Supplementary Information (SI)

SI1: To equation (2)

With $\dot{\varepsilon}(t)=\alpha\exp(\alpha t)$ and $\alpha>0$, $\sigma(0)=0$ and $\beta=1+\alpha\dfrac{D}{K}$:

$$\begin{aligned}\sigma(t)&=K\int_0^t\alpha\exp(\alpha\tau)\exp\left(\frac{K}{D}(\tau-t)\right)d\tau=K\alpha\exp\left(-\frac{K}{D}t\right)\int_0^t\exp\left(\frac{K}{D}\beta\tau\right)d\tau\\&=\frac{D}{\beta}\alpha\exp\left(-\frac{K}{D}t\right)\left(\exp\left(\frac{K}{D}\beta t\right)-1\right)=\frac{D}{\beta}\alpha\exp(\alpha t)\left(1-\exp\left(-\frac{K}{D}\beta t\right)\right)\\&=\frac{D}{\beta}\dot{\varepsilon}(t)\left(1-q_\beta^t\right)\end{aligned}$$

where $q_\beta^t=\exp\left(-\dfrac{K}{D}\beta t\right)$. Introducing the dimensionless time $n=\dfrac{t}{\Delta t}$ (where $\Delta t=1\,y$), equation (2) takes the form

$$\sigma(n)=\frac{D}{\beta}\frac{\alpha_n}{\Delta t}\exp(\alpha_n n)\left(1-q_\beta^n\right)=\frac{D}{\beta}\dot{\varepsilon}(n)\left(1-q_\beta^n\right)\tag{SI1-1}$$

where $\alpha_n:=\alpha\Delta t$, $q_\beta=q_\alpha q$, $q_\alpha=\exp(-\alpha\Delta t)$, $q=\exp\left(-\dfrac{K}{D}\Delta t\right)$, and $q_\beta^t=\left(\exp\left(-\dfrac{K}{D}\beta\Delta t\right)\right)^n$.

SI2: To equation (3)

We start from equation (2b) in the form $\sigma_D(q,n):=\dfrac{1}{D}\sigma(n)=\dot{\varepsilon}(n)\dfrac{\ln q}{\ln q-\alpha_n}\left(1-q_\alpha^n q^n\right)$ with $\dfrac{1}{\beta}=\dfrac{\ln q}{\ln q-\alpha_n}$:

$$\begin{aligned}q\frac{\partial}{\partial q}\sigma_D(q,n)&=\dot{\varepsilon}(n)q\left\{\frac{\partial}{\partial q}\left(\frac{\ln q}{\ln q-\alpha_n}\right)\left(1-q_\beta^n\right)-\frac{\ln q}{\ln q-\alpha_n}q_\alpha^n\frac{\partial}{\partial q}q^n\right\}\\&=\dot{\varepsilon}(n)\left\{q\frac{\partial}{\partial q}\left(\frac{\ln q}{\ln q-\alpha_n}\right)\left(1-q_\beta^n\right)-\frac{1}{\beta}q_\beta^n n\right\}\end{aligned}\tag{SI2-1a}$$

where we can avoid the effort of writing out the 1st derivative on the right side; and $q\,q_\alpha^n\dfrac{\partial}{\partial q}q^n=q\,q_\alpha^n q^{n-1}n=q_\beta^n n$.

On the other hand, with $S_n = \frac{1-q_\beta^n}{1-q_\beta}$ and the help of equation (SI3-4a):

$$
\begin{aligned}
q\frac{\partial}{\partial q}\sigma_D(q,n) &= \dot{\varepsilon}(n)\,q\frac{\partial}{\partial q}\left(\frac{\ln q}{\ln q-\alpha_n}\left(1-q_\beta\right)S_n\right) \\
&= \dot{\varepsilon}(n)\,q\left\{\frac{\partial}{\partial q}\left(\frac{\ln q}{\ln q-\alpha_n}\right)\left(1-q_\beta\right)S_n - \frac{1}{\beta}\frac{\partial q_\beta}{\partial q}S_n + \frac{1}{\beta}\left(1-q_\beta\right)\frac{\partial q_\beta}{\partial q}\frac{\partial S_n}{\partial q_\beta}\right\} \\
&= \dot{\varepsilon}(n)\left\{q\frac{\partial}{\partial q}\left(\frac{\ln q}{\ln q-\alpha_n}\right)\left(1-q_\beta\right)S_n - \frac{1}{\beta}q_\beta S_n + \frac{1}{\beta}\left(1-q_\beta\right)S_n\frac{q_\beta}{S_n}\frac{\partial S_n}{\partial q_\beta}\right\} \\
&= \dot{\varepsilon}(n)\left\{q\frac{\partial}{\partial q}\left(\frac{\ln q}{\ln q-\alpha_n}\right)\left(1-q_\beta^n\right) - \frac{1}{\beta}q_\beta S_n + \frac{1}{\beta}\left(1-q_\beta^n\right)T\right\}
\end{aligned}
\tag{SI2-1b}
$$

Balancing equations (SI2-1a) and (SI2-1b) yields equation (3):

$$
T = -\frac{q_\beta^n}{1-q_\beta^n}n + \frac{q_\beta}{1-q_\beta}\,. \tag{SI2-2}
$$

$T = T(q,n)$ is a characteristic function of the Maxwell body (MB).

SI3: Justifying T as delay time

We can let any function f of time t (dimensionless throughout SI3 and $\in \mathbb{N}_0$ without restricting generality) depend increasingly on previous times by applying the approach of a simple weighted average and a weighting fading away exponentially backward in time $(q<1)$:

$$
y_1(t) = f\left(\frac{q^0 t}{q^0}\right)
$$

$$
y_2(t) = f\left(\frac{q^0 t + q^1(t-1)}{q^0+q^1}\right)
$$

…

$$
y_k(t) = f\left(\frac{q^0 t + q^1(t-1) + q^2(t-2) + \ldots + q^{k-1}\left(t-(k-1)\right)}{\sum_{i=0}^{k-1} q^i}\right) = f(t-T) \tag{SI3-1}
$$

$(t \ge k \in \mathbb{N}_0)$ with T appearing as delay time in the argument of the function f. The denominator of the argument in the middle is given by

$$S_k = \sum_{i=0}^{k-1} q^i = \frac{1-q^k}{1-q}\,; \tag{SI3-2}$$

while the numerator can be transformed with the help of[1]

$$\sum_{k=a}^{b-1} k^m z^k = \left(z\frac{d}{dz}\right)^m \frac{z^b - z^a}{z-1} \qquad (z \neq 1),$$

here with i instead of k, and q instead of z, and $a = 0$, $b = k$, and $m = 1$

$$\sum_{i=0}^{k-1} i\,q^i = q\frac{d}{dq}\frac{q^k - q^0}{q-1} = q\frac{d}{dq}\frac{1-q^k}{1-q} = q\frac{d}{dq}S_k \qquad (q \neq 1) \tag{SI3-3}$$

to derive T:

$$T = \frac{q}{S_k}\frac{d}{dq}S_k\,. \tag{SI3-4a}$$

Similar to and in accordance with equation (3), carrying out the derivation by q on the right side yields

$$T = \frac{q}{S_k}\frac{1}{1-q}\left(-q^{k-1}k + S_k\right) = -\frac{q^k}{1-q^k}k + \frac{q}{1-q}\,. \tag{SI3-4b}$$

It is straightforward to show by applying l'Hospital that $\lim_{k\to\infty}\left(q^k k\right) = 0$. Thus:

$$T_{k\to\infty}\left(= T_\infty\right) = \frac{q}{1-q} = q\frac{1}{1-q} = q\,S_{k\to\infty}\,. \tag{SI3-5}$$

To strengthen the justification of T as delay time for the exponential function $y(t) = 1 - \exp(c\,t) = 1 - q^t$ with $c = \ln(q)$, it is useful to consider the power-law case $y(t) = c\,t^q$. Here, the ratio $\frac{y}{\dot{y}}$ with $T = q$ functions as a linearizer such that $\frac{y-\dot{y}}{\dot{y}}b = a(t-T)$ $\Leftrightarrow \frac{y-\dot{y}}{\dot{y}} = \frac{1}{T}(t-T)$; where $b = a\,T$ is the intercept, $T = \frac{\dot{y}}{y}t$ is the intersection with the time axis, and the difference $t - T$ can be expressed as well as weighted (w) (or moving weighted) average $(t-T) = \sum_{i=0}^{k-1} w_{k-i}(t-i) \Big/ \sum_{i=0}^{k-1} w_{k-i}$. T being constant is in line with the finding (not shown here) that the change in memory can be considered Gaussian backward in time.

Similar for the exponential function $y(t) = 1 - q^t$. Here, q and t appear mirrored to the power-law case. Nonetheless, T (reduced by T_∞) in equation (SI3-5) can also be expressed, in principle (i.e., apart from additional factors), by the operation

$$T_{red} = T - T_{\infty} = \frac{1}{\ln(q)} \frac{\dot{y}}{y} t \,. \tag{SI3-6}$$

However, despite this agreement, the change in memory here is exponential backward in time. Equation (SI3-6) generalizes to $T - T_{\infty} = \frac{1-\beta}{\beta} \frac{\dot{\sigma} - \alpha\sigma}{\alpha\sigma} t$ in the case of equation (2a).

SI4: To equation (4) reflecting the history of the MB

Rewriting equation (4) shows that it reflects the history of the MB:

$$\begin{aligned} S_n &= \frac{q_\beta^n - 1}{q_\beta - 1} = \frac{1}{q_\beta - 1}\left\{\left(q_\beta^n - q_\beta^{n-1}\right) + \left(q_\beta^{n-1} - q_\beta^{n-2}\right) + q_\beta^{n-2} - + \ldots - q_\beta + \left(q_\beta - 1\right)\right\} \\ &= \frac{1}{q_\beta - 1}\left\{q_\beta^{n-1}\left(q_\beta - 1\right) + q_\beta^{n-2}\left(q_\beta - 1\right) + \ldots + q_\beta^{0}\left(q_\beta - 1\right)\right\} = \sum_{i=0}^{n-1} q_\beta^i = \text{Past} \end{aligned} \tag{SI4-1}$$

SI5: To monitoring ln(M · P)

According to equations (3)–(5):

$$M_{\infty} = \frac{1}{1 - q_\beta} = \frac{T_{\infty}}{q_\beta} \text{ and } P_{\infty} = \frac{1}{T_{\infty}} \,. \tag{SI5-1,2}$$

Hence:

$$\frac{1}{M_{\infty} P_{\infty}} = q_\beta = \exp\left(-\left(\frac{K}{D} + \alpha\right)\Delta t\right) = \exp\left(-\frac{K}{D}\beta\,\Delta t\right) \quad \Leftrightarrow \quad \ln\left(M_{\infty} P_{\infty}\right) = \frac{K}{D}\beta\,\Delta t = \lambda_\beta = \lambda\beta$$

with q_β and q as defined under Methods, and $\lambda_\beta = \lambda\beta$ with $\lambda = \frac{K}{D}\Delta t$. Thus, the ratio $\frac{\lambda}{\ln(MP)}$ allows indicating how much smaller the system's natural rate of change in the numerator turns out compared to the system's rate of change in the denominator under continued increase in stress. This gradual build-up relative to λ (with K/D constant) is limited by β^{-1}.

SI6: Overview of data and conversion factors

Tab. SI6-1: Overview of the data used in the paper. All data refer to the global scale (or are assumed to be globally representative).

Data	Source	Time range	Brief description
Atmospheric CO_2 concentration (in ppm)	2 Degrees Institute, Canada[2]	1750–1955	Ice core data (75-year smoothed); Law Dome, Antarctica
		1959–1979	Atmospheric measurements (annual means); Mouna Loa, Hawaii
	Global Monitoring Laboratory, NOAA, USA[3]	1980–2018	
CO_2 emissions from fossil-fuel combustion and cement production (in PgC y^{-1})	Global Carbon Project[4]	1751–1958	Global estimates derived from energy statistics by nation and year
		1959–2015	
Land-use change emissions (in PgC y^{-1})		1850–1958	Global mean values derived from multiple models
		1959–2015	
Net primary production (in PgC y^{-1})	O'Sullivan et al. (2019)[5]	1900–2016	Model-based global mean values (Community Land Model; CLIM4.5-BGC)
Dissolved organic carbon (in μmol kg^{-1})	Bates et. al. (2014)[6]	1983–2012 (max. range)	Shipboard observations (annual means); from 7 sites (2 in the subpolar North Atlantic and 5 in the tropical/subtropical/temperate waters of the North Atlantic and Pacific

Tab. SI6-2: Overview of the conversion factors used in the paper.

From	To	Value	Unit	Source
C	CO_2	3.664	$gCO_2\ (gC)^{-1}$	CDIAC (2012: Tab. 3)[7]
ppmv CO_2	PgC	2.120	PgC $ppmv^{-1}$	Ciais et al. (2014: Tab. 6.1)[8]
ppmv CO_2	Pa	0.101325	Pa $(10^6\ ppmv)^{-1}$	CDIAC (2012: Tab. 3)[7] and Dalton's law[9]

SI7: Use of equation (9) to estimate the photosynthetic carbon flux ratio $\Delta Ph_i / Ph$

The leaf-level factor L denotes the relative leaf photosynthetic response to a 1 ppmv change in the atmospheric concentration of CO_2. The photosynthetic limits L_1 (photosynthesis limited by electron transport) and L_2 (photosynthesis limited by rubisco activity) are determined by using equations (7) and (9) in Luo et al. (1996).[10]

We follow equation (9) to derive the photosynthetic carbon flux ratio $\Delta Ph_i / Ph$ by the change in L_i, which we describe by means of a geometric sequence (with the common ratio $1-q_{L_i}$). We demonstrate the quality of this approximation by comparing our results (to the extent possible) with those cited by Luo et al. (1996). Dropping index i:

$$\begin{aligned} & L_{high} - L_{low} \\ & = \Delta L = L_{high} + L_{high}\left(1-q_L\right) + \ldots + L_{high}\left(1-q_L\right)^{\left(\Delta CO_2 - 1\right)} \\ & \quad = L_{high} \sum_{k=0}^{\Delta CO_2 - 1} \left(1-q_L\right)^k = L_{high} \frac{1-\left(1-q_L\right)^{\Delta CO_2}}{1-\left(1-q_L\right)} \end{aligned} \tag{SI7-1}$$

where $q_L = \Delta L / \left(L_{high}\, \Delta CO_2\right)$. (We follow the authors and express L_i in units of % [and not in % ppmv^{-1}]. To express q_L in units of 1, we consider ΔCO_2 dimensionless [equivalent to multiplying ΔCO_2 with ppmv^{-1}].) The term L_{high} has to be replaced by the term $L_{high}\, f_{ppm}$ if ΔL is not calculated per 1-ppmv step but per 1-year step (when the change in ppmv is not necessarily 1 ppmv; see also SD1). With the values in Table SI7-1, equation (SI7-1) allows accumulated ΔL_i values to be derived which can be compared with the $\Delta Ph_i / Ph$ values reported by Luo et al. (1996) in their Table 1.[10] The agreement is sufficient for our purposes (Tab. SI7-2).

Tab. SI7-1: Limits of the relative leaf photosynthetic response to a 1 ppm change in the atmospheric concentration of CO_2 using equations (7) and (9) in Luo et al. (1996).

Time	**CO_2**	**L_1**	**L_2**
y	ppmv	%	%
preindustrial	280	0.1827	0.3520
1958	315	0.1457	0.2969
1992	355.5	0.1155	0.2495
1993	357	0.1146	0.2479

Tab. SI7-2: Comparison of ΔL_i (accumulated) derived with equation (SI7-1) with $\Delta Ph_i / Ph$ as listed in Table 1 in Luo et al. (1996).

Period	**ΔCO_2**	**q_{L1}**	**q_{L2}**	**ΔL_1**	**ΔL_2**	**$\Delta Ph_i/Ph$**
y	ppmv	1	1	%	%	%
1992–1993	1.5	0.005358	0.004031	0.17	0.37	0.17–0.37
1958–1993	42	0.005080	0.003929	5.6	11.5	5.6–12.1
preindustrial –1993	77	0.004839	0.003840	11.8	23.5	11.8–25.5

SI8: The compression module referring to a tropospheric expansion of 20 m (standard atmosphere)

The standard atmosphere assigns a temperature gradient of -6.5 °C/1000 m up to the tropopause at 11 km. The isentropic coefficient of expansion γ varies with temperature and atmospheric CO_2 concentration: γ increases with decreasing T and decreases with increasing atmospheric CO_2.[11] However, in the case of dry air and no change in its chemical composition, the compression module K_{ad} can be expected to stay constant. Here we provide an overview of the altitudes different isentropic coefficients of expansion refer to assuming a tropospheric expansion of 20 m;[12,13] and, thereupon, determine K_{ad}.

Combining equations (19) and (20b):

$$K_{ad} = \gamma\, p = -\frac{\Delta p}{\Delta V/V} \tag{SI8-1}$$

where the difference in pressure for a difference in altitude $\Delta h = h_2 - h_1$ is given by

$$\Delta p = p_2 - p_1 = p_0\left[\left(1 - a\left(h_1 + \Delta h\right)\right)^b - \left(1 - a\, h_1\right)^b\right]$$

according to equation (7) in Cavcar (2000)[14] with $p_0 = 1013.25\,\text{hPa}$, $a = 0.0065/T_0$, $T_0 = 288.15\,\text{K}$, $b = 5.2561$, and h the altitude in units of meter;

and the difference in volume by

$$\frac{\Delta V}{V} = \frac{V_2 - V_1}{V_1 - V_{Earth}} = \frac{\left(r_{Earth} + \left(h_1 + \Delta h\right)/1000\right)^3 - \left(r_{Earth} + h_1/1000\right)^3}{\left(r_{Earth} + h_1/1000\right)^3 - r_{Earth}^3}$$

with $r_{Earth} = 6371\,\text{km}$.

Letting p refer to p_1 in equation (SI8-1) and solving for γ:

$$\gamma = \frac{1 - \left\{\left(1 - a\left(h_1 + \Delta h\right)\right)/\left(1 - a\, h_1\right)\right\}^b}{\Delta V/V}\ . \tag{SI8-2}$$

Setting $\Delta h = 20\,\text{m}$ in agreement with observations, equation (SI8-2) allows calculating γ in dependence of h_1 (see Tab. SI8-1). As can also be seen from the table, the value of K_{ad} ranges between 400 and 412 hPa.

Tab. SI8-1: Standard atmosphere: isentropic coefficient of expansion γ and compression module K_{ad} for a tropospheric expansion of 20 m at different altitudes.

h_1	γ	p_1	K_{ad}
m	1	hPa	hPa
Input	Eq. (SI8-2)	Eq. (7) in Cavcar (2000)	Eq. (SI8-1)
7,100	1.000	404.8	404.8
7,685	1.100	372.5	409.6
8,255	1.200	343.0	411.6
8,810	1.300	316.2	411.1
8,865	1.310	313.6	411.0
9,345	1.400	292	408.8
9,360	1.403	291.3	408.7
9,865	1.500	269.9	404.9
10,370	1.600	249.7	399.6

SI9: Equation (1a) with strain given by a second-order polynomial

We start from $\varepsilon(t)=c_2\,t^2+c_1\,t$. Inserting $\dot{\varepsilon}(t)=2c_2\,t$ into equation (1a) with $\sigma(0)=0$ and $\int x\,e^{cx}\,dx=e^{cx}\,\frac{cx-1}{c^2}$:[15]

$$\sigma(t)=K\int_0^t\dot{\varepsilon}(\tau)\exp\left(\frac{K}{D}(\tau-t)\right)d\tau=2c_2\,K\exp\left(-\frac{K}{D}t\right)\int_0^t\tau\exp\left(\frac{K}{D}\tau\right)d\tau$$

$$=2c_2\,K\exp\left(-\frac{K}{D}t\right)\left\{\exp\left(\frac{K}{D}\tau\right)\frac{\frac{K}{D}\tau-1}{\left(\frac{K}{D}\right)^2}\right\}_0^t=2c_2\,K\exp\left(-\frac{K}{D}t\right)\left\{\exp\left(\frac{K}{D}t\right)\frac{\frac{K}{D}t-1}{\left(\frac{K}{D}\right)^2}+\frac{1}{\left(\frac{K}{D}\right)^2}\right\}$$

$$=2c_2\,\frac{D^2}{K}\left\{\frac{K}{D}t-1+\exp\left(-\frac{K}{D}t\right)\right\}\xrightarrow[t\to\infty]{}2c_2\,D\left(t-\frac{D}{K}\right)$$

SI10: Overview of parameters in experiments B and C

Table SI10-1 provides an overview of the parameters which result from the set of stress and strain explicit experiments B and C. They can be understood as a repetition of the 1959–2015 Case 0 experiment (see A.1 in the Results section), but with the difference that now upstream emissions as of 1900 (B) or 1850 (C), respectively, are considered; thus allowing initial conditions for 1959 other than zero as in the Case 0 experiment to be taken into account:

Case 0: 1959–2015
B: 1900–1958 (upstream emissions), 1959–2015
C: 1850–1958 (upstream emissions), 1959–2015.

The experiments are ordered consecutively in term of time. By way of contrast, Table SI10-2 comprises the parameters of the three 1959–2015 periods in the form of min–max intervals. Except for the exponential growth factor α, these intervals are dominated by Case 0 and B (1959–2015) parameters (as shown by the background color of the cells); mirroring the fact that we had difficulties with describing the entire upstream period 1850–1958 by means of a single exponential growth factor (0.0151 y^{-1}).

Nonetheless, Table SI10-2 allows drawing a number of robust results:

- The compression modulus K increased between 1850 and 1959–2015 from ~2 to 10–13 Pa (the atmosphere became less compressible);
- while the damping constant D decreased between 1850 and 1959–2015 from ~468 to 459–462 Pa y (the uptake of carbon by land and oceans became less viscous);
- with the consequence that the ratio $\lambda=K/D$ increased between 1850 and 1959–2015 from ~0.004–0.005 y^{-1} to 0.021–0.028 y^{-1} (i.e., by a factor of 4 to 6).

- Delay time T_∞ decreased (hence persistence P_∞ increased) between 1850 and 1959–2015 from ~51 (~0.02) to 18–21 (0.047–0.055) on the dimensionless timescale;
- while memory M_∞ decreased between 1850 and 1959–2015 from ~52 to 19–22 on the dimensionless timescale.

Tab. SI10-1: Overview of parameters in experiments B and C.

Parameters		**Case 0**	**B**		**C**	
		1959–2015	**1900–1958**	**1959–2015**	**1850–1958**	**1959–2015**
		stress explicit				
σ(0)	Pa	0	0	5.8	0	7.8
K	Pa	9.9	2.4	12.7	2.1	11.6
D	Pa y	461.5	467.7	459.2	467.9	460.1
$\lambda^{a,b}$	y^{-1}	0.0214	0.0051	0.0276	0.0045	0.0253
λ^{-1}	y	46.8	196.3	36.3	223.5	39.6
α^{a}	y^{-1}	0.0247	0.0228	0.0262	0.0151	0.0281
β	1	2.158	5.475	1.951	4.371	2.112
$\lambda_\beta{}^{a}$	y^{-1}	0.0461	0.0279	0.0538	0.0196	0.0533
λ_β^{-1}	y	21.7	35.9	18.6	51.1	18.7
q_β	1	0.9549	0.9725	0.9476	0.9806	0.9481
T_∞	1	21.2	35.4	18.1	50.6	18.3
$M_\infty = T_\infty/q_\beta$	1	22.2	36.4	19.1	51.6	19.3
$P_\infty = 1/T_\infty$	1	0.0472	0.0283	0.0553	0.0197	0.0548
$\lambda/\lambda_\beta = 1/\beta$	%	46.3	18.3	51.3	22.9	47.3
SUMXMY2	Pa^2	1.400	1.399	21.000	1.100	60.902
		strain explicit				
ε(0)	1	0	0	2.5	0	4.3
α^{a}	y^{-1}	0.0247	0.0214	0.0257	0.0162	0.0270

[a] Given in y^{-1}.

[b] Derived for K and D deviating from their respective mean values equally in relative terms.

Tab. SI10-2: Like Table SI10-1; with the difference that Table SI10-2 comprises the parameters of the three 1959–2015 periods in terms of min-max intervals. The background colors of the cells in Table SI10-1 are preserved.

Parameters		**C**	**B**	**Min–Max: Case 0 and B and C**	
		1850–1958	**1900–1958**	**1959–2015**	
		stress explicit			
$\sigma(0)$	Pa	0	0	---	---
K	Pa	2.1	2.4	9.9	12.7
D	Pa y	467.9	467.7	459.2	461.5
$\lambda^{a,b}$	y^{-1}	0.0045	0.0051	0.0214	0.0276
λ^{-1}	y	223.5	196.3	36.3	46.8
α^{a}	y^{-1}	0.0151	0.0228	0.0247	0.0281
β	1	4.371	5.475	1.951	2.158
$\lambda_\beta{}^{a}$	y^{-1}	0.0196	0.0279	0.0461	0.0538
$\lambda_\beta{}^{-1}$	y	51.1	35.9	18.6	21.7
q_β	1	0.9806	0.9725	0.9476	0.9549
T_∞	1	50.6	35.4	18.1	21.2
M_∞ $=T_\infty/q_\beta$	1	51.6	36.4	19.1	22.2
P_∞ $=1/T_\infty$	1	0.0197	0.0283	0.0472	0.0553
$\lambda/\lambda_\beta = 1/\beta$	%	22.9	18.3	46.3	51.3
SUMXMY2	Pa^2	1.100	1.399	---	---
		strain explicit			
$\varepsilon(0)$	1	0	0	---	---
α^{a}	y^{-1}	0.0162	0.0214	0.0247	0.0270

Acronyms and Nomenclature (used in Ms No. esd-2021-27 and in this SI)

ad	adiabatic
C	carbon
comb	combined
CO_2	carbon dioxide (chemical formula)
CO_2	atmospheric CO_2 concentration (in ppmv; parameter)
D	damping constant (in Pa y)
DIC	dissolved inorganic carbon (in μmol kg^{-1})
E	Young's modulus (in Pa)
GHG	greenhouse gas
h	altitude (in m)
it	isothermal
K	compression modulus (in Pa)
L	land (index)
L	leaf-level factor (in $ppmv^{-1}$; parameter)
M	memory (in units of 1)
MB	Maxwell body
n.a.	not assessable
NPP	net primary productivity (in PgC y^{-1})
O	oceans
p	atmospheric pressure (in hPa)
pCO_2	partial pressure of atmospheric CO_2 (in μatm)
P	persistence (in units of 1)
Ph	global photosynthetic carbon influx (in PgC y^{-1})
q	auxiliary quantity (in units of 1)
red	reduced
R	Revelle (buffer) factor (in units of 1)
SD	supplementary data
SE	sensitivity experiment
SI	supplementary information
t	time (in y)
T	delay time (in units of 1)
TOA	top of the atmosphere
w	weight(ed)
α	exponential growth factor of the strain (in y^{-1})
α_{ppm}	exponential growth factor of the atmospheric CO_2 concentration (in y^{-1})
β	auxiliary quantity (in units of 1)
β_b	biotic growth factor (in units of 1)
β_{Ph}	photosynthetic beta factor (in units of 1)
ε	strain (referring to atmospheric expansion by volume and CO_2 uptake by sinks; in units of 1)
γ	isentropic coefficient of expansion (in units of 1)

κ compressibility (in Pa^{-1})

σ stress (atmospheric CO_2 emissions from fossil fuel burning and land use; in Pa)

References

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