.

In GR, the conditions for regular matching, or alternatively, the equations of motion for the shell are given by Israel's junction conditions (Israel, 1966), valid for either timelike or spacelike shells. For a regular transition, the Israel junction conditions prescribe the continuity of the induced metric and extrinsic curvature on the hypersurface. For thin shells however, the continuity of the extrinsic curvature may be violated, in which case the distributional energy-momentum tensor is related to the jump of the extrinsic curvature via the Lanczos equation.

The breakdown of the Israel formalism for hypersurfaces with null points is due to the fact that the normal vector at that point becomes tangential and no orthogonal 3+1 split can be performed there.

The junction conditions valid for hypersurfaces with arbitrary but fixed causal character have been developed by Barrabès and Israel (1991), where an arbitrary transversal vector realizes a *non-orthogonal* 3+1 split. This has been further generalized to signature-changing hypersurfaces by Mars and Senovilla (1993), and specialized to purely null hypersurfaces in a much simpler manner by Poisson (2002).

The case of null shells and junction conditions are physically important, as impulsive electromagnetic and gravitational waves can be characterized this way, as well as event horizons, concentrated layers of null dust, cosmological phase transitions and bubbles that propage at the speed of light (Barrabès and Hogan, 2003).

Junction conditions and boundary terms for the full Horndeski theory have been found for timelike and spacelike shells (Padilla and Sivanesan, 2012), however the null case had been left undeveloped. We have previously derived the null junction conditions in the Jordan frame Brans-Dicke theory (Racskó and Gergely, 2018) and we have found that null shells must be pressureless.

In section 2 we review the standard thin shell formalism in GR, with a special emphasis for null shells. Then we generalize the discussion for null shells in a subclass of Horndeski theory compatible with both gravitational wave measurements and cosmological evolution in Section 3. The details of these results were presented in our recent paper (Racskó and Gergely, 2019). In Section 4 we proceed further to analyze the possibility of vacuum shells, generated purely by the scalar field. In the last section we summarize our findings.

2. THIN SHELLS IN GENERAL RELATIVITY

Here we review the thin shell formalism in GR.

2.1. THE LANCZOS EQUATION

Suppose that Σ is a hypersurface in four dimensional spacetime M, partitioning M into the domains M^+ and M^-. An arbitrary coordinate system in $M^\pm$ is denoted as $x^\mu_\pm$, while an arbitrary intrinsic (three dimensional) coordinate system in Σ is ξ^a.

The induced metric on the hypersurface is defined as

$$h_{ab} = g_{\mu\nu} E^\mu_a E^\nu_b, \tag{1}$$

where $E^\mu_a = \partial x^\mu / \partial \xi^a$ and for notational clarity we have omitted the $\pm$ signs. If Σ is timelike or spacelike, h_{ab} is either indefinite or positive definite (but nondegenerate).

The first junction condition is the "continuity" of the induced metric on Σ, thus its jump vanishes:

$$[h_{ab}] = h^+_{ab} - h^-_{ab} = 0. \tag{2}$$

In gaussian normal coordinates, which are smooth across Σ (Clarke and Dray, 1987), the *bulk* line element can be written as

$$\mathrm{d}s^2_\pm = \epsilon \mathrm{d}n^2 + h^\pm_{ab} \mathrm{d}\xi^a \mathrm{d}\xi^b, \tag{3}$$

where

$$\epsilon = \begin{cases} 1, & \text{for } \Sigma \text{ timelike} \\ -1, & \text{for } \Sigma \text{ spaceike,} \end{cases} \tag{4}$$

and the normal coordinate n is *differentiable* through Σ. We see that the invariant line elements agree on Σ if and only if the induced metrics agree on Σ, thus the condition $[h_{ab}] = 0$ is also sufficient to ensure the continuity of the full metric through the hypersurface. This however holds only in smooth coordinate systems. For charts

x_+^μ and x_-^μ which are either discontinuous or continuous but not differentiable across Σ, the metric tensor may show discontinuities through Σ.

The stress-energy tensor gives $T^{\mu\nu} = \overline{T}^{\mu\nu} + \mathscr{T}^{ab} E_a^\mu E_b^\nu \delta(n)$, where $\overline{T}^{\mu\nu}$ is a regular but possibly discontinuous function, δ is the Dirac delta distribution, n is a differentiable scalar field in M measuring geodesic distance from Σ (due to (3), this function is differentiable across Σ) and

$$\mathscr{T}^{ab} = -\frac{1}{8\pi}\left([K^{ab}] - [K]\, h^{ab}\right) \tag{5}$$

is the singular part of the energy-momentum tensor.

Equation (5) is known as the Lanczos equation and relates the distributional source to the jump of the extrinsic curvature, defined as

$$K_{ab} = E_a^\mu E_b^\nu \nabla_\mu N_\nu, \tag{6}$$

where N_μ is the unit normal field along Σ, assumed to point from M^- to M^+. The Lanczos equation is expressed in an intrinsic coordinate chart of Σ, which is the same when viewed from both sides, thus this equation may be used even if discontinuous bulk charts are used on the two sides.

We may rearrange the equation by taking the trace to obtain

$$[K^{ab}] = -8\pi\left(\mathscr{T}^{ab} - \frac{1}{2}\mathscr{T} h^{ab}\right), \tag{7}$$

showing that the Einstein field equation is regular if and only if $[K_{ab}] = 0$.

2.2. THE NULL CASE

Rather than using the generic Barrabès-Israel prescription, we follow Poisson (2002) implementing a much simpler formalism, valid only for the null case.

The normal field N_μ along Σ is a null vector satisfying $N_\mu N^\mu = 0$, thus is not uniquely determined. The normal field being also tangential its integral curves run on Σ, being *null generators* of the hypersurface. The normal field is also geodesic, obeying

$$N^\nu \nabla_\nu N^\mu = \kappa N^\mu \tag{8}$$

for some parameter κ. When the hypersurface defines a thin shell of lightlike matter, the null field N^μ is the tangent vector to the flow lines of the matter.

It is useful to choose the parameter n along the null generators as one of the coordinates of Σ and pick two additional spacelike coordinates σ^2, σ^3 which label the null generators. They generate the coordinate basis elements

$$E_1^\mu \equiv N^\mu = \frac{\partial x^\mu}{\partial n}, \quad E_2^\mu = \frac{\partial x^\mu}{\partial \sigma^2}, \quad E_3^\mu = \frac{\partial x^\mu}{\partial \sigma^3}. \tag{9}$$

We complete the basis by adding a null vector field L^μ along Σ satisfying

$$L^\mu L_\mu = 0, \quad L^\mu N_\mu = 1, \quad L_\mu E^\mu_A = 0, \tag{10}$$

with the capital latin indices having the values $A, B, ... = 2, 3$.

The induced metric on a null Σ is degenerate, and in the special coordinates defined above we can write the intrinsic line element of Σ in the form

$$\mathrm{d}s^2_\Sigma = q_{AB}\mathrm{d}\sigma^A \mathrm{d}\sigma^B, \tag{11}$$

where $q_{AB} = g_{\mu\nu}E^\mu_A E^\nu_B$ is a two-dimensional spacelike metric on the foliation of Σ by two-surfaces transversal to the null generators. We will take q_{AB} as the induced metric. Using the induced metric and the frame vectors, the resolution of the identity can be written along Σ as

$$g^{\mu\nu} = L^\mu N^\nu + N^\mu L^\nu + E^\mu_A E^\nu_B q^{AB}, \tag{12}$$

where q^{AB} is the inverse of the two-dimensional induced metric.

We also introduce the *transverse curvature* for null hypersurfaces as a generalization of the extrinsic curvature as

$$\mathcal{K}_{nn} = N^\mu N^\nu \nabla_\mu L_\nu, \quad \mathcal{K}_{nA} = N^\mu E^\nu_A \nabla_\mu L_\nu, \quad \mathcal{K}_{AB} = E^\mu_A E^\nu_B \nabla_\mu L_\nu, \tag{13}$$

which is symmetric, thus $\mathcal{K}_{nA} = \mathcal{K}_{An}$ and $\mathcal{K}_{AB} = \mathcal{K}_{BA}$.

With these, one may write the energy-momentum tensor as

$$T^{\mu\nu} = \overline{T}^{\mu\nu} + \mathscr{T}^{\mu\nu}\delta(\lambda), \tag{14}$$

where $\overline{T}^{\mu\nu}$ is regular, and

$$\mathscr{T}^{\mu\nu} = \rho N^\mu N^\nu + j^A \left(N^\mu E^\nu_A + E^\mu_A N^\nu\right) + p q^{AB} E^\mu_A E^\nu_B, \tag{15}$$

where

$$\rho = -\frac{1}{8\pi}[\mathcal{K}_{AB}]q^{AB} \tag{16}$$

$$j^A = \frac{1}{8\pi}[\mathcal{K}_{nB}]q^{AB} \tag{17}$$

$$p = -\frac{1}{8\pi}[\mathcal{K}_{nn}] \tag{18}$$

are the surface energy density, energy current and isotropic pressure respectively (Poisson, 2002). These expressions for ρ, j^A and p given in terms of the jump of the transverse curvature are the analogues of the Lanczos equation, valid for purely null shells. As they were expressed in terms of intrinsic coordinates for the hypersurface Σ, they can be calculated even if discontinuous coordinates are used in the bulk.

3. NULL SHELLS IN GENERALIZED KINETIC GRAVITY BRAIDING THEORIES WITH LINEAR DEPENDENCE ON THE KINETIC TERM

3.1. HORNDESKI'S THEORY

The most general scalar-tensor theory that produce second order field equations in four dimensions has been formulated by Horndeski (1974). Here we give the "DGSZ reformulation" (Deffayet *et al.*, 2011), which is equivalent to Horndeski's in four dimensions, but valid for arbitrary spacetime dimensions.

The Lagrangian has the form $L = L_2 + L_3 + L_4 + L_5$ with

$$L_2 = G_2(\phi, X) \tag{19}$$

$$L_3 = -G_3(\phi, X)\Box\phi \tag{20}$$

$$L_4 = G_4(\phi, X)R + 2G_{4X}\phi^{\mu}_{[\mu}\phi^{\nu}_{\nu]} \tag{21}$$

$$L_5 = G_5(\phi, X)G_{\mu\nu}\phi^{\mu\nu} - G_{5X}\phi^{\mu}_{[\mu}\phi^{\nu}_{\nu}\phi^{\kappa}_{\kappa]}, \tag{22}$$

where ϕ is the dynamical scalar field, $X = -\frac{1}{2}\phi^{\mu}\phi_{\mu}$ is the kinetic term, $\phi_{\mu} \equiv \nabla_{\mu}\phi$, $\phi_{\mu\nu} \equiv \nabla_{\mu}\nabla_{\nu}\phi$, the $G_2, ..., G_5$ are unspecified (smooth) functions of the scalar field and its kinetic term, and a subscript on these functions denotes derivative with respect to the given argument, eg. $G_{iX} = \partial G_i / \partial X$.

The equations of motion for this theory are second order in both the metric and the scalar field. Recent observations of gravitational waves and corresponding electromagnetic counterparts confirm to high precision that gravitational waves propagate at the speed of light. Horndeski's theory is only compatible with this requirement if $G_4 = G_4(\phi)$ and G_5 is a constant (Baker *et al.*, 2017). Then the entire quintic sector can be thrown out since

$$G_{\mu\nu}\phi^{\mu\nu} = \nabla_{\mu}(G^{\mu\nu}\phi_{\nu}) - \nabla_{\mu}G^{\mu\nu}\phi_{\nu} = \nabla_{\mu}(G^{\mu\nu}\phi_{\nu}) \tag{23}$$

is a boundary term. (We have used $\nabla_{\mu}G^{\mu\nu} = 0$.)

3.2. GENERALIZED KINETIC GRAVITY BRAIDING THEORY

The most general theory from the Horndeski class compatible with gravitational wave observations is

$$S_{GKGB} = \int \mathrm{d}^4x\sqrt{-\mathfrak{g}}\left(G_2(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi)R\right), \tag{24}$$

a generalized kinetic gravity braiding theory due to the term $G_3(\phi, X)\Box\phi$ in the action. This nonlinear derivative coupling makes it impossible to use conformal transformations and field redefinitions to bring the action into the Einstein frame (Kase and Tsujikawa, 2019). The scalar field can be suppressed at short distance scales via the Vainshtein mechanism, moreover it has been shown that it contains models compatible with cosmological evolution provided the dependence on the kinetic term is at most quadratic (Kase *et al.*, 2016).

As the quadratic dependence on X violates the conditions for propagation with the speed of light of the tensorial modes, we keep only the linear dependence in the action:

$$S = \int \mathrm{d}^4x\sqrt{-\mathfrak{g}}\left\{\underbrace{B(\phi)X + V(\phi)}_{L_2}\underbrace{-2\xi(\phi)\Box\phi X}_{L_3}\underbrace{+\frac{1}{2}F(\phi)R}_{L_4}\right\}. \tag{25}$$

The equations of motion then are given by the vanishing of the Euler-Lagrange expressions:

$$E^{(2)}_{\mu\nu} = -\frac{1}{2}B(\phi)\left(Xg_{\mu\nu} - \phi_\mu\phi_\nu\right) - \frac{1}{2}V(\phi)g_{\mu\nu}, \tag{26}$$

$$\begin{aligned} E^{(3)}_{\mu\nu} &= \xi(\phi)\Box\phi\phi_\mu\phi_\nu + 2\xi'(\phi)X\left(\phi_\mu\phi_\nu + Xg_{\mu\nu}\right)\\ &\quad + 2\xi(\phi)X_{(\mu}\phi_{\nu)} - \xi(\phi)X_\kappa\phi^\kappa g_{\mu\nu}, \end{aligned} \tag{27}$$

$$\begin{aligned} E^{(4)}_{\mu\nu} &= \frac{1}{2}\left\{F(\phi)G_{\mu\nu} + \left(F'(\phi)\Box\phi - 2F''(\phi)X\right)g_{\mu\nu}\right.\\ &\quad \left.-F'(\phi)\phi_{\mu\nu} - F''(\phi)\phi_\mu\phi_\nu\right\}, \end{aligned} \tag{28}$$

$$E^{(2)}_{\phi} = B(\phi)\Box\phi - B'(\phi)X + V'(\phi), \tag{29}$$

$$E^{(3)}_{\phi} = \xi(\phi)\left\{(\Box\phi)^2 - \phi_{\mu\nu}\phi^{\mu\nu} - R^{\mu\nu}\phi_\mu\phi_\nu\right\} - 2\xi''(\phi)X^2, \tag{30}$$

$$E^{(4)}_{\phi} = \frac{1}{2}F'(\phi)R, \tag{31}$$

where

$$E^{(i)}_{\mu\nu} = \frac{\delta}{\delta g^{\mu\nu}}\int \mathrm{d}^4x\sqrt{-\mathfrak{g}}L_i, \quad E^{(i)}_{\phi} = \frac{\delta}{\delta\phi}\int \mathrm{d}^4x\sqrt{-\mathfrak{g}}L_i. \tag{32}$$

3.3. EQUATIONS OF MOTION FOR NULL SHELLS

Suppose that Σ is a null hypersurface in spacetime. We present the null shell equations of motion for the kinetic gravity braiding theory with linear dependence on the kinetic term, given by (25), by using the notation of Section 2.

In a coordinate system which passes smoothly through Σ we have $[g_{\mu\nu}] = 0$ and $[\phi] = 0$. One reason we assume these conditions is that in this way the singularities in the energy-momentum tensor are proportional to $\delta(\lambda)$ at most, avoiding derivatives of the delta distribution.

We then obtain (Racskó and Gergely, 2019) the expressions

$$\rho = F(\phi)[\mathcal{K}_{AB}]q^{AB} + F'(\phi)[\phi_L] - 2\xi(\phi)\phi_N[\phi_L^2], \tag{33}$$

$$j^A = -\left(F(\phi)[\mathcal{K}_{nB}] - 2\xi(\phi)[\phi_L]\phi_N\phi_B\right)q^{AB}, \tag{34}$$

$$p = F(\phi)[\mathcal{K}_{nn}] - 2\xi(\phi)[\phi_L]\phi_N^2 \tag{35}$$

for the components of the distributional energy-momentum tensor, where $\phi_L = L^\mu\phi_\mu$, $\phi_N = N^\mu\phi_\mu$ and $\phi_A = E^\mu_A\phi_\mu$. We also obtain the equation

$$\begin{aligned} 0 &= \xi(\phi)\phi_N^2 q^{AB}[\mathcal{K}_{AB}] - 2\xi(\phi)\phi_N\phi^A[\mathcal{K}_{nA}] \\ &+ \left(F'(\phi) + 2\xi(\phi)\phi_N\langle\phi_L\rangle\right)[\mathcal{K}_{nn}] - 2\xi(\phi)[\phi_L]\left(\phi_{NN} - \langle\mathcal{K}_{nn}\rangle\phi_N\right), \end{aligned} \tag{36}$$

where $\langle\cdot\rangle$ denotes the average of a quantity taken over Σ, eg. $\langle Y\rangle = \frac{1}{2}(Y^+ + Y^-)$. This equation provides a constaint on the distributional sources and can be rewritten in a form avoiding averages as

$$\begin{aligned} 0 = {}& \xi(\phi)\phi_N^2 q^{AB}[\mathcal{K}_{AB}] - 2\xi(\phi)\phi_N\phi^A[\mathcal{K}_{nA}] + F'(\phi)[\mathcal{K}_{nn}] \\ & - 2\xi(\phi)[\phi_L]\phi_{NN} + 2\xi(\phi)\phi_N[\phi_L\mathcal{K}_{nn}]. \end{aligned} \tag{37}$$

4. VACUUM SHELLS

In GR setting ρ, j^A and p simultaneously to zero in (16-18) will set $[\mathcal{K}_{nn}]$, $[\mathcal{K}_{nA}]$ and $[\mathcal{K}_{AB}]q^{AB}$ to zero as well, thus the transverse curvature does not necessarily vanish, even if the dristributional stress-energy momentum vanishes. The traceless part

$$[\overline{\mathcal{K}}_{AB}] = [\mathcal{K}_{AB}] - \frac{1}{2}[\mathcal{K}]q_{AB} \tag{38}$$

can be nonzero even if the energy-momentum tensor of the shell vanishes (here $[\mathcal{K}] = [\mathcal{K}_{AB}]q^{AB}$). In this case we refer to Σ as a vacuum shell. This is related to the fact that even if the energy-momentum tensor of the shell vanishes, the shell can still represent an *impulsive gravitational wave*, which must have vanishing energy-momentum tensor, but when concentrated over a shell can still cause jumps in the geometry (Barrabès and Hogan, 2003).

As the necessary and sufficient condition for the metric to be C^1 at Σ (in coordinates that are smooth across Σ) is $[\mathcal{K}_{AB}] = [\mathcal{K}_{nA}] = [\mathcal{K}_{nn}] = 0$, imposing the requirement for the metric to be C^1 at Σ, the jump of the full transverse curvature vanishes, implying that the distributional energy-momentum tensor will also vanish.

In the generalized kinetic gravity braiding model we have considered, there are richer possibilities. Supposing that $F(\phi) \neq 0$, after setting ρ, j^A and p to zero, we obtain

$$[\mathcal{K}_{AB}]q^{AB} = 2\frac{\xi(\phi)}{F(\phi)}\phi_N[\phi_L^2] - (\ln F)'[\phi_L] \tag{39}$$

$$[\mathcal{K}_{nA}] = 2\frac{\xi}{F}[\phi_L]\phi_N\phi_A \tag{40}$$

$$[\mathcal{K}_{nn}] = 2\frac{\xi}{F}[\phi_L]\phi_N^2. \tag{41}$$

Inserting this into the scalar equation gives the constraint

$$0 = 2\frac{\xi^2}{F}\phi_N^2\left(\phi_N - 2[\phi_L]\phi_A\phi^A\right) + (\ln F)'\xi[\phi_L]\phi_N^2 - 2\xi[\phi_L]\phi_{NN} + 2\xi\phi_N[\phi_L\mathcal{K}_{nn}]. \tag{42}$$

The existence of such vacuum shells satisfying this equation has not been explored yet, however if the nonlinear derivative coupling $\xi(\phi)$ vanishes as it does for Brans-Dicke type theories, then the constraint equation is identically satisfied and the jumps of $\mathcal{K}_{nn}$ and $\mathcal{K}_{nA}$ also vanish. Thus, for theories with $\xi = 0$, it is possible to have the entire $[\mathcal{K}_{AB}]$, including its trace non-vanishing even if the shell has a zero energy-momentum tensor.

Unlike in GR, here it might also be possible that the metric is C^1 through Σ, but the shell has nonzero energy-momentum. Suppose that all components of the jump of the transverse curvature vanish. Then the equations for the sources are

$$\rho = F'(\phi)[\phi_L] - 2\xi(\phi)\phi_N[\phi_L^2] \tag{43}$$

$$j_A = 2\xi(\phi)[\phi_L]\phi_N\phi_A \tag{44}$$

$$p = -2\xi(\phi)[\phi_L]\phi_N^2, \tag{45}$$

and the scalar equation takes the form

$$0 = 2\xi(\phi)[\phi_L](\mathcal{K}_{nn}\phi_N - \phi_{NN}). \tag{46}$$

Once again, the $\xi = 0$ case is the easiest to analyze, where the constraint equation is identically satisfied, and the shell is allowed to have energy density $\rho = F'(\phi)[\phi_L]$ even if the metric is C^1.

5. CONCLUDING REMARKS

We have investigated the equations of motion and properties of null shells in a wide class of scalar-tensor theories compatible with observations. We have derived the analogues of the Lanczos equation relating distributional sources to jumps of geometric quantities along the shell and a constraint equation. We have also shown that under certain circumstances, the transverse curvature can be discontinuous with the shell having a zero intrinsic energy-momentum tensor and it is also possible to have a distributional energy-momentum tensor supported on the shell with C^1 metric and a non-vanishing jump of the transverse derivative of the scalar field.

Further generalisations of this work we intend to pursue are i) the calculation of the equations of motion of null shells in the full generalized kinetic gravity braiding theory (24) and ii) exploring the formalism for vacuum shells for the study of the impulsive gravitational waves.

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