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Tracing Uncertainty Contributors in the Multi-Hazard Risk Analysis for Compound Extremes

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Key Points:

- An iterative factorial multimodel Bayesian copula framework was developed for tracing uncertainty contributors in multi-hazard risk analysis
- The contributions of marginals, copulas, and parameters to predictive risks were quantified through an iterative factorial analysis
- Results suggested marginal and dependence structures, and parameter uncertainties would have different impacts on different risk indices

Supporting Information:

Supporting Information may be found in the online version of this article.

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Abstract

In this study, an iterative factorial multimodel Bayesian copula (IFMBC) framework was developed for revealing uncertainties in risk inferences of compound extremes under the consideration of diverse model structures and parameter sets. Particularly, an iterative factorial analysis (IFA) method would be advanced in IFMBC to track the dominant contributors to the imprecise predictions of multi-hazard risks. The developed IFMBC framework was applied for the risk assessment of compound floods at two estuarine systems (i.e., Washington and Philadelphia) in US. The results indicate that the most likely compound events, under predefined return periods, exhibit noticeable uncertainties. Those uncertainties also present multiple hotspots which may be attributed to different impacts from different factors. By applying the IFA method, the results suggest the copula structure would likely be ranked as one of the top 2 impact factors for predictions of failure probabilities (FPs) in the scenarios of AND, and Kendall, with its contributions higher than 30% for FP in Kendall (more than 40% at Washington) and more than 25% for FP in Kendall (larger than 40% at Philadelphia). In comparison, the copula structure may not pose a visible effect on the predictive uncertainty for FP in OR, with its contribution possibly less than 5% under long-term service time periods. However, the marginal distributions would have higher effects on FP in OR than the effects on the other two FPs. Particularly, the marginal distribution for the extreme variable with high skewness and kurtosis values tends to be ranked as one of the most significant impact factors for FP in OR. Also, the overall impacts from parametric uncertainties in both marginal and dependence models cannot be neglected for the predictions of all three FPs with their contributions probably larger than 20% under a short service time period. Compared with the traditional multilevel factorial analysis, the IFA method can provide more reliable characterization for uncertainty contributors in multi-hazard risk analyses, since the traditional method seems to significantly overestimate the contributions from parameter uncertainties.

Plain Language Summary

The risk analysis for compound extremes, consisting of concurrent or consecutive hazard drivers, is of great importance for disaster resilience and infrastructure designs, in which extensive uncertainties are inevitable issues embedded in various components such as model structure and parameters. Overlook of these uncertainties may cause undesired resilience strategies for compound extremes and further lead to unpredictable damages or fatalities. This study developed an innovative framework to generate the critical thresholds for compound floods as well as their predictive regions under consideration of uncertainties in model structures and parameters. Moreover, the dominant contributors to the predictive uncertainties in multi-hazard risk inferences were revealed through a reliable analysis technique in the developed framework. Such a framework can help generate desired design thresholds with their predictive confidence for compound extreme events, and also direct the most efficient pathway to enhance/improve the risk inferences. Moreover, the developed framework can be extended to high-dimensional compound extremes and have a wide application potential.

1. Introduction

The compound extremes in the hydroclimatic system, resulting from combinations of two or more (not necessarily) extreme events, may trigger significant consequences much larger than the sum of impacts from individual extremes alone (Leonard et al., 2014; Mehran et al., 2017; Seneviratne et al., 2021; Wahl et al., 2015). The compound extremes can be classified as (a) preconditioned events, (b) multivariate events, (c) temporally compounding events, and (d) spatially compounding events (Zscheischler et al., 2020). The impacts of concurrent extremes on land area have increased with a high confidence (Seneviratne et al., 2021). Without consideration of multiple hazards, the risk reduction efforts targeting one type of hazard may increase exposure and vulnerability to other hazards in the present and future (IPCC, 2012). This necessitates the development of effective modeling tools for reliable and accurate characterization for multi-hazard risk analyses of compound hazards.

There have been a number of studies to address the frequency and severity of compound events. De Michele and Salvadori (2003) first adopted the copula method for addressing dependence between storm duration and rainfall intensity. This kind of approaches has been extensively developed to explore the interdependence among different hydroclimatic extremes. Some typical studies include compound floods consisting of sea level rise and river discharge (Moftakhari et al., 2017; Sadegh et al., 2018), storm surge and river discharge (Bevacqua et al., 2017; Muñoz et al., 2020; Wahl et al., 2015), dry-hot extremes (Alizadeh et al., 2020; Sun et al., 2019, 2021), multivariate standardized drought index characterized by precipitation and soil moisture (Hao & AghaKouchak, 2013, 2014), and flood peak and volume (Fan et al., 2018; Xu et al., 2017). The copula approaches have been widely used for multi-hazard risk analyses of compound extremes due to their attractive capabilities of (a) effectively characterizing complex dependence among correlated random variables, (b) flexibly choosing different marginal and dependence models, and (c) separately estimating parameters in marginal and dependence functions (AghaKouchak et al., 2012; Fan, Huang, Huang, & Li, 2020; Huang et al., 2017; Salvadori et al., 2007).

Uncertainty analysis has been one critical challenge for risk inferences of both individual and compound extremes, which has received great attention in different branches of hydrology and climate science (Dung et al., 2015; Fan et al., 2018; Sadegh et al., 2017, 2018; Shi et al., 2008; Zhang et al., 2020). For instance, Sadegh et al. (2017) developed a multivariate copula analysis toolbox to describe dependence and underlying uncertainty through a Bayesian framework. Dung et al. (2015) have handled uncertainty in bivariate quantile estimation for flood hazard analysis in the Mekong Delta through bootstrap-based algorithms. In addition, some studies have been reported to reveal contributions of uncertain parameters to the resulting risk inferences of multi-hazards. For example, a Bayesian information-theoretical approach has been developed by Guo et al. (2020) to explore uncertainty propagation from model parameters to design flood estimations. The effects of parameter uncertainties on multivariate risk analyses have also been addressed by Fan, Huang, Huang, and Li (2020) and Fan, Huang, Huang, Li, and Wang (2020). However, most previous studies merely focused on parameter uncertainties whilst uncertainties in model structures are somewhat overlooked possibly due to some technical difficulties. For instance, the model parameters can be considered as continuous variables while the model structures would only be denoted as discrete variables with limited choices/samples, thus some sampling-based methods (e.g., the mutual information partitioning method in Guo et al., 2020, Sobol's global sensitivity analysis in Huang & Fan, 2021) may not be applicable to deal with uncertainties in model structures. For the multi-hazard risk analysis of compound extremes, uncertainties may exist in different components such as model parameters and structures of marginal and dependence functions. The interactions among those uncertainties may also challenge the robustness of multi-hazard risk analyses for compound extremes. Consequently, it is desirable to effectively reflect those uncertainties in the process of multi-hazard risk analyses and further reveal their effects on the resulting risk predictions for specific extreme events.

Therefore, an iterative factorial multimodel Bayesian copula (IFMBC) framework was developed in this study. The developed IFMBC framework is able to (a) generate ensemble inferences for the multi-hazard risks of compound extremes under consideration of multiple uncertainties, and (b) characterize the main effects of different uncertain factors and their interactions on the predictive variabilities in multi-hazard risk predictions. The proposed IFMBC framework consists of different modules including (a) the copula models with diverse marginal and dependence structures, (b) Bayesian estimation approach and (c) iterative factorial analysis (IFA). In IFMBC, the interdependence of compound extremes was characterized through copula models made of various marginal and dependence functions. The parameter uncertainties in each copula model were quantified by using a Bayesian (e.g., MCMC) method. An IFA method was finally advanced for characterizing the main effects and interactions of uncertain factors on the predictive uncertainties in multi-hazard risk inferences. The developed IFMBC framework was then applied for compound flood risk analyses at two coastal sites in US.

2. Methodology

2.1. Multi-Hazard Risk Analysis for Compound Extremes

The basic theorem of copula was first introduced by Sklar (1959), which defined the multivariate distribution function on $\boldsymbol{I}^d = [0, 1]^d$ with uniform marginals (Salvadori et al., 2011). Copulas can be employed to establish the joint distributions for multiple random variables with diverse correlation and dependence structures, regardless of their margins (Laux et al., 2011; Liu et al., 2015). Let F be the d-dimensional joint probability distribution for a random vector $\boldsymbol{X} = [X_1, X_2, \ldots, X_d]^T$. There would exist a copula function such that:

$$P\{X_1 \le x_1, \ldots, X_d \le x_d\} = F(x_1, \ldots, x_d) = C(u_1, \ldots, u_d|\theta) \tag{1}$$

where C is the copula function with θ being its parameters. $u_i = F_i(x_i|\gamma_i)$ $(i = 1, 2, \ldots, d)$ and F_i is the marginal distribution for X_i with γ_i being its parameters. The copula C is unique if F_i $(i = 1, 2, \ldots, d)$ are continuous.

Based on the joint probability function expressed through the copula method, risk analyses for compound extremes can be conducted. There are several indices for multi-hazards risk analysis such as multivariate return period (RP), multivariate failure probability (FP), and the most-likely compound events (Joe, 2014; Read & Vogel, 2015; Salvadori et al., 2013, 2016). Moreover, there are three scenarios to characterize hydroclimatic risks within a multivariate context, which are denoted as "AND", "OR", and "Kendall" cases. In detail, a general form for the multivariate RP can be expressed as:

$$T^* = \frac{\mu}{\Pr(\boldsymbol{x} \in R_*^d)} \tag{2}$$

where the * indicate the three scenarios (i.e., "AND", "OR", and "Kendall") for multivariate RP, and R_*^d denote the hazardous regions under these three scenarios. Similarly, the multivariate FP can be expressed in a general form as:

$$p_M^* = 1 - (1 - \Pr(\boldsymbol{x} \in R_*^d))^M \tag{3}$$

where p_M^* indicates the FP in "AND", "OR" or "Kendall", and M denotes the given time period in years. The most likely compound extreme describes the event with the highest joint probability density among all the feasible combinations with equal RPs (Guo et al., 2020; Sadegh et al., 2018). Consequently, such a most likely compound extreme can be derived as:

$$\boldsymbol{x}^q = \underset{((x_1, x_2, \ldots, x_d) \in L_q^F)}{\operatorname{argmax}} h_C(u_1, u_2, \ldots, u_d|\theta) \tag{4}$$

where $h(.)$ denotes the joint probability density function derived based on the copula function. The critical layer L_q^F is defined as:

$$L_q^F = \left\{\boldsymbol{x} = (x_1, x_2, \ldots, x_d) : \Pr(\boldsymbol{x} \in R_*^d) = q\right\} \tag{5}$$

The detailed formulations for the multivariate RPs, multivariate FPs, and the most-likely compound events under "AND," "OR," and "Kendall" scenarios are presented in Section S1 in Supporting Information S1.

2.2. Uncertainties in the Multi-Hazard Risk Analysis of Compound Extremes

2.2.1. Model Structural Uncertainty

The copula-based multi-hazard risk analysis would be affected by the structures in both marginal distributions (i.e., $F_i(x_i|\gamma_i)$ in Equation 1) and the dependence model (i.e., C(.) in Equation 1). For the individual attributes of compound extremes, there are many distributions available to quantify their probabilistic features, including both parametric and non-parametric distributions. Typical parametric distributions mainly include normal family (e.g., two- and three-parameter lognormal distribution), GEV family (e.g., Gumbel, GEV, Weibull distribution), Pearson Type III family (e.g., Pearson Type III and log-Pearson Type III distribution), generalized logistic family (e.g., generalized logistic and log-logistic distribution), and so on (Ahmed et al., 1988; Li et al., 2018; Lin & Dong, 2019; Longfield et al., 2019; Singh, 1998; Stedinger et al., 1993). Some non-parametric methods have also been adopted for modeling the distributions of individual extreme variables such as maximum entropy probability distribution (e.g., Kong et al., 2015), kernel distribution (e.g., Karmakar & Simonovic, 2008, 2009) and Gaussian mixture model (e.g., Sun et al., 2019). In general, the probabilistic distribution for individual extreme variables is chosen mainly through three methods including official recommendation, experience knowledge, and statistical test (Qi et al., 2016). However, there may be many candidate distributions for one variable that pass statistical tests, leading to uncertainty resulting from probability distribution selection (Qi et al., 2016). In this study, three candidate distributions, including Pearson Type III (P3), 3-parameter lognormal (LN3), 3-parameter log-logistic (LLOGIS3) distributions, would be adopted for individual extreme variables, with their formulae presented in Table S1 in Supporting Information S1.

Moreover, to model the interdependence among extreme variables, a number of copula functions have been proposed, which are grouped into diverse families such as Archimedean and elliptical families (Brechmann & Schepsmeier, 2013). Each family also has several copulas with different expressions. For instance, the elliptical family has Gaussian and Student *t* copulas, while the Archimedean family would have copulas such as Gumbel, Frank, and Joe. Different copulas have been employed to quantify the risks of compound extremes (Corbella & Stretch, 2012; Li et al., 2018; Montes-Iturrizaga & Heredia-Zavoni, 2015), and the selection of the copula function may significantly influence the resulting risks or environmental contours (Montes-Iturrizaga & Heredia-Zavoni, 2016). In this study, the Frank, Joe, and Gumbel copulas, as presented in Table S2 in Supporting Information S1, were considered as the candidate models for quantifying the interdependence of compound extremes.

2.2.2. Parametric Uncertainty

Once the copula model has been formulated with pre-specified marginal and dependence structures, the parameter uncertainties (i.e., γ_i and θ in Equation 1) would also produce noticeable impacts on the predictive variabilities for the resulting risk analyses of compound extremes (e.g., Fan et al., 2018; Guo et al., 2020; Sarhadi et al., 2016). There are several methods for quantifying parameter uncertainties in copula models, such as Monte Carlo simulation (e.g., Montes-Iturrizaga & Heredia-Zavoni, 2017), bootstrapping methods (e.g., Dung et al., 2015; Fan, Huang, Huang, Li, & Wang, 2020) and Bayesian inferences (e.g., Fan et al., 2018; Guo et al., 2020; Sadegh et al., 2017).

In this study, a Bayesian inference approach will be employed for uncertainty quantification of parameters in the multi-hazard risk models with prescribed marginal and dependence structures. Bayesian analysis has been successfully applied in different fields for model inference and uncertainty quantification purposes, including multivariate hydrologic risk analyses (e.g., Fan, Huang, Huang, & Li, 2020; Fan et al., 2018; Sadegh et al., 2017). The Bayes' theorem updates the prior probability of a certain hypothesis when new information is available, and then derives the posterior distributions of model parameters as follows:

$$\pi(\theta|\tilde{\mathbf{Y}}) = \frac{L(\theta|\tilde{\mathbf{Y}})\pi_0(\theta)}{\int L(\theta|\tilde{\mathbf{Y}})\pi_0(\theta)d\theta} \tag{6}$$

where $\pi_0(\theta)$ and$\pi(\theta|\tilde{\mathbf{Y}})$ respectively represents the prior and posterior distributions of model parameters, $L(\theta|\tilde{\mathbf{Y}})$ denotes the likelihood function, $\int L(\theta|\tilde{\mathbf{Y}})\pi_0(\theta)d\theta$ is the normalization constant, and $\tilde{\mathbf{Y}} = \{\tilde{y}_1, \tilde{y}_2, ..., \tilde{y}_m\}$ are the observations. In practical applications, Equation 6 can hardly be solved analytically and numerical methods such as Markov Chain Monte Carlo (MCMC) algorithms are employed to approximately estimate the posterior distribution of $\pi(\theta|\tilde{\mathbf{Y}})$. The major purpose of this study is to reveal the major contributors to predictive uncertainties in multi-hazard risk analysis for compound extremes and thus the adaptive Metropolis (AM; Haario et al., 2001) algorithm, which is one of the most common MCMC methods, was adopted. Detailed descriptions for AM-based MCMC method can be referred to relevant studies (e.g., Haario et al., 2001).

2.3. Iterative Factorial Analysis

As described in Sections 2.2, the multi-hazard risk inferences for compound extremes would be influenced by various uncertain factors such as model structures and the associated parameter uncertainties. Consequently, it is of great importance to characterize both the main effects and their interactions of those uncertain factors on predictive variabilities in multi-hazard risk analyses. An IFA method was developed in this study to address this challenge.

The proposed IFA approach improves upon traditional factorial analysis (FA) method, in which a subsampling procedure would be adopted to generate a series of two-level experimental designs. The main effects from the chosen factors and their interactions are obtained by averaging the results from all the two-level experimental designs. Such a process would be illustrated through a generic bivariate example with two marginals and one copula. Consider a bivariate risk model with two correlated extremes, the multi-hazard risk inferences, subject to two marginal distributions and one copula function, can be generically formulated as:

$$\mathrm{RI} = F(A, B, C, \varepsilon) \tag{7}$$

Here A, B, and C are the choices for two marginal distributions (A and B) and the copula function (i.e., C). ε is the random error in the process of multi-hazard risk inferences. RI is the risk indices of interest such as multivariate RPs (Equation 2) and FPs (Equation 3).

One critical process in IFA is to conduct a subsampling procedure for the uncertain factors to decompose the multiple levels for one factor into a number of two-level pairs. If each factor (i.e., A, B, and C) has T levels, these T levels for each factor can be decomposed into a total number of $\binom{T}{2}$ two-level pairs, expressed as a $2 \times \binom{T}{2}$ matrix as follows:

$$g_A(h_A, j_A) = \begin{pmatrix} A_1 & A_1 & \ldots & A_1 & A_2 & A_2 & \ldots & A_{T-2} & A_{T-2} & A_{T-1} \\ A_2 & A_3 & \ldots & A_T & A_3 & A_4 & \ldots & A_{T-1} & A_T & A_T \end{pmatrix} \tag{8a}$$

$$g_B(h_B, j_B) = \begin{pmatrix} B_1 & B_1 & \ldots & B_1 & B_2 & B_2 & \ldots & B_{T-2} & B_{T-2} & B_{T-1} \\ B_2 & B_3 & \ldots & B_T & B_3 & B_4 & \ldots & B_{T-1} & B_T & B_T \end{pmatrix} \tag{8b}$$

$$g_C(h_C, j_C) = \begin{pmatrix} C_1 & C_1 & \ldots & C_1 & C_2 & C_2 & \ldots & C_{T-2} & C_{T-2} & C_{T-1} \\ C_2 & C_3 & \ldots & C_T & C_3 & C_4 & \ldots & C_{T-1} & C_T & C_T \end{pmatrix} \tag{8c}$$

Such a subsampling procedure denoted by Equations 8a–8c would lead to $\binom{T}{2} \times \binom{T}{2} \times \binom{T}{2}$ iterations in IFA with each iteration consisting of a two-level factorial design. Based on these iterations, both the individual and interactive effects of the uncertain factors on the resulting risk can be obtained (See Section S2 in Supporting Information S1).

2.4. Development of the IFMBC Framework

The developed IFMBC framework in this study consists of three main modules including (a) the copula model for multi-hazard risk analysis, the AM-based MCMC method for parameter estimation, and the IFA method for impact quantification of studied factors. Such a framework can provide ensemble inferences for multi-hazard risks under consideration of various uncertain factors and also reveal the dominant contributors to predictive uncertainties in those risk analyses. As presented in Figure 1, the procedures for the IFMBC framework are described below:

Step 1: For one set of historical records for compound extremes, select potential candidate marginals and dependence structures (i.e., copula functions).

Step 2: Subsample the candidate marginals and copulas to formulate the possible pair combinations. For instance, P3, LN3, and LLOGIS3 (i.e., 3 levels in total) would be considered as the candidate marginals in this study, and

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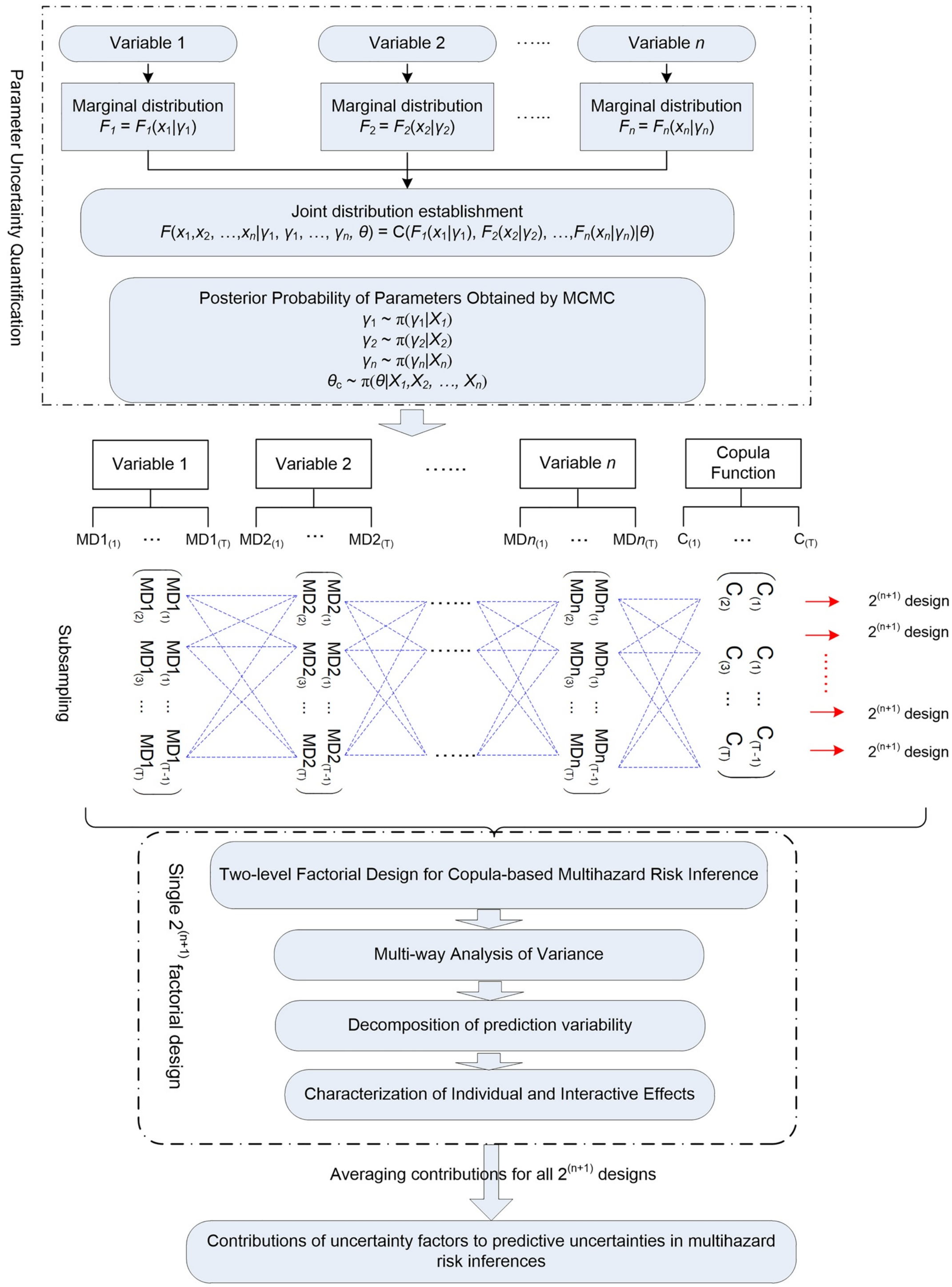

Figure 1. Framework of the IFMBC framework.

thus all possible 2-level pairs can be formulated as: $\begin{pmatrix} P3 & P3 & LN3 \\ LN3 & LLOGIS3 & LLOGIS3 \end{pmatrix}$. A similar decomposition matrix can be obtained for the candidate copulas.

Step 3: For each column in the decomposition matrices for candidate marginals and copulas, formulate the $2^{(n+1)}$ (here n is the number of extremes under consideration) factorial design matrix. In this study, the fluvial discharges and seal levels were considered (i.e., $n = 2$). Thus one sample $2^{(n+1)}$ factorial design matrix can be formulated as Table S3 in Supporting Information S1, in which each row in the matrix specifies the marginal and dependence structures for multi-hazard risk analysis.

Step 4: For each multi-hazard risk analysis model consisting of prescribed marginal and dependence structures, quantify the posterior distributions for its parameters through the AM-based MCMC method.

Step 5: For one prescribed critical event $\boldsymbol{x}^*$ (i.e., $\boldsymbol{x}^* = \{x_1^*, x_2^*, ..., x_d^*\}$), randomly sample parameter sets for the multi-hazard risk analysis model from the obtained posteriors in Step 4, and then derive the risk inferences (e.g., multivariate PRs or FPs) for this event.

Step 6: Reiterate Step 4 and Step 5 for all combinations of the candidate marginal and dependence structures in the $2^{(n+1)}$ factorial design matrix.

Step 7: Generate the total variability (i.e., SS_T) for the risk index of interest and its decomposition components expressed in Equations S17–S19 in Supporting Information S1.

Step 8: Based on Equation S20 in Supporting Information S1, obtain the single and interactive effects of the uncertain factors (i.e., marginals, copula function, and parameters) on the results of multi-hazard risk analysis within the $2^{(n+1)}$ factorial design.

Step 9: Repeat Steps 3–8 for all the combinations of the columns in the decomposition matrices, and generate the associated individual and interactive effects for the uncertain factors based on the $2^{(n+1)}$ factorial designs.

Step 10: Generate the overall single and interactive contributions for the studied factors (i.e., marginals, copula function, and parameters in this study) to the predictive variabilities in multi-hazard risk inferences through averaging the corresponding results from all the $2^{(n+1)}$ factorial designs.

In the developed IFMBC framework, the marginal distribution and copula functions can be selected independently, which are considered as the factors in IFA. However, the parameters are associated with the pre-specification for marginal and copula functions. Different marginals and copulas would have different parameters. Consequently, a multi-hazard risk model is unable to set the parameters to be factors independent from choices of marginal and dependence structures. In the proposed IFMBC framework, the contribution of parameter uncertainties to multi-hazard risk analysis would be characterized as the component of random error as expressed by Equations S20d and S21d in Supporting Information S1. Specifically, a number of parameter sets are sampled from their posteriors derived by the AM algorithm. The inferences for different risk indices are then obtained based on these parameter sets as described in Step 5. The parameters' contribution to multi-hazard risk analysis in a single $2^{(n+1)}$ factorial design is derived by Equation S20d in Supporting Information S1 and the overall contribution is finally generated by Equation S21d in Supporting Information S1.

3. Case Study

There are a number of compound events in the hydroclimatic system resulting from multiple hazards or drivers (Ridder et al., 2020; Zscheischler et al., 2018). The compound flooding events, as the combinations of high river discharges and high sea levels, have attracted great attention especially for the coastal regions and deltas (e.g., Ganguli & Merz, 2019; Hendry et al., 2019; Sadegh et al., 2018; Ward et al., 2018). Consequently, the proposed IFMBC framework was applied for compound floods to demonstrate its capability of tracing dominant contributors to the uncertainties in multi-hazard risk inferences. In detail, the compound flood events at two estuarine systems (i.e., Philadelphia, PA and Washington, DC) along the eastern coasts of United States were selected as the demonstrative cases since the river discharges and coastal water levels are statistical dependent there (Moftakhari et al., 2017).

Table 1 presents the data information for the river discharges and coastal water levels at the chosen estuarine systems. The river discharges are obtained from the United States Geological Survey (USGS) while the observations of coastal water levels are obtained from the National Oceanic and Atmospheric Administration (NOAA). The compound floods, consisting of high river discharges and high coastal water levels, can be identified through different ways (Ward et al., 2018). In this study, the compound extremes are referred to as the coastal water levels conditional on high discharges, in which the annual maximum discharge would be first identified each year, and then the highest sea level

Table 1
Data Sources and Information for the Studied Cases

Location			
Variable		Washington, DC	Philadelphia, PA
River flow	River name	Potomac	Delaware
	USGS ID	1646500	1463500
	Duration	1931–2018	1913–2018
	Mean (m^3/s)	3,480	2,500
	Skewness	2.05	1.66
	Kurtosis	7.78	6.98
	Range (m^3/s)	[824, 12,063]	[875, 7,900]
Water level	Tide station	Washington	Philadelphia
	NOAA ID	8594900	8545530
	Duration	1931–2018	1913–2018
	Mean (m)	2.77	3.5
	Skewness	1.96	0.40
	Kurtosis	7.45	2.65
	Range (m)	[2.12, 4.76]	[2.84, 4.32]
	Vertical datum	Station datum	Station datum

would be selected within ±1 day of the fluvial flood event. The future predictions under climate change will not be considered even though the sea levels are predicted to rise by IPCC since this study mainly focused on the impacts of structural and parametric uncertainties on multi-hazard risk analyses. Table S4 in Supporting Information S1 presents the performances of different univariate distributions, including P3, LLOGIS3, LN3, and GEV, on modeling the probabilistic features of discharges and sea levels at Philadelphia and Washington. The results indicate that all those four distributions would have acceptable performances at the two coastal sites. In detail, the GEV distribution would perform best for river discharges but worst for sea levels at Philadelphia. In comparison, the LLOGIS3 performed best for both river discharges and sea levels at Washington. These results are slightly different from the conclusions in Moftakhari et al. (2017) since different distributions were compared (e.g., Birnbaum-Saunders, Exponential, Gamma, GEV, Generalized Pareto, Inverse Gaussian, Loglogistic, Lognormal, and Weibull were chosen in Moftakhari et al., 2017). In this study, the P3, LLOGIS3, and LN3 distributions are selected as the marginal distributions since they would have acceptable performances as presented in Table S4 in Supporting Information S1 and also these distributions have been widely used in relevant studies (e.g., Li et al., 2018; Lin & Dong, 2019; Longfield et al., 2019).

4. Results Analysis

4.1. Risk Inferences Under Structural and Parametric Uncertainties

In IFMBC framework, a number of copula-based multi-hazard risk analysis models, with diverse marginal and dependence structures, are involved and the associated model parameters are quantified through the AM-based MCMC approach. Consequently, reliable risk inferences and their variabilities can be obtained through ensembling all modeling results considering both structural (marginal and dependence) and parametric uncertainties. For the same design standard, different risk indices would lead to different thresholds for the variables of interest. Table 2 presents the ensemble means for the most likely compound events of river discharge and sea level under a multivariate RP of 100 yr. It can be observed that the thresholds of water level and discharge would be significantly varied if different risk indices are adopted. In detail, the multivariate RP in OR ($T^{OR} = 100$ corresponds to $\Pr(\boldsymbol{x} \in R^d_{OR}) = 0.01$ in Equation 5) would lead to the highest thresholds of the compound flood event, followed by the multivariate RP in Kendall and multivariate PR in AND. For instance, at Washington, the most likely compound flood would have a discharge rate of 10,160 m^3/s and a water level of 4.19 m under $T^{OR} = 100$. In comparison, a discharge rate of 6,716 m^3/s and a water level of 3.41 m would be likely observed under $T^{Kendall} = 100$, and the discharge rate and water level may likely reduce to 6,202 m^3/s and 3.29 m respectively when T^{AND} is employed. Similar features are also observed at Philadelphia. These differences are mainly due to the different implications for the multivariate RPs in AND, OR, and Kendall. The OR case indicates that either the discharge rate or water level exceeds their corresponding thresholds, the AND case implies that both the discharge rate and water level should exceed their corresponding thresholds, and the Kendall case suggests that the joint probability of the two variables would exceed the threshold associated with the predefined RP.

Table 2
The Ensemble Means for the Most Likely Compound Event for High River Discharges and High Sea-Levels Under a Joint Return Period of 100 yr

Return period	Variables	Washington	Philadelphia
T^{AND}	Water level (m)	3.29	4.02
	Discharge (m^3/s)	6202.00	5325.45
T^{OR}	Water level (m)	4.19	4.49
	Discharge (m^3/s)	10160.69	9077.59
$T^{Kendall}$	Water level (m)	3.41	4.09
	Discharge (m^3/s)	6715.92	5806.40

In the proposed IFMBC framework, there are total 27 models for each risk index consisting of three copulas and three marginals respectively for discharges and water levels, and the model parameters are estimated through the AM-based MCMC algorithm. Consequently, it is straightforward that uncertainties would present in the inferences for the most likely compound floods. Figure 2 exhibits the uncertainties embedded in the forecasts for the most likely compound floods at two studied sites under multivariate RPs (T^{AND}, T^{OR}, $T^{Kendall}$) being 100 yr. The gray points show the most likely compound events generated under structural and parametric uncertainties, the orange and green contours respectively present the corresponding 90% and 50% predictive regions, and the red stars indicate the corresponding ensemble means. It is apparent that extensive uncertainties may exist

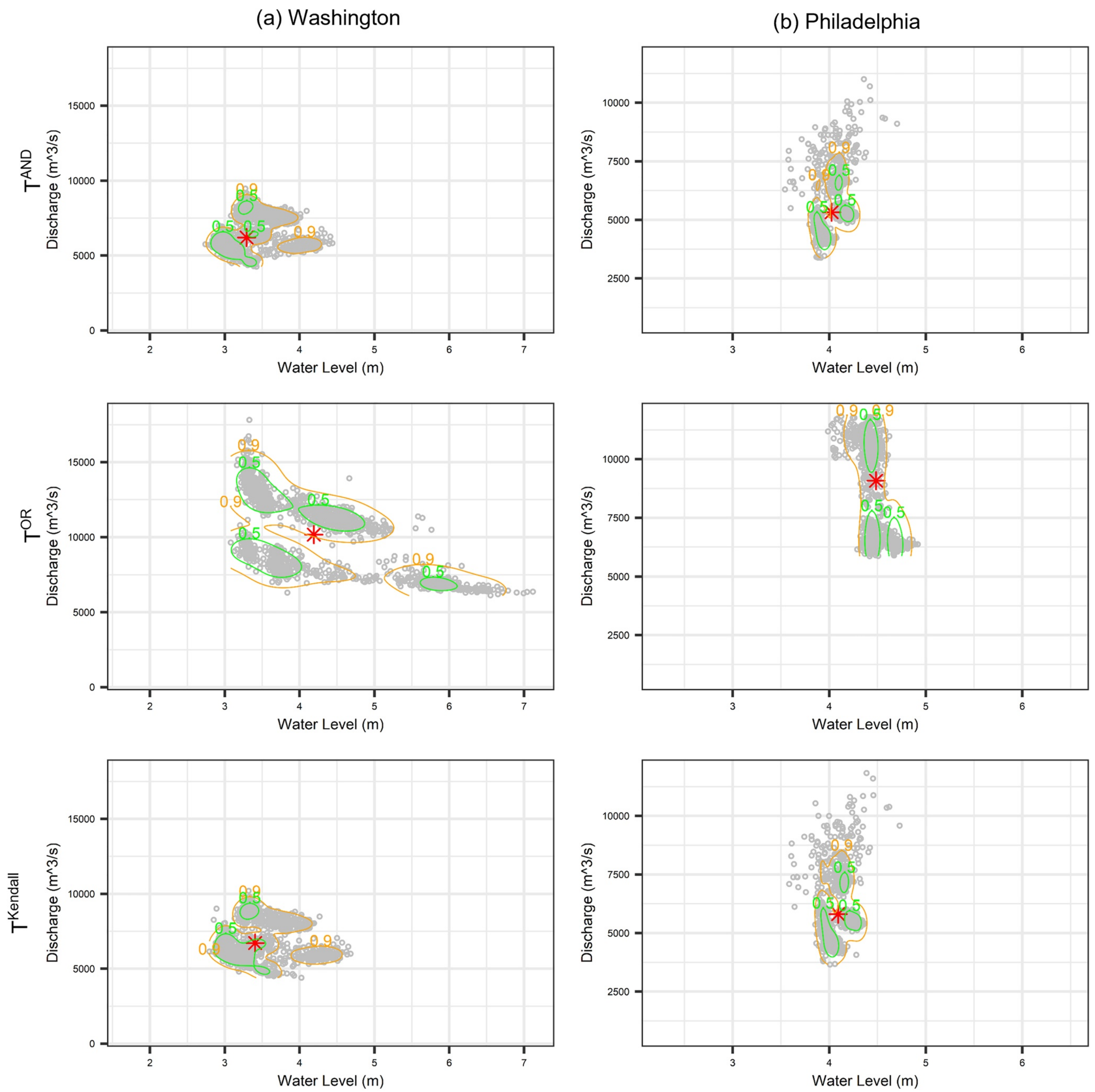

Figure 2. Predictive quantiles for the most likely compound extremes under a 100 yr multivariate RP. The green and yellow contours respectively exhibit the 50% and 90% predictive quantiles, and the red stars indicate the ensemble means.

for the inferences of the most likely compound events based on all three multivariate RPs. Also, for different risk indices, those predictions present different uncertainty degrees, but the estimates for the most likely compound floods under T^{OR} tend to have larger uncertainties than the inferences under the other two multivariate RPs. More specifically, it is noticeable that multiple clusters are observed for the compound flood predictions from all the three multivariate RPs, especially for the 50% predictive regions. Such a phenomenon may be due to the fact that the predictive uncertainties in the inferences of compound floods are resulted from different sources such as model structures and parameters, and different sources would pose distinguishable impacts on the estimates of compound floods under diverse risk indices.

Figure 3 presents the inference uncertainties for the compound floods under the T^{AND} being 100 yr at Washington, in which the dashed contours indicate the ensemble means of T^{AND} for different combinations

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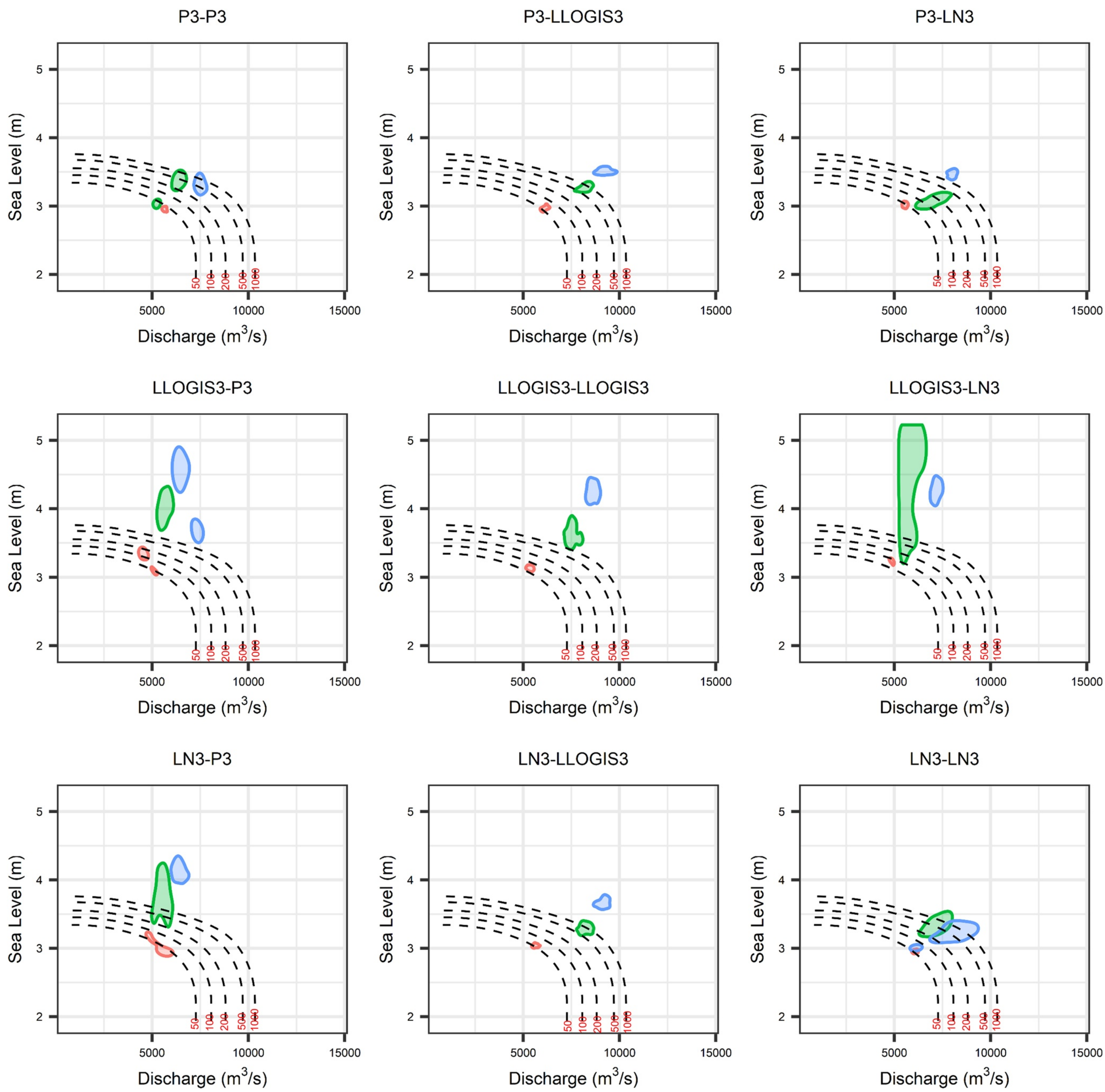

Figure 3. Uncertainties in the estimates for the most likely events of compound floods with a 100 yr multivariate RP in AND at Washington.

of discharges and water levels. In each subfigure, the colored contours exhibit the 50% predictive regions for the most likely compound floods obtained from the risk model with specified marginals (denoted as the title of the subfigure) and copulas (specified as the legend). In other words, the uncertain predictions enclosed in the colored contours would mainly be resulted from parameter uncertainties of the risk model. In comparison, the discrepancies among the colored contours in each subfigure would be resulted from the differences in copula structures, while the contours with the same color but in different subfigures exhibit the impacts from the changes of marginals. As presented in Figure 3, the parameters would lead to different predictive uncertainties in the predictions of compound floods from the risk models with different marginal and dependence structures. For instance, even though the 50% predictive regions from the Frank copula (i.e., red contours) are relatively small for most cases, we can still observe noticeable uncertainties in the

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compound flood inferences from other copula functions. This is mainly due to the inconsistent parameter quantifications resulting from (a) discrepancies of parameter posteriors in different runs of MCMC and (b) correlations among parameters in both marginal and dependence structures. Moreover, it is apparent that, in each subfigure, the compound flood predictions from different copulas are distinguishable among each other, implying significant impacts of the dependence structure on the risk inference under T^{AND}. However, the inferences of compound floods from the model with the same copula but different marginals seem to have chaotic features. The red contours (i.e., the Frank copula is adopted) would not show visible differences in different subfigures while the green contours from Joe copula and the blue contours from Gumbel copulas present explicit differences in different subfigures. Consequently, the impacts of marginal distributions need to be further addressed.

Figures 4 and 5 present the uncertainties in the estimates of the most likely compound floods with a 100 yr multivariate RP for T^{OR} and T^{Kendall} at Washington. Similar to the predictions under T^{AND}, noticeable uncertainties can be observed in the colored regions, suggesting significant effects of parametric uncertainties on the estimates of compound floods under T^{OR} and T^{Kendall}. Nevertheless, the 50% predictive regions of compound floods, generate from the risk model with specified marginal and dependence structures, would present different uncertain degrees under different multivariate RPs. For instance, we can observe significant predictive uncertainty (in Figure 3) for the compound floods under T^{AND} from the model with the marginals of LLOGIS3 and LN3 and the dependence structure of Gumbel copula, while the predictive uncertainty under T^{OR} is much smaller (e.g., around [4, 5] for water level) from the same model structure as shown in Figure 4. This is mainly due to the different expressions for T^{AND} and T^{OR}, but also suggests that one specific factor (e.g., parameter uncertainties) would have diverse impacts on predictions of compound floods under different risk indices. Similar features can also be observed for the impact of the dependence structure (i.e., copula function). Compared with Figure 3, the colored contours in each subfigure of Figure 4 seem to present some overlaps among each other for most cases, which indicates that the dependence structure may have a less impact for the compound flood inferences under T^{OR}. However, Figure 5 shows analogous features of the predictive uncertainties with those exhibited in Figure 3, implying a similar impact pattern for those uncertain factors on the compound flood predictions under T^{Kendall} with the predictions under T^{AND}.

Figures S1–S3 in Supporting Information S1 exhibit the estimates for the most likely compound flood with a 100 yr RP for T^{AND}, T^{OR}, and T^{Kendall} at Philadelphia. It can be observed that the predictive uncertainties under different risk indices respectively present similar features with the uncertain inferences for Washington. The uncertain degrees resulting from model parameters are different for diverse risk models with different candidate marginals and copulas. Moreover, the compound floods generated through risk models with diverse copulas under T^{AND} and T^{Kendall} seem to be more distinguishable than the results under T^{OR}, indicating more possible impacts for the dependence structure on the risk indices of T^{AND} and T^{Kendall}.

4.2. Characterization of Dominant Contributors to Uncertainties in Multi-Hazard Risk Analyses

As stated in Section 4.1, remarkable uncertainties would be present in the risk analyses for compound floods, which may be resulted from various sources such as structural (e.g., marginal and copula) and parametric uncertainties. More specifically, the effects, stemmed from different uncertainties, on one risk index (e.g., T^{AND}, T^{Kendall}, or T^{OR}) would be distinguishable, whilst one uncertain factor may also pose different impacts on different risk indices. Nevertheless, Figures 3–5 merely provided qualitative descriptions for the impacts of different uncertain sources on different risk indices. Consequently, the IFA method was developed to track the dominant contributors to predictive uncertainties in the multi-hazard risk inferences of compound floods.

In this study, three uncertain factors were considered in IFA, including two marginal distributions and one copula function. Also, each factor has three levels consisting of different marginal (i.e., P3, LLOGIS3, LN3) or dependence (i.e., Gumbel, Frank, and Joe copula) structures. Consequently, each factor was decomposed into three (i.e., $\binom{3}{2}$) two-level pairs as stated in Step 2 in Section 2.4, and finally formed a total number of 27 two-level experimental designs in the IFA process. Moreover, for each two-level experimental design, 10 parameter sets, as

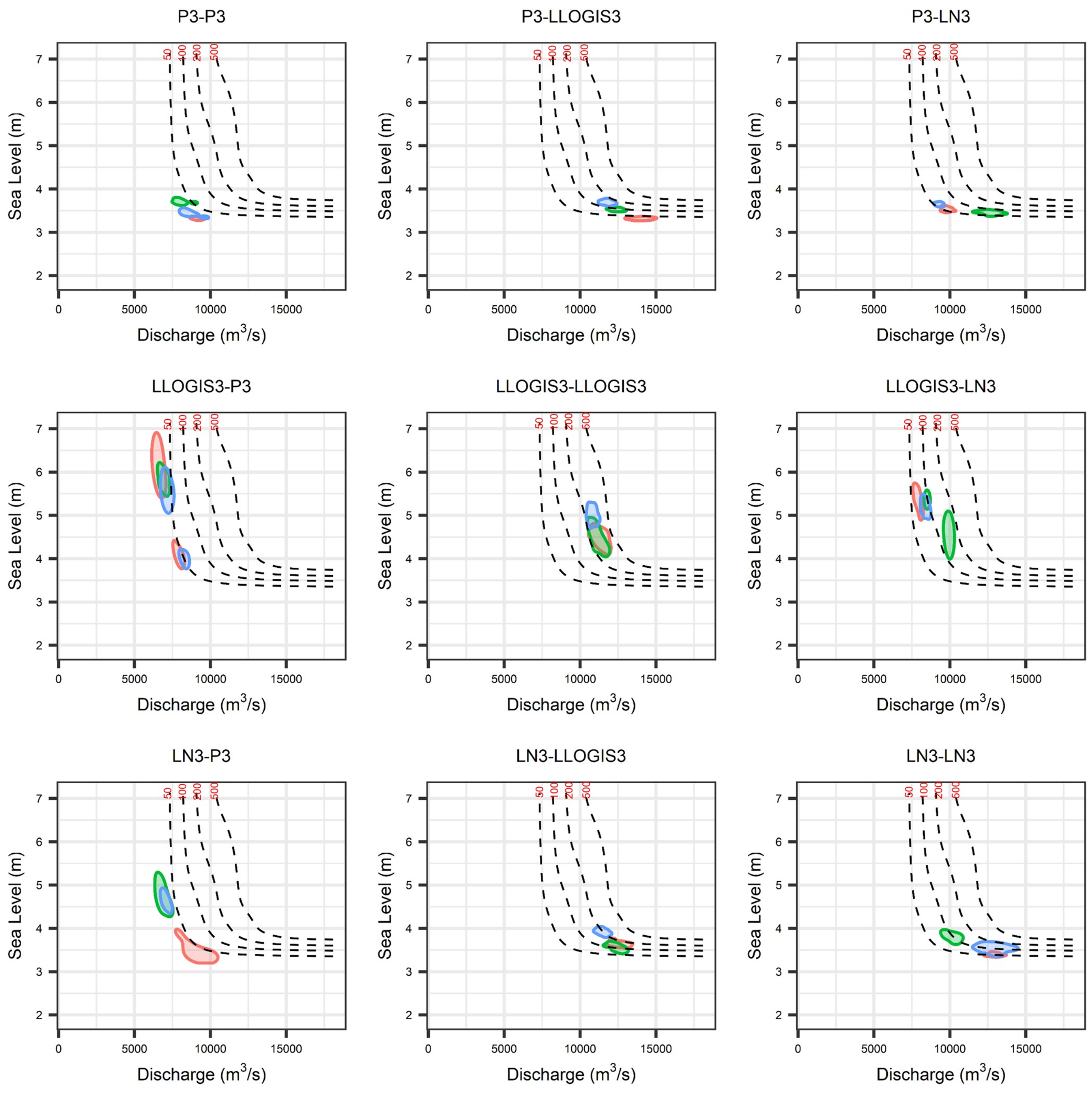

Figure 4. Uncertainties in the estimates of the most likely compound floods with a 100 yr multivariate RP in OR at Washington.

recommended in Fan et al. (2021), would be randomly sampled from their posteriors to derive the contribution from parametric uncertainties. In this section, the FPs in "AND", "OR", and "Kendall" (expressed as p_T^{AND}, p_T^{OR}, andp_T^{Kendall} respectively) are considered as the responses (i.e., risk inferences) in IFA since these indices also reflect the impact of infrastructure service time.

The effects of various uncertain factors on p_T^{AND} are presented in Figure 6, in which *A*, *B*, and *C* respectively represent the marginal for sea level, the marginal for river discharge and the dependence structure (i.e., Copula). It shows that diverse factors would pose distinguishable effects on p_T^{AND} inference. However, it can be concluded that the copula structure would have a highest contribution to the predictive uncertainties in p_T^{AND}, with the contribution approaching 30% for p_T^{AND} with a 70 yr service time at Washington. In addition, the parameter uncertainties

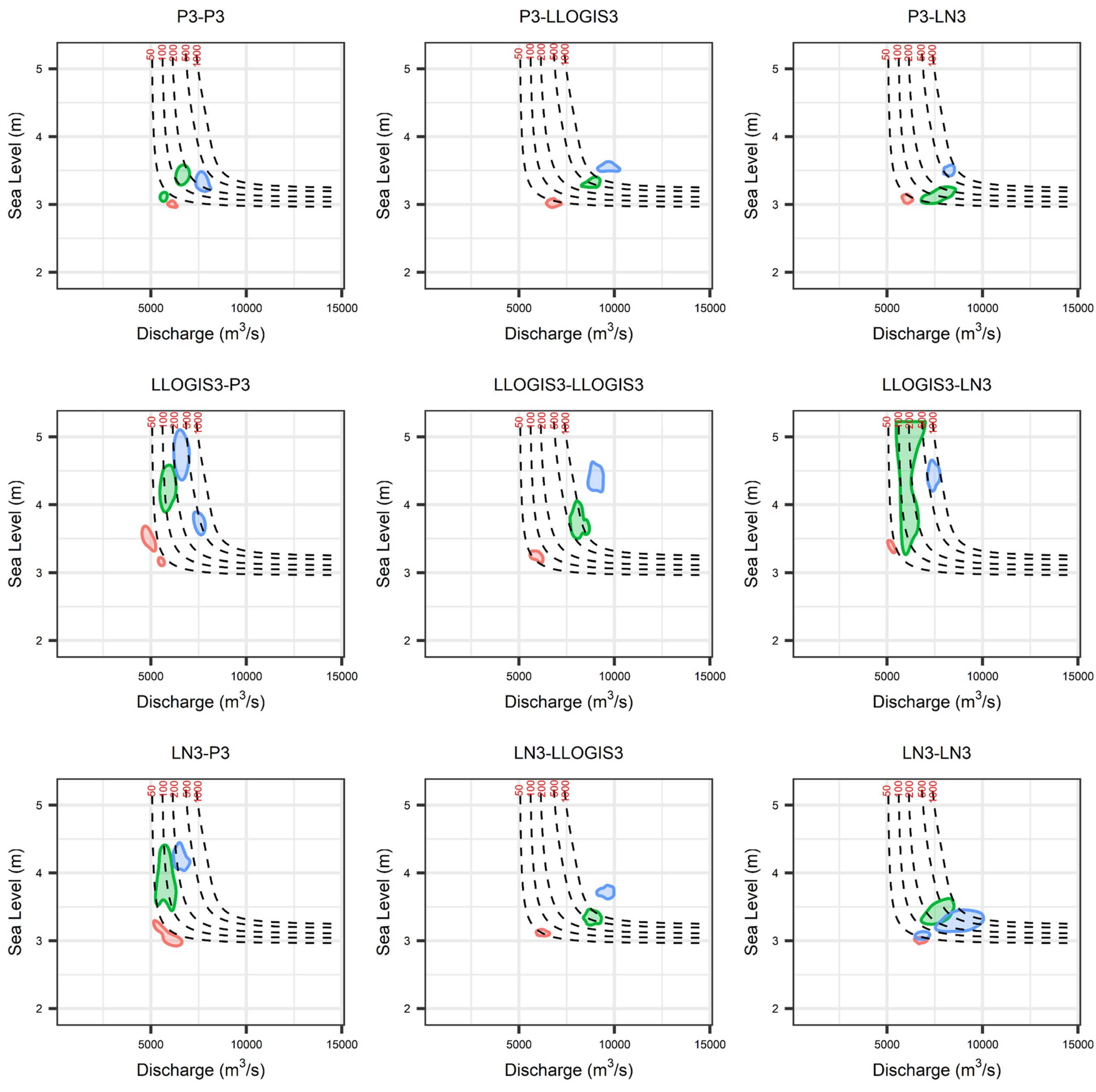

Figure 5. Uncertainties in the estimates of the most likely compound floods with a 100 yr multivariate RP in Kendall at Washington.

also pose noticeable impacts on the prediction of p_T^{AND}, with their contributions being around 20% at both sites. Nevertheless, the contributions from model parameters would decrease as the service time increases, which is opposite to the contribution trend of the copula function. In this study, the model parameters are considered as the component of random error in IFA, which implies the overall contribution from all parameters in marginals and copula function. For the contributions from different parameters, the parametric uncertainties in marginals (especially for the shape parameter in a distribution) would tend to have much higher contributions than the uncertainty in copula parameter (Fan, Huang, Huang, Li, & Wang, 2020). For the marginal distributions for individual variables, they are also likely to have visible effects on the predictions of p_T^{AND} for a specific design threshold. But the marginal for river discharge is expected to have a greater effect (e.g., more than 20% at Philadelphia) than the marginal for sea level (e.g., about 4% at Philadelphia). This may be due to the probabilistic features for the

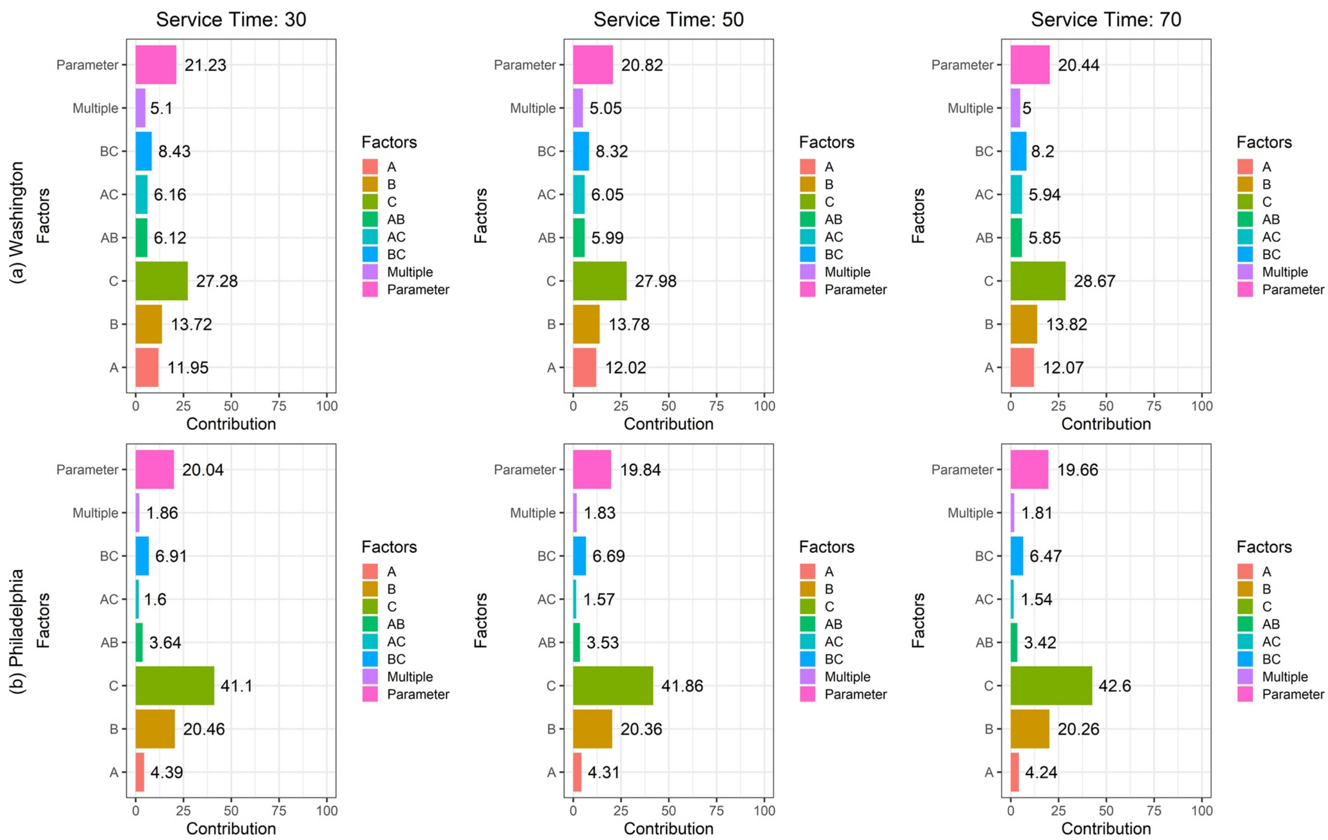

Figure 6. The main effects of uncertain factors and their interactions on the predictive uncertainties for FP in AND.

historical extreme records. In current study, the annual maximum discharge of river flow was first identified, and then the maximum sea level within ±1 day of the annual max discharge was selected. Therefore, the compound extreme in this study were considered as the discharge dominated events. Also, from Table 1, the sea level records at Philadelphia have much smaller skewness (i.e., 0.40) and kurtosis (i.e., 2.65) than those from river discharges (i.e., 1.66 for skewness and 6.98 for kurtosis). These may lead to a less impact for the marginal distribution of sea level. The interactions among marginal and dependence structures do not have significant effects on p_T^{AND} and some of them seem to be negligible especially at Philadelphia. The obtained results suggest that an appropriate dependence structure may be prioritized to get reliable risk inferences for p_T^{AND}, followed by robust quantification for model parameters and proper selection for the marginals especially for the variable for river discharges.

For the risk inference of p_T^{OR}, it is also obvious that the marginals, dependence structure, and model parameters would have different effects on its predictive uncertainties, as exhibited in Figure 7. There are both similarities and dissimilarities between the effects on p_T^{AND} and p_T^{OR} from the studied uncertain factors. First, visible effects from marginals and parameter uncertainties are observed on the inferences of p_T^{OR}, which is similar to their effects on p_T^{AND}. Nevertheless, the impact from parameter uncertainties seems to dominate the predictive variability of p_T^{OR} with the highest contribution more than 30%, even though such an impact would decrease as the increase of service time. Moreover, the marginal distributions tend to have higher contributions to p_T^{OR} predictions than the contributions to the inference of p_T^{AND}. For instance, at the site of Washington, the marginals respectively for sea level and river discharge would approximately have a contribution of 13.7% and 12.0% to the predictive uncertainty of p_T^{AND} with a service time of 30 yr, while such contributions to the inference of p_T^{OR} would increase to 14.6% and 28.6%. For certain long-term service time scenario, the marginal distribution from one variable may have the highest impact. In comparison, the copula structure has shown a much smaller contribution to the prediction of p_T^{OR} than its contribution to p_T^{AND}, which is possibly less than 5% for a service time larger than 30 yr at both sites. These results imply that appropriate marginal distributions and well parameter quantification would be the key factors to get reliable inferences for p_T^{OR}.

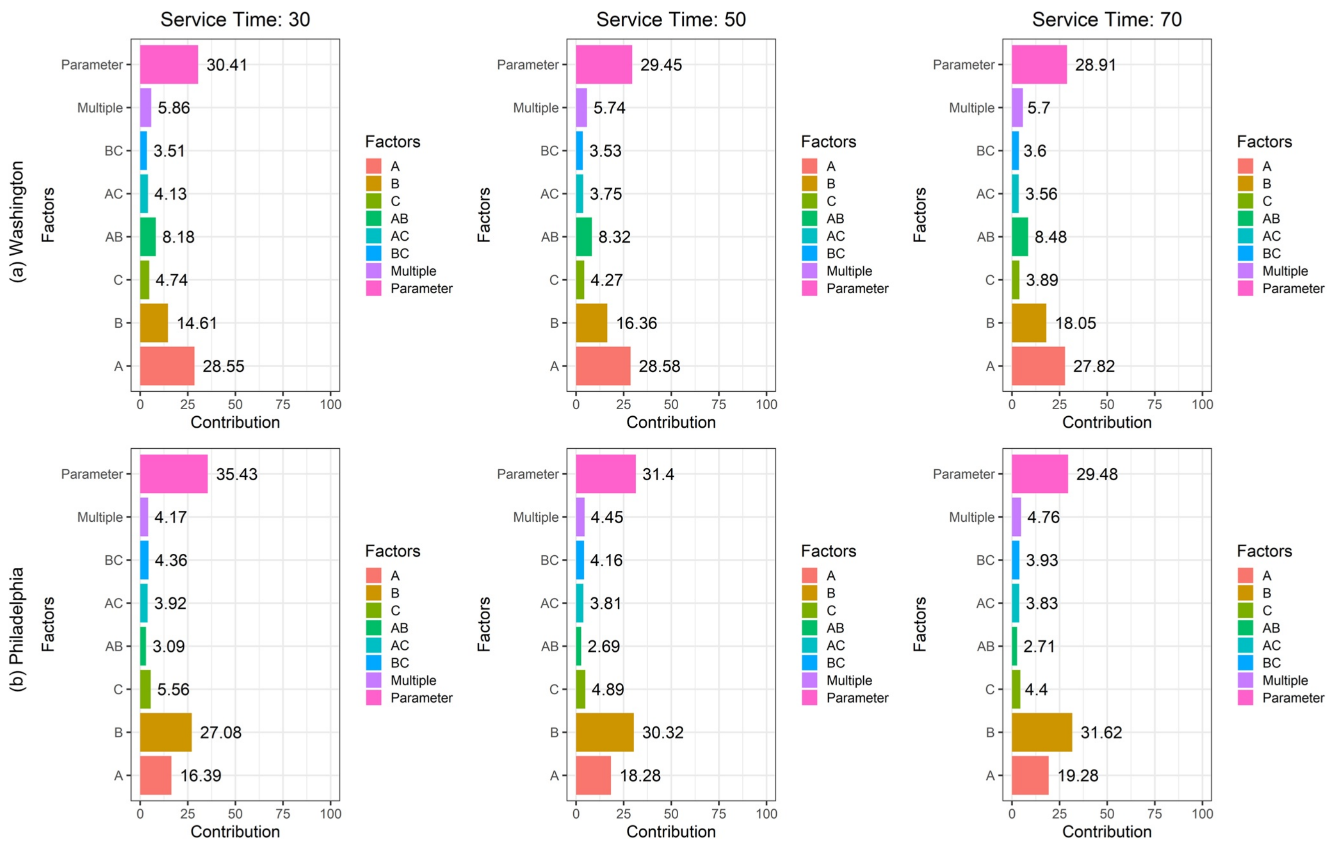

Figure 7. The main effects of uncertain factors and their interactions on the predictive uncertainties for FP in OR.

Figure 8 exhibits the individual and interactive contributions to the predictions of p_T^{Kendall} resulted from both structural (i.e., both marginals and copula) and parametric uncertainties. These contributions present similar features with the impacts on the inference of p_T^{AND}, in which uncertainties in both dependence structure (i.e., copula) and model parameters uncertainties have greater effects than the other factors. Nevertheless, the parametric uncertainty seems to pose a higher contribution to the predictive uncertainty in p_T^{Kendall} than its contribution top_T^{AND} inference. For instance, under a 30 yr service time, the parametric uncertainty would respectively have a contribution of 21.2% and 20.0% to the predictive variability of p_T^{AND} at the two stations, while its contribution top_T^{Kendall} would respectively be 24.4% and 40.0%. The copula structure tends to pose a higher effect on p_T^{Kendall} than its effect on p_T^{AND} at Washington, but this factor would have a less impact on p_T^{Kendall} than its impact on p_T^{AND} at Philadelphia. However, the impacts of copula structure on the inference of p_T^{Kendall} at both sites are noticeable with the contributions higher than 30%.

In comparison, there are slight increases for the impacts of the marginal for sea level between predictions of p_T^{AND} and p_T^{Kendall} at the two sites, whilst explicit decreases occur for the impacts from the marginal of discharge. Moreover, compared to the impacts of the marginals onp_T^{OR}, the two marginals present much smaller contributions to the p_T^{Kendall} prediction with the highest value less than 15%. In general, for the prediction of p_T^{Kendall}, the parameter uncertainties and copula structure are the two prioritized factors to be well considered, followed by the marginal distributions.

Table 3 summarizes the first two significant contributors to the uncertainties in FP inferences with a 30 yr service time. It can be concluded that the parameter uncertainties significantly influence the inferences of all the three FPs except the FP in AND at Philadelphia. In comparison, the copula structure tends to be prioritized for the FP in AND and Kendall whilst the marginal distributions would have more effects than the dependence structure on the FP prediction in OR. Those results can also be descriptively implied by Figures 3–5 and S1–S3 in Supporting Information S1.

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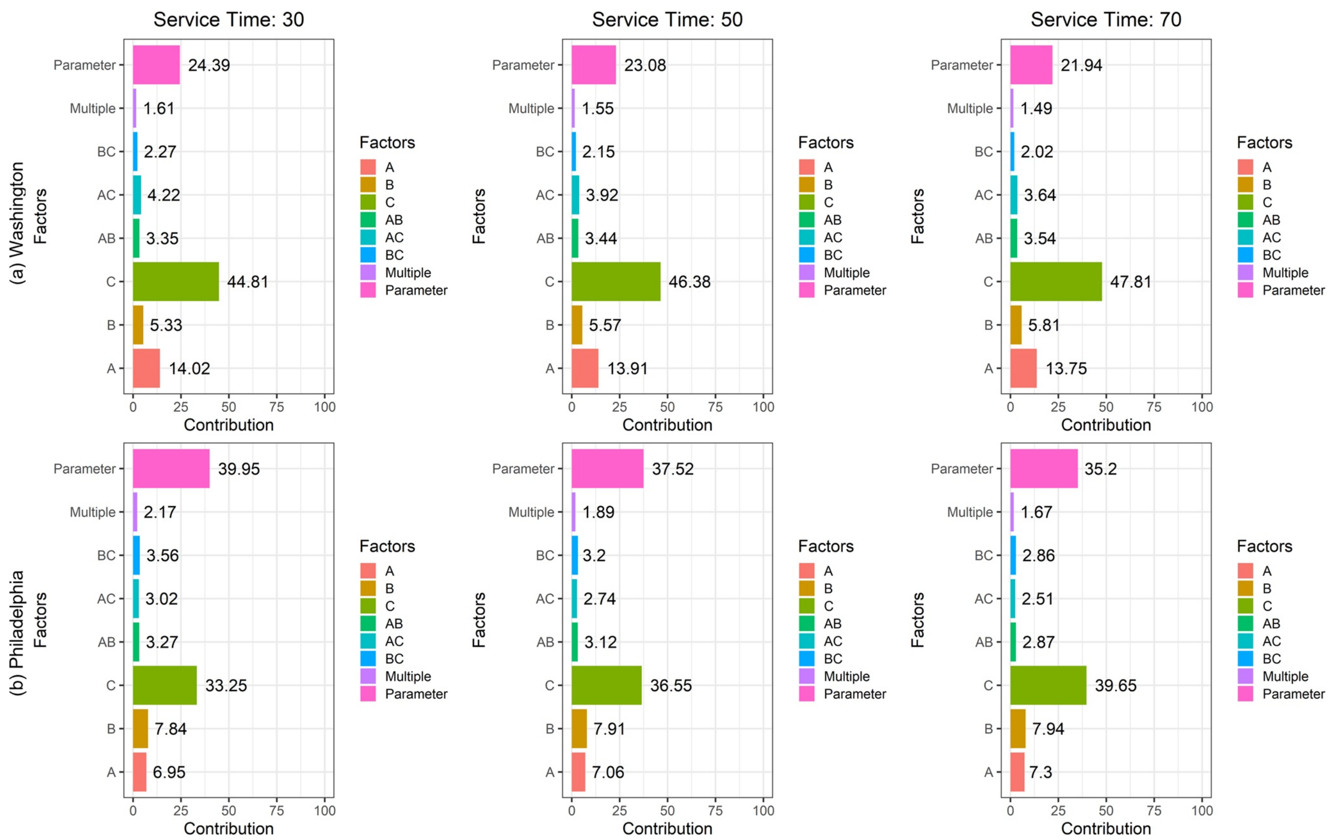

Figure 8. The main effects of uncertain factors and their interactions on the predictive uncertainties for FP in Kendall.

5. Discussion

5.1. Comparison With Traditional Multilevel Factorial Analysis

One of the major innovations for the proposed IFMBC framework is the development of the IFA method, in which a subsampling procedure has been adopted to mitigate the biased variance estimators in traditional FA methods. A factorial multimodel Bayesian copula (FMBC) method was recently developed to characterize the impacts of various uncertain factors on the predictive uncertainties in multivariate flood risk analyses, in which a traditional multilevel FA approach is adopted to characterize the dominant factors (Fan et al., 2021; Montgomery, 2013). Such an FMBC approach can also be employed to track the dominant contributors to the multi-hazard risk inferences for compound floods consisting of extreme river discharges and sea levels.

Table 3
The First Two Contributors for the Predictive Uncertainties in the Multi-Hazard Risk Inferences for Different Failure Probabilities With a 30 yr Service Time

Stations		Washington	Philadelphia
FP in AND	1st	Copula	Copula
	2nd	Parameter	Marginal for discharge
FP in OR	1st	Parameter	Parameter
	2nd	Marginal for sea level	Marginal for discharge
FP in Kendall	1st	Copula	Parameter
	2nd	Parameter	Copula

Figure 9 presents the individual and interactive effects for the uncertainties in marginals, dependence, and model parameters on the inferences of FPs with different service times at Washington. Even though we can see visible effects from the copula structure on the FPs in AND and Kendall as well as the effect from the marginal of sea level on the FP in OR, the effects from parameter uncertainties on all the three FPs seem to be significantly overestimated. This also leads to underestimations for the effects from marginals and the copula structure. For instance, the effect of the dependence structure on the p_T^{AND}, obtained by the IFA method, would be more than 27% at Washington as presented in Figure 6a, whilst such an effect is around 7% obtained by the traditional multilevel FA method (Figure 9a). Figure S4 in Supporting Information S1 exhibits the effects of the uncertain factors on the prediction of FPs

Figure 9. The main effects of uncertain factors and their interactions on the predictive uncertainties for FPs obtained by the FMBC method at Washington.

at the station of Philadelphia, which also presents overestimations for the contributions from model parameters but underestimations for the effects from other factors. Such results may be due to the biased variance estimators in the traditional multilevel FA method as stated in some studies (e.g., Bosshard et al., 2013). Thus, we argue that the developed IFA method in the IFMBC framework can provide more reliable characterizations for the dominant contributors to predictive uncertainties in the multi-hazard risk inference of compound extremes.

5.2. Comparison With More Options for Marginals and Copulas

In the developed IFMBC framework, three options were adopted for both marginal and copula functions which led to a total number of 27 two-level factorial designs in the IFA process. To further demonstrate the robustness of the results from IFMBC, more options were adopted for both marginal and copula functions at the site of

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Washington. As recommended in Moftakhari et al. (2017), the GEV distribution was introduced to model the distributions of river discharges and sea levels and the survival Clayton copula was employed to quantify the dependence for these two extremes. Therefore, both marginal and dependence models would have four levels (i.e., P3, LN3, LLOGIS3, and GEV for marginals; Gumbel, Frank, Joe, and survival Clayton copula for dependence structure), which would lead to 6 two-level pairs (i.e., $\binom{4}{2}$) for each factor, and further produce 216 two-level experimental designs (i.e., 6*6*6) in IFA.

Figure 10 shows contribution partition for the studied uncertain factors on FP inferences with different service times from a 4-level IFA case at Washington. Compared with the results presented in Figures 6a–8a, the quantification for factor contributions to different FPs shows a similar pattern especially for those dominant contributors. For instance, the first two impact factors (i.e., copula structure and model parameters) would present a contribution of 27.0% and 20.2% respectively, as presented in Figure 10a, to the predictive uncertainty in p_T^{AND} with a 30 yr service time, compared with the corresponding contribution of 27.3% and 21.1% from the 3-level IFA case presented in Figure 6a. For the FP in OR, some discrepancies are observed for the detailed contributions between the 3-level IFA (e.g., 30.4% for parameter uncertainties and 28.6% for marginal of sea level) and 4-level IFA (e.g., 37.9% and 18.6% for parameter uncertainties and marginal of sea level respectively) cases. However, the same major contributors (e.g., parameter uncertainties and two marginals are the first three factors) are identified in both 3-level and 4-level IFA cases. Similarly, the 4-level IFA case would also identify the same dominant contributors to the FP in Kendall with those found in the 3-level IFA case. These results indicate that in the proposed IFMBC framework, three levels would be sufficient to trace the significant contributors to the predictive uncertainties in multi-hazard risk analyses of compound extremes.

6. Conclusions

The multi-hazard risk analysis for compound extremes is of great importance since it can give more insightful characterization for the interdependence among different extremes and thus provide support for developing effective resilience strategies. However, the risk inferences for compound events are challenged by uncertainties existing in model structures and parameters, in which the contributions from those uncertain factors to multi-hazard risk inferences have not been sufficiently addressed. Consequently, the IFMBC framework has been developed to help track the dominant contributors to uncertainties in the inferences of multi-hazard risk analyses. In IFBMC, an IFA method, coupled with the AM-based MCMC algorithm, was integrated into copula-based models consisting of multiple marginals and dependence structures. The AM method was adopted to estimate parameter uncertainties, whilst the IFA method was then used to reveal the main effects of the uncertain factors and their interactions on different risk indices.

The applicability of IFMBC framework has been demonstrated through the risk inference problems for compound floods consisting of extreme river discharges and sea levels at Washington and Philadelphia. The most likely compound floods under a 100 yr RP were generated to reveal uncertainties in multivariate risk analyses. The FPs in the scenarios of AND, OR, and Kendall were adopted to reveal the dominant uncertainty contributors. Based on the obtained results from the two cases, some conclusions can be summarized.

1. For a pre-defined RP, the most likely compound floods would generally have the highest magnitudes under the multivariate RP in OR, followed by the most likely compound events under the multivariate RPs in Kendall and AND. This is mainly due to the different implications for those three multivariate RPs under consideration. Nevertheless, extensive uncertainties exist in the predictions for the most likely compound floods. More specifically, the 90% predictive regions present multiple clusters or hotspots, which may be attributed to different impacts from different uncertain factors.
2. The contribution characterization from IFA indicates that the copula structure tends to pose the highest impact on the FP inferences in AND, which would be higher than 25% and increase as the increase of service time. Moreover, the copula structure would also have noticeable effects on the inferences of FP in Kendall, ranked as the highest one or the second impact factor just after model parameters. Parameter uncertainties seem to make significant contributions to the predictive uncertainties in inferences for all the three FPs, which is always ranked as one of the top 2 contributors. In comparison, the marginal distributions would possibly

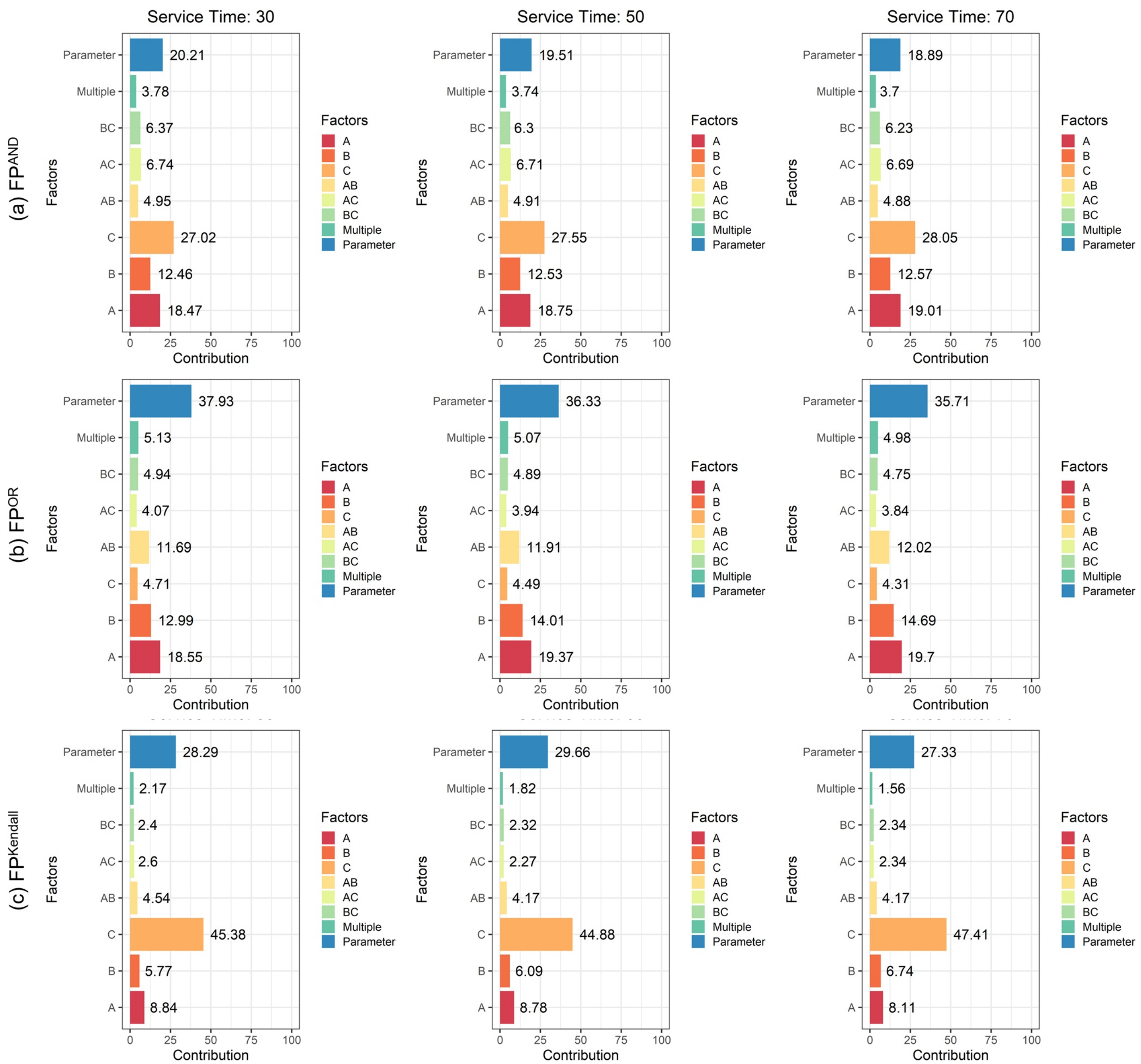

Figure 10. The main effects of uncertain factors and their interactions on the predictive uncertainties for FPs obtained by the 4-level IFMBC case at Washington.

have more effects on FP predictions in OR than their effects on the other two FP inferences, whilst the copula structure would pose a much smaller impact on FP in OR with its contribution only around 5%.

3. The developed IFA method in this study would generate more robust characterization for the contributions of uncertain factors than traditional multilevel FA method. The traditional factorial method would also recognize visible effects from dependence structure on FP predictions in AND and Kendall, and explicit effects of the marginal distributions on the PF inferences in OR. Nevertheless, this traditional method would significantly overestimate the contribution of parameter uncertainties and thus underestimate the contributions from other factors. Thus, the IFA method involved in IFBMC framework tends to give more reliable tracking for the dominant contributors to the uncertainty in multi-hazard risk inferences only based on limited factor levels (e.g., 3 levels), which can help direct effective pathways to improve risk inference practices.

Even though the applicability of IFMBC has been illustrated through the multi-hazard risk inference for compound floods with extreme discharges and sea levels, such a framework can also be applied for other hydroclimatic extremes. Also, the developed IFMBC framework can be extended to high-dimensional extremes rather than bivariate cases. In addition, uncertainties in model structures and parameters were addressed in the proposed study where the impact from data observations, which has been recognized as another uncertain source (e.g., Ajami et al., 2007; Qi et al., 2016), was not considered. However, a further study is ongoing now to characterize the impacts of uncertainties in data, model structures, and parameters on the multi-hazard risk inferences of compound extremes. Based on the developed IFMBC framework, long-term extreme observations (ideally larger than 90 yr) will randomly be divided into three groups. Coupled with three marginals and copula functions, a 3-level IFMBC model consisting of four independent factors (i.e., data, two marginals, and copula) will be established to reveal the major impact factors on multivariate risk inferences of compound extremes.

Data Availability Statement

The daily river discharges are obtained from the United States Geological Survey website (USGS), in which the river discharge for Washington is accessed by https://waterdata.usgs.gov/nwis/dv/?site_no=01646500, and the discharge at Philadelphia can be collected at https://waterdata.usgs.gov/nwis/dv/?site_no=01463500. The hourly water level data are collected from the National Oceanic and Atmospheric Association (NOAA). The water level data at Washington can be accessed at https://tidesandcurrents.noaa.gov/stationhome.html?id=8594900, and the water level at Philadelphia can be obtained at https://tidesandcurrents.noaa.gov/stationhome.html?id=8545530.

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References

AghaKouchak, A., Easterling, D., Hsu, K., Schubert, S., & Sorooshian, S. (2012). *Extremes in a changing climate: detection, analysis, and uncertainty* (Vol. 65). Springer Science & Business Media.

Ahmed, M. I., Sinclair, C. D., & Werritty, A. (1988). Log-logistic flood frequency analysis. *Journal of Hydrology*, *98*(3–4), 205–224. https://doi.org/10.1016/0022-1694(88)90015-7

Ajami, N. K., Duan, Q., & Sorooshian, S. (2007). An integrated hydrologic Bayesian multimodel combination framework: Confronting input, parameter, and model structural uncertainty in hydrologic prediction. *Water Resources Research*, *43*(1), W01403. https://doi.org/10.1029/2005WR004745

Alizadeh, M. R., Adamowski, J., Nikoo, M. R., AghaKouchak, A., Dennison, P., & Sadegh, M. (2020). A century of observations reveals increasing likelihood of continental-scale compound dry-hot extremes. *Science Advances*, *6*, eaaz4571. https://doi.org/10.1126/sciadv.aaz4571

Bevacqua, E., Maraun, D., Hobæk Haff, I., Widmann, M., & Vrac, M. (2017). Multivariate statistical modeling of compound events via pair-copula constructions: Analysis of floods in Ravenna (Italy). *Hydrology and Earth System Sciences*, *21*(6), 2701–2723. https://doi.org/10.5194/hess-21-2701-2017

Bosshard, T., Carambia, M., Georgen, K., Kotlarski, S., Krahe, P., Zappa, M., & Schar, C. (2013). Quantifying uncertainty sources in an ensemble of hydrological climate-impact projections. *Water Resources Research*, *49*, 1523–1536. https://doi.org/10.1029/2011wr011533

Brechmann, E. C., & Schepsmeier, U. (2013). Modeling dependence with C- and D-Vine Copulas: The R package CDVine. *Journal of Statistical Software*, *52*(3), 1–27. https://doi.org/10.18637/jss.v052.i03

Corbella, S., & Stretch, D. (2012). Predicting coastal erosion trends using non-stationary statistics and process-based models. *Coastal Engineering*, *70*, 40–49. https://doi.org/10.1016/j.coastaleng.2012.06.004

De Michele, C., & Salvadori, G. (2003). A generalized Pareto intensity-duration model of storm rainfall exploiting 2-copulas. *Journal of Geophysical Research*, *108*(D2), 4067. https://doi.org/10.1029/2002JD002534

Dung, N. V., Merz, B., Bardossy, A., & Apel, H. (2015). Handling uncertainty in bivariate quantile estimation–An application to flood hazard analysis in the Mekong delta. *Journal of Hydrology*, *527*, 704–717. https://doi.org/10.1016/j.jhydrol.2015.05.033

Fan, Y., Huang, K., Huang, G., & Li, Y. (2020). A factorial Bayesian copula framework for partitioning uncertainties in multivariate risk inference. *Environmental Research*, *183*, 109215. https://doi.org/10.1016/j.envres.2020.109215

Fan, Y., Huang, K., Huang, G., Li, Y., & Wang, F. (2020). An uncertainty partition approach for inferring interactive hydrologic risks. *Hydrology and Earth System Sciences*, *24*(9), 4601–4624. https://doi.org/10.5194/hess-24-4601-2020

Fan, Y., Huang, G. H., Zhang, Y., & Li, Y. P. (2018). Uncertainty quantification for multivariate eco-hydrological risk in the Xiangxi River within the Three Gorges Reservoir Area in China. *Engineering*, *4*(5), 617–626. https://doi.org/10.1016/j.eng.2018.06.006

Fan, Y., Yu, L., & Shi, X. (2021). Uncertainty quantification and partition for multivariate risk inferences through a factorial multimodel Bayesian copula (FMBC) system. *Journal of Hydrology*, *598*, 126406. https://doi.org/10.1016/j.jhydrol.2021.126406

Ganguli, P., & Merz, B. (2019). Trends in compound flooding in northwestern Europe during 1901–2014. *Geophysical Research Letters*, *46*, 10810–10820. https://doi.org/10.1029/2019gl084220

Guo, A. J., Chang, J. X., Wang, Y. M., Huang, Q., & Li, Y. Y. (2020). Uncertainty quantification and propagation in bivariate design flood estimation using a Bayesian information-theoretic approach. *Journal of Hydrology*, *584*, 124677. https://doi.org/10.1016/j.jhydrol.2020.124677

Haario, H., Saskman, E., & Tamminen, J. (2001). An adaptive Metropolis algorithm. *Bernoulli*, *7*(2), 223–242. https://doi.org/10.2307/3318737

Hao, Z., & AghaKouchak, A. (2013). Multivariate standardized drought index: A parametric multi-index model. *Advances in Water Resources*, *57*, 12–18. https://doi.org/10.1016/j.advwatres.2013.03.009

Hao, Z., & AghaKouchak, A. (2014). A nonparametric multivariate multi-index drought monitoring framework. *Journal of Hydrometeorology*, *15*, 89–101. https://doi.org/10.1175/JHM-D-12-0160.1

Hendry, A., Haigh, I. D., Nicholls, R. J., Winter, H., Neal, R., Wahl, T., et al. (2019). Assessing the characteristics and drivers of compound flooding events around the UK coast. *Hydrology and Earth System Sciences*, *23*, 3117–3139. https://doi.org/10.5194/hess-23-3117-2019

Huang, K., Dai, L. M., Yao, M., Fan, Y. R., & Kong, X. M. (2017). Modeling dependence between traffic noise and traffic flow through an entropy-copula method. *Journal of Environmental Informatics*, *29*(2), 134–151. https://doi.org/10.3808/jei.201500302

Huang, K., Fan, Y. R., & Fan, Y. R. (2021). Parameter uncertainty and sensitivity evaluation of copula-based multivariate hydroclimatic risk assessment. *Journal of Environmental Informatics*, *38*(2), 131–144. https://doi.org/10.3808/jei.202100462

IPCC. (2012). In C. B. Field, V. Barros, T. F. Stocker, D. Qin, D. J. Dokken, K. L. Ebi, et al. (Eds.), *Managing the risks of extreme events and disasters to advance climate change adaptation. A Special Report of Working Groups I and II of the Intergovernmental Panel on Climate Change* (p. 582). Cambridge University Press.

Joe, H. (2014). *Dependence modeling with copulas* (1st ed.). Chapman and Hall/CRC.

Karmakar, S., & Simonovic, S. P. (2008). Bivariate flood frequency analysis: Part 1. Determination of marginals by parametric and nonparametric techniques. *Journal of Flood Risk Management*, *1*, 190–200. https://doi.org/10.1111/j.1753-318X.2008.00022.x

Karmakar, S., & Simonovic, S. P. (2009). Bivariate flood frequency analysis. Part 2: A copula-based approach with mixed marginal distributions. *Journal of Flood Risk Management*, *2*, 32–44. https://doi.org/10.1111/j.1753-318x.2009.01020.x

Kong, X. M., Huang, G. H., Fan, Y. R., & Li, Y. P. (2015). Maximum entropy-Gumbel-Hougaard copula method for simulation of monthly streamflow in Xiangxi River, China. *Stochastic Environmental Research and Risk Assessment*, *29*, 833–846. https://doi.org/10.1007/s00477-014-0978-0

Laux, P., Vogl, S., Qiu, W., Knoche, H. R., & Kunstmann, H. (2011). Copula-based statistical refinement of precipitation in RCM simulations over complex terrain. *Hydrology and Earth System Sciences*, *15*, 2401–2419. https://doi.org/10.5194/hess-15-2401-2011

Leonard, M., Westra, S., Phatak, A., Lambert, M., van den Hurk, B., McInnes, K., et al. (2014). A compound event framework for understanding extreme impacts. *WIREs Climate Change*, *5*, 113–128. https://doi.org/10.1002/wcc.252

Li, F., Zhou, J., & Liu, C. (2018). Statistical modelling of extreme storms using copulas: A comparison study. *Coastal Engineering*, *142*, 52–61. https://doi.org/10.1016/j.coastaleng.2018.09.007

Lin, Y., & Dong, S. (2019). Wave energy assessment based on trivariate distribution of significant wave height, mean period, and direction. *Applied Ocean Research*, *87*, 47–63. https://doi.org/10.1016/j.apor.2019.03.017

Liu, Z., Zhou, P., Chen, X., & Guan, Y. (2015). A multivariate conditional model for streamflow prediction and spatial precipitation refinement. *Journal of Geophysical Research: Atmospheres*, *120*, 10116–10129. https://doi.org/10.1002/2015JD023787

Longfield, S. A., Faulkner, D., Kjeldsen, T. R., Macklin, M. G., Jones, A. F., Foulds, S. A., et al. (2019). Incorporating sedimentological data in UK flood frequency estimation. *Journal of Flood Risk Management*, *12*(1), e12449. https://doi.org/10.1111/jfr3.12449

Mehran, A., AghaKouchak, A., Nakhjiri, N., Stewardson, M. J., Peel, M. C., Phillips, T. J., et al. (2017). Compounding impacts of human-induced water stress and climate change on water availability. *Scientific Reports*, *7*, 6282. https://doi.org/10.1038/s41598-017-06765-0

Moftakhari, H. R., Salvadori, G., AghaKouchak, A., Sanders, B. F., & Matthew, R. A. (2017). Compounding effects of sea level rise and fluvial flooding. *Proceedings of the National Academy of Sciences*, *114*(37), 9785–9790. https://doi.org/10.1073/pnas.1620325114

Montes-Iturrizaga, R., & Heredia-Zavoni, E. (2015). Environmental contours using copulas. *Applied Ocean Research*, *52*, 125–139. https://doi.org/10.1016/j.apor.2015.05.007

Montes-Iturrizaga, R., & Heredia-Zavoni, E. (2016). Multivariate environmental contours using C-vine copulas. *Ocean Engineering*, *118*, 68–82. https://doi.org/10.1016/j.oceaneng.2016.03.011

Montes-Iturrizaga, R., & Heredia-Zavoni, E. (2017). Assessment of uncertainty in environmental contours due to parametric uncertainty in models of the dependence structure between metocean variables. *Applied Ocean Research*, *64*, 86–104. https://doi.org/10.1016/j.apor.2017.02.006

Montgomery, D. C. (2013). *Design and analysis of experiments* (8th ed.). John Wiley & Sons, Inc.

Muñoz, D. F., Moftakhari, H., & Moradkhani, H. (2020). Compound effects of flood drivers and wetland elevation correction on coastal flood hazard assessment. *Water Resources Research*, *56*(7), e2020WR027544. https://doi.org/10.1029/2020WR027544

Qi, W., Zhang, C., Fu, G., & Zhou, H. (2016). Imprecise probabilistic estimation of design floods with epistemic uncertainties. *Water Resources Research*, *52*, 4823–4844. https://doi.org/10.1002/2015WR017663

Read, L. K., & Vogel, R. M. (2015). Reliability, return periods, and risk under nonstationarity. *Water Resources Research*, *51*, 6381–6398. https://doi.org/10.1002/2015WR017089

Ridder, N. N., Pitman, A. J., Westra, S., Ukkola, A., Do, H. X., Bador, M., et al. (2020). Global hotspots for the occurrence of compound events. *Nature Communications*, *11*, 5956. https://doi.org/10.1038/s41467-020-19639-3

Sadegh, M., Moftakhari, H., Gupta, H. V., Ragno, E., Mazdiyasni, O., Sanders, B., et al. (2018). Multi-hazard scenarios for analysis of compound extreme events. *Geophysical Research Letters*, *45*, 5470–5480. https://doi.org/10.1029/2018GL077317

Sadegh, M., Ragno, E., & AghaKouchak, A. (2017). Multivariate copula analysis toolbox (MvCAT): Describing dependence and underlying uncertainty using a Bayesian framework. *Water Resources Research*, *53*, 5166–5183. https://doi.org/10.1002/2016WR020242

Salvadori, G., De Michele, C., & Durante, F. (2011). On the return period and design in a multivariate framework. *Hydrology and Earth System Sciences*, *15*, 3293–3305. https://doi.org/10.5194/hess-15-3293-2011

Salvadori, G., De Michele, C., Kottegoda, N. T., & Rosso, R. (2007). *Extremes in Nature: An Approach Using Copulas* (Vol. 56). Springer Science & Business Media.

Salvadori, G., Durante, F., & De Michele, C. (2013). Multivariate return period calculation via survival functions. *Water Resources Research*, *49*, 2308–2311. https://doi.org/10.1002/wrcr.20204

Salvadori, G., Durante, F., De Michele, C., Bernardi, M., & Petrella, L. (2016). A multivariate copula-based framework for dealing with hazard scenarios and failure probabilities. *Water Resources Research*, *52*, 3701–3721. https://doi.org/10.1002/2015WR017225

Sarhadi, A., Burn, D. H., Ausín, M. C., & Wiper, M. P. (2016). Time-varying nonstationary multivariate risk analysis using a dynamic Bayesian copula. *Water Resources Research*, *52*, 2327–2349. https://doi.org/10.1002/2015wr018525

Seneviratne, S. I., Zhang, X., Adnan, M., Badi, W., Dereczynski, C., Di Luca, A., et al. (2021). Weather and climate extreme events in a changing climate. In V. Masson-Delmotte, P. Zhai, A. Pirani, S. L. Connors, C. Péan, S. Berger, et al. (Eds.), *Climate Change 2021: The physical science basis. Contribution of Working Group I to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change*. Cambridge University Press. Retrieved from https://www.ipcc.ch/report/ar6/wg1/downloads/report/IPCC_AR6_WGI_Chapter_11.pdf

Shi, X., Wood, A. W., & Lettenmaier, D. P. (2008). How essential is hydrologic model calibration to seasonal streamflow forecasting? *Journal of Hydrometeorology*, *9*(6), 1350–1363. https://doi.org/10.1175/2008jhm1001.1

Singh, V. P. (1998). *Entropy-based parameter estimation in hydrology*. Kluwer Academic Publishers.

Sklar, A. (1959). *Fonctions de Répartition à n Dimensions et Leurs Marges* (Vol. 8, pp. 229–231). Publications de l'Institut Statistique de l'Université de Paris.

Stedinger, J. R., Vogel, R. M., & Foufoula-Georgiou, E. (1993). Frequency analysis of extreme events. In D. R. Maidment (Ed.), *Handbook of hydrology*. McGraw-Hill. Chapter 18.

Sun, C. X., Huang, G. H., Fan, Y. R., Zhou, X., Lu, C., & Wang, X. Q. (2019). Drought occurring with hot extremes: Changes under future climate change on Loess Plateau, China. *Earth's Future*, *7*(6), 587–604. https://doi.org/10.1029/2018ef001103

Sun, C. X., Huang, G. H., Fan, Y. R., Zhou, X., Lu, C., & Wang, X. Q. (2021). Vine copula ensemble downscaling for precipitation projection over the Loess Plateau based on high-resolution multi-RCM outputs. *Water Resources Research*, *57*(1), e2020WR027698. https://doi.org/10.1029/2020WR027698

Wahl, T., Jain, S., Bender, J., Meyers, S. D., & Luther, M. E. (2015). Increasing risk of compound flooding from storm surge and rainfall for major US cities. *Nature Climate Change*, *5*(12), 1093–1097. https://doi.org/10.1038/nclimate2736

Ward, P. J., Couasnon, A., Eilander, D., Haigh, I. D., Hendry, A., Muis, S., et al. (2018). Dependence between high sea level and high river discharge increases flood hazard in global deltas and estuaries. *Environmental Research Letters*, *13*, 084012. https://doi.org/10.1088/1748-9326/aad400

Xu, Y., Huang, G. & Fan, Y. (2017). Multivariate flood risk analysis for Wei River. *Stochastic Environmental Research and Risk Assessment*, *31*, 225–242. https://doi.org/10.1007/s00477-015-1196-0

Zhang, D., Shi, X., Xu, H., Jing, Q., Pan, X., Liu, T., et al. (2020). A GIS-based spatial multi-index model for flood risk assessment in the Yangtze River Basin, China. *Environmental Impact Assessment Review*, *83*, 106397. https://doi.org/10.1016/j.eiar.2020.106397

Zscheischler, J., Martius, O., Westra, S., Bevacqua, E., Raymond, C., Horton, R. M., et al. (2020). A typology of compound weather and climate events. *Nature Reviews Earth & Environment*, *1*, 333–347. https://doi.org/10.1038/s43017-020-0060-z

Zscheischler, J., Westra, S., van den Hurk, B. J. J. M., Seneviratne, S. I., Ward, P. J., Pitman, A., et al. (2018). Future climate risk from compound events. *Nature Climate Change*, *8*, 469–477. https://doi.org/10.1038/s41558-018-0156-3