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# Average Rate Analysis of Cooperative NOMA aided Underwater Optical Wireless Systems 

Kapila W. S. Palitharathna, Student Member, IEEE, Himal A. Suraweera, Senior Member, IEEE, Roshan I. Godaliyadda, Senior Member, IEEE, Vijitha R. Herath, Senior Member, IEEE, and John S. Thompson, Fellow, IEEE


#### Abstract

In this paper, we consider a cooperative non-orthogonal multiple access (NOMA) aided underwater optical wireless system in which the source transmits to two users where the near user serves as a relay node to the far user. Our proposed system consists of multiple narrow-angle light-emitting diode (LED)/photodiode (PD) elements at the source, near user, and far user. In order to achieve communication, our system selects a single LED/PD at each node. We propose several low complexity LED/PD selection schemes that aim to maximize the link throughput and in addition consider optimal and random LED/PD selection for benchmarking. In order to characterize the performance of each scheme, bounds and closed-form tight approximations on the average achievable sum rates are presented. The use of multi element nodes and NOMA increase the average sum rate significantly over conventional orthogonal access. Moreover, near-optimal throughput can be achieved using channel gain based and line-of-sight based LED/PD selection schemes in the medium-to-high transmit power regimes. The derived expressions are also useful to investigate the impact of key system and channel parameters such as the source transmit power, power allocation factor, node placement, and the number of elements at each node.


Index Terms-Underwater optical wireless communication, cooperative non-orthogonal multiple access (NOMA), performance analysis, lower bound, achievable sum rate, LED/PD selection.

## I. Introduction

Recent advances in oceanographic research such as observation of marine life, earthquake prediction, water pollution monitoring, and oil/gas exploration have shown the need to develop robust and high data rate underwater communication systems. Optical, and acoustic waves are more commonly used than radio frequency (RF) in underwater applications due to the very high attenuation of RF waves [1]. Optical waves boast higher bandwidths and low latency levels and hence provide a promising solution for short-range high data rate underwater wireless applications [2], [3].

Underwater optical wireless communication (UOWC) systems are subject to performance degradation due to various channel effects. The communication of UOWC systems is limited by absorption, scattering, and turbulence effects [2]. In the literature, several techniques have been studied to combat the channel effects encountered in UOWC systems including the use of pointed transmitters [4], transmit laser selection [5], and relay-based multi-hop systems [6], [7]. A long-distance UOWC system using a single-photon avalanche photodiode (SPAD) has been proposed in [4]. The half-power angle of a light-emitting diode (LED) is narrowed to enhance the optical intensity at the transmitter, and a SPAD is used at the receiver to improve the detection sensitivity. The limitations of such methods are misalignment effects and additional hardware complexity. In addition, transmit laser selection combined with optical spatial modulation is proposed for weak turbulence based UOWC systems to enhance the performance in [5].
K. W. S. Palitharathna is with the Centre for Telecommunication Research, School of Engineering, Sri Lanka Technological Campus, Padukka, 10500, Sri Lanka. (Email: kapilap@sltc.ac.lk)
H. A. Suraweera, R. I. Godaliyadda, and V. R. Herath are with the Department of Electrical and Electronic Engineering, University of Peradeniya, Peradeniya, 20400, Sri Lanka. (Email: \{himal, roshangodd, vijitha\}@ee.pdn.ac.lk)
J. S. Thompson is with the School of Engineering, Institute for Digital Communications, The University of Edinburgh, Edinburgh, EH9 3JL, UK. (Email: j.s.thompson@ed.ac.uk)

A preferred approach to increase the range and reliability of UOWC systems is to employ relays. The end-to-end error performance has been investigated in [6] for multi-hop configurations. In [8], the maximum achievable distance for a multi-hop system and optimum relay placement for decode-and-forward (DF) and amplify-and-forward relaying have been studied. In [9], it was shown that attenuation and blocking effects due to suspended particles degrade the performance of relay-based UOWC systems. In [10], a parallel amplify-and-forward relay system has been designed to mitigate the performance loss due to suspended particles.

More recently, non-orthogonal multiple access (NOMA) has been studied as a promising method to increase the spectral efficiency of UOWC systems [11]. NOMA allows multiple users to use the same frequency and time resources to improve the performance over the traditional orthogonal multiple access concept [12], [13]. In particular, the power domain NOMA uses different power levels to multiplex signals of multiple users. Subsequently, users with better channel gains decode and subtract out messages of the other users before decoding their own messages [13]. Some papers have studied the performance of over air optical wireless communication (OWC) systems. In [14], NOMA is considered in the context of OWC under different channel uncertainty models. Some papers have studied the performance of different NOMA-based OWC systems. In [15], analytical expressions for coverage probability and ergodic sum rate are presented for two cases; quality-of-service guaranteed and opportunistic best-effort service provisioning. In [16], NOMA has been used to enhance the achievable throughput in OWC. Some papers have also applied NOMA for UOWC systems. In [11], the performance of a NOMA aided UOWC system has been studied. In [17], the coverage probability and cell capacity of a NOMA aided UOWC system has been presented and it is shown that NOMA can increase the number of users within a cell. In [17], it has been shown that NOMA can be used to increase the number of users within a cell. A NOMA aided UOWC system with a
photon counting receiver has been presented in [18]. In [19], power domain NOMA is introduced for UOWC to improve the spectrum efficiency and sum rate. Further, two LEDs with different colors have been used to improve the efficiency of communication. In [20], an underwater asymmetric clipped optical orthogonal frequency division multiplexing NOMA system capable of energy savings and massive device access has been proposed and optimized. Some studies have experimentally demonstrated the feasibility of underwater NOMA systems. In [21], a NOMA aided high-speed system using green and blue polarization multiplexing has been proposed. The combination of NOMA and relay-based communication is referred to as cooperative NOMA. Compared to the traditional NOMA, cooperative NOMA systems deliver additional gains. In cooperative NOMA, strong users are used as relays to improve the performance of weak users by forwarding decoded messages [22]. In [23], a full-duplex cooperative relay aided underwater NOMA UOWC system has been proposed. In [24], the impact of receiver orientation on a full-duplex relay assisted NOMA aided UOWC system has been studied. Despite recent research on the application of NOMA for UOWC systems many knowledge gaps remain to be addressed. Some of these include quantifying the NOMA gains under diverse underwater conditions, multi-element designs for improved performance, beamforming, scheduling and resource allocation in multi-user UOWC systems and lightwave power transfer performance, and analysis of hybrid NOMA underwater/over water communication systems based on both optical wireless and RF links. Often, these challenges and corresponding solutions are distinct from the widely investigated NOMA aided RF wireless systems presented in the existing literature. Moreover, how emerging machine learning techniques can be applied to better design NOMA aided UOWC systems is an open research question.
In OWC, multi-element deployment is a well-known method to increase the performance [25]-[28]. Several papers have presented multiple LED and/or photodiode (PD) OWC systems. In [25], a multiple LED/PD OWC system with LED grouping to achieve maximum throughput, fairness among users, and quality of service has been presented. In [26], a multi-transceiver spherical free space optics structure has been presented as a basic building block for enabling optical-based ad hoc networking. However, using more LED/PD elements results in a higher cost and complexity in implementation. Fortunately, to reduce the practical burden of operating multiple elements at the same time, LED/PD selection methods can be used [27]. For example in [27], a reduced complexity LED selection scheme has been proposed to study the secrecy probability for an OWC system with spatially distributed eavesdroppers. In [28], a specific LED layout that maximizes the signal-to-noise ratio (SNR) has been used to study the performance of an OWC system. In [29], a sub-optimal LED selection algorithm for distributed multiple-input multipleoutput OWC system is presented. In [30], a transmit laser selection for a diver-to-diver UOWC link where the transmitter has multiple laser sources and the receiver has a one PD has been proposed. However in the existing body of literature, there are no papers that have investigated the LED/PD selec-
tion schemes for cooperative NOMA aided UOWC systems.
In UOWC systems laser diodes or narrow-angle LEDs can be used as transmitters to achieve longer transmission distances [4], [5]. However, high turbid conditions and swaying of underwater devices due to water flow can cause misalignment effects in laser based communications. In addition, when the node locations change with time, for example due to waveinduced movement of the water, lasers need to be steered preciously to avoid alignment errors. As an alternative, some papers such as [4], [31], [32] have considered narrow-angle LED transmitters for underwater communication.

In this paper, a cooperative NOMA aided UOWC setup is considered, where an overhead access point (source) located at the surface level transmits to two underwater sensors (users) in weak turbulence conditions. In this system, the near user is anchored above the seabed and assists the far user communication. In order to enhance the communication distance and coverage of the system, multiple narrow-angle LED/PD elements are deployed at each node. In particular, we consider a single LED/PD selection at each node such that NOMA operation can be performed. In general, it is important to stress that element selection schemes for UOWC systems need to consider unique aspects (e.g., directional lightwave travel, blockage due to aquatic life, suspended particles, bubbles, and light color) and requires different design and analysis as compared with widely studied antenna selection schemes in traditional RF wireless systems. The proposed schemes based on channel/lightwave directions are very general since they are applicable to all types of transmitters and receivers. A transceiver of a specific kind used in the design (e.g., narrow angle/wide angle field of view) will only change the numerical values.
Our contributions are summarized as follows:

- We present several new low-complexity LED/PD selection schemes namely, optimal, channel state information (CSI) based, and orientation of LEDs/PDs based methods. We analyze bounds on the average achievable sum rate of the proposed schemes. In particular, we present mathematical expressions and accurate approximations that are useful to obtain design insights.
- We present simulation results applicable to the proposed schemes for different key system parameters which enable a detailed study of the performance trade-offs for different schemes. Our results reveal that near-optimal results can be obtained using both the CSI and orientation of LEDs/PDs based schemes. The proposed schemes are of high practical value and are suitable in order to deliver performance gains in UOWC systems.

The rest of the paper is organized as follows. In Section II system model and weak turbulence propagation model are presented. Section III presents the lower bound of the system and LED/PD selection schemes that can achieve high performance. Analysis of the average achievable sum rate of each LED/PD selection scheme is presented in Section IV. Numerical results for various system and channel parameters of all proposed schemes are presented in Section V. Finally, conclusions are drawn in Section VI.


Fig. 1. Cooperative NOMA aided UOWC system in which the source communicates to two underwater sensors.

## II. System and Channel Models

## A. System Model

As shown in Fig. 1 we consider a cooperative NOMA aided UOWC system, which consists of a source $(S)$, a near user $(U 1)$, and a far user $(U 2)$. We assume that $S, U 1$, and $U 2$ are located at the three dimensional coordinates $\left(x_{s}, y_{s}, z_{s}\right)$, $\left(x_{1}, y_{1}, z_{1}\right)$, and $\left(x_{2}, y_{2}, z_{2}\right)$ respectively. $S$ transmits two message streams; a broadcast message $x_{1}[n]$ intended for both $U 1$ and $U 2$, and a message $x_{2}[n]$ intended only for $U 2$. In addition to direct transmission from $S$ to $U 2, U 1$ forwards $x_{2}[n]$ from $S$ to $U 2$, thus serving as a DF relay ${ }^{1}$. We consider a situation where $S$ floats on the water surface at a height $z_{s} \mathrm{~m}$ from the seabed and is equipped with $N_{I}$ optical transmitters placed on a hemisphere with equal spacing. $U 1$ is equipped with $N_{J}$ PDs to receive from $S$ and $N_{K}$ LEDs for forwarding signals to $U 2 . U 2$ has $N_{L}$ PDs for data reception. At each node, the LEDs and PDs are placed according to a spherical design. Specifically, LEDs are equally spaced on the lower hemisphere, while PDs are equally spaced on the upper hemisphere. Other LED/PD placement options such as planar, hexagonal, and cylindrical arrangement etc. could also deliver viable choices for certain underwater systems depending on the device constraints.

According to the cooperative NOMA principle, $S$ simultaneously transmits two messages $x_{1}[n]$ and $x_{2}[n]$ at the same time and frequency by allocating different power levels to the respective messages [33]. The superimposed signal of $x_{1}[n]$ and $x_{2}[n]$ is transmitted from a selected LED $i^{*}$ from $S$ in the first time slot according to a specific LED/PD selection scheme. The signal transmitted from $S$ is received at $U 1$ by a selected PD $j^{*}$. Next, $U 1$ decodes and re-transmits the signal $x_{2}[n]$ in the second time slot towards $U 2$ using a selected transmitter $k^{*}$ according to one of the LED/PD selection schemes that will be discussed in Section III. Then the selected $\mathrm{PD} l^{*}$ at $U 2$ receives the signal and $x_{2}[n]$ is decoded. In addition, $x_{1}[n]$ is decoded at the selected $\mathrm{PD}, l^{*}$ at $U 2$ in the first time slot using the direct signal coming from $S$.

[^0]The source $S$ transmits a superposition of messages $x_{1}[n]$ and $x_{2}[n]$. Hence, the transmitted optical signal is

$$
\begin{equation*}
x_{S}[n]=a_{1} P_{S} x_{1}[n]+a_{2} P_{S} x_{2}[n], \tag{1}
\end{equation*}
$$

where $P_{S}$ is the optical transmit power, and $a_{1}$ and $a_{2}$ are the power allocation coefficients for $x_{1}[n]$ and $x_{2}[n]$, respectively, such that $a_{1}+a_{2}=1$. The received current signal at $U 1$ in the first time slot can be written as

$$
\begin{equation*}
y_{R}[n]=R h_{i^{*} j^{*}}^{S 1}\left(a_{1} P_{S} x_{1}[n]+a_{2} P_{S} x_{2}[n]\right)+\eta_{1}[n] \tag{2}
\end{equation*}
$$

where $R$ is the responsivity of the PD, $h_{i^{*} j^{*}}^{S 1}$ is the composite channel gain between the selected transmitter $i^{*}$ of $S$ and selected PD $j^{*}$ of $U 1$, and $\eta_{1}[n]$ is the zero mean additive white Gaussian noise (AWGN) with variance $\sigma_{1}^{2}$ at the PD at $U 1$.

The near user $U 1$ performs successive interference cancellation (SIC) according to the NOMA concept and first decodes $x_{1}[n]$ treating $x_{2}[n]$ as interference. Next, $U 1$ removes the decoded message $x_{1}[n]$ from the received signal to detect $x_{2}[n]$. After a short processing delay, the near user $U 1$ forwards the decoded message $x_{2}[n]$ towards $U 2$. Hence, the received current signal at $U 2$ in the second phase of communication in $n$-th frame can be written as

$$
\begin{equation*}
y_{2}[n]=R h_{k^{*} l^{*}}^{12} P_{1} x_{2}[n]+\eta_{2}[n], \tag{3}
\end{equation*}
$$

where $P_{1}$ is the optical transmit power at $U 1, h_{k^{*} l^{*}}^{12}$ is the composite channel gain between the $U 1$ and $U 2$, and $\eta_{2}[n]$ is zero mean AWGN with variance $\sigma_{2}^{2}$ at the $U 2$ receiver. In addition, $x_{1}[n]$ is decoded at $U 2$ in the first time slot ${ }^{2}$.

## B. Channel Model

In this subsection, we present the underwater light propagation model which will be used to analyze the performance of the system in Section IV. The channel gain from transmitter to receiver $h$ can be described by the product of three terms as

$$
\begin{equation*}
h=h_{g} h_{l} h_{t}, \tag{4}
\end{equation*}
$$

where $h_{g}$ is the geometric loss, $h_{l}$ is the path loss coefficient, and $h_{t}$ is the weak turbulence induced fading [17], [34], [35].

1) Geometric Loss

The geometric loss of the line-of-sight (LoS) channel between the transmitter and receiver is the loss due to the geometric placement of transmitter and receiver. It can be determined through [17], [31]

$$
h_{g}= \begin{cases}\frac{(m+1) A_{p}}{2 \pi d^{2}} \cos ^{m}(\theta) \cos (\psi) T c(\psi), & -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}  \tag{5}\\ 0, & \text { otherwise }\end{cases}
$$

where $m=-\ln (2) / \ln \left(\cos \left(\theta_{1 / 2}\right)\right)$ is the Lambertian order of the transmitter, $\theta_{1 / 2}$ is the half power angle of the transmitter, $A_{p}$ is the receiver aperture area, $d$ is the Euclidean distance between the transmitter and the receiver, $\theta$ is the irradiance

[^1]

Fig. 2. Geometry of LED/PD pairs; (a) $\left|\theta_{1}\right| \leq \pi / 2$; (b) $\left|\theta_{2}\right|>\pi / 2$.
angle at the transmitter, $\psi$ is the incident angle at the receiver, $T$ is the gain of the trans-impedance amplifier (TIA) at the receiver, and $c(\psi)$ is the gain of the optical concentrator which is defined as

$$
c(\psi)= \begin{cases}\frac{n^{2}}{\sin ^{2}(\Psi)}, & 0 \leq \psi \leq \Psi  \tag{6}\\ 0, & \psi>\Psi\end{cases}
$$

where $n$ is the internal refraction index and $\Psi$ is the concentrator field-of-view (FOV). In our system model, $\theta$ is tied to the orientation of the LED. The possible range of values of $\theta$ should be within $-\pi / 2$ and $\pi / 2$ as shown in the Fig. 2(a) as otherwise communication between the LED/PD pair becomes impossible - see Fig. 2(b).

## 2) Path Loss

Optical propagation through water induces interactions between each photon and the seawater particles [9], [31]. As a result of this phenomenon, absorption and scattering effects reduce the mean irradiance of the light beam. Beer's law provides the simplest and most widely used model for describing the absorption and scattering effects as follows:

$$
\begin{equation*}
h_{l}=\exp (-c(\lambda) d), \tag{7}
\end{equation*}
$$

where $c(\lambda)$ is the attenuation coefficient of the underwater environment which can be expressed as

$$
\begin{equation*}
c(\lambda)=a(\lambda)+b(\lambda), \tag{8}
\end{equation*}
$$

where $a(\lambda)$ and $b(\lambda)$ represent the absorption and scattering coefficients, respectively [9], [31].
3) Oceanic Turbulence-Induced Fading

In weak turbulence conditions, the log-normal distribution is widely used in the literature to model the channel fading coefficient. In particular, the probability density function (pdf) of the log-normal distributed channel coefficient can be written as [34], [35]

$$
\begin{equation*}
f_{h_{t}}\left(h_{t}\right)=\frac{1}{2 h_{t} \sqrt{2 \pi \sigma_{x}^{2}}} \exp \left(-\frac{\left(\ln \left(h_{t}\right)-2 \mu_{x}\right)^{2}}{8 \sigma_{x}^{2}}\right) \tag{9}
\end{equation*}
$$

where $h_{t}$ denotes the weak turbulence-induced fading coefficient, $\mu_{x}$ and $\sigma_{x}^{2}$ are the mean and variance of the Gaussian distributed log-amplitude factor, $X=\frac{1}{2} \ln \left(h_{t}\right)$. In order to preserve the energy of the fading coefficient, we normalize the fading amplitude such that $\mathbb{E}\left\{h_{t}\right\}=1$ where $\mathbb{E}\{$.$\} is the$
expectation operator giving $\mu_{x}=-\sigma_{x}^{2}$ [35]. The variance of the log-amplitude factor $\sigma_{x}^{2}$ is related to the scintillation index of the propagating signal $\sigma_{I}^{2}$ as $\sigma_{x}^{2}=\frac{1}{4} \ln \left(\sigma_{I}^{2}+1\right)$. In a weak turbulence regime, $\sigma_{I}^{2}<1$.

Finally, collecting the effects of the geometric loss, path loss and weak turbulence condition, the composite channel gain can be written as

$$
\begin{equation*}
h=G h_{t}, \tag{10}
\end{equation*}
$$

where $G=h_{g} h_{l}$ is the deterministic part of the channel gain. The distribution of the transformed $h^{2}$ can be obtained with the help of (11) as

$$
\begin{equation*}
f_{h^{2}}\left(h^{2}\right)=\frac{1}{4 h^{2} \sqrt{2 \pi \sigma_{x}^{2}}} \exp \left(-\frac{\left(\ln \left(\frac{h^{2}}{G^{2}}\right)-4 \mu_{x}\right)^{2}}{32 \sigma_{x}^{2}}\right) \tag{11}
\end{equation*}
$$

## III. Sum Rate and LED/PD Selection Schemes

In this section we present several LED/PD selection schemes to optimize the performance of the proposed cooperative NOMA aided UOWC system. So far in the literature, the exact capacity of UOWC systems have not been reported. In order to circumvent this difficulty, bounds reported in the literature are used [36] to establish the system's achievable sum rate.

## A. Lower Bound on the Average Achievable Sum Rate

First, we present separate expressions for the lower bound on the instantaneous achievable rates for message streams $x_{1}[n]$ and $x_{2}[n]$ separately. Next, they are averaged and the sum is obtained. The instantaneous achievable rates for decoding the message $x_{1}[n]$ at $U 1$ and $U 2$ can be lower bounded as [36]

$$
C_{1}^{S 1} \geq \frac{1}{4} \log _{2}\left(\frac{2 \pi \sigma_{1}^{2}+\left(R P_{S} h_{i * j *}^{S 1}\right)^{2} \sum_{i=1}^{2} e_{i}\left(\epsilon_{i}\right) a_{i}^{2}}{2 \pi \sigma_{1}^{2}+2 \pi\left(a_{2} R P_{S} h_{i * j *}^{S 1}\right)^{2} \epsilon_{2}}\right)
$$

and

$$
C_{1}^{S 2} \geq \frac{1}{4} \log _{2}\left(\frac{2 \pi \sigma_{2}^{2}+\left(R P_{S} h_{i * l *}^{S 2}\right)^{2} \sum_{i=1}^{2} e_{i}\left(\epsilon_{i}\right) a_{i}^{2}}{2 \pi \sigma_{2}^{2}+2 \pi\left(a_{2} R P_{S} h_{i * l *}^{S 2}\right)^{2} \epsilon_{2}}\right)
$$

where $e_{i}\left(\epsilon_{i}\right)=e^{1+2\left(\mu_{i}+\nu_{i} \epsilon_{i}\right)}, \mathbb{E}\left\{x_{i}^{2}\right\}=\epsilon_{i}, \mu_{i}$ and $\nu_{i}$ are constants that depends on the input distribution and can be obtained solving equation in [36, Eq. (12)], $h_{i * l *}^{S 2}$ is the composite channel gain between the selected transmitter $i^{*}$ of $S$ and selected PD $l *$ of $U 2$ respectively. Hence, the lower bound on the instantaneous achievable rate for decoding message stream $x_{1}[n]$ can be expressed as [33]

$$
\begin{equation*}
C_{1} \geq \min \left\{C_{1}^{S 1}, C_{1}^{S 2}\right\} \tag{12}
\end{equation*}
$$

Decoding message stream $x_{2}[n]$ can be lower bounded as [33]

$$
\begin{align*}
& C_{2} \geq \frac{1}{4} \min \left\{\log _{2}\left(1+\frac{e_{2}\left(\epsilon_{2}\right)\left(a_{2} R P_{S} h_{i^{*} j^{*}}^{S 1}\right)^{2}}{2 \pi \sigma_{1}^{2}}\right),\right. \\
& \log _{2}\left(1+\frac{e_{2}\left(\epsilon_{2}\right)\left(R P_{R} h_{k^{*} l^{*}}^{12}\right)^{2}}{2 \pi \sigma_{2}^{2}}\right) . \tag{13}
\end{align*}
$$

The lower bound on the instantaneous achievable sum rate of the proposed system, can be expressed using (12) and (13). Further, by using the expectation operation, we have

$$
\begin{equation*}
\mathbb{E}\left\{C_{L}^{S U M}\right\}=\mathbb{E}\left\{C_{L, 1}\right\}+\mathbb{E}\left\{C_{L, 2}\right\} \leq \mathbb{E}\left\{C_{1}\right\}+\mathbb{E}\left\{C_{2}\right\}, \tag{14}
\end{equation*}
$$

where $\mathbb{E}\left\{C_{L, 1}\right\}$ and $\mathbb{E}\left\{C_{L, 2}\right\}$ are the lower bounds on the average achievable rates for message streams $x_{1}[n]$ and $x_{2}[n]$ separately, $\mathbb{E}\left\{C_{L}^{S U M}\right\}$ is the average achievable sum rate. Eqs. (12), (13), and (14) are used to find approximate expressions for each LED/PD selection schemes.

## B. LED/PD Selection Schemes

In this subsection we propose (1) optimal, (2) max $S-U 1-$ $U 2$ channel gain based, (3) best $S-U 1-U 2$ LoS based, and (4) random LED/PD selection schemes as follows.

## 1) Optimal LED/PD Selection

The optimal LED/PD selection scheme should be decided to maximize the sum rate in (18). In order to maximize $\mathbb{E}\left\{C_{L}^{S U M}\right\}$ we select the $i^{*}$-th LED at $S$ from $N_{I}$ LEDs, the $j^{*}$-th PD at $U 1$ from $N_{J}$ PDs, the $k^{*}$-th LED at $U 1$ from $N_{K}$ LEDs, and the $l^{*}$-th PD at $U 2$ from $N_{L}$ PDs such that

$$
\begin{equation*}
\left\{i^{*}, j^{*}, k^{*}, l^{*}\right\}=\arg \max _{i, j, k, l}\left\{\mathbb{E}\left\{C_{L}^{S U M}\right\}\right\} \tag{15}
\end{equation*}
$$

The optimal LED/PD selection scheme has high implementation complexity as explained in Section IV-F.
2) Max $S-U 1-U 2$ Channel Gain Based Selection

The max $S-U 1-U 2$ channel gain based selection scheme relies on channel state information (CSI) of $S-U 1$ and $U 1-U 2$ links. First, the $i^{*}$-th LED at $S$ and the $j^{*}$-th PD at $U 1$ are selected such that $\left(h_{i j}^{S 1}\right)^{2}$ is maximized which can be expressed as

$$
\begin{equation*}
\left\{i^{*}, j^{*}\right\}=\arg \max _{i, j}\left\{\left(h_{i j}^{S 1}\right)^{2}\right\} \tag{16}
\end{equation*}
$$

where $h_{i j}^{S 1}$ is the composite channel gain between the transmitter $i$ of $S$ and PD $j$ of $U 1$. Similarly, the $k^{*}$-th LED at $U 1$ and the $l^{*}$-th PD at $U 2$ is selected such that $\left(h_{k l}^{12}\right)^{2}$ is maximized which can be expressed as

$$
\begin{equation*}
\left\{k^{*}, l^{*}\right\}=\arg \max _{k, l}\left\{\left(h_{k l}^{12}\right)^{2}\right\} \tag{17}
\end{equation*}
$$

where $h_{k l}^{12}$ is the composite channel gain between the transmitter $k$ of $U 1$ and PD $l$ of $U 2$.
3) Best $S-U 1-U 2$ Line-of-Sight Based Selection

In certain underwater implementations, obtaining CSI at all nodes may not be practical. Hence, we present a lowcomplexity method based on the best LoS links. According to this selection, the LEDs and PDs at $S, U 1$, and $U 2$ are selected exploiting position information. First, the $i^{*}$-th LED at $S$ is selected such that the $i^{*}$-th LED is the closest LED to the line connecting $S$ and $U 1$ among all LEDs. Hence, $i^{*}$ is given by

$$
\begin{equation*}
\left\{i^{*}\right\}=\arg \min _{i}\left|\theta_{S, i}-\alpha\right| \tag{18}
\end{equation*}
$$

where $\theta_{S, i}$ is the angle between vertical axis and $i$-th LED at $S$, and $\alpha$ is the angle between vertical axis and the line
connecting $S$ and $U 1$. Considering the placement of $S$ and $U 1$, (18) can be explicitly expressed as

$$
\begin{array}{r}
\left\{i^{*}\right\}=\arg \min _{i} \left\lvert\, \frac{\pi}{2}-\tan ^{-1}\left(\frac{\sqrt{\left(x_{S}-x_{1}\right)^{2}+\left(y_{S}-y_{1}\right)^{2}}}{z_{S}-z_{1}}\right)\right. \\
\left.-\frac{\pi i}{N_{I}+1} \right\rvert\, \tag{19}
\end{array}
$$

Next, the $j^{*}$-th PD at $U 1$ is selected such that the $j^{*}$-th PD is the closest PD to the line connecting $S$ and $U 1$ among all PDs.

In this scheme, LED/PD selection for the $S-U 1$, and $U 1-U 2$ links will be performed at $S$ and $U 1$ respectively. In practice, it may not always be feasible to select PDs that are in exact alignment with the respective LED due to imperfect position information. An incorrect selection of PDs will result in LoS misalignment errors. To model such an error at $U 1$, we consider that a single PD $j^{1}$ is selected randomly from a set $J^{1}$ which lies inside a cone around the line connecting $S$ and $U 1$ given by

$$
\begin{equation*}
-\Delta \Phi_{1} \leq \alpha-\varphi_{1, j^{1}} \leq \Delta \Phi_{1} \tag{20}
\end{equation*}
$$

where $\Delta \Phi_{1}$ is the half angle of the cone, $\varphi_{1, j^{1}}$ is the angle between vertical axis and the $j^{1}$-th PD at $U 1$. Next, $U 1$ selects the $k^{*}$-th LED which is closest to the line connecting $U 1$ and $U 2$. Hence, selection of $k^{*}$ can be expressed as

$$
\begin{equation*}
\left\{k^{*}\right\}=\arg \min _{k}\left|\beta-\theta_{1, k}\right| \tag{21}
\end{equation*}
$$

where $\beta$ is the angle between vertical axis and the line connecting $U 1$ and $U 2, \theta_{1, k}$ is the angle between vertical axis and the $k$-th LED. Considering the placement of $U 1$ and $U 2$, (21) can be modified as

$$
\begin{array}{r}
\left\{k^{*}\right\}=\arg \min _{k} \left\lvert\, \frac{\pi}{2}+\tan ^{-1}\left(\frac{\sqrt{\left(x_{1}-x_{2}\right)^{2}+\left(y_{1}-y_{2}\right)^{2}}}{z_{1}-z_{2}}\right)\right. \\
\left.-\frac{\pi k}{N_{K}+1} \right\rvert\, \tag{22}
\end{array}
$$

Similar to the selection of the PD at $U 1$, the selection of the $l^{*}$-th PD at $U 2$ will also be impaired by PD selection errors. The PD selection error at $U 2$ is modeled by randomly selecting a single PD $l^{1}$ from a set $L^{1}$ which lies inside a cone around line connecting $U 1$ and $U 2$ given by

$$
\begin{equation*}
-\Delta \Phi_{2} \leq \beta-\varphi_{2, l^{1}} \leq \Delta \Phi_{2} \tag{23}
\end{equation*}
$$

where $\Delta \Phi_{2}$ is the half angle of the cone, $\varphi_{2, l^{1}}$ is the angle between vertical axis and the $l^{1}$-th PD at $U 2$.

## 4) Random LED/PD Selection

In order to benchmark the channel based and LoS based schemes, a random LED/PD selection approach is also described. Specifically according to this scheme at each node, we arbitrarily select a LED/PD to initiate communication. First, the $i^{*}$-th LED at $S$ is selected randomly from the LED set $\left\{1,2, \ldots, N_{I}\right\}$. The $j^{*}$-th PD at $U 1$ is selected randomly from the PD set $\left\{1,2, \ldots, N_{J}\right\}$. The $k^{*}$-th LED at $U 1$ is selected randomly from the PD set $\left\{1,2, \ldots, N_{K}\right\}$. Similarly, the $l^{*}$-th PD at $U 2$ is selected randomly from the PD set $\left\{1,2, \ldots, N_{L}\right\}$.

## IV. Performance Analysis

In this section, after establishing a general expression, we study the lower bounds on $\mathbb{E}\left\{C_{1}\right\}$ and $\mathbb{E}\left\{C_{2}\right\}$ as applicable to the selection schemes described in Section III above. Moreover, accurate closed-form approximations to complement them are also derived.

With the help of (12), a lower bound on $C_{1}$ can be expressed as

$$
\begin{gather*}
C_{1} \geq \frac{1}{4} \log _{2}\left(\operatorname { m i n } \left\{\frac{2 \pi \sigma_{1}^{2}+\left(R P_{S} h_{i * j *}^{S 1}\right)^{2} \sum_{i=1}^{2} e_{i}\left(\epsilon_{i}\right) a_{i}^{2}}{2 \pi \sigma_{1}^{2}+2 \pi\left(a_{2} R P_{S} h_{i * j *}^{S 1}\right)^{2} \epsilon_{2}},\right.\right. \\
\left.\left.\frac{2 \pi \sigma_{2}^{2}+\left(R P_{S} h_{i * l *}^{S 2}\right)^{2} \sum_{i=1}^{2} e_{i}\left(\epsilon_{i}\right) a_{i}^{2}}{2 \pi \sigma_{2}^{2}+2 \pi\left(a_{2} R P_{S} h_{i * l *}^{S 2}\right)^{2} \epsilon_{2}}\right\}\right) . \tag{24}
\end{gather*}
$$

Eq. (24) can be re-expressed as

$$
\begin{equation*}
C_{1} \geq \frac{1}{4} \log _{2}\left(\frac{2 \pi \sigma^{2}+\left(R P_{S}\right)^{2} \sum_{i=1}^{2} e_{i}\left(\epsilon_{i}\right) a_{i}^{2} X}{2 \pi \sigma^{2}+2 \pi\left(a_{2} R P_{S}\right)^{2} \epsilon_{2} X}\right) \tag{25}
\end{equation*}
$$

where $\sigma=\sigma_{1}=\sigma_{2}$, and $X=\min \left\{\left(h_{i^{*} j^{*}}^{S 1}\right)^{2},\left(h_{i^{*} l^{*}}^{S 2}\right)^{2}\right\}$. The distribution of random variable (RV), $X$, depends on the adopted LED/PD selection schemes and respective statistics for each schemes are reported in subsections B, C, and D respectively. Eq. (25) can further be simplified as

$$
\begin{equation*}
C_{1} \geq \frac{1}{4} \log _{2}\left(\frac{1+A X}{1+B X}\right) \tag{26}
\end{equation*}
$$

where $A=\left(\frac{\sum_{i=1}^{2} e_{i}\left(\epsilon_{i}\right) a_{i}^{2}}{2 \pi \sigma^{2}}\right)\left(R P_{S}\right)^{2}$ and $B=\frac{\left(a_{2} R P_{S}\right)^{2} \epsilon_{2}}{\sigma^{2}}$. To find the lower bound on $\mathbb{E}\left\{C_{1}\right\}$, (26) is averaged over the pdf of $\mathrm{RV} X$ as

$$
\begin{equation*}
\mathbb{E}\left\{C_{1}\right\} \geq \frac{1}{4} \int_{0}^{\infty} \log _{2}\left(\frac{1+A x}{1+B x}\right) f_{X}(x) d x \tag{27}
\end{equation*}
$$

where $f_{X}(x)$ is the pdf of RV $X$. To simplify (27), $f_{X}(x)$ is expressed using order statistics and with the realistic assumption that squared channel gains $\left(h_{i^{*} j^{*}}^{S 1}\right)^{2}$ and $\left(h_{i^{*} l^{*}}^{S 2}\right)^{2}$ are RVs that are statistically independent of each other which is valid for all LED/PD selection schemes. Hence, $f_{X}(x)$ is expressed as

$$
\begin{align*}
f_{X}(x)=(1- & \left.F_{\left(h_{i^{*} j^{*}}^{S 1}\right)^{2}}(x)\right) f_{\left(h_{i^{*} l^{*}}^{S 2}\right)^{2}}(x) \\
& +\left(1-F_{\left(h_{i^{*} l^{*}}^{S 2}\right)^{2}}(x)\right) f_{\left(h_{i^{*} j^{*}}^{S 1}\right)^{2}}(x) . \tag{28}
\end{align*}
$$

where $F_{X}(x)$ is the cumulative distribution function (cdf) of RV X. Now, using (27) and (28) an expression for the lower bound on $\mathbb{E}\left\{C_{1}\right\}$ can be derived as

$$
\begin{align*}
& \mathbb{E}\left\{C_{1}\right\} \geq \frac{1}{4} \int_{0}^{\infty} \log _{2}\left(\frac{1+A x}{1+B x}\right)\left(1-F_{\left(h_{i^{*} j^{*}}^{S 1}\right)^{2}}(x)\right) \times \\
& f_{\left(h_{i^{*} l^{*}}^{S 2}\right)^{2}}(x) d x+\frac{1}{4} \int_{0}^{\infty} \log _{2}\left(\frac{1+A x}{1+B x}\right)\left(1-F_{\left(h_{i^{*} l^{*}}^{S}\right)^{2}}(x)\right) \\
& \times f_{\left(h_{i^{*} j^{*}}^{S 1}\right)^{2}}(x), \tag{29}
\end{align*}
$$

In order to derive a general expression for the lower bound on $\mathbb{E}\left\{C_{2}\right\}$ we first express (13) as

$$
\begin{align*}
C_{2} \geq \frac{1}{4} \log _{2}(1+\min \{ & \frac{e_{2}\left(\epsilon_{2}\right)\left(a_{2} R P_{S} h_{i^{*} j^{*}}^{S 1}\right)^{2}}{2 \pi \sigma^{2}} \\
& \left.\left.\frac{e_{2}\left(\epsilon_{2}\right)\left(R P_{R} h_{k^{*} l^{*}}^{12}\right)^{2}}{2 \pi \sigma^{2}}\right\}\right) \tag{30}
\end{align*}
$$

After simplifying (30) yields

$$
\begin{equation*}
C_{2} \geq \frac{1}{4} \log _{2}\left(1+\frac{e_{2}\left(\epsilon_{2}\right) R^{2} Y}{2 \pi \sigma^{2}}\right) \tag{31}
\end{equation*}
$$

where $Y=\min \left\{\left(a_{2} P_{S} h_{i^{*} j^{*}}^{S 1}\right)^{2},\left(P_{R} h_{k^{*} l^{*}}^{12}\right)^{2}\right\}$. The statistics of $Y$ are described below. The lower bound on $\mathbb{E}\left\{C_{2}\right\}$ can be obtained by averaging (31) over the pdf of RV $Y$ as

$$
\begin{equation*}
\mathbb{E}\left\{C_{2}\right\} \geq \frac{1}{4} \int_{0}^{\infty} \ln (1+D y) f_{Y}(y) d y \tag{32}
\end{equation*}
$$

where $D=\frac{e_{2}\left(\epsilon_{2}\right)}{2 \pi}\left(\frac{R}{\sigma}\right)^{2}$, and $f_{Y}(y)$ is the pdf of $Y$. In order to further simplify (32), $f_{Y}(y)$ is expressed using order statistics and with the realistic assumption that squared channel gains $\left(h_{i^{*} j^{*}}^{S 1}\right)^{2}$ and $\left(h_{k^{*} l^{*}}^{12}\right)^{2}$ are RVs that are statistically independent of each other which is valid for all LED/PD selection schemes. Hence, $f_{Y}(y)$ can be expressed as

$$
\begin{align*}
f_{Y}(y)= & \left(1-F_{\left(a_{2} P_{S} h_{i^{*} j^{*}}^{S 1}\right)^{2}}(y)\right) f_{\left(P_{R} h_{k^{*} l^{*}}^{12}\right)^{2}}(y) \\
& +\left(1-F_{\left(P_{R} h_{k^{*} l^{*}}^{12}\right)^{2}}(y)\right) f_{\left(a_{2} P_{S} h_{i^{*} j^{*}}^{S}\right)^{2}}(y) \tag{33}
\end{align*}
$$

The derived general expression for the lower bound on $\mathbb{E}\left\{C_{2}\right\}$ using (32) and (33) is

$$
\begin{align*}
& \mathbb{E}\left\{C_{2}\right\} \geq\left.\frac{1}{4} \int_{0}^{\infty} \log _{2}(1+D y)\left(1-F_{\left(a_{2} P_{S} h_{i^{*} j^{*}}^{S 1}\right.}\right)^{2}(y)\right) \times \\
& f_{\left(P_{R} h_{k^{*} l^{*}}^{12}\right)^{2}}(y) d y+\frac{1}{4} \int_{0}^{\infty} \log _{2}(1+D y) \times \\
&\left(1-F_{\left(P_{R} h_{k^{*} l^{*}}^{12}\right)^{2}}(y)\right) f_{\left(a_{2} P_{S} h_{i^{*} j^{*}}^{S 1}\right)^{2}}(y) d y \tag{34}
\end{align*}
$$

Using (29), and (34) we derive expressions for the lower bounds on $\mathbb{E}\left\{C_{1}\right\}$, and $\mathbb{E}\left\{C_{2}\right\}$ for optimal, max $S-U 1-U 2$ channel gain based, and best $S-U 1-U 2$ LoS based LED/PD selection scheme in the sequel.

## A. Optimal LED/PD Selection

In the case of optimal LED/PD selection, the pdf and cdf of the SNR and SINR required to derive the average achievable sum rate is extremely difficult to find. However, in Section V through simulations the corresponding performance of the optimal scheme is shown.

## B. Max $S-U 1-U 2$ Channel Gain Based Selection

In order to obtain an exact expression for the lower bound on the average achievable sum rate, first we derive cdfs and pdfs of max $\left\{\left(h_{i j}^{S 1}\right)^{2}\right\},\left(h_{i^{*} l^{*}}^{S 2}\right)^{2}$, $\max \left\{\left(a_{2} P s h_{i j}^{S 1}\right)^{2}\right\}$, and $\max \left\{\left(P_{r} h_{k l}^{12}\right)^{2}\right\}$ where $h_{i l}^{S 2}$ is the composite channel gain between the transmitter $i$ of $S$ and PD $l$ of $U 2$. Next, (14), (29), and (34) are used to obtain exact expressions.

Let us focus on deriving expressions for the lower bound on $\mathbb{E}\left\{C_{1}\right\}$. The cdf of the distribution of $X=\max \left\{\left|h_{i j}^{S 1}\right|^{2}\right\}$ can be obtained with the help of [34, Eq. (8)] and the use of order statistics as

$$
\begin{equation*}
F_{X}(x)=\prod_{\substack{1 \leq i \leq N_{I}, 1 \leq j \leq N_{J}}} \frac{1}{2} \operatorname{erfc}\left(\frac{\ln \left(\frac{\left(G_{i j}^{S 1}\right)^{2}}{x}\right)+4 \mu_{x}}{4 \sqrt{2} \sigma_{x}}\right) \tag{35}
\end{equation*}
$$

where $\operatorname{erfc}(x)=1-\frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-t^{2}} d t$ is the complementary error function. By differentiating (35) with respect to (w.r.t.) $x$ the corresponding pdf can be written as

$$
\begin{align*}
& f_{X}(x)=\sum_{i=1}^{N_{I}} \sum_{j=1}^{N_{J}} \exp \left(-\frac{\left(\ln \left(\frac{x}{\left(G_{i j}^{S 1}\right)^{2}}\right)-4 \mu_{x}\right)^{2}}{32 \sigma_{x}^{2}}\right) \times \\
& \frac{1}{4 x \sqrt{2 \pi \sigma^{2}}} \prod_{\substack{1 \leq i_{1} \leq N_{I}, 1 \leq j_{1} \leq N_{J}, i_{1} \neq i \& j_{1} \neq j}} \frac{1}{2} \operatorname{erfc}\left(\frac{\ln \left(\frac{\left(G_{i_{1} j_{1}}^{S 1}\right)^{2}}{x}\right)+4 \mu_{x}}{4 \sqrt{2} \sigma_{x}}\right) \tag{36}
\end{align*}
$$

However, after the selection of $i^{*}, j^{*}, k^{*}$, and $l^{*}$, the distribution of $\left(h_{i^{*} l^{*}}^{S 2}\right)^{2}$ can not be obtained straightforwardly. Hence, we obtain the weighting factor of $\left(h_{i l}^{S 2}\right)^{2}$ given by $w_{i, l}$ from offline simulations for the use in the analytical expression. The cdf of $X=\left(h_{i^{*} l^{*}}^{S 2}\right)^{2}$ can be expressed as

$$
\begin{equation*}
F_{X}(x)=\sum_{i=1}^{N_{I}} \sum_{l=1}^{N_{L}} \frac{1}{2} w_{i, l} \operatorname{erfc}\left(\frac{\ln \left(\frac{\left(G_{i l}^{S 2}\right)^{2}}{x}\right)+4 \mu_{x}}{4 \sqrt{2} \sigma_{x}}\right) \tag{37}
\end{equation*}
$$

By differentiating w.r.t. $x$, the pdf of $\left(h_{i^{*} l^{*}}^{S 2}\right)^{2}$ is given by
$f_{X}(x)=\sum_{i=1}^{N_{I}} \sum_{l=1}^{N_{L}} \frac{w_{i, l}}{4 x \sqrt{2 \pi \sigma^{2}}} \exp \left(-\frac{\left(\ln \left(\frac{x}{\left(G_{i l}^{S^{2}}\right)^{2}}\right)-4 \mu_{x}\right)^{2}}{32 \sigma_{x}^{2}}\right)$.

Substituting (35), (36), (37), and (38) into (29), the lower bound on $\mathbb{E}\left\{C_{1}\right\}$ is given by (39) as shown in the top of the next page.

According to the authors' knowledge, the above integral is difficult to solve and hence finding a closed-form solution is not possible. However, (39) can be evaluated numerically using popular software such as Matlab or Mathematica as demonstrated in Section V. In addition, a closed-form accurate approximation to evaluate (39) is presented. To this end, a key result reported in [37, Eq. (5-7)] is exploited as

$$
\begin{equation*}
\mathbb{E}[F(\theta)] \approx \frac{2}{3} F(\mu)+\frac{1}{6} F(\mu+\sqrt{3} \sigma)+\frac{1}{6} F(\mu-\sqrt{3} \sigma) \tag{41}
\end{equation*}
$$

where $\theta$ be normally distributed with mean $\mu$ and variance $\sigma^{2}$ and $F$ be a real function of $\theta$. We observe that with a simple variable substitution $\theta_{1}=\frac{1}{4} \ln \left(x /\left(G_{i l}^{S 2}\right)^{2}\right)$, and $\theta_{2}=$ $\frac{1}{4} \ln \left(x /\left(G_{i j}^{S 1}\right)^{2}\right)$, (39) reduces to the form given in (41) with $\theta_{1}$, and $\theta_{2}$ Gaussian distributed. Hence, the approximate lower bound on the average achievable rate for message stream $x_{1}[n]$
is expressed as (40) as shown on the top of the page, where $L(n)=\exp \left(4 \mu_{x}+4 \sqrt{3} n \sigma_{x}\right)$.

Let us consider the derivation of the lower bound on $\mathbb{E}\left\{C_{2}\right\}$. The cdf and pdf of the RV $Y=\max \left\{\left(a_{2} P_{s} h_{i j}^{S 1}\right)^{2}\right\}$ can be expressed using the result given in (35) and (36) replacing $X$ with $Y$, and $G_{i j}^{S 1}$ with $a_{2} P_{s} G_{i j}^{S 1}$. Similarly, the cdf and pdf of $Y=\max \left\{\left(P_{r} h_{k l}^{12}\right)^{2}\right\}$ can be expressed using (35) and (36) replacing $X$ with $Y$, and $G_{i j}^{S 1}$ with $P_{r} h_{k l}^{R D}$. Substituting the corresponding pdfs and cdfs into (34), the lower bound on $\mathbb{E}\left\{C_{2}\right\}$ can be expressed as (42) as shown on the top of the page 9 . Next, with the use of (41), an approximate lower bound on $\mathbb{E}\left\{C_{2}\right\}$ can be derived as (43) as shown on the top of the page 9 .

Finally, with the help of (39) and (42) in (14), the lower bound on the average achievable sum rate can be established. Similarly, with the help of (40) and (43) yields an approximate expression for the lower bound on the average achievable sum rate.

## C. Best $S-U 1-U 2$ LoS Based Selection

To obtain an expression for the lower bound on the average achievable sum rate in best $S-U 1-U 2 \operatorname{LoS}$ based selection, first we obtain cdfs and pdfs for the RVs $\left(h_{i^{*} j^{*}}^{S 1}\right)^{2},\left(h_{i^{*} l^{*}}^{S 2}\right)^{2}$, $\left(a_{2} P_{s} h_{i^{*} j^{*}}^{S 1}\right)^{2}$, and $\left(P_{r} h_{k l}^{12}\right)^{2}$. Next, using (14), (29), and (34) expressions for lower bounds are obtained.

The cdf of $\mathrm{RV}\left(h_{i^{*} j^{*}}^{S 1}\right)^{2}$ is given by

$$
\begin{equation*}
F_{\left(h_{i^{*} j^{*}}^{S 1}\right)^{2}}(x)=\frac{1}{2} \operatorname{erfc}\left(\frac{\ln \left(\frac{\left(G_{i^{*} j^{*}}\right)^{2}}{x}\right)+4 \mu_{x}}{4 \sqrt{2} \sigma_{x}}\right) \tag{46}
\end{equation*}
$$

Using the cdf given in [34, Eq. (8)] and by differentiating (46) w.r.t. $x$ the pdf is expressed as

$$
\begin{equation*}
f_{\left(h_{i^{*} j^{*}}\right)^{2}}(x)=\frac{1}{4 x \sqrt{2 \pi \sigma^{2}}} \exp \left(-\frac{\left(\ln \left(\frac{x}{\left(G_{i^{*} j^{*}}\right)^{2}}\right)-4 \mu_{x}\right)^{2}}{32 \sigma_{x}^{2}}\right) \tag{47}
\end{equation*}
$$

The cdf and pdf of $\left(h_{i^{*} l^{*}}^{S 2}\right)^{2}$ can be expressed by replacing $G_{i^{*} j^{*}}^{S 1}$ with $G_{i^{*} l^{*}}^{S 2}$ in (46) and (47). Inserting this pdf and cdf into (29) and averaging over possible combinations of PDs, the lower bound on $\mathbb{E}\left\{C_{1}\right\}$ can be established as (44) shown on the page 9 , where $J_{s}$ and $J_{e}$ are the indices of starting and finishing PDs at $U 1$ from the PD set given in (20). Further, with the help of (41), equation (44) can be approximated as (45).

Consider the lower bound on $\mathbb{E}\left\{C_{2}\right\}$. The cdf and pdf of RV $\left(a_{2} P_{s} h_{i^{*} j^{*}}^{S 1}\right)^{2}$ can be found by replacing $G_{i^{*} j^{*}}^{S 1}$ with $a_{2} P_{s} G_{i^{*} j^{*}}^{S 1}$ in (46) and (47) respectively. Similarly, the cdf and pdf of $\left(P_{r} h_{i^{*} l^{*}}^{S 1}\right)^{2}$ can be expressed by replacing $G_{i^{*} j^{*}}^{S 1}$ with $P_{r} G_{i^{*} l^{*}}^{S 1}$ in (46) and (47) respectively. Inserting these pdfs and cdfs into (34) and averaging over possible random selection of PDs at $U 2$ given by (23), the expression for the lower bound on $\mathbb{E}\left\{C_{2}\right\}$ can be written as (48) where $L_{s}$ and $L_{e}$ are indexes of starting and finishing PDs at $U 2$ from the PD set given in (23). Further, with the help of (41), (48) can be approximated as (49) as shown on the top of the page 10.

Finally, with the help of (44) and (48) in (14) the lower bound on the average achievable sum rate can be established. Similarly, with the help of (45) and (49) yields an approximate

$$
\begin{align*}
& \mathbb{E}\left\{C_{1}\right\} \geq \frac{1}{16 \sqrt{2 \pi \sigma^{2}}}\left\{\sum_{i=1}^{N_{I}} \sum_{l=1}^{N_{L}} w_{i, l} \int_{0}^{\infty} \log _{2}\left(\frac{1+A x}{1+B x}\right)\left(1-\prod_{\substack{1 \leq i_{1} \leq N_{I}, 1 \leq j_{1} \leq N_{J}}} \frac{1}{2} \operatorname{erfc}\left(\frac{\ln \left(\frac{\left(G_{i_{1} j_{1}}^{S 1}\right)^{2}}{x}\right)+4 \mu_{x}}{4 \sqrt{2} \sigma_{x}}\right)\right) \times\right. \\
& \frac{1}{x} \exp \left(-\frac{\left(\ln \left(\frac{x}{\left(G_{i l}^{S 2}\right)^{2}}\right)-4 \mu_{x}\right)^{2}}{32 \sigma_{x}^{2}}\right) d x+\sum_{i=1}^{N_{I}} \sum_{j=1}^{N_{J}} \int_{0}^{\infty} \log _{2}\left(\frac{1+A x}{1+B x}\right) \times\left(1-\sum_{i_{1}=1}^{N_{I}} \sum_{l_{1}=1}^{N_{L}} \frac{1}{2} w_{i_{1}, l_{1}} \operatorname{erfc}\left(\frac{\ln \left(\frac{\left(G_{i_{1} l_{1}}^{S 2}\right)^{2}}{x}\right)+4 \mu_{x}}{4 \sqrt{2} \sigma_{x}}\right)\right) \\
& \left.\left\{\prod_{\substack{1 \leq i_{2} \leq N_{I}, 1 \leq j_{2} \leq N_{J}, i_{2} \neq i \& j_{2} \neq j}} \frac{1}{2} \operatorname{erfc}\left(\frac{\ln \left(\frac{\left(G_{i_{2} j_{2}}^{S 1}\right)^{2}}{x}\right)+4 \mu_{x}}{4 \sqrt{2} \sigma_{x}}\right)\right\} \times \frac{1}{x} \exp \left(-\frac{\left(\ln \left(\frac{x}{\left(G_{i j}^{S 1}\right)^{2}}\right)-4 \mu_{x}\right)^{2}}{32 \sigma_{x}^{2}}\right) d x\right\} .  \tag{39}\\
& \mathbb{E}\left\{C_{L, 1}\right\} \approx \frac{1}{6} \sum_{i=1}^{N_{I}}\left\{\sum_{l=1}^{N_{L}} w_{i, l} \sum_{n=-1}^{1} \frac{1}{4|n|} \log _{2}\left(\frac{1+A\left(G_{i l}^{S 2}\right)^{2} L(n)}{1+B\left(G_{i l}^{S 2}\right)^{2} L(n)}\right)\left(1-\prod_{\substack{1 \leq i_{1} \leq N_{I}, 1 \leq j_{1} \leq N_{J}}} \frac{1}{2} \operatorname{erfc}\left(\frac{\ln \left(\frac{\left(G_{i 1 j_{1}}^{S)^{2}}\right)-4 \sqrt{3} n \sigma_{x}}{\left(G_{i l}^{S 2}\right)^{2}}\right)-4 \sqrt{2} \sigma_{x}}{4}\right)\right)\right. \\
& +\sum_{j=1}^{N_{J}} \sum_{n=-1}^{1} \frac{1}{4^{|n|}} \log _{2}\left(\frac{1+A\left(G_{i j}^{S 1}\right)^{2} L(n)}{1+B\left(G_{i j}^{S 1}\right)^{2} L(n)}\right) \prod_{\substack{1 \leq i_{2} \leq N_{I}, 1 \leq j_{2} \leq N_{J}, i_{2} \neq i \notin j_{2} \neq j}} \frac{1}{2} \operatorname{erfc}\left(\frac{\ln \left(\frac{\left(G_{\left.i_{2} j_{2}\right)^{2}}^{S 1}\right.}{\left(G_{i j}^{S 1}\right)^{2}}\right)-4 \sqrt{3} n \sigma_{x}}{4 \sqrt{2} \sigma_{x}}\right) \times \\
& \left.\left(1-\sum_{i_{3}=1}^{N_{I}} \sum_{l_{3}=1}^{N_{L}} \frac{1}{2} w_{i_{3}, l_{3}} \operatorname{erfc}\left(\frac{\ln \left(\frac{\left(G_{i_{3} l_{3}}^{S 2}\right)^{2}}{\left(G_{i j}^{S 1}\right)^{2}}\right)-4 \sqrt{3} n \sigma_{x}}{4 \sqrt{2} \sigma_{x}}\right)\right)\right\}, \tag{40}
\end{align*}
$$

expression for the lower bound on the average achievable sum rate.

## D. Random LED/PD Selection

The lower bound on the average achievable sum rate in random LED/PD selection method can be found by first obtaining the lower bounds for $\mathbb{E}\left\{C_{1}\right\}$ and $\mathbb{E}\left\{C_{2}\right\}$ for a selected LED/PD set as discussed in previous scheme and then averaging it over all possible $N_{I} N_{J} N_{K} N_{L}$ link combinations.

Using a similar approach as in above schemes, the lower bound on $\mathbb{E}\left\{C_{1}\right\}$ can be expressed as (50). Using (41) the approximate lower bound on $\mathbb{E}\left\{C_{1}\right\}$ under random LED/PD selection can be expressed as (51). Similarly, the expression for the lower bound on $\mathbb{E}\left\{C_{2}\right\}$ can be expressed as (52) shown on the top of the page 11 . The approximate lower bound on $\mathbb{E}\left\{C_{2}\right\}$ can be established with the help of (41) and is given by (53).

Finally, with the help of (50) and (52) in (14) the lower bound on the average achievable sum rate can be established. Similarly, with the help of (51) and (53) yields an approximate expression for the lower bound on the average achievable sum rate.

## E. Upper Bound on the Average Achievable Rate

In this subsection we present expressions which helps to derive the upper bound on the average achievable sum rate. Establishing an upper bound allows to reaffirm the value of the derived lower bound as a meaningful measure of the exact achievable sum rate. In Section V, numerical results are presented to illustrate the usefulness of lower and upper bounds such that possible exact sum rate margins offered by the LED/PD selection schemes can be well understood.

We present separate expressions for the upper bound on the instantaneous achievable rate for message streams $x_{1}[n]$ and $x_{2}[n]$ separately. Next, they are averaged and the sum is obtained. The instantaneous achievable rates for decoding message stream $x_{1}[n]$ at $U 1$ and $U 2$ are upper bounded by [36]

$$
C_{1}^{S 1} \leq \frac{1}{4} \log _{2}\left(\frac{2 \pi \sigma_{1}^{2}+2 \pi\left(R P_{S} h_{i * j *}^{S 1}\right)^{2} \sum_{m=1}^{2} \epsilon_{m} a_{m}^{2}}{2 \pi \sigma_{1}^{2}+e_{2}\left(\epsilon_{2}\right)\left(a_{2} R P_{S} h_{i * j *}^{S 1}\right)^{2}}\right)
$$

and

$$
C_{1}^{S 2} \leq \frac{1}{4} \log _{2}\left(\frac{2 \pi \sigma_{2}^{2}+2 \pi\left(R P_{S} h_{i * l *}^{S 2}\right)^{2} \sum_{m=1}^{2} \epsilon_{m} a_{m}^{2}}{2 \pi \sigma_{2}^{2}+e_{2}\left(\epsilon_{2}\right)\left(a_{2} R P_{S} h_{i * l *}^{S 2}\right)^{2}}\right) .
$$

The upper bound on the instantaneous achievable rate for decoding message stream $x_{1}[n]$ can be expressed with the help of results in [33] and after some mathematical manipulation, as

$$
\begin{align*}
C_{1} \leq \min \left\{\begin{array}{l}
\frac{1}{4} \log _{2}\left(\frac{1+E\left(h_{i * j *}^{S 1}\right)^{2}}{1+F\left(h_{i * j *}^{S 1}\right)^{2}}\right) \\
\frac{1}{4} \log _{2}\left(\frac{1+E\left(h_{i * l *}^{S 2}\right)^{2}}{1+F\left(h_{i * l *}^{S 2}\right)^{2}}\right)
\end{array},\right.
\end{align*}
$$

where $E=2 \pi\left(R P_{S}\right)^{2} \sum_{m=1}^{2} \epsilon_{m} a_{m}^{2}$, and $F=$ $e_{2}\left(\epsilon_{2}\right)\left(a_{2} R P_{S} h_{i * j *}^{S 1}\right)^{2}$. Using a similar approach given in Section IV, the upper bound on the average achievable rate for message stream $x_{1}[n], \mathbb{E}\left\{C_{U, 1}\right\}$, can be derived using (29) by replacing $A$ with $E$, and $B$ with $F$.

Now, we present the upper bound on the instantaneous achievable rates for decoding message stream $x_{2}[n]$ at $U 1$ and $U 2$ is established as [36]

$$
C_{2}^{S 1} \leq \frac{1}{2} \log _{2}\left(1+\frac{\epsilon_{2}\left(a_{2} R P_{S} h_{i^{*} j^{*}}^{S 1}\right.}{\sigma_{1}^{2}}\right)
$$

$$
\mathbb{E}\left\{C_{L, 2}\right\} \approx \frac{1}{6} \sum_{n=-1}^{1} \frac{1}{4^{|n|}}\left\{\sum_{k=1}^{N_{K}} \sum_{l=1}^{N_{L}} \log _{2}\left(1+D\left(P_{r} G_{k l}^{12}\right)^{2} L(n)\right) \prod_{\substack{1 \leq k_{2} \leq N_{K}, 1 \leq l_{2} \leq N_{L}, k_{2} \neq k \& j_{2} \neq j}} \frac{1}{2} \operatorname{erfc}\left(\frac{\ln \left(\frac{\left.\left(G_{k_{2}}^{12}\right)^{2}\right)^{2}}{\left(G_{k}^{12}\right)^{2}}\right)-4 \sqrt{3} n \sigma_{x}}{4 \sqrt{2} \sigma_{x}}\right)\right.
$$

$$
\begin{equation*}
\times\left(1-\prod_{\substack{1 \leq i_{1} \leq N_{I}, 1 \leq j_{1} \leq N J}} \frac{1}{2} \operatorname{erfc}\left(\frac{\ln \left(\frac{\left(a_{2} P_{s} G_{11_{j}}^{S_{1}}\right)^{2}}{\left(P_{r} G_{k l}^{12}\right)^{2}}\right)-4 \sqrt{3} n \sigma_{x}}{4 \sqrt{2} \sigma_{x}}\right)\right)+\sum_{i=1}^{N_{I}} \sum_{j=1}^{N_{J}} \log _{2}\left(1+D\left(a_{2} P_{s} G_{i j}^{S 1}\right)^{2} L(n)\right) \tag{43}
\end{equation*}
$$

$$
\mathbb{E}\left\{C_{1}\right\} \geq \frac{1}{4\left(J_{e}-J_{s}\right)\left(L_{e}-L_{s}\right)} \sum_{j=J_{s}}^{J_{e}} \sum_{l=L_{s}}^{L_{e}} \int_{0}^{\infty} \log _{2}\left(\frac{1+A x}{1+B x}\right)\left\{\left(1-\frac{1}{2} \operatorname{erfc}\left(\frac{\ln \left(\frac{\left(G_{i^{*}}^{S_{1}}\right)^{2}}{x}\right)+4 \mu_{x}}{4 \sqrt{2} \sigma_{x}}\right)\right)\right.
$$

$$
\times \frac{1}{4 x \sqrt{2 \pi \sigma^{2}}} \exp \left(-\frac{\left(\ln \left(\frac{x}{\left(G_{i+1}^{\left.S_{2}^{2}\right)^{2}}\right)}\right)-4 \mu_{x}\right)^{2}}{32 \sigma_{x}^{2}}\right) d x+\left(1-\frac{1}{2} \operatorname{erfc}\left(\frac{\ln \left(\frac{\left(G_{i=1}^{S}\right)^{2}}{x}\right)+4 \mu_{x}}{4 \sqrt{2} \sigma_{x}}\right)\right)
$$

$$
\begin{equation*}
\left.\times \frac{1}{4 x \sqrt{2 \pi \sigma^{2}}} \exp \left(-\frac{\left(\ln \left(\frac{x}{\left(G_{\left.i i^{R}\right)^{2}}\right)^{2}}\right)-4 \mu_{x}\right)^{2}}{32 \sigma_{x}^{2}}\right)\right\} d x \tag{44}
\end{equation*}
$$

$$
\begin{align*}
& \mathbb{E}\left\{C_{L, 1}\right\} \approx \frac{1}{6\left(J_{e}-J_{s}\right)\left(L_{e}-L_{s}\right)} \sum_{j=J_{s}}^{J_{e}} \sum_{l=L_{s}}^{L_{e}} \sum_{n=-1}^{1} \frac{1}{4^{|n|}}\left\{\log _{2}\left(\frac{\left.1+A\left(G_{i^{*}}\right)^{2}\right)^{2} L(n)}{1+B\left(G_{i^{*} l}^{S}\right)^{2} L(n)}\right)\right. \\
& \times\left(1-\frac{1}{2} \operatorname{erfc}\left(\frac{\ln \left(\frac{\left(G_{i^{*} j}^{S 1}\right)^{2}}{\left(G_{i^{*} i}\right)^{2}}\right)-4 \sqrt{3} n \sigma_{x}}{4 \sqrt{2} \sigma_{x}}\right)\right)+\log _{2}\left(\frac{1+A\left(G_{\left.i^{*}\right)^{2}}^{S}\right)^{2} L(n)}{1+B\left(G_{i^{*} j}^{S_{j}}\right)^{2} L(n)}\right) \\
& \left.\times\left(1-\frac{1}{2} \operatorname{erfc}\left(\frac{\ln \left(\frac{\left(G_{S_{2}}^{S}\right)^{2}}{\left(G_{i_{i}^{1}}^{S_{1}}\right)^{2}}\right)-4 \sqrt{3} n \sigma_{x}}{4 \sqrt{2} \sigma_{x}}\right)\right)\right\} . \tag{45}
\end{align*}
$$

$$
\begin{align*}
& \mathbb{E}\left\{C_{2}\right\} \geq \frac{1}{16 \sqrt{2 \pi \sigma^{2}}}\left\{\sum_{k=1}^{N_{K}} \sum_{l=1}^{N_{L}} \int_{0}^{\infty} \frac{1}{y} \exp \left(-\frac{\left(\ln \left(\frac{y}{\left(P_{r} G_{k l^{2}}^{12}\right)^{2}}\right)-4 \mu_{y}\right)^{2}}{32 \sigma_{y}^{2}}\right)\left\{\prod_{\substack{1 \leq k_{2} \leq N_{K}, 1 \leq l_{2} \leq N_{L}, k_{2} \neq k<j_{2} \neq j}} \frac{1}{2} \operatorname{erfc}\left(\frac{\ln \left(\frac{\left(P_{r} G_{k_{2}}^{12} l_{2}\right)^{2}}{y}\right)+4 \mu_{y}}{4 \sqrt{2} \sigma_{y}}\right)\right\} \times\right. \\
& \log _{2}(1+D y)\left(1-\prod_{\substack{1 \leq i_{1} \leq N_{N}, 1 \leq j_{1} \leq N_{J}}} \frac{1}{2} \operatorname{erfc}\left(\frac{\ln \left(\frac{\left(a_{2} P_{s} G_{i_{1} j_{1}}^{S S_{1}}\right)^{2}}{y}\right)+4 \mu_{y}}{4 \sqrt{2} \sigma_{y}}\right)\right) d y+\sum_{i=1}^{N_{I}} \sum_{j=1}^{N_{J}} \int_{0}^{\infty} \log _{2}(1+D y) \\
& \left(1-\prod_{\substack{1 \leq k_{3} \leq N_{K}, 1 \leq l_{3} \leq N_{L}}} \frac{1}{2} \operatorname{erfc}\left(\frac{\ln \left(\frac{\left(P_{r} G_{k_{3}}^{12} l_{3}\right)^{2}}{y}\right)+4 \mu_{y}}{4 \sqrt{2} \sigma_{y}}\right)\right)\left\{\prod_{\substack{1 \leq i_{4} \leq N_{I}, 1 \leq 4 \leq N_{J}, i_{4} \neq i<i j_{4} \neq j}} \frac{1}{2} \operatorname{erfc}\left(\frac{\ln \left(\frac{\left(a_{2} P_{s} G_{2} G_{2} j_{2}\right)^{2}}{y}\right)+4 \mu_{y}}{4 \sqrt{2} \sigma_{y}}\right)\right\} \\
& \left.\times \frac{1}{y} \exp \left(-\frac{\left(\ln \left(\frac{y}{\left(a_{2} P_{s} G_{i j}^{S 1}\right)^{2}}\right)-4 \mu_{y}\right)^{2}}{32 \sigma_{y}^{2}}\right) d y\right\} . \tag{42}
\end{align*}
$$

$$
\begin{align*}
& \mathbb{E}\left\{C_{2}\right\} \geq \frac{1}{4\left(J_{e}-J_{s}\right)\left(L_{e}-L_{s}\right)} \sum_{j=J_{s}}^{J_{e}} \sum_{l=L_{s}}^{L_{e}} \int_{0}^{\infty} \log _{2}(1+D y) \times \\
& \left\{\left(1-\frac{1}{2} \operatorname{erfc}\left(\frac{\ln \left(\frac{\left(a_{2} P_{s} G_{i^{*} j}^{S 1}\right)^{2}}{y}\right)+4 \mu_{y}}{4 \sqrt{2} \sigma_{x}}\right)\right) \frac{1}{4 y \sqrt{2 \pi \sigma^{2}}} \exp \left(-\frac{\left(\ln \left(\frac{y}{\left(P_{r} G_{k^{*}}^{12}\right)^{2}}\right)-4 \mu_{y}\right)^{2}}{32 \sigma_{y}^{2}}\right)+\right. \\
& \left.\left(1-\frac{1}{2} \operatorname{erfc}\left(\frac{\ln \left(\frac{\left(P_{r} G_{k^{*} l}^{12}\right)^{2}}{y}\right)+4 \mu_{y}}{4 \sqrt{2} \sigma_{y}}\right)\right) \frac{1}{4 y \sqrt{2 \pi \sigma^{2}}} \exp \left(-\frac{\left(\ln \left(\frac{y}{\left(a_{2} P_{s} G_{i^{*} j}^{S 1}\right)^{2}}\right)-4 \mu_{y}\right)^{2}}{32 \sigma_{y}^{2}}\right)\right\} d y,  \tag{48}\\
& \mathbb{E}\left\{C_{L, 2}\right\} \approx \frac{1}{6\left(J_{e}-J_{s}\right)\left(L_{e}-L_{s}\right)} \sum_{j=J_{s}}^{J_{e}} \sum_{l=L_{s}}^{L_{e}} \sum_{n=-1}^{1} \frac{1}{4^{|n|}}\left\{\log _{2}\left(1+D\left(P_{r} G_{k^{*} l}^{12}\right)^{2} L(n)\right)\right. \\
& \times\left(1-\frac{1}{2} \operatorname{erfc}\left(\frac{\ln \left(\frac{\left(a_{2} P_{s} G_{i^{*} j}^{S 1}\right)^{2}}{\left(P_{r} G_{k^{*}}^{1 *}\right)^{2}}\right)-4 \sqrt{3} n \sigma_{y}}{4 \sqrt{2} \sigma_{y}}\right)\right)+\log _{2}\left(1+D\left(a_{2} P_{s} G_{i^{*} j}^{S 1}\right)^{2} L(n)\right) \\
& \left.\times\left(1-\frac{1}{2} \operatorname{erfc}\left(\frac{\ln \left(\frac{\left(P_{r} G_{\left.k^{*}\right)}^{12}\right)^{2}}{\left(a_{2} P_{s} G_{i^{*} j}^{S 1}\right)^{2}}\right)-4 \sqrt{3} n \sigma_{x}}{4 \sqrt{2} \sigma_{y}}\right)\right)\right\},  \tag{49}\\
& \mathbb{E}\left\{C_{1}\right\} \geq \frac{1}{6 N_{I} N_{J} N_{K} N_{L}} \sum_{i=1}^{N_{I}} \sum_{j=1}^{N_{J}} \sum_{k=1}^{N_{K}} \sum_{l=1}^{N_{L}} \int_{0}^{\infty} \log _{2}\left(\frac{1+A x}{1+B x}\right) \times \\
& \left\{\left(1-\frac{1}{2} \operatorname{erfc}\left(\frac{\ln \left(\frac{\left(G_{i j}^{S 1}\right)^{2}}{x}\right)+4 \mu_{x}}{4 \sqrt{2} \sigma_{x}}\right)\right) \times \frac{1}{4 x \sqrt{2 \pi \sigma^{2}}} \exp \left(-\frac{\left(\ln \left(\frac{x}{\left(G_{i l}^{S 2}\right)^{2}}\right)-4 \mu_{x}\right)^{2}}{32 \sigma_{x}^{2}}\right)+\right. \\
& \left.\left(1-\frac{1}{2} \operatorname{erfc}\left(\frac{\ln \left(\frac{\left(G_{i l}^{S 2}\right)^{2}}{x}\right)+4 \mu_{x}}{4 \sqrt{2} \sigma_{x}}\right)\right) \times \frac{1}{4 x \sqrt{2 \pi \sigma^{2}}} \exp \left(-\frac{\left(\ln \left(\frac{x}{\left(G_{i j}^{S 1}\right)^{2}}\right)-4 \mu_{x}\right)^{2}}{32 \sigma_{x}^{2}}\right)\right\} d x .  \tag{50}\\
& \mathbb{E}\left\{C_{L, 1}\right\} \approx \frac{1}{6 N_{I} N_{J} N_{K} N_{L}} \sum_{i=1}^{N_{I}} \sum_{j=1}^{N_{J}} \sum_{k=1}^{N_{K}} \sum_{l=1}^{N_{L}} \sum_{n=-1}^{1} \frac{1}{4|n|}\left\{\left(1-\frac{1}{2} \operatorname{erfc}\left(\frac{-\ln \left(\frac{\left(G_{i j}^{S 1}\right)^{2}}{\left(G_{i l}^{S 2}\right)^{2}}\right)-4 \sqrt{3} n \sigma_{x}}{4 \sqrt{3} \sigma_{x}}\right)\right)\right. \\
& \left.\times \log _{2}\left(\frac{1+A\left(G_{i l}^{S 2}\right)^{2} L(n)}{1+B\left(G_{i l}^{S 2}\right)^{2} L(n)}\right)+\left(1-\frac{1}{2} \operatorname{erfc}\left(\frac{-\ln \left(\frac{\left(G_{i j}^{S 1}\right)^{2}}{\left(G_{i j}^{S 1}\right)^{2}}\right)-4 \sqrt{3} n \sigma_{x}}{4 \sqrt{3} \sigma_{x}}\right)\right) \times \log _{2}\left(\frac{1+A\left(G_{i j}^{S 1}\right)^{2} L(n)}{1+B\left(G_{i j}^{S 1}\right)^{2} L(n)}\right)\right\} . \tag{51}
\end{align*}
$$

and

$$
C_{2}^{12} \leq \frac{1}{2} \log _{2}\left(1+\frac{\epsilon_{2}\left(R P_{R} h_{k^{*} l^{*}}^{S 1}\right)^{2}}{\sigma_{2}^{2}}\right)
$$

Hence the upper bound on the instantaneous achievable rate for decoding message stream $x_{2}[n]$ can be expressed as [33]

$$
\begin{equation*}
C_{2} \leq \frac{1}{4} \log _{2}\left(1+G_{1} Y\right) \tag{55}
\end{equation*}
$$

where $Y=\min \left\{\left(a_{2} P_{S} h_{i^{*} j^{*}}^{S 1}\right)^{2},\left(R P_{R} h_{k^{*} l^{*}}^{S 1}\right)^{2}\right\}$ and $G_{1}=$ $\epsilon_{2} R^{2} / \sigma^{2}$. Using a similar approach as in Section IV, the upper bound on the average achievable rate for message stream $x_{2}[n], \mathbb{E}\left\{C_{U, 2}\right\}$, can be derived using (34) by replacing $U 2$ with $R / \sigma^{2}$. The upper bound on the average achievable sum rate of the proposed system can be expressed as

$$
\begin{equation*}
\mathbb{E}\left\{C_{U}^{S U M}\right\}=\mathbb{E}\left\{C_{U, 1}\right\}+\mathbb{E}\left\{C_{U, 2}\right\} \geq \mathbb{E}\left\{C_{1}\right\}+\mathbb{E}\left\{C_{2}\right\} \tag{56}
\end{equation*}
$$

where $\mathbb{E}\left\{C_{U, 1}\right\}$ and $\mathbb{E}\left\{C_{U, 2}\right\}$ are the upper bounds on the average achievable rates for message streams $x_{1}[n]$ and $x_{2}[2]$
separately. Upper bounds on the average achievable rates of optimal, max $S-U 1-U 2$ channel gain based, and best $S-U 1-U 2$ LoS based selection schemes can be found by following a similar approach as in Section IV. Due to the limited space we do not proceed further. However, they are presented in Fig. 4(b) for comparison with the lower bound results.

## F. Complexity of LED/PD Selection Schemes

This subsection summarizes the selection and implementation complexity of each LED/PD selection scheme. The optimal LED/PD selection scheme examine all the possible combinations and therefore the highest complexity is equal to $\left(N_{I} N_{J} N_{K} N_{L}\right)$ operations. The proposed max $S-U 1-U 2$ channel gain based selection scheme selects LEDs/PDs such that highest gains of $S-U 1$ and $U 1-U 2$ links are chosen. Such a selection results in a complexity of $\left(N_{I} N_{J}+N_{K} N_{L}\right)$ operations. The best $S-U 1-U 2$ LoS based selection scheme

$$
\begin{align*}
& \mathbb{E}\left\{C_{2}\right\} \geq \frac{1}{6 N_{I} N_{J} N_{K} N_{L}} \sum_{i=1}^{N_{I}} \sum_{j=1}^{N_{J}} \sum_{k=1}^{N_{K}} \sum_{l=1}^{N_{L}} \int_{0}^{\infty} \log _{2}(1+D y) \times \\
& \left\{\left(1-\frac{1}{2} \operatorname{erfc}\left(\frac{\ln \left(\frac{\left(a_{2} P_{s} G_{i j}^{S 1}\right)^{2}}{y}\right)+4 \mu_{x}}{4 \sqrt{2} \sigma_{x}}\right)\right) \times \frac{1}{4 y \sqrt{2 \pi \sigma^{2}}} \exp \left(-\frac{\left(\ln \left(\frac{y}{\left(G_{k l}^{12}\right)^{2}}\right)-4 \mu_{x}\right)^{2}}{32 \sigma_{y}^{2}}\right)+\right. \\
& \left.\left(1-\frac{1}{2} \operatorname{erfc}\left(\frac{\ln \left(\frac{\left(P_{r} G_{k l}^{12}\right)^{2}}{y}\right)+4 \mu_{y}}{4 \sqrt{2} \sigma_{y}}\right)\right) \frac{1}{4 y \sqrt{2 \pi \sigma^{2}}} \exp \left(-\frac{\left(\ln \left(\frac{y}{\left(a_{2} P_{s} G_{i j}^{S 1}\right)^{2}}\right)-4 \mu_{y}\right)^{2}}{32 \sigma_{y}^{2}}\right)\right\} d y .  \tag{52}\\
& \mathbb{E}\left\{C_{L, 2}\right\} \approx \frac{1}{6 N_{I} N_{J} N_{K} N_{L}} \sum_{i=1}^{N_{I}} \sum_{j=1}^{N_{J}} \sum_{k=1}^{N_{K}} \sum_{l=1}^{N_{L}} \sum_{n=-1}^{1} \frac{1}{4|n|}\left\{\left(1-\frac{1}{2} \operatorname{erfc}\left(\frac{-\ln \left(\frac{\left(a_{2} P_{s} G_{i j}^{S 1}\right)^{2}}{\left(P_{r} G_{k l}^{12}\right)^{2}}\right)-4 \sqrt{3} n \sigma_{y}}{4 \sqrt{3} \sigma_{y}}\right)\right)\right. \\
& \times \log _{2}\left(1+D\left(a_{2} P_{s} G_{i j}^{S 1}\right)^{2} L(n)\right)+\left(1-\frac{1}{2} \operatorname{erfc}\left(\frac{-\ln \left(\frac{\left(P_{r} G_{k l}^{12} y\right)^{2}}{\left(a_{2} P_{s} G_{i j}^{S 1}\right)^{2}}\right)-4 \sqrt{3} n \sigma_{y}}{4 \sqrt{3} \sigma_{y}}\right)\right. \\
& \left.\times \log _{2}\left(1+D\left(P_{r} G_{k l}^{12}\right)^{2} L(n)\right)\right\} . \tag{53}
\end{align*}
$$

selects LEDs/PDs at each node independently, hence a reduced complexity of $\left(N_{I}+N_{J}+N_{K}+N_{L}\right)$ operations is observed.

In terms of implementation complexity, the optimal scheme has the highest complexity since it requires the knowledge of all $N_{I} N_{J}+N_{K} N_{L}+N_{I} N_{L}$ channel gains and LED/PD selection is performed at $S$. The proposed "max $S-U 1-U 2$ channel gain based selection" scheme selects LED/PD pairs for transmission and reception at $S$ and $U 1$ respectively, and $N_{I} N_{J}+N_{K} N_{L}$ channels gains are necessary. The best $S-$ $U 1-U 2$ LoS based selection scheme does not require the knowledge of channel gains. Instead, the LED/PD selection is performed at $S$ and $U 1$ using the position information of $N_{I}+N_{J}+N_{K}+N$ LED/PD links. Such information can be collected prior to the communication phase and would be remain valid until completion of communication. The random selection scheme uses random numbers for LED/PD selection and does not need the prior knowledge of channel gains or position information.

Typically in communication systems, pilot signals are transmitted for CSI acquisition prior to the data communication phase. These pilots can be used with an estimation scheme such as least squares estimation to acquire the CSI at node level. The nodes can feedback the CSI values or corresponding index of a LED/PD pair in order to compute the LED/PD pairs that should be activated at each node. For example, in the case of the optimal selection scheme the procedure for selection can be explained as follows. Pilot signals are sent from $S$ to $U 1$, $S$ to $U 2$, and $U 1$ to $U 2$. Sequential channel estimation is done at $U 1$ and $U 2$. Using low-speed feedback channel from $U 1$ to $S$ and $U 2$ to $S$, all CSI are sent to $S$. The source $S$ computes LED-PD pairs and the respective indices are notified to $U 1$ and $U 2$.

## V. Numerical Results and Discussion

In this section, numerical results are presented to show the performance of the proposed LED/PD selection schemes and

TABLE I
Parameter values

| Parameter | Symbol | Value |
| :---: | :---: | :---: |
| Refractive index | $n$ | 1.25 |
| Absorption coefficient | $a$ | $0.114 \mathrm{~m}^{-1}$ |
| Scattering coefficient | $b$ | $0.037 \mathrm{~m}^{-1}$ |
| Attenuation coefficient | $c$ | $0.151 \mathrm{~m}^{-1}$ |
| Noise variance at receivers | $\sigma_{1}^{2}=\sigma_{2}^{2}$ | $5 \times 10^{-12}$ |
| Mean of the log amplitude factor | $\mu_{x}^{2}=\mu_{y}^{2}$ | -0.1 |
| Variance of the log amplitude factor | $\sigma_{x}^{2}=\sigma_{y}^{2}$ | 0.1 |
| Effective area of the PD | $A_{P D}$ | $10^{-3} \mathrm{~m}^{2}$ |
| FOV of the concentrator | $\Psi$ | $90^{0}$ |
| Divergence angle of the transmitter | $\theta$ | $10^{0}$ |
| Power allocation factor for $x_{1}[n]$ | $a_{1}$ | 0.8 |
| Power allocation factor for $x_{2}[n]$ | $a_{2}$ | 0.2 |
| Responsivity of PDs | $R$ | $0.5 \mathrm{~A} / \mathrm{W}$ |
| Number of LEDs at the source | $N_{I}$ | 5 |
| Number of PDs at $U 1$ | $N_{J}$ | 5 |
| Number of LEDs at $U 1$ | $N_{K}$ | 5 |
| Number of PDs at $U 2$ | $N_{L}$ | 5 |
| Half angle of the uncertainty cone | $\Delta \varphi_{1}$ | $5^{0}$ |

the impact of key system and channel parameters. In particular, we compare the derived expressions and approximations on the average achievable rate of the LED/PD selection schemes with the simulation results to verify the correctness of our analysis. As shown in Fig. 3, a configuration where the $S, U 1$, and $U 2$ are placed at coordinates $(0,0,20),(-2,0,10)$, and $(2,0,0)$, respectively is considered. Unless stated explicitly otherwise in all simulations the employed parameter values are given in Table I. Moreover, the transmission power at $U 1$ is set as $P_{1}=0.1 P_{S}$.

To highlight the performance of LED/PD selection schemes with cooperative NOMA, Fig. 4(a) shows the lower bound on the average achievable sum rate versus source transmit power, $P_{S}$ using different LED/PD selection schemes for NOMA and OMA. Specifically in the OMA protocol, $x_{1}[n]$ is sent from $S$ to $U 1$ and $U 2$ in time slot 1 while $x_{2}[n]$ is sent from $S$ to $U 1$ and from $U 1$ to $U 2$ in time slot 2 and 3, respectively. The results show that cooperative NOMA aided


Fig. 3. Cooperative NOMA aided UOWC setup.
UOWC systems outperform conventional OMA in terms of achievable rate. As expected, the optimal solution shows the best lower bound on the achievable rate in the considered setup. Max $S-U 1-U 2$ channel gain based selection shows near-optimal performance, while best $S-U 1-U 2 \operatorname{LoS}$ based selection shows better performance than the random LED/PD selection scheme. Since our selection criteria is based on data rates, a scheme that maximizes the channel gains will have a superior performance. Due to the channel fluctuations, there is a finite probability that in LoS based scheme, the best channel is not selected. As such, it yields a inferior performance as compared to the channel gain based scheme. Random LED/PD selection has poorer performance when compared to other schemes. This is expected as random LED/PD selection does not exploit knowledge such as the channel gains or the orientation of elements. However, for applications which demand low-complexity implementations and low data rates, the random LED/PD selection is a possible option. Further, our results show that exact expressions and approximations for the lower bound on the average achievable sum rate closely match each other. Fig. 4(b) shows lower and upper bounds on the average achievable sum rate of the proposed LED/PD selection schemes. The results show that lower and upper bounds provided are fairly tight and are useful to understand the exact achievable sum rate. The rankings of the schemes using the upper bounds is also found to be the same as for the lower bound, confirming that either approach gives a useful indication of system performance. As the transmit power increases, the gap between lower and upper bounds further reduces. In the remainder, curves corresponding to the upper bound and OMA are not shown in the figures to avoid excessive clutter.

Fig. 5 shows the lower bound on the average achievable sum rate versus the source $S$ 's transmit power, $P_{S}$, using
best $S-U 1-U 2$ LoS based selection scheme for different $\Delta \Phi_{1}$ values and $x_{1}$. Results show that when $U 1$ is in the vicinity of $S$ and $U 2$, higher performance can be obtained from best $S-U 1-U 2 \operatorname{LoS}$ based selection scheme. When $\Delta \Phi_{1}$ is increased, the performance degrades due to random selection among a higher number of PDs at $U 1$ and $U 2$. Further, it can be noted that performance degradation due to imperfect position information is higher when $U 1$ is not in the vicinity of $S$ and $U 2$. Hence, accurate position information and placement of $U 1$ in the vicinity of $S$ and $U 2$ will lead to better performance.

Results shown in the rest of the figures are for a source transmit power of $P_{S}=30 \mathrm{~W}$. Fig. 6 shows the lower bound on the average achievable sum rate versus power allocation factor $a_{2}$, for the message stream $x_{2}[n]$. Max $S-U 1-U 2$ channel gain based selection scheme and best $S-U 1-U 2$ LoS based selection scheme show near-optimal performance while the performance of random LED/PD selection is inferior. Moreover, the variation of the lower bound on the average achievable sum rate with the power allocation factor is limited. There exists an optimal power allocation factor for each LED/PD selection. This result can be used to select the power allocation factor for different LED/PD selection schemes.

Fig. 7 shows the lower bound on the average achievable sum rate as the placement of $U 2$ is varied. The use of multiple LED/PD elements results in a range of rate pairs for $x_{1}[n]$ and $x_{2}[n]$. On the other hand, when a single LED/PD is employed, the values of $\mathbb{E}\left\{C_{1}\right\}$ and $\mathbb{E}\left\{C_{2}\right\}$ spans a confined region as seen from Fig. 7(a). The max $S-U 1-U 2$ channel gain based selection scheme and best $S-U 1-U 2$ LoS based selection scheme show near-optimal performance. The gap between the lower bounds of max $S-U 1-U 2$ channel gain based selection scheme and best $S-U 1-U 2$ LoS based selection scheme increases when $U 1$ is placed away from the line connecting $S$ and $U 2$. Further, there is an optimal $U 1$ location for each LED/PD selection scheme. Simulation results show that optimal $U 1$ placement for optimal, max $S-U 1-U 2$ channel gain based, best $S-U 1-U 2$ LoS based, and random selection schemes are $(2,0,6),(0,0,5),(0,0,4)$, and $(2,0,1)$ respectively. The corresponding lower bounds on the average achievable sum rate are $5.42 \mathrm{bits} / \mathrm{Hz} / \mathrm{sec}, 5.23 \mathrm{bits} / \mathrm{Hz} / \mathrm{sec}, 4.52$ bits $/ \mathrm{Hz} / \mathrm{sec}$, and $1.67 \mathrm{bits} / \mathrm{Hz} / \mathrm{sec}$ respectively. Moreover, the maximum value in the case of the single LED/PD configuration is $4.34 \mathrm{bits} / \mathrm{Hz} / \mathrm{sec}$. Depending on the availability of CSI or orientation of LEDs/PDs an appropriate LED/PD selection scheme can be selected and for each scheme, an optimal placement of $U 1$ can be found. Further, our results show that by using the LED/PD selection scheme better performance can be obtained for a wide range of $U 1$ positions when compared to a single LED/PD configuration.

Fig. 8(a) shows the lower bound on the average achievable sum rate versus the number of LEDs/PDs at each node, $N$. When the number of LEDs/PDs is increased, the performance of the system increases. However, the performance improvement beyond six LEDs/PDs is negligible. Hence, $N=6$ can be considered as a practical design choice for the purposed system. Further, the performance of the random LED/PD selection reduces with the number of LEDs/PDs and saturates due to the


Fig. 4. Bounds on the average achievable sum rate versus source transmit power. (a) lower bounds for NOMA and OMA. (b) lower and upper bounds for NOMA.


Fig. 5. The lower bound on the average achievable sum rate versus source transmit power.
increase of non-LoS LEDs/PDs. The average achievable sum rate fluctuates in configurations where the number of LED/PD elements is low. When the number of LEDs/PDs is increased, the fluctuations reduce since the possibility of finding an LoS LED/PD pair increases. The performance gap between the optimal scheme and the channel gain based scheme shows a fluctuating behavior for the considered parameter values. This behavior can be explained by recalling that our node design is hemispherical. The number of effective LED/PD pairs actively participating in the selection of the highest channel gain most of the time fluctuate with $N$. As the half power angle of the LEDs, $\theta_{1 / 2}$ increases the gap becomes consistent which is not shown in the figure due to space constraints. This is due to the fact that the effective number of LED/PD pairs actively contributing to the selection pool comes closer to


Fig. 6. The lower bound on the average achievable sum rate versus $a_{2}$.
the cardinality of the total selection set when when $\theta_{1 / 2}$ is increased. In addition, simulations were carried out to observe which node has the highest impact of increasing the number of LEDs/PDs. Our results verified that increasing the number of LEDs at $U 1$ has the highest impact on performance as shown in Fig. 8(b). Also, an extremely poor performance was observed when $U 1$ has only a single LED.

The results above show that the number of LEDs/PDs and the local arrangement of LEDs and PDs within a specific node can affect the performance. Hence when demanded by a specific application requirement, there is scope for optimal design of the number of LEDs/PDs and their best arrangement within the node for additional performance improvement.


Fig. 7. The lower bound on the average achievable sum rate versus position of $U 1$ using different LED/PD selection schemes; (a) single LED/PD configuration; (b) optimal selection; (c) max $S-U 1-U 2$ channel gain based selection; (d) best $S-U 1-U 2$ LoS based selection; and (e) random selection.


Fig. 8. The lower bound on the average achievable sum rate versus number of LEDs/PDs; (a) $N$ LEDs/PDs at each node; (b) $N_{k}$ LEDs at $U 1$.

## VI. Conclusions

In this paper, we have analyzed a cooperative NOMA aided UOWC system consisting of multiple LED/PD elements at the source, relay, and destination. We have presented several LED/PD selection schemes that exhibit different implementation complexity. The lower bounds and approximate expressions of the average sum rate for maximum channel gain based, best LoS based, and random selection schemes were presented. Our system exhibits significant performance gains over a single LED/PD system as well as orthogonal transmission, specially in the medium-to-high power region.

Maximum channel gain based and best LoS based selection schemes achieve results close to the optimum LED/PD selection scheme. Moreover, the performance of random LED/PD selection is significantly inferior to that of optimal selection. This observation clearly highlights the importance of acquiring channel or orientation knowledge to deliver performance gains in low-complexity UOWC systems. For all of the selection schemes, the results reveal that the number of LED/PD elements at the relay and the use of optimal relay placement has a significant impact on the average sum rate.

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Kapila W. S. Palitharathna (Student Member, IEEE) received the B.Sc. Eng. degree (Hons.) in electrical and electronic engineering from the University of Peradeniya, Sri Lanka, in 2016. He is currently pursuing the Ph.D. degree at the University of Peradeniya, Sri Lanka. He is working as a Research Assistant with Sri Lanka Technological Campus, Sri Lanka, since 2018. His research interests include visible light communication, non-orthogonal multiple access, and energy harvesting communications.


Himal A. Suraweeera (Senior Member, IEEE) received the B.Sc. Eng. degree (Hons.) in electrical and electronic engineering from the University of Peradeniya, Sri Lanka, in 2001, and the Ph.D. degree from Monash University, Australia, in 2007. He is currently a Senior Lecturer with the Department of Electrical and Electronic Engineering, University of Peradeniya. He was a recipient of the 2017 IEEE ComSoc Leonard G. Abraham Prize, the IEEE ComSoc Asia Pacific Outstanding Young Researcher Award in 2011, the WCSP Best Paper Award in 2013, and the SigTelCom Best Paper Award in 2017. He served as an Editor for the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS (2014-2019) and the IEEE COMMUNICATIONS LETTERS (2010-2015). He is currently serving as an Editor for the IEEE TRANSACTIONS ON COMMUNICATIONS and the IEEE TRANSACTIONS ON GREEN COMMUNICATIONS. Dr. Suraweera was a Co-Chair of the Signal Processing for Communications Symposium of IEEE Globecom 2015 and a Co-Chair of the Emerging Technologies, Architectures and Services Track of IEEE WCNC 2018. His research interests include cooperative relay networks, full-duplex communications, multiple-input multiple-output systems, energy harvesting communications and cognitive radio systems.


Roshan I. Godaliyadda (Senior Member, IEEE) is a Senior Lecturer at the Department of Electrical and Electronics Engineering, University of Peradeniya. He obtained his B.Sc. Eng. degree (Hons.) in electrical and electronic engineering from University of Peradeniya, Sri Lanka, in 2005 and Ph.D. from the Department of Electrical and Computer Engineering, National University of Singapore in 2011. He has published extensively in high impact journals and presented and published his works in numerous international and national conferences. His research work has received multiple presidential awards and best paper awards. His current research interests include artificial intelligence, computer vision, image and signal processing, pattern recognition, machine learning, remote sensing and smart grid applications.


Vijitha R. Herath (Senior Member, IEEE) received the B.Sc. Eng. degree (Hons.) in electrical and electronic engineering from the University of Peradeniya, Sri Lanka, in 1998, the M.Sc. degree in electrical and computer engineering with the award of academic merit from the University of Miami, FL, USA, in 2002, and the Ph.D. degree from the University of Paderborn, Germany, in 2009. In 2009, he joined the Department of Electrical and Electronic Engineering, University of Peradeniya, as a Senior Lecturer. Since 2020 he is a professor of electrical and electronic engineering at the same institution. His research interests include hyperspectral imaging for remote sensing, multispectral imaging for good quality assessment, coherent optical communications and integrated electronics. Dr. Herath was a member of one of the teams that successfully demonstrated coherent optical transmission with QPSK and polarization multiplexing for the first time. He is currently a senior member of IEEE, a member of the Institution of Engineers, Sri Lanka and a member of the Optica. He was a recipient of the Sri Lanka President's Award for Scientific Research in 2013 and the Engineering Faculty Research Excellence award in 2020. He received the Paper Award in the International Conference on Advances in ICT for Emerging Regions (ICTer) 2017 Conference held in Colombo, Sri Lanka. He was the General Chair of the IEEE International Conference on Industrial and Information Systems (ICIIS) 2013 held in Kandy, Sri Lanka.


John S. Thompson (Fellow, IEEE) received the Ph.D. degree in electrical engineering from University of Edinburgh, Edinburgh, U.K., in 1995. He is currently a Professor with the School of Engineering, University of Edinburgh. He currently participates in two U.K. research projects, which study new concepts for signal processing and next-generation wireless communications. He has authored or coauthored in excess of 350 papers on the topics of his research interests, which include antenna array processing, cooperative communications systems, energy efficient wireless communications, and their applications. He was the Co-Chair of the IEEE Smart Gridcomm Conference held in Aalborg, Denmark, in 2018. In January 2016, he was elevated to Fellow of the IEEE for contributions to antenna arrays and multihop communications. From 2015 to 2018, he was recognized by Thomson Reuters as a Highly Cited Researcher.


[^0]:    ${ }^{1}$ Beyond single relay deployment, multiple relays can be implemented in underwater cooperative communications for coverage extension. Hence studying the NOMA performance under multi-hop transmission is an interesting and worthwhile future research direction.

[^1]:    ${ }^{2}$ In certain maritime applications privacy issues could be important. An effective solution to guarantee privacy in such applications is to use upper layer encryption techniques. Privacy-preserving analysis of upper layer encryption schemes applicable for such cases therefore stands out as an interesting and worthwhile future direction.

