Basic Studies on the Design of Simple Adaptive Control Systems

by

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Abstract

Simple Adaptive Control (SAC) is a direct model reference adaptive control (based on the command generator tracker theory) which has robustness with regard to disturbances, unmodelled dynamics and non-linearities. Since the structure of adaptive controller in SAC is very simple compared with conventional adaptive schemes, one can easily apply the method to practical plants. However, there are some severe constraints to implement the method, such as the requirement of the almost strictly positive realness (ASPR-ness) of the controlled plant. Thus, somewhat serious problems have remained with regard to the applicability of the SAC to the wider class of the controlled plant.

The objective of this research is to expand the applicable class of SAC methods to a wider class of controlled plants, including both minimum and non-minimum phase non-ASPR plants with unmodelled dynamics, large-scale systems, plants with unknown disturbances, and so on.

In this thesis, first of all, a basic concept of SAC for ASPR plants is reviewed in Chapter 2 for the sake of brevity of discussions in the following chapters. The command generator tracker (CGT) theory and the ASPR-ness of the plant are discussed. A basic algorithm of the SAC and stability of the control system are also given in this chapter. In Chapter 3, to expand the applicable class of the SAC method to non-ASPR plants, design schemes of compensators (which make non-ASPR plants ASPR in the sense that the resulting augmented plant with compensators is ASPR) are presented. Systematic design schemes of a parallel feedforward compensator (PFC) for both single-input/single-output (SISO) and multi-input/multi-output (MIMO) minimum phase plants with unknown orders but known relative degrees are given. Further, robust design schemes of compensators (PFC and pre-compensator) using frequency domain analysis are also presented for plants with multiplicative plant uncertainties. Chapter 4 presents a robust SAC algorithm for plants with state-dependent disturbances. By adding a robust adaptive control term to the original SAC algorithm, the control performance of the SAC system will be significantly improved. In Chapter 5, decentralized SAC schemes for large-scale systems with unknown interconnections are presented. The stability conditions corresponding to M-matrix condition and range condition are derived. A modified SAC algorithm with a derivative control term aimed at robust performance in transient state is presented in Chapter 6. In each chapter, the effectiveness of proposals are confirmed through numerical simulations.

Contents

1 Introduction

1.1 Historical Review

Adaptive control is a direct aggregation of a control methodology with some form of recursive system identification. It combines the design of controllers based on plant system models with the on-line estimation of the model parameter or controller parameters using input and output data measurements.

Most conventional control techniques are based on a certain knowledge of the controlled plant. However, in practice, there are uncertainties on practical plants and parameters of the plant frequently vary and change. In this situation, one is required to design a controller which works well during the whole operation. The adaptive control is an effective one that adjusts controller parameters automatically so as to obtain good control performance in the presence of uncertainties and parameter changes (Landau 1979, Narendra and Monopoli 1980, Sastry and Bodson 1989, Bitmead *et al.* 1990, Isermann *et al.* 1991).

There are two extremes in adaptive controls. One is an indirect adaptive control and the other is a direct adaptive control. A typical scheme of the indirect adaptive control is the Self-tuning Regulator which was originally proposed by Kalman (1958). Since then, the method was further established by Astrom and Wittenmark (1973). In this method, the controller structure is first determined using a conventional controller under the assumption that plant parameters are known. After that, unknown parameters of the plant are identified at on-line, and the controller parameters are adjusted using these estimated parameters. MRAC (Model Reference Adaptive Control) is a typical direct adaptive control. This method was first developed by Whitaker and co-workers (1958). The method was based on the so-called M.I.T. rule. Unfortunately, closed loop stability or convergence of the error signal could not be ensured by this rule. Since then, Parks (1966) showed a proof of stability using the Lyapunov theory with the Kalman-Yakubovich lemma, but only for the case where the plant transfer function is positive definite. A more general and fundamental strategy for MRAC was developed by Monopoli (1974) using augmented error signal. As mentioned above, the basic formulations of adaptive controls were built up during the late 1950's and the first 1970's.

During the late 1970's, the global stability of adaptive control systems was resolved, mainly for MRAC. Many interesting proofs of stability for several types of adaptive control systems appeared. Narendra and Valavani (1980) and Morse (1980) showed proofs of stability based on the Lyapunov theorem. Landau (1979) and Egardt (1979) adopted the Popov hyperstability theorem to prove stability. For discrete time control systems, Goodwin and co-workers (1984) also showed proofs of the stability of adaptive control systems. This period marked the development of the fundamental field of adaptive control.

In the above-mentioned works, it was assumed that some prior information regarding the plant transfer function (order of the plant, relative degree of the plant, and sign of high frequency gain) were available and that no external disturbances were present. However, these assumptions are seldom valid in practice. At this point, Rohrs and his co-workers (1982) first pointed out that the presence of unmodelled dynamics or disturbances often very much degrades the control performance of the system and sometimes makes the control system unstable. With these points as background, during the late 1980's a great deal of attention was devoted to analyzing the robustness of the adaptive controller. Many modified adaptive control algorithms aimed at improving the robustness of the control systems were proposed (Ioannou and Kokotovic 1983, 1984a, 1984b, Ioannou and Tsakalis 1986, Kreisselmeier and Anerson 1986, Narendra and Annaswamy 1987, Tao and Ioannou 1988, Ortega and Tang 1989). At present, the robustness of the adaptive control system is still an important problem for practical application of adaptive schemes, and several teams of researchers are trying to complete this problem (Tao and Ioannou 1991, Chen 1992, Bartolini and Ferrara 1992, Tao 1992, Chien and Fu 1992, Lee and Anderson 1993, Zang and Bitmead 1994, Tao Kokotovic 1994).

The general on-line adaptive control algorithms depend on a large number of estimated parameters within a given mathematical model structure. Thus the structure of the adaptive controller sometimes becomes extremely complicated compared with that of usual conventional controllers. As a result, a slight miscounting of the order between the mathematical model and the real plant due to presence of unmodelled dynamics often degrades the control performance of the system and sometimes makes the control system unstable. Research of robust adaptive controls was begun with these points in mind and several interesting modifications in adaptive algorithms countered to plant uncertainties and external disturbances have been proposed as mentioned above. However, most of these modifications caused complications of the adaptive controller structure.

The simplicity of controller structure is extremely fascinating for practicing engineers since they are able to understand and easily implement the control schemes. With this in mind, a new strategy to direct model reference adaptive control, which makes it possible to construct the adaptive control system regardless of the plant order, was first proposed by Sobel, Kaufman and Mabius (1979, 1982). From the simplicity of the controller, this adaptive scheme is called *Simple Adaptive Control* (SAC). The basic idea of this adaptive method is to ensure the stability of the control system by using the output feedback under the ASPR (almost strictly positive real) condition on the plant (the plant is said to be ASPR if there exists a static output feedback such that the resulting closed-loop transfer function is SPR (strictly positive real)) and to attain the model output following by forward compensation based on the Command Generator Tracker (CGT) theory (Broussard and O'Brien 1980). Thus, the structure of the adaptive controller of the method consists of a linear combination of the reference model states, reference inputs, and output error feedback between the plant and reference model outputs. Since the order of the reference model can be chosen irrespective of the order of the plant, the number of adaptive gain parameters to be identified can be decreased in the adaptive controller if one chooses a low-order reference model. That is, we can obtain a simple form of adaptive controller with a low-order reference model even if the plant has higher order. This is why the method is called *Simple Adaptive Control.*

The original SAC algorithm was modified by incorporating σ- modification term (Ioannou and Kokotovic 1983) into the parameter adjusting laws (Bar-Kana 1987b, Bar-Kana and Kaufman 1985a). This algorithm is applicable to control systems with not only step reference input but also arbitrary time varying reference inputs and applicable to controlled plants with bounded disturbances. The SAC schemes have also been developed for discrete-time systems (Bar-Kana and Kaufman 1983, Bar-Kana 1986, 1989, Ohtsuka *et al.* 1992, Shibata and Kurebayashi 1995), time-varying systems (Bar-Kana 1988, 1990), and non-linear systems (Bar-Kana and Guez 1990).

In spite of the simplicity of the controller, SACs have much robustness with regard to disturbances, unmodelled dynamics and non-linearities because the ASPR characteristics of the plant enable us to stabilize the plant robustly with hight gain output feedback (Steinberg and Corless 1985, Steinberg 1988, Zeheb 1986, and Gu 1990). These robust performances have been confirmed through several numerical simulations and practical experiments on large flexible structures (Bar-Kana 1987b, Bar-Kana, Kaufman and Balas 1983, Ih *et al.* 1985, 1987, Lee *et al.* 1988, Sanchez 1986, Iwai *et al.* 1993, 1995, Hino *et al.* 1995), robotic manipulators (Bar-Kana 1987a, Meldrum and Balas 1986, Bar-Kana and Guez 1991, Mizumoto *et al.* 1993,), servo systems (Bar-Kana and Kaufman 1988, Ohtsuka and co-workers 1992, 1993, 1994, Ohtomo *et al.* 1992, Iwai *et al.* 1992), automated guided vehicles (Kawasaki, Iwai and Haramaki 1994), inverted pendulums (Kawasaki *et al.* 1993, 1994), and drug infusions (Kaufman, Roy and Xu 1984). However, the

ASPR condition on the controlled plants is a severe restriction in practical plants because most practical plants do not satisfy the ASPR condition. As a countermeasure to this problem, the introduction of a parallel feedforward compensator (PFC) to non-ASPR plants was suggested by Bar-Kana and Kaufman (1985b). It has also been shown that if there exists a known dynamic output feedback compensator: $H(s)$ which stabilizes the closed-loop system, then the augmented plant with $H(s)^{-1}$ in parallel is ASPR (Bar-Kana 1987a). Unfortunately, this approach does not guarantee perfect output following in general because of a bias effect from the additional PFC. However, it is argued that if the gain of the PFC can be chosen small enough, we can approximately attain the control objective for original controlled plants. In SAC system designs, how to design PFCs is the most important problem to ensure the robust performance of the control system.

Recently, new SAC system configurations for asymptotic output tracking are suggested by Kaufman and Neat (1993) and Su and Sobel (1992). Kaufman's approach incorporates the PFC dynamics into the reference model in a manner such that asymptotic tracking of the original plant and reference model outputs is ensured subject to asymptotic tracking of the augmented plant and reference model outputs. Su's method introduces an adaptive gain to supplementary dynamics, which is implemented in the plant in parallel to guarantee asymptotic tracking only for the case where the reference model input is constant.

1.2 Outline of the Dissertation

The objective of this research is to expand the applicable class of SAC methods to a wider class of controlled plants, including both minimum and non-minimum phase non-ASPR plants with unmodelled dynamics, large-scale systems, plants with unknown disturbances, and so on. The obtained schemes for SAC system design are very useful and powerful for practical plants with several uncertainties.

The contents are organized as follows. Chapter 2 presents a basic concept of SAC for ASPR plants for the sake of brevity of discussions in the following chapters. In this chapter, the command generater tracker (CGT) theory (which is a basic theory on the SAC scheme) and ASPR conditions of the plant are discussed. A basic algorithm of the SAC for ASPR plants and stability of the control system are also given in this chapter.

In Chapter 3, design schemes of compensators, which make non-ASPR plants ASPR in the sense that the resulting augmented plant with compensators is ASPR, are presented to expand the applicable class of the SAC method to non-ASPR plants. Systematic design schemes of a parallel feedforward compensator (PFC) for both single-input/single-output (SISO) and multi-input/multi-output

(MIMO) minimum phase plants with unknown orders but known relative degrees are given. Further, robust design schemes of compensators (PFC and precompensator) using frequency domain analysis are presented for plants with multiplicative plant uncertainties which might be non-minimum phase. Chapter 4 presents a robust SAC algorithm for plants with state-dependent disturbances. SAC has robustness with regard to disturbances in general. However, in the case where *large* external disturbances or state-dependent disturbances are present, of course, the control performance might become worse. By adding a robust adaptive control term to the original SAC algorithm, the control performance of the SAC system will be significantly improved. In Chapter 5, decentralized SAC schemes for large-scale systems with unknown interconnections are presented. The stability conditions corresponding to M-matrix and range conditions are clarified. It is also shown that the use of the robust SAC scheme given in Chapter 4 in decentralized SAC systems is effective in eliminating the affects of interconnections. A modified SAC algorithm with a derivative control term aimed at robust performance in transient state is presented in Chapter 6. In each chapter, the effectiveness of proposals are confirmed through numerical simulations.

2 Basic Concept of Simple Adaptive Control

2.1 Introduction

In this chapter, a basic concept of the simple adaptive control (SAC) is reviewed for the sake of brevity of discussions in the following chapters. First, the command generator tracker (CGT) theory which is a basic theory on the SAC scheme is discussed. A sufficient condition, under which the signals generated by the CGT are bounded, is considered. Almost strictly positive real (ASPR) conditions for both multi-input/multi-output (MIMO) and single-input/single-output (SISO) plants are also derived. Finally, a basic algorithm of the SAC and stability of the control system are given.

The SAC is a direct model reference adaptive control based on the CGT theory. If the plant is known, then we can attain perfect model output tracking only by using the ideal control input generated by the CGT. However, there are uncertainties on practical plants in general. The SAC adaptively adjusts parameters in the CGT to have the ideal control input for unknown plants. The SAC also is a control strategy based on the almost strictly positive realness (ASPR-ness) of the controlled plant. Under the ASPR condition, one can ensure the stability of the closed-loop system with output feedback. In the SAC system, the feedback gain to ensure the stability of the control system is also adaptively adjusted.

2.2 Command Generator Tracker Theory

The *command generator tracker* is an ideal control input which achieves perfect model output tracking. This idea was proposed by Broussard and O'Brien (1980) with regard to a feedforward control problem.

Let consider the following continuous linear time-invariant (LTI) plant:

$$\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + B\boldsymbol{u}(t) \tag{2.2.1a}$$

$$\boldsymbol{y}(t) = C\boldsymbol{x}(t) \tag{2.2.1b}$$

where $\boldsymbol{x}$ is a vector of dimension n and $\boldsymbol{u}$ and $\boldsymbol{y}$ are m-dimensional vectors. The constant matrices A, B and C are of appropriate size. It is assumed that the pair (A, B) is controllable and the pare (A, C) is observable. Further, consider the reference model which the plant is required to follow:

$$\dot{\boldsymbol{x}}_m(t) = A_m \boldsymbol{x}_m(t) + B_m \boldsymbol{u}_m(t) \tag{2.2.2a}$$

$$\boldsymbol{y}_m(t) = C_m \boldsymbol{x}_m(t) \tag{2.2.2b}$$

Now, we make the following assumptions.

Assumption 2.1:

(1)

$$det \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \neq 0 \tag{2.2.3}$$

(2) Ω_i are solutions of the matrix equation:

$$\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \begin{bmatrix} \Omega_1 & \Omega_2 \\ \Omega_3 & \Omega_4 \end{bmatrix} = I \tag{2.2.4}$$

and no eigenvalue of Ω_1 is equal to the inverse of an eigenvalue of A_m.

Under this assumption we get the following theorem concerning the perfect model output following.

Theorem 2.1: (*Command Generator Tracker (CGT) Theory*)
Assumption 2.1 holds. Then, assuming that $\boldsymbol{y}(0) = \boldsymbol{y}_m(0)$, the optimal input $\boldsymbol{u}^*(t)$ and optimal state $\boldsymbol{x}^*(t)$, which attain the perfect model output following:

$$\boldsymbol{e}_y(t) = \boldsymbol{y}(t) - \boldsymbol{y}_m(t) \equiv 0 \ , \quad \forall t \geq 0$$

are given by

$$\begin{bmatrix} \boldsymbol{x}^*(t) \\ \boldsymbol{u}^*(t) \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_m(t) \\ \boldsymbol{u}_m(t) \end{bmatrix} + \begin{bmatrix} \Omega_1 \\ \Omega_3 \end{bmatrix} \boldsymbol{v}(t) \tag{2.2.5}$$

where

$$\begin{cases} S_{11} = \Omega_1 S_{11} A_m + \Omega_2 C_m \\ S_{12} = \Omega_1 S_{11} B_m \\ S_{21} = \Omega_3 S_{11} A_m + \Omega_4 C_m \\ S_{22} = \Omega_3 S_{11} B_m \end{cases} \tag{2.2.6}$$

and

$$\Omega_1\dot{\boldsymbol{v}}(t) = \boldsymbol{v}(t) - S_{12}\dot{\boldsymbol{u}}_m(t) \tag{2.2.7}$$

Proof: The detailed proof has been shown by Broussard and O'Brien (1980). Here we derive the result directly.

From (2.2.4), we have

$$\begin{cases} A\Omega_1 + B\Omega_3 = I_n \\ A\Omega_2 + B\Omega_4 = 0 \\ C\Omega_1 = 0 \\ C\Omega_2 = I_m \end{cases} \tag{2.2.8}$$

Using these facts, it follows from (2.2.5) - (2.2.7) that

$$\begin{aligned} \dot{\boldsymbol{x}}^*(t) &= S_{11}\dot{\boldsymbol{x}}_m(t) + S_{12}\dot{\boldsymbol{u}}_m(t) + \Omega_1\dot{\boldsymbol{v}}(t) \\ &= S_{11}A_m\boldsymbol{x}_m(t) + S_{11}B_m\boldsymbol{u}_m(t) + \boldsymbol{v}(t) \\ &= A(S_{11}\boldsymbol{x}_m(t) + S_{12}\boldsymbol{u}_m(t)) + B(S_{21}\boldsymbol{x}_m(t) + S_{22}\boldsymbol{u}_m(t)) \\ &\quad +(A\Omega_1 + B\Omega_3)\boldsymbol{v}(t) \\ &= A\boldsymbol{x}^*(t) + B\boldsymbol{u}^*(t) \end{aligned} \tag{2.2.9}$$

and

$$\begin{aligned} \boldsymbol{y}^*(t) &= C\boldsymbol{x}^*(t) \\ &= C(S_{11}\boldsymbol{x}_m(t) + S_{12}\boldsymbol{u}_m(t) + \Omega_1\boldsymbol{v}(t)) \\ &= C_m\boldsymbol{x}_m(t) \\ &= \boldsymbol{y}_m(t) \end{aligned} \tag{2.2.10}$$

Thus the desired result is obtained. □

When one tries to apply CGT theory to practical plants, the signal $\boldsymbol{v}(t)$ in (2.2.5) has to be bounded. Noting that (2.2.7) for $\boldsymbol{v}(t)$ has the so-called descriptor form, we need some explanation with regard to the boundedness of $\boldsymbol{v}(t)$.

Lemma 2.1: *Suppose that the plant is strictly minimum phase, i.e. the zero polynomial of the plant is a Hurwiz polynomial, and has the relative MacMillan degree of $(n-m)/n$. Further suppose $\boldsymbol{u}_m^{(i)}(t)$, $i = 0, 1, \cdots, m$, denoting the i-th derivative of $\boldsymbol{u}_m(t)$, exists and is uniformly bounded, and $\boldsymbol{v}(0)$. Then $\boldsymbol{v}(t)$ is bounded.*

Proof: Consider the fact that the inverse of the eigenvalues of Ω_1 are equal to the transmission zeros ($(n-m)$ zeros) of the plant (Broussard and O'Brien 1980) and they have negative real parts under the assumption of plant zeros. Then the solution of the descriptor system (2.2.7) can be constructed from the stable exponential function mode and a linear combination of $\boldsymbol{u}_m^{(i)}(t)$, $i = 0, 1, \cdots, m$ (Rosenbrock 1976). Therefore, $\boldsymbol{v}(t)$ is bounded as far as the $\boldsymbol{u}_m^{(i)}(t)$ are bounded. □

Remark 2.1: If $\boldsymbol{u}_m(t)$ is a step input vector function, $\boldsymbol{v}(t)$ vanishes in (2.2.5).

2.3 Almost Strict Positive Realness

The almost strict positive realness of the plant is defined as follows.

Definition: *The plant is said to be almost strictly positive real (ASPR) if there exists a static output feedback such that the resulting closed-loop transfer function is strictly positive real (SPR)*
(concerning SPR, see Taylor 1974, Ioannou and Tao 1987, Wen 1988, Tao and Ioannou 1988b, Lozano-Leal and Joshi 1990)

In the SAC, we require a particular plant to be SPR in the construction of the Lyapnov function to ensure stability of the entire adaptive system. The ASPR-ness of the plant is a suitable weaker requirement than SPR-ness. It is important to note that the ASPR conditions do not require the plant to be stable.

The sufficient condition for an m-input/m-output n-dimensional plant to be ASPR has been provided by Bar-Kana (1991) as follows:

ASPR condition:

(1) The plant is minimum phase

(2) The relative MacMillan degree of the plant is $(n-m)/n$

(3) Let the minimum realization of the plant be (A, B, C)*, then* $CB > 0$ *(positive definite)*

For SISO plants, the condition can be rewritten as follows:

ASPR condition: (*for SISO plants*)

(1) The plant transfer function is inversely stable

(2) The relative degree of the plant transfer function is 1

(3) The leading coefficient of the plant transfer function is positive

This condition for SISO plants has also been shown by Zeheb (1986) and Steinberg (1988).

Remark 2.2: In practice, proper plants also are able to be ASPR. This is the reason why the above conditions are the sufficient condition for a plant to be ASPR.

2.4 Basic Simple Adaptve Control Algorithm

2.4.1 Problem Setup

Consider the controllable and observable LTI continuous system:

$$\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + B\boldsymbol{u}(t) + \boldsymbol{g}(t) \tag{2.4.1a}$$

$$\boldsymbol{y}(t) = C\boldsymbol{x}(t) \tag{2.4.1b}$$

where $A \in \mathrm{R}^{n\times n}$, $B \in \mathrm{R}^{n\times m}$ and $C \in \mathrm{R}^{m\times n}$ are constant matrices, $\boldsymbol{x}(t) \in \mathrm{R}^n$, $\boldsymbol{y}(t) \in \mathrm{R}^m$ and $\boldsymbol{u}(t) \in \mathrm{R}^m$ are vector functions denoting state vector, output vector and control vector, respectively, and $m \leq n$. $\boldsymbol{g}(t) \in \mathrm{R}^n$ is an unknown but bounded disturbance vector. Further consider the asymptotically stable reference model that the plant output is required to follow:

$$\dot{\boldsymbol{x}}_m(t) = A_m\boldsymbol{x}_m(t) + B_m\boldsymbol{u}_m(t) \tag{2.4.2a}$$

$$\boldsymbol{y}_m(t) = C_m\boldsymbol{x}_m(t) \tag{2.4.2b}$$

where $A_m \in \mathrm{R}^{n_m\times n_m}$, $B_m \in \mathrm{R}^{n_m\times m}$, $C_m \in \mathrm{R}^{m\times n_m}$, $\boldsymbol{x}_m(t) \in \mathrm{R}^n_m$, $\boldsymbol{y}_m(t) \in \mathrm{R}^m$ and $\boldsymbol{u}_m(t) \in \mathrm{R}^m$.

Here we make the following assumptions on the plant and reference model.

Assumption 2.2:

(1) Plant (2.4.1) *is ASPR, i.e. there exists a constant gain matrix K_e^* such that the following transfer matrix:*

$$G_s(s) = C(sI - A_c)^{-1}B \tag{2.4.3}$$

is SPR. Where

$$A_c = A + BK_e^*C \tag{2.4.4}$$

(2)

$$det \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \neq 0 \,. \tag{2.4.5}$$

(3) Ω_i are solutions of the matrix equation:

$$\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \begin{bmatrix} \Omega_1 & \Omega_2 \\ \Omega_3 & \Omega_4 \end{bmatrix} = I \tag{2.4.6}$$

and no eigenvalue of Ω_1 is equal to the inverse of an eigenvalue of A_m.

(4) $\boldsymbol{u}_m^{(i)}(t)$, $i = 0, 1, \cdots, m$, denoting the i-th derivative of $\boldsymbol{u}_m(t)$, exist and are uniformly bounded.

The control objective is to find, without explicit knowledge of the plant parameters, the control input $\boldsymbol{u}(t)$ such that the output $\boldsymbol{y}(t)$ of (2.4.1b) tracks to the output $\boldsymbol{y}_m(t)$ of the reference model.

According to the CGT theory, if the plant parameters are known and $\boldsymbol{g}(t) \equiv 0$, then the optimal input $\boldsymbol{u}^*(t)$ and optimal state $\boldsymbol{x}^*(t)$, which attain the perfect model output following: $\boldsymbol{e}_y(t) = \boldsymbol{y}(t) - \boldsymbol{y}_m(t) \equiv 0$, $\forall t \geq 0$ are given by

$$\boldsymbol{x}^*(t) = S_{11}\boldsymbol{x}_m(t) + S_{12}\boldsymbol{u}_m(t) + S_{13}(t) \tag{2.4.7a}$$

$$\boldsymbol{u}^*(t) = S_{21}\boldsymbol{x}_m(t) + S_{22}\boldsymbol{u}_m(t) + S_{23}(t) \tag{2.4.7b}$$

where S_{ij} $(i, j = 1, 2)$ are appropriate dimensional matrices, which can be determined from the solution of (2.4.6), and vector functions $S_{13}(t)$ and $S_{23}(t)$ can be determined from the command input $\boldsymbol{u}_m(t)$ and are uniformly bounded from Assumption 2.2.

In practice, it is not possible to realize the ideal control input (2.4.7b) because we have assumed that plant parameters are generally unknown.

2.4.2 Control Algorithm

The control input is given as follows in the SAC.

$$\boldsymbol{u}(t) = K(t)\boldsymbol{z}(t) \tag{2.4.8}$$

where

$$\boldsymbol{z}(t) = [\boldsymbol{e}_y(t)^T, \boldsymbol{x}_m(t)^T, \boldsymbol{u}_m(t)^T]^T \tag{2.4.9}$$

$$\boldsymbol{e}_y(t) = \boldsymbol{y}(t) - \boldsymbol{y}^*(t) = \boldsymbol{y}(t) - \boldsymbol{y}_m(t)$$

$$K(t) = [K_e(t), K_x(t), K_u(t)] \tag{2.4.10}$$

Here the gain matrix $K(t)$ is adaptively adjusted by the following parameter adjusting law:

$$\begin{cases} K(t) = K_I(t) + K_P(t) \\ \dot{K}_I(t) = -e_y(t)z(t)^T\Gamma_I - \sigma_I(t)K_I(t) \\ K_P(t) = -e_y(t)z(t)^T\Gamma_P \\ \sigma_I(t) = \sigma_1 \frac{e_y(t)^T e_y(t)}{1+e_y(t)^T e_y(t)} + \sigma_2 \end{cases} \tag{2.4.11}$$

where

$$\Gamma_I = \Gamma_I^T > 0,\ \Gamma_P = \Gamma_P^T > 0,\ \sigma_1, \sigma_2 > 0$$

The constructed SAC system is shown in figure 2.1. As shown in figure 2.1, the SAC has a structure which ensures stability by an adaptive output feedback with gain $K_e(t)$, and attains the model output following using a feedforward with adaptively adjusted gains $K_x(t)$ and $K_u(t)$ instead of CGT gains S_{21} and S_{22}, respectively.

2.4.3 Stability of the Control System

Suppose that $g(t) \equiv 0$ in (2.4.1) and further suppose that the perfect model output following has been attained between the plant (2.4.1) and the reference model (2.4.2). Then using (2.4.7), we have

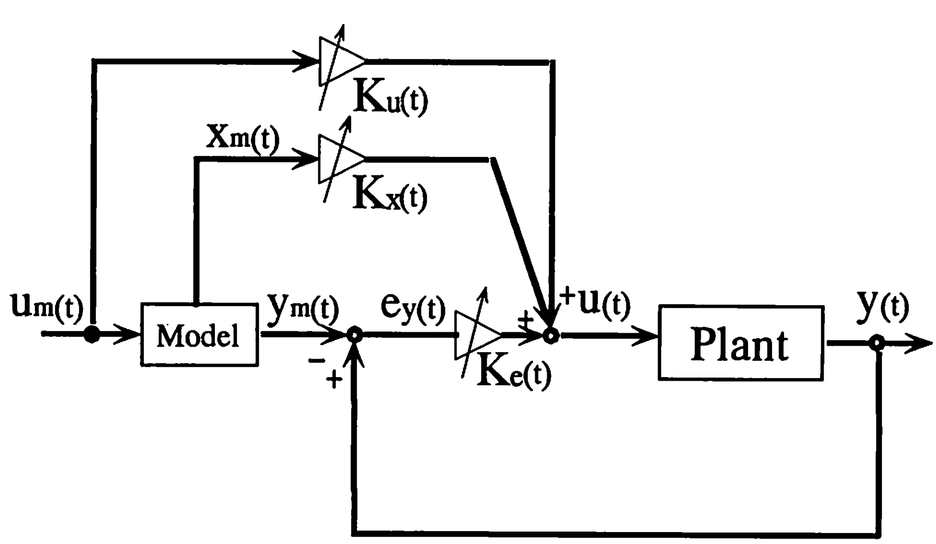


Figure 2.1 Overall block-diagram of a SAC

$$\dot{\boldsymbol{x}}^*(t) = A\boldsymbol{x}^*(t) + B\boldsymbol{u}^*(t) \tag{2.4.12a}$$
$$\boldsymbol{y}^*(t) = C\boldsymbol{x}^*(t) = \boldsymbol{y}_m(t) \tag{2.4.12b}$$

From (2.4.1), (2.4.8) and (2.4.12), we can obtain the following error system:

$$\dot{\boldsymbol{e}}_x(t) = A_c\boldsymbol{e}_x(t) + B(\Delta K(t)\boldsymbol{z}(t) - S_{23}(t)) + \boldsymbol{g}(t) \tag{2.4.13a}$$
$$\boldsymbol{e}_y(t) = C\boldsymbol{e}_x(t) \tag{2.4.13b}$$

where

$$\begin{aligned} \boldsymbol{e}_x(t) &= \boldsymbol{x}(t) - \boldsymbol{x}^*(t),\ \boldsymbol{e}_y(t) = \boldsymbol{y}(t) - \boldsymbol{y}^*(t) = \boldsymbol{y}(t) - \boldsymbol{y}_m(t) \\ \Delta K(t) &= K(t) - K^*,\ K^* = [K_e^*, S_{21}, S_{22}] \end{aligned} \tag{2.4.14}$$

and A_c has been defined by (2.4.4). Furthermore, From Assumption 2.2 (1), there exist positive matrices P and Q satisfying the Kalman-Yakubovich Lemma:

$$\begin{aligned} A_c^T P + PA_c &= -Q \\ B^T P &= C \end{aligned} \tag{2.4.15}$$

Using these results, we have the following theorem concerning the boundedness of all the signals in the control system.

Theorem 2.2: *Suppose that Assumption* 2.2 *holds. Then the use of the control input* (2.4.8) *guarantees the ultimate uniform boundedness of all the signals in the control system.*

Proof: Consider the positive definite function

$$V(t) = \boldsymbol{e}_x(t)^T P\boldsymbol{e}_x(t) + \mathrm{tr}\left\{\Delta K_I(t)\Gamma_I^{-1}\Delta K_I(t)^T\right\} \tag{2.4.16}$$

where $\Delta K_I(t) = K_I(t) - K^*$. From (2.4.13)-(2.4.16), we have

$$\begin{aligned} \frac{dV(t)}{dt} &= -\boldsymbol{e}_x(t)^T Q\boldsymbol{e}_x(t) + 2(\Delta K(t)\boldsymbol{z}(t) - S_{23}(t))^T\boldsymbol{e}_y(t) + 2\boldsymbol{g}(t)^T\boldsymbol{e}_y(t) \\ &\quad + \mathrm{tr}\left\{\Delta\dot{K}_I(t)\Gamma_I^{-1}\Delta K_I(t)^T + \Delta K_I(t)\Gamma_I^{-1}\Delta\dot{K}_I(t)^T\right\} \end{aligned} \tag{2.4.17}$$

It follows from (2.4.11) that

$$\begin{aligned} &\mathrm{tr}\left\{\Delta\dot{K}_I(t)\Gamma_I^{-1}\Delta K_I(t)^T + \Delta K_I(t)\Gamma_I^{-1}\Delta\dot{K}_I(t)^T\right\} \\ &\quad \le -2\boldsymbol{e}_y(t)^T\Delta K(t)\boldsymbol{z}(t) - 2\sigma_I(t)\mathrm{tr}\left\{\Delta K_I(t)\Gamma_I^{-1}\Delta K_I(t)^T\right\} \\ &\quad\quad -2\sigma_I(t)\mathrm{tr}\left\{\Delta K_I(t)\Gamma_I^{-1}K^{*T}\right\} \end{aligned} \tag{2.4.18}$$

Then from (2.4.17) and (2.4.18) we have

$$\frac{dV(t)}{dt} \leq -\lambda_{min}[Q] \parallel \boldsymbol{e}_x(t) \parallel^2 + a_g \parallel \boldsymbol{e}_x(t) \parallel$$
$$-2\sigma_I(t)\lambda_{min}[\Gamma_I^{-1}]\left\{\sum_{i=1}^{m} \parallel \Delta \boldsymbol{k}_{Ii}(t) \parallel^2\right\}$$
$$+2\sigma_I(t)\lambda_{max}[\Gamma_I^{-1}]\left\{\sum_{i=1}^{m} \parallel \boldsymbol{k}_i^* \parallel \parallel \Delta \boldsymbol{k}_{Ii}(t) \parallel\right\} \quad (2.4.19)$$

where

$$a_g = 2(\max_t \parallel S_{23}(t) \parallel + \max_t \parallel \boldsymbol{g}(t) \parallel) \parallel C \parallel \quad (2.4.20)$$

and vectors $\Delta \boldsymbol{k}_{Ii}(t)$ and $\boldsymbol{k}_i^*$ are denoted as the ith row of matrices $\Delta K_I(t)$ and K^*, respectively. Hence it is apparent from (2.4.19) that $\boldsymbol{e}_x(t)$ and $\Delta K_I(t)$ are uniformly ultimately bounded (Corless and Leitmann 1981, Chen 1986). It can be verified from this conclusion that $\boldsymbol{e}_y(t)$, $\boldsymbol{z}(t)$ and $K(t)$ are also uniformly ultimately bounded and proof is complete.

2.5 Conclusions

In this chapter, the basic concept of SAC has been reviewed. The SAC is a direct model reference adaptive control method based on the CGT theory and has a simpler adaptive controller structure than conventional adaptive control schemes. The boundedness of all the signals in the control system is ensured under the ASPR condition.

3 Simple Adaptive Control for Plants not Satisfying Almost Strictly Positive Real Condition

3.1 Introduction

The boundedness of all the signals in the SAC system is guaranteed under the ASPR condition. However, most actual plants do not satisfy the ASPR condition. Hence this condition imposes a severe restriction on the plant with respect to the practical applicability of SAC. With this in mind Bar-Kana (1987a) first suggested that the non-ASPR plant can be made ASPR by implementing a parallel feedforward compensator (PFC) $H(s)$ on the plant. That is, if the plant can be stabilized by a suitable dynamic output compensator $H(s)^{-1}$ and the gain of $H(s)$ can be chosen *small* enough, then the augmented plant with PFC becomes ASPR and we can apply SAC for the thus obtained augmented plant. Recently, Su and Sobel (1992) proposed a unified theory to apply the SAC method to non-ASPR plants. However, their methods require *a priori* knowledge of the dynamic feedback compensator that stabilizes the plant, and it may be difficult to find such a compensator for a plant that is unknown and of high order.

In this chapter the problem of designing a PFC is discussed. New PFC design approaches which are concrete and systematic will be proposed.

3.2 Parallel Feedforward Compensator

Let us consider a non-ASPR plant $G(s)$ and introduce a PFC $F(s)$ as shown in Figure 3.1. Then we have the following augmented plant:

$$G_a(s) = G(s) + F(s) \tag{3.2.1}$$

If the obtained augmented plant satisfies the ASPR condition, and, if, $F(s)$ is '*small*' enough compared with the plant $G(s)$ (that is, the augmented plant output $y_a(t)$ is supposed to be approximately equal to the output $y(t)$ of the original plant), then we can attain the control objective approximately by applying the SAC method to the augmented plant $G_a(s)$ instead of the original plant $G(s)$.

However the problem remains unsolved unless we can offer some concrete and systematic approaches to realizing such an augmented plant with the PFC $F(s)$. The purpose of this section is to express a concrete method of realizing such a PFC.

3.2.1 Parallel Feedforward Compensator Design for Single-Input Single-Output Plants

The problem is to find a PFC $F(s)$ satisfying the following conditions.

Condition 3.1:

(c1) $G_a(s)$ *is ASPR.*

(c2) $G_a(s) \simeq G(s)$

(c3) $F(s)$ *is physically realizable.*

However, Condition (c2) raises the difficult question of whether it can be realized over the whole frequency range or not. Hence we replace it with a more realistic condition

(c2)' There exists an $\omega_0 > 0$ *such that, for given* $\varepsilon > 0$,

$$||G_a(j\omega)| - |G(j\omega)|| \le \varepsilon$$

holds on $0 \le \omega \le \omega_0$.

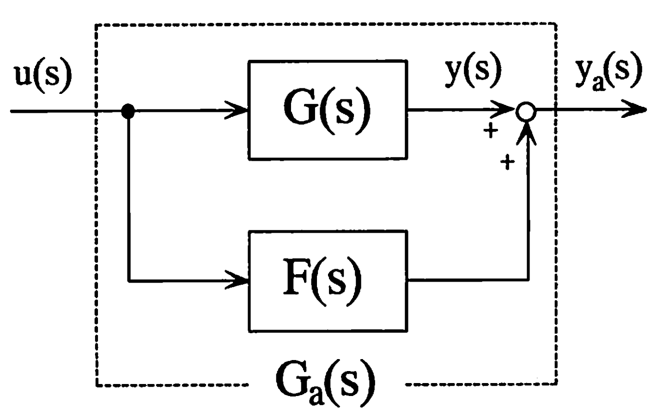


Figure 3.1 Augmented plant with PFC

Now, we give the controlled plant transfer function as follows:

$$G(s) = k_p \frac{B(s)}{A(s)}, \quad k_p > 0 \tag{3.2.2}$$

where $A(s)$ and $B(s)$ are nth and mth order monic polynomials, respectively. Further, the following assumptions are imposed on (3.2.2).

Assumption 3.1:

(1) The upper bound γ^* of the relative degree of the plant $\gamma_p = n - m$ is known.

(2) $B(s)$is a Hurwitz polynomial.

(3) The approximate values of the leading coefficient k_p and $|G(j0)|$ are known.

In preparation for investigating the design method of PFC, we give the following lemma.

Lemma 3.1: *Consider the following augmented plant $G_{a1}(s)$ for the plant* (3.2.2)*:*

$$G_{a1}(s) = G(s) + F_1(s) \tag{3.2.3}$$

$$F_1(s) = f_1/D_1(s), \quad k_p \gg f_1 > 0 \tag{3.2.4}$$

$$D_1(s): \quad \gamma_1 th \text{ order monic stable polynomial.}$$

Then, subject to Assumption 3.1*, It follows that*

(a) if $\gamma_1 \geq \gamma_p$, then $G_{a1}(s)$ is inversely stable, i.e. the numerator polynomial of $G_{a1}(s)$ is a stable polynomial, and its relative degree is γ_p.

(b) if $\gamma_1 = \gamma_p - 1$, then $G_{a1}(s)$ is inversely stable and its relative degree is $\gamma_p - 1$.

Proof: From (3.2.2)–(3.2.4), we have

$$\begin{aligned} G_{a1} &= \{k_p B(s) D_1(s) + f_1 A(s)\} / A(s) D_1(s) \\ &= B_{a1}(s)/A_{a1}(s) \end{aligned} \tag{3.2.5}$$

Part (a): Let $\gamma_1 = \gamma_p + \alpha$, $\alpha \geq 0$. Then $\deg A_{a1}(s) = n + \gamma_p + \alpha$ and $\deg B_{a1}(s) = n + \alpha$. Hence, the relative degree of $G_{a1}(s)$ is γ_p. From (3.2.5), the zeros of $G_{a1}(s)$ are determined from the equation

$$B(s)D_1(s) + f_1' A(s) = 0, \quad f_1' = f_1/k_p \tag{3.2.6}$$

Since $\deg B(s)D_1(s) \geq \deg A(s)$, every solution of the algebraic equation (3.2.6) approaches the solution of $B(s)D_1(s) = 0$ as f_1' tends to zero. Taking into account that $B(s)$ and $D_1(s)$ are stable polynomials, we may conclude that all solutions of (3.2.6) are located in the left half-plane for sufficiently small f_1'. That is, $B_{a1}(s)$ is a stable polynomial for f_1 such that $k_p \gg f_1 > 0$.

Part (b): If $\gamma_1 = \gamma_p - 1$, then $\deg A_{a1}(s) = n + \gamma_p - 1$ and $\deg B_{a1}(s) = n$. Therefore, the relative degree of $G_{a1}(s)$ is $\gamma_p - 1$. Further, the zeros of $G_{a1}(s)$ are given to be the solution of (3.2.6). Rewriting (3.2.6) leads to

$$1 + f_1'' \{B(s)D_1(s)/A(s)\} = 0, \; f_1'' = k_p/f_1 \tag{3.2.7}$$

Equation (3.2.7) means that zeros of $G_{a1}(s)$ coincide with the characteristic roots of the closed-loop system whose loop transfer function is

$$G_{a1}(s)' = f_1'' \{B(s)D_1(s)/A(s)\} \tag{3.2.8}$$

Hence, as in the root locus method, the loci of $(n-1)$ characteristic roots of (3.2.7) on the s-plane move from the poles of $G_{a1}(s)'$ (roots of $A(s) = 0$) to $(n-1)$ zeros of $G_{a1}(s)'$ (roots of $B(s)D_1(s) = 0$) and the remaining characteristic root moves along the real axis to minus infinity as the gain f_1'' increases from zero to infinity. Since $B(s)D_1(s)$ is a stable polynomial, it is apparent that all the characteristic roots are located in the left half-plane for sufficiently large gain f_1''. That is, $G_{a1}(s)$ is inversely stable for f_1 satisfying $k_p \gg f_1 > 0$. □

We now establish the following extension of Lemma 3.1 concerning the design method of the PFC.

Theorem 3.1: *Consider the following augmented system for the plant* (3.2.2)*:*

$$G_a(s) = G(s) + F(s) \tag{3.2.9}$$

where

$$F(s) = \sum_{i=1}^{\gamma^*-1} F_i(s), \; \gamma \geq 2 \tag{3.2.10}$$

$$F_i(s) = f_i/D_i(s), \; i = 1, \ldots, \gamma^* - 1 \tag{3.2.11}$$

$$D_i(s) : \; (\gamma^* - i)th \text{ order stable polynomial.}$$

and

$$(i) k_p \gg f_1 \gg \cdots \gg f_{\gamma^*-1} > 0; \;\; (ii) |G(j0)| \gg |F(j0)|$$

Then $G_a(s)$ is ASPR and satisfies Condition (c2)'.

Proof: According to Zeheb (1986), Steinberg and Corless (1985) and Steinberg (1988), an SISO plant is ASPR if (i) the plant transfer function is inversely stable, (ii) its leading coefficient is positive, and (iii) its relative degree is one. Define

$$\begin{aligned} G_{ai}(s) &= G_{ai-1}(s) + F_i(s),\ i = 1, \ldots, \gamma^* - 1 \\ G_{a0} &= G(s),\ f_0 = k_p \end{aligned} \tag{3.2.12}$$

Then the leading coefficient of $G_{aj}(s)$ is f_0 for $j = 1, \ldots, \gamma^* - \gamma_p$, and f_i for $j = \gamma^* - \gamma_p + 1, \ldots, \gamma^* - 1$. Thus, from the assumption, the leading coefficient of $G_a(s) = G_{a\gamma^*-1}(s)$ is positive. Further, regarding $G_{ai}(s)$ as a new plant generated successively from $G_{ai-1}(s)$ with $F_i(s)$, we can conclude from Lemma 3.1 that $G_a(s) = G_{a\gamma^*-1}(s)$ is inversely stable and its relative degree is equal to one. Hence, $G_a(s)$ is ASPR according to Zeheb's ASPR conditions stated above.

In the following, we will show that the condition (c2)' will be satisfied under the above stated assumptions. From the definition, the $D_i(s)$ are stable polynomials. Hence $|F_i(j0)|$ always takes a finite value. Further $\lim_{\omega \to 0} |G(j\omega)| = 0$ does not hold since $G(s)$ is inversely stable. First, let us consider the case where $G(j0)$ has a finite value. Then, from the assumption, there exist some positive integers ε and ε_1 such that

$$|G(j0)| \gg \varepsilon_1 \geq \varepsilon > |F(j0)|$$

Taking into account the continuity of $|G(j\omega)|$ with respect to ω, there always exists a frequency range: $0 \leq \omega \leq \omega_1$ ($\omega_1 > 0$), satisfying $|G(j\omega)| \geq \varepsilon_1$. Similarly, there exists a frequency range: $0 \leq \omega \leq \omega_2$ ($\omega_2 > 0$), satisfying $|F(j\omega)| \leq \varepsilon$. Since parameters included in $F(s)$ are design parameters, we can construct $F(s)$ so as to satisfy the relation $F(j\omega) \leq \varepsilon$ for sufficiently small ε. In other words, we can assume $\omega_1 > \omega_2$ without loss of generality. Thus, by choosing $\omega_0 = \omega_2$, we can conclude that there exists an ω_0 such that $|G(j\omega)| > |F(j\omega)|$ on $0 \leq \omega \leq \omega_0$. It follows that

$$\begin{aligned} ||G_a(j\omega)| - |G(j\omega)|| &\leq ||G(j\omega) + F(j\omega)| - |G(j\omega)|| \\ &\leq |F(j\omega)| \\ &\leq \varepsilon \end{aligned} \tag{3.2.13}$$

holds on $0 \leq \omega \leq \omega_0$. In the case where $\lim_{\omega \to 0} |G(j\omega)| = \infty$, recognizing $G(s)$ has low-pass filter characteristics, we can easily see that there exists a frequency range $0 \leq \omega \leq \omega_0$, satisfying $|G(j\omega)| \leq \varepsilon$ for given $\varepsilon > 0$. Hence a consideration similar to that stated above holds. □

Remark 3.1: In the case where $\gamma^* = \gamma_p = 1$, $G(s)$ is ASPR under Assumption 3.1.

Remark 3.2: It is possible to design PFCs which cover the condition (c2)' taking the frequency gain of the expected responses into consideration.

Remark 3.3: In the case of actual implementation of PFCs, it is convenient to reduce the order of the compensators. For example, if we choose $D_i(s)$ such that

$$\begin{aligned} D_{\gamma^*-i}(s) &= (s+\alpha_i)D_{\gamma^*-i+1}(s) \\ D_{\gamma^*}(s) &= 1, \ \alpha_i > 0, \ i = 1, \ldots, \gamma^* - 1 \end{aligned} \tag{3.2.14}$$

then the compensators are constructed by using $(\gamma^* - 1)$ integrators as shown in Figure 3.2. it should be noted that these compensators are implemented in the control algorithm only. That is, there is no need to construct any kind of hardware.

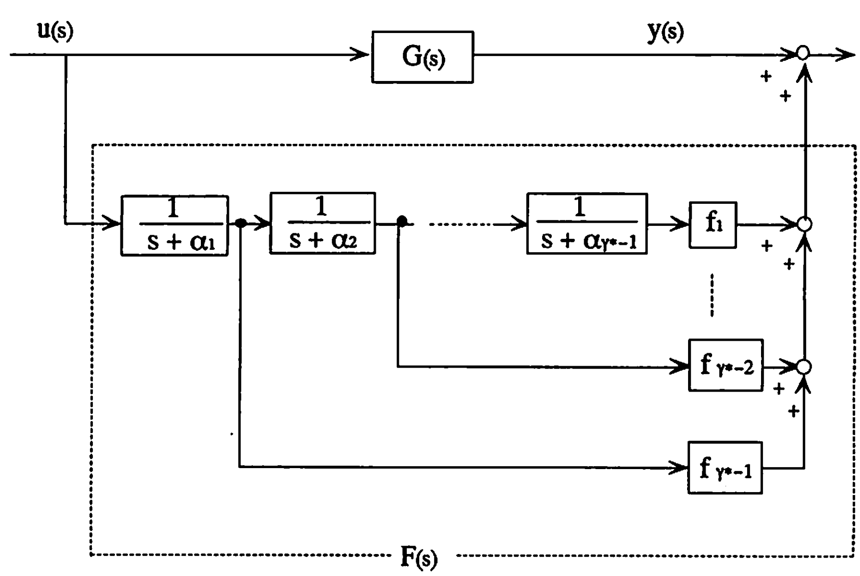


Figure 3.2 A practical realization of a PFC

3.2.2 Parallel Feedforward Compensator Design for Multi-Input Multi-Output Plants

The design method of the PFC proposed for a SISO plant is extended for MIMO systems.

Let G(s) be

$$\begin{aligned} G(s) &= [g_{ij}(s)] = C(sI - A)^{-1}B \\ &= \frac{1}{p(s)}\Phi(s) \end{aligned} \tag{3.2.15}$$

where

$$p(s) = \det(sI - A) \tag{3.2.16}$$

$$\Phi(s) = C\text{adj}(sI - A)B = [\phi_{ij}(s)] \tag{3.2.17}$$

Here $G(s)$ is the transfer function matrix of the plant with a minimal realization $\{A, B, C\}$ and $p(s)$ corresponds to the pole polynomial of the plant (Kouvaritakis and MacFarlane 1976, MacFarlane and Karcanias 1976). Further, we introduce a parallel feedforward compensator (PFC) matrix $F(s)$ as shown in Figure 3.1. Then we have the following augmented plant transfer function matrix:

$$G_a(s) = [g_{aij}(s)] = G(s) + F(s) \tag{3.2.18}$$

As stated in the SISO case, the problem is to find a PFC $F(s)$ satisfying Condition 3.1. In MIMO case, it is also uncertain whether or not we can find $F(s)$ that satisfies Condition (c2) over the whole frequency range. Hence, we replace this condition with a more realistic one as follows:

*(c2)** *There exists an* $\omega_0 > 0$ *such that, for given* $\varepsilon > 0$, *the following relation holds on* $0 \leq \omega \leq \omega_0$*:*

$$\sum_{i=1}^{m}\sum_{k=1}^{m} \left| |g_{aik}(j\omega)| - |g_{ik}(j\omega)| \right| \leq \varepsilon \tag{3.2.19}$$

where $g_{ik}(j\omega)$ *and* $g_{aik}(j\omega)$ *are elements of transfer function matrices* $G(j\omega)$ *and* $G_a(j\omega)$, *respectively.*

Now, we impose the following assumptions on the plant (3.2.15).

Assumption 3.2:

(1) Plant (3.2.15) *is strictly minimum phase, i.e. the zero polynomial denoted as* $z(s)$ *(monic) is a Hurwitz polynomial.*

(2) *The leading coefficient of* $\Phi[i_1, i_2, \ldots, i_h]$ $(1 \leq i_1 < i_2 < \cdots < i_h \leq m,\ h = 1, 2, \ldots, m)$ *is positive, where* $\Phi[i_1, i_2, \ldots, i_h]$ *denotes the principal minor of order* h *formed from* $\Phi(s)$ *by deleting all rows except rows* $i_1, i_2, \ldots, i_h$ *and all columns except columns* $i_1, i_2, \ldots, i_h$.

(3) *The relative degree of* $g_{ij}(s)$ *denoted as* γ_{ij} *is known and* $\gamma_{ij,i\neq j} \geq 2$.

(4) *The following relation is satisfied between the relative order of pole and zero polynomials and the relative degree of* $g_{ii}(s)$, *the diagonal elements of* $G(s)$.

$$\deg p(s) - \deg z(s) = d \leq \sum_{i=1}^{m} \gamma_{ii} = d_0 \tag{3.2.20}$$

(5) *The approximate values of the leading coefficient of* $g_{ij}(s)$ *and* $|g_{ii}(j0)|$ *are known.*

In preparation for investigating the design method of PFC, we give the following lemma which gives the basic procedure of the design scheme.

Lemma 3.2: *Suppose that the plant* (3.2.15) *satisfies Assumption* 3.2(1), (2) *and* (4). *Further, consider the following augmented plant* $G_{af}(s)$:

$$G_{af}(s) = [g_{afij}(s)] = G(s) + G_f(s) \tag{3.2.21}$$

$$G_f(s) = \frac{1}{p_f(s)} diag[\rho_1 f_1(s), \rho_2 f_2(s), \ldots, \rho_m f_m(s)] \tag{3.2.22}$$

where

$p_f(s)$: *Hurwitz polynomial of order* n_f *(pole polynomial of* $G_f(s)$*)*

$f_i(s)$: *monic polynomial of order* $(n_f - \gamma_{ii} + 1)$

and ρ_i, $i \in M = \{1, \ldots, m\}$ *are positive and satisfy the following relation*

$$\tilde{b} = \tilde{\tilde{b}}_m \gg \tilde{\tilde{b}}_{m-1} \gg \cdots \gg \tilde{\tilde{b}}_1 \gg \tilde{\tilde{b}}_0 = \sum_{i=1}^{m} \rho_i \tag{3.2.23}$$

where $\tilde{\tilde{b}}_h$ *is a leading coefficient of the polynomial*

$$\sum_{1 \leq i_1 < \ldots < i_h \leq m} \left\{ \left(\prod_{\substack{j=1 \\ j \neq i_1 \ldots i_h}}^{m} \rho_j f_j(s) \right) \Phi[i_1, \ldots, i_h] \right\} \tag{3.2.24}$$

here, $\sum_{1\le i_1<\ldots<i_h\le m}$ *denotes the sum of all combinations of index* $1 \le i_1 < \ldots < i_h \le m$.

Then, on condition that no pole and zero cancellations occur on the augmented plant $G_{af}(s)$ *defined by* (3.2.21), $G_{af}(s)$ *satisfies Assumption* 3.2(1), 3.2(2) *and* 3.2(4) *and the relative degree of its* i*th diagonal elements* $g_{afii}(s)$ *becomes* $\gamma_{ii} - 1$.

Proof: The augmented plant $G_{af}(s)$ is given as follows:

$$G_{af}(s) = \frac{1}{p_{af}(s)}\Phi_{af}(s) \tag{3.2.25}$$

$$p_{af}(s) = p(s)p_f(s): \quad \text{pole polynomial of augmented plant} \tag{3.2.26}$$

where

$$\Phi_{af}(s) = [\phi_{afij}(s)] \tag{3.2.27a}$$

$$\phi_{afij}(s) = \begin{cases} \phi_{ii}(s)p_f(s) + \rho_i f_i(s)p(s), & i = j \\ \phi_{ij}(s)p_f(s), & i \neq j \end{cases} \tag{3.2.27b}$$

From (3.2.26) and (3.2.27), it is clear that the relative degree of the ith diagonal element of $G_{af}(s)$ becomes $\gamma_{ii} - 1$. Next, we have the following equation (see Appendix 3.A).

$$\begin{aligned}
&\det \Phi_{af}(s) \\
&= \left(\prod_{j=1}^{m} \rho_j f_j(s)\right) p(s)^m \\
&\quad + \sum_{i_1=1}^{m} \left\{ \left(\prod_{\substack{j=1 \\ j\neq i_1}}^{m} \rho_j f_j(s)\right) \Phi[i_1] \right\} p_f(s)p(s)^{m-1} \\
&\quad + \sum_{1\le i_1<i_2\le m} \left\{ \left(\prod_{\substack{j=1 \\ j\neq i_1,i_2}}^{m} \rho_j f_j(s)\right) \Phi[i_1,i_2] \right\} p_f(s)^2 p(s)^{m-2} \\
&\quad + \cdots \\
&\quad + \sum_{1\le i_1<\ldots<i_{m-1}\le m} \left\{ \left(\prod_{\substack{j=1 \\ j\neq i_1,\ldots,i_{m-1}}}^{m} \rho_j f_j(s)\right) \Phi[i_1,\ldots,i_{m-1}] \right\} p_f(s)^{m-1}p(s) \\
&\quad + \det \Phi(s)p_f(s)^m
\end{aligned} \tag{3.2.28}$$

Since $p(s)$ and $p_f(s)$ are the pole polynomials of $G(s)$ and $G_f(s)$, respectively, there exists monic polynomial $q_h(s)$ satisfying the following equation:

$$\sum_{1\le i_1<\dots<i_h\le m}\left\{\left(\prod_{\substack{j=1\\ j\ne i_1,\dots,i_h}}^{m}\rho_j f_j(s)\right)\Phi[i_1,\dots,i_h]\right\}=\tilde{\tilde{b}}_h q_h(s)p_f(s)^{m-h-1}p(s)^{h-1} \tag{3.2.29}$$

Further, we have (MacFarlane and Karcanias 1976)

$$\prod_{i=1}^{m} f_i(s) = z_f(s)p_f(s)^{m-1} \tag{3.2.30}$$

$z_f(s)$: zero polynomial of $G_f(s)$

$$\det\Phi(s) = \tilde{b}z(s)p(s)^{m-1} = \tilde{\tilde{b}}_m z(s)p(s)^{m-1} \tag{3.2.31}$$

It follows from (3.2.29), (3.2.30) and (3.2.31) that $\det\Phi_{af}(s)$ can be rewritten as follows:

$$\det\Phi_{af}(s) = \left[\left(\prod_{i=1}^{m}\rho_i\right)z_f(s)p(s)+\sum_{h=1}^{m-1}\tilde{\tilde{b}}_h q_h(s)+\tilde{b}z(s)p_f(s)\right]p_f(s)^{m-1}p(s)^{m-1} \tag{3.2.32}$$

Hence, taking into account the fact that $p_f(s)p(s) = p_{af}(s)$ is the pole polynomial of $G_{af}(s)$, we get

$$\tilde{b}_a z_{af}(s) = \left(\prod_{i=1}^{m}\rho_i\right)z_f(s)p(s)+\sum_{h=1}^{m-1}\tilde{\tilde{b}}_h q_h(s)+\tilde{b}z(s)p_f(s) \tag{3.2.33}$$

where

$z_{af}(s)$: zero polynomial of $G_{af}(s)$
$\tilde{b}_a$: leading coefficient of $\det\Phi_{af}(s)$

Using the fact that

$$\deg\Phi[i_1,\dots,i_h] \ge \deg\Phi[i_1,\dots,i_{s-1},i_{s+1},\dots,i_h]+n-\gamma_{i_s i_s} \tag{3.2.34}$$

and the definition of $G_f(s)$, we have (see Appendix 3.B)

$$\deg q_{h+1}(s)+1 \ge \deg q_h(s),\quad h=1,\dots,m-1 \tag{3.2.35a}$$

$$q_m(s) = z(s)p_f(s) \tag{3.2.35b}$$

Further, we obtain

$$\deg z_f(s)p(s) = n+n_f-d_0+m \tag{3.2.36}$$

$$\deg q_1(s) = n+n_f-d_0+m-1 \tag{3.2.37}$$

$$\deg z(s)p_f(s) = n+n_f-d \tag{3.2.38}$$

Now let us define the functions $R_h(s)$, $h \in M = \{1, \ldots, m\}$, such that

$$R_h(s) = R_{h-1}(s) + \tilde{\tilde{b}}_{m-h} q_{m-h}(s), \; h \in M = \{1, \ldots, m\} \qquad (3.2.39a)$$

$$R_0(s) = \tilde{b} z(s) p_f(s) \qquad (3.2.39b)$$

$$\tilde{\tilde{b}}_0 q_0(s) = \prod_{i=1}^{m} \rho_i z_f(s) p(s) \qquad (3.2.39c)$$

It is also apparent that $R_m(s) = \tilde{b}_a z_{af}(s)$ holds in (3.2.39a). From Assumption 3.2(4) and (3.2.34)–(3.2.39), it follows that

$$\deg R_{h-1}(s) - \deg q_{m-h}(s) \geq -1, \; h \in M \qquad (3.2.40)$$

Since relation (3.2.40) holds for all $h \in M$, if we put $\tilde{r}_{h-1}$ as a leading coefficient of $R_{h-1}(s)$, then the roots of $R_h(s) = 0$ tend to the roots of $R_{h-1}(s) = 0$ and minus infinity (in the case where the equality holds in (3.2.40)) as $\tilde{r}_{h-1}/\tilde{\tilde{b}}_{m-h}$ tends to infinity. Thus if the roots of $R_{h-1}(s) = 0$ are located in the left half-plane and $\tilde{r}_{h-1} \gg \tilde{\tilde{b}}_{m-h}$ holds, then it follows that the roots of $R_h(s) = 0$ are also located in the left half-plane. By applying the above results for $h = 1, \ldots, m$ successively, we therefore reach the important conclusion that the roots of $R_m(s) = 0$ are located in the left half-plane if the all roots of $R_0(s) = 0$ are located in the left half-plane and relations

$$\tilde{r}_{h-1} \gg \tilde{\tilde{b}}_{m-h}, \; \tilde{r}_0 = \tilde{b} = \tilde{\tilde{b}}_m \qquad (3.2.41)$$

hold for all $h \in M$. Taking into account that the zeros of plant (3.2.15) are asymptotically stable from Assumption 3.2(1) and $p_f(s)$ was given as an asymptotically stable polynomial, we can easily see that $R_0(s) = 0$ is also an asymptotically stable polynomial. Further, from (3.2.23) and the definition of $R_h(s)$, we can obtain the following relations:

$$\tilde{r}_h \geq \tilde{\tilde{b}}_{m-h}, \; h = 0, \ldots, m-1 \qquad (3.2.42)$$

and

$$\tilde{r}_{h-1} \geq \tilde{\tilde{b}}_{m-h+1} \gg \tilde{\tilde{b}}_{m-h}, \; h \in M \qquad (3.2.43)$$

Therefore we can conclude that every root of $R_m(s) = \tilde{b}_a z_{af}(s) = 0$ is located in the left half-plane. That is, the augmented plant $G_{af}(s)$ satisfies Assumption 3.2(1).

Next, using the same operation to get (3.2.28) (see Appendix 3.B) the principal minor of $\Phi(s)$ of order r can be expanded as

$$
\begin{aligned}
&\det \Phi_{af}[i_1,\ldots,i_r] \\
&= \left(\prod_{j=i_1,\ldots,i_r} \rho_j f_j(s)\right) p(s)^r \\
&\quad + \sum_{k_1=i_1,\ldots,i_r} \left\{ \left(\prod_{\substack{j=i_1,\ldots,i_r \\ j\neq k_1}} \rho_j f_j(s) \right) \Phi[k_1] \right\} p_f(s) p(s)^{r-1} \\
&\quad + \sum_{\substack{k_1<k_2 \\ k_1,k_2=i_1,\ldots,i_r}} \left\{ \left(\prod_{\substack{j=i_1,\ldots,i_r \\ j\neq k_1,k_2}} \rho_j f_j(s) \right) \Phi[k_1,k_2] \right\} p_f(s)^2 p(s)^{r-2} \\
&\quad + \cdots \\
&\quad + \sum_{\substack{k_1<\cdots<k_{r-1} \\ k_1,\ldots,k_{r-1}=i_1,\ldots,i_r}} \left\{ \left(\prod_{\substack{j=i_1,\ldots,i_r \\ j\neq k_1,\ldots,k_{r-1}}} \rho_j f_j(s) \right) \Phi[k_1,\ldots,k_{r-1}] \right\} p_f(s)^{r-1} p(s) \\
&\quad + \Phi[i_1,\ldots,i_r] p_f(s)^r
\end{aligned} \tag{3.2.44}
$$

where

$$
\sum_{\substack{k_1<\cdots<k_h \\ k_1,\ldots,k_h=i_1,\ldots,i_r}}
$$

denotes the sum of all combinations of indices $\{k_1, k_2, \ldots, k_h\} \subset \{i_1, i_2, \ldots, i_r\}$ and $k_1 < k_2 < \cdots < k_h$. Then, from Assumption 3.2(2) and $\rho_j > 0$, $j = 1, \ldots, m$, it is apparent that the leading coefficient of $\Phi_{af}[i_1, \ldots, i_r]$ is positive, i.e. $\Phi_{af}[i_1, \ldots, i_r]$ generated from the augmented plant $\Phi_{af}(s)$ satisfies Assumption 3.2(2). Further, using (3.2.34)-(3.2.39), we get

$$
n + n_f - d_0 + m \leq \deg R_m(s) = \deg z_{af}(s) \tag{3.2.45}
$$

Therefore, we have

$$
\deg p_{af}(s) - \deg z_{af}(s) \leq d_0 - m \tag{3.2.46}
$$

Thus, Assumption 3.2(4) is also satisfied. □

Further, the following Lemma is given.

Lemma 3.3: *In Lemma 3.2, let ρ_i, $i \in L$, $\rho_i = 0$, $i \in N$, where $L + N = M$. Then the augmented plant $G_{af}(s)$ satisfies Assumption 3.2(1), 3.2(2) and 3.2(4), and the relative degree of its diagonal element $g_{afii}(s)$, $i \in L$, becomes $\gamma_{ii} - 1$.*

Proof: The proof is given in the same manner used in the proof of Lemma 3.2 only by setting $\rho_i = 0$, $i \in N$ in the proof of Lemma 3.2 and Appendix 3.B. □

The above-mentioned Lemmas are summarized as follows. The augmented plant $G_{af}(s)$ satisfies Assumption 3.2(1), (2) and (4) by adding $G_f(s)$ in parallel to the plant satisfying Assumption 3.2(1), (2) and (4). Moreover, the relative degrees of the diagonal elements of $G_{af}(s)$ become $\gamma_{ii} - 1$. From these results, by considering the obtained augmented plant as a new controlled plant, we are able to compose the new minimum phase augmented plant of which the relative the degrees of diagonal elements are successively reduced while keeping the characteristics given by Assumption 3.2(1), (2) and (4).

We can now obtain the following theorem for establishing the design method for the parallel feedforward compensator.

Theorem 3.2: *Suppose that plant* (3.2.15) *is non-ASPR and satisfies Assumption* 3.2. *Consider the following augmented plant* (Figure 3.3) *for the plant* (3.2.15):

$$G_a(s) = G(s) + F(s) \tag{3.2.47}$$

$$F(s) = \sum_{i=1}^{\gamma_M - 1} F_i(s), \quad \gamma_M = \max_{i \in M} (\gamma_{ii}) \tag{3.2.48}$$

where

$$F_i(s) = \text{diag}\,[\rho_{i1}/d_{i1}(s), \ldots, \rho_{im}/d_{im}(s)] \tag{3.2.49}$$

$$\begin{gathered} d_{ij}(s)\text{: monic stable polynomial of order } (\gamma_{jj} - i) \\ i = 1, \ldots, \gamma_M - 1, \; j \in M \end{gathered}$$

$$\begin{cases} \rho_{ij} > 0, & \text{if } \gamma_{jj} - i > 0 \\ \rho_{ij} = 0, & \text{if } \gamma_{jj} - i \le 0 \end{cases} \tag{3.2.50}$$

and ρ_{ij} holds relation (3.2.23) *which is defined from $G_{ai-1}(s)$ and $F_i(s)$ for all $i \in M_\gamma$, $M_\gamma = \{1, \ldots, \gamma_M - 1\}$ by replacing $G(s)$ with $G_{ai-1}(s)$ and $G_f(s)$ with $F_i(s)$ in* (3.2.21)*, where*

$$G_{ai}(s) = G_{ai-1}(s) + F_i(s) \tag{3.2.51a}$$

$$G_{a0}(s) = G(s) \tag{3.2.51b}$$

Further, we assume that $F_i(s)$ are chosen so as to satisfy

$$|g_{kk}(j0)| \gg \sum_{i=1}^{\gamma_M - 1} |\rho_{ik}/d_{ik}(j0)|, k \in M \tag{3.2.52}$$

Then, the augmented plant $G_a(s) = G_{a\gamma_M - 1}(s)$ satisfies Condition 3.1 (c1), (c2)* and (c3).

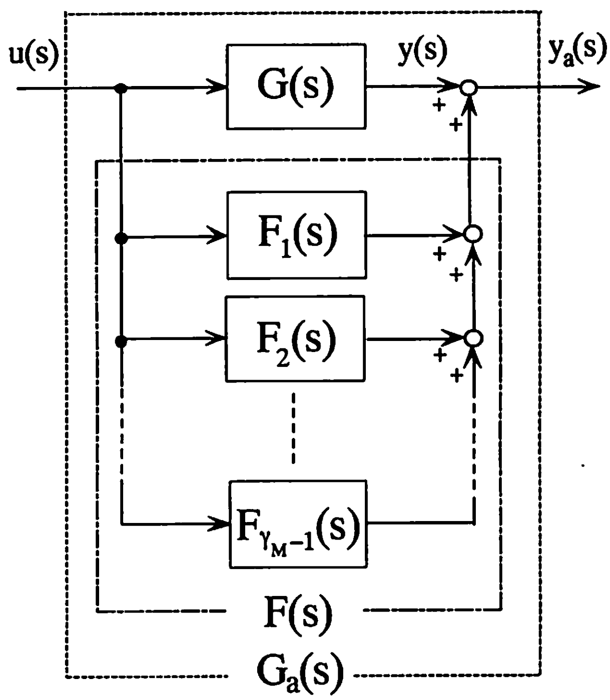


Figure 3.3 Augmented plant including multi-parallel feedforward compensator

3.2.3 Augmented Control System

As shown in the preceding subsections, we can make non-ASPR plants virtually ASPR using a parallel feedforward compensator. That is, the SAC can be applied to the ASPR augmented plant with the PFC instead of the non-ASPR original plant. This is summarized as follows:

Let the minimum realizations of the plant and the PFC be (A, B, C) and (A_f, B_f, C_f), respectively. The augmented plant with PFC, which is made ASPR, is expressed as follows:

$$\dot{\boldsymbol{x}}_a(t) = A_a \boldsymbol{x}_a(t) + B_a \boldsymbol{u}(t) \tag{3.2.53a}$$

$$\boldsymbol{y}_a(t) = C_a \boldsymbol{x}_a(t) \tag{3.2.53b}$$

where

$$\boldsymbol{x}_a(t) = [\boldsymbol{x}(t)^T, \boldsymbol{x}_f(t)^T]^T \tag{3.2.54}$$

$$A_a = \begin{bmatrix} A & 0 \\ 0 & A_f \end{bmatrix}, \quad B_a = \begin{bmatrix} B \\ B_f \end{bmatrix}$$
$$C_a = [C, C_f] \tag{3.2.55}$$

Here, $\boldsymbol{x}(t)$ and $\boldsymbol{x}_f(t)$ are state vectors of the plant and the PFC and $\boldsymbol{y}_a(t)$ is the output of the augmented plant.

Denoting the augmented error signal $\boldsymbol{e}_a(t)$ as

$$\boldsymbol{e}_{ay}(t) = \boldsymbol{y}_a(t) - \boldsymbol{y}_m(t) \tag{3.2.56}$$

we have the following control input according to (2.3.8)-(2.3.11).

$$\boldsymbol{u}(t) = K_a(t)\boldsymbol{z}_a(t) \tag{3.2.57}$$
$$\boldsymbol{z}_a(t) = [\boldsymbol{e}_{ay}(t)^T, \boldsymbol{x}_m(t)^T, \boldsymbol{u}_m(t)^T]^T$$

$$\begin{cases} K_a(t) = K_{Ia}(t) + K_{Pa}(t) \\ \dot{K}_{Ia}(t) = -\boldsymbol{e}_{ay}(t)\boldsymbol{z}_a(t)^T\Gamma_{Ia} - \sigma_{Ia}(t)K_{Ia}(t) \\ K_{Pa}(t) = -\boldsymbol{e}_{ay}(t)\boldsymbol{z}_a(t)^T\Gamma_{Pa} \\ \sigma_{Ia}(t) = \sigma_{a1}\frac{\boldsymbol{e}_{ay}(t)^T\boldsymbol{e}_{ay}(t)}{1+\boldsymbol{e}_{ay}(t)^T\boldsymbol{e}_{ay}(t)} + \sigma_{a2} \end{cases} \tag{3.2.58}$$

$$\Gamma_{Ia} = \Gamma_{Ia}^T > 0, \ \Gamma_{Pa} = \Gamma_{Pa}^T > 0, \ \sigma_{a1}, \sigma_{a2} > 0$$

Figure 3.4 illustrates the overall block-diagram of the augmented SAC system.

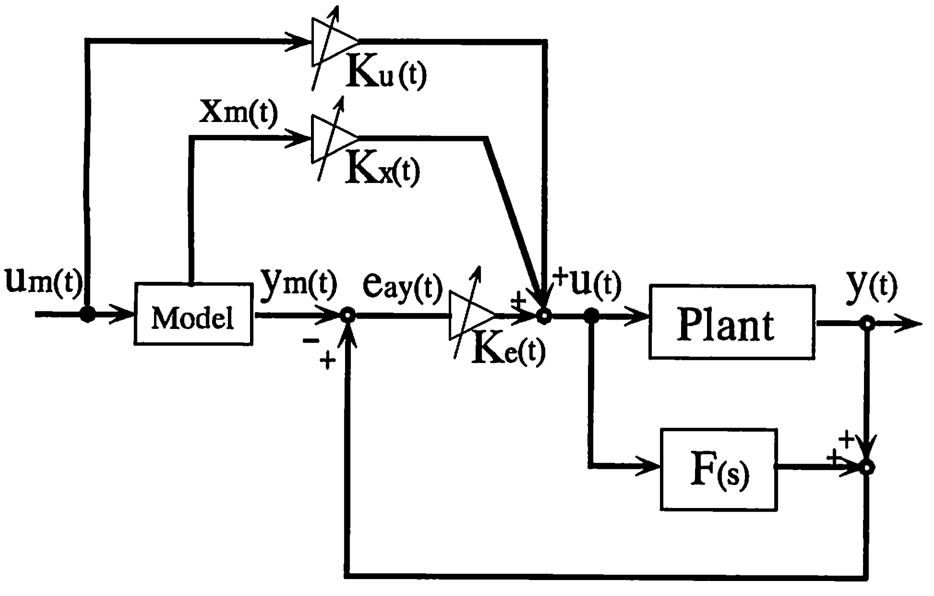


Figure 3.4 The overall block-diagram of the augmented SAC system

3.2.4 Numerical Simulations

The effectiveness of the proposed methods in this section (the design methods of the PFCs) will be confirmed through numerical simulations.

i) Simulation results for SISO plants

Case 1: *Rohrs' example*

In this simulation, the effectiveness of the PFC for the SISO plant with parasitics is shown by using Rohrs' example (Rohrs *et al.* 1982). The simulations were executed using a nominally first-order plant $G_0(s) = 2/(s+1)$ with a pair of complex unmodelled poles, described by

$$G(s) = G_0(s)G_1(s),\ G_1(s) = 229/(s^2 + 30s + 229) \qquad (3.2.59)$$

and a reference model

$$G_m(s) = 3/(s+3) \qquad (3.2.60)$$

According to the literature (Rohrs *et al.* 1982), the command input and the sensor noise are chosen such that

$$u_m(t) = 0.3 + 1.85 \sin 16.1t,\ d(t) = 5.59 \times 10^{-6} \sin 16.1t$$

In the plant (3.2.59), the relative degree γ_p is three. But in this simulation an overestimated value $\gamma^* = 5$ is used. Then, from Theorem 3.1, the PFC $F(s)$ is chosen such that

$$\begin{cases} F(s) = \sum_{i=1}^{4} F_i(s),\ F_i(s) = f_i/(s+\alpha)^{5-i} \\ f_i = 100/10^{i-1},\ \alpha = 20,\ i = 1, \ldots, 4 \end{cases} \qquad (3.2.61)$$

Adaptation parameters in (3.2.58) are given as follows:

$$\Gamma_{Ia} = \mathrm{diag}[2 \times 10^8, 10, 10],\ \Gamma_{Pa} = \mathrm{diag}[3 \times 10^7, 1, 1]$$

$$\sigma_{a1} = 1,\ \sigma_{a2} = 0.3,\ K_a(0) = 0$$

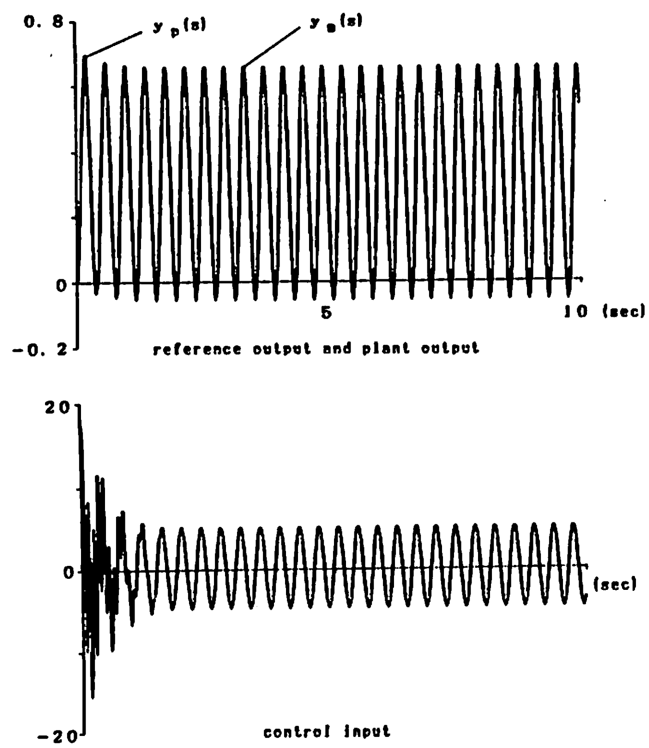


Figure 3.5 Rohrs' example: the use of the SAC algorithm with a PFC

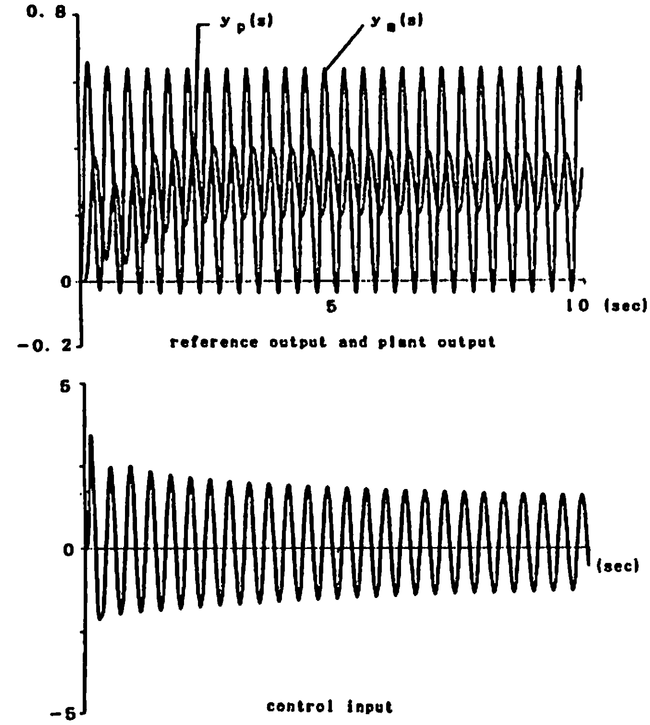


Figure 3.6 Rohrs' example: the use of the σ-modification algorithm

The simulation result is shown in Figure 3.5. Compared with the simulation result shown in Figure 3.6 which was obtained using the σ-modification method (Ioannou and Tsakalis 1986), it is apparent that the former gives a better tracking performance.

Case 2: *Stress test example*

The proposed method is applied to the stress testing example given as a showcase example by Masten and Cohen (1990). The fundamental statements of the example are as follows:

Nominal and model plant:

$$G_0(s) = G_m(s) = 1/(s^2 + 1.4s + 1)$$

Unmodelled dynamics:

$$u(s)/u_i(s) = 1/(0.33s + 1)$$

Unmodelled zeros:

$$0.3s + 1$$

Sensor noise $n(t)$:

Zero-mean with a 10 Hz band-limited gaussian distribution and a specified RMS value (RMS=0.2)

Command input and disturbance:

These are defined in Figure 3.7 and specified for 20 seconds in duration.

Here $u(s)$ is the actual control signal and $u_i(s)$ is the ideal signal generated by the adaptive controller. Note that the above-stated conditions correspond to the most severe level-3 case in the showcase example. In this simulation, we assumed that the actual plant had the following form:

$$G(s) = 3/(s^2 - 0.6s - 1)$$

The upper bound of relative degree γ^* was assumed to be three and, based on Theorem 3.1, PFCs were given as

$$F_1(s) = 0.8/(s + 10)^2, \ F_2(s) = 0.016/(s + 10)$$

The design parameters in the adjusting laws (3.2.58) are as follows:

$$\Gamma_{Ia} = \Gamma_{Pa} = \text{diag}[10^8, 10^3 I_3], \sigma_{a1} = 0.01, \sigma_{a2} = 0.001, K_a(0) = 0$$

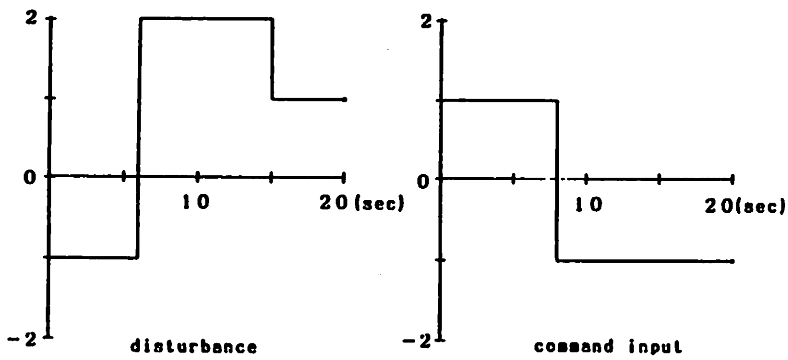


Figure 3.7 Command input and disturbance waveforms.

The typical output and control input responses without unmodelled zeros and sensor noise are shown in Figure 3.8. Figure 3.9 shows the results when we take all the constraints into consideration. In Figure 3.10, the simulation result is given for case where $G_m(s) = 1/(s+1)$. These results suggest the practical efficiency of the proposed method.

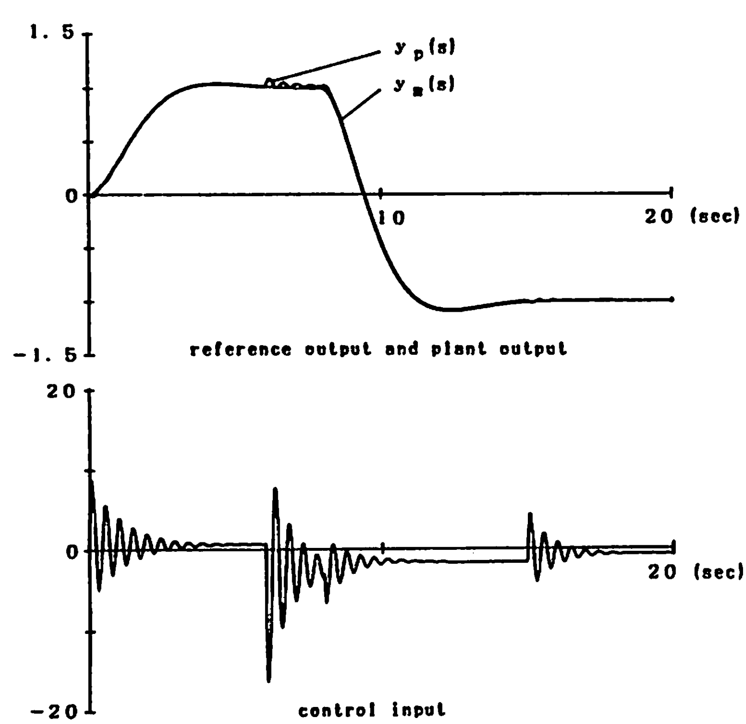


Figure 3.8 Stress test example: time response without unmodelled zeros and sensor noise.

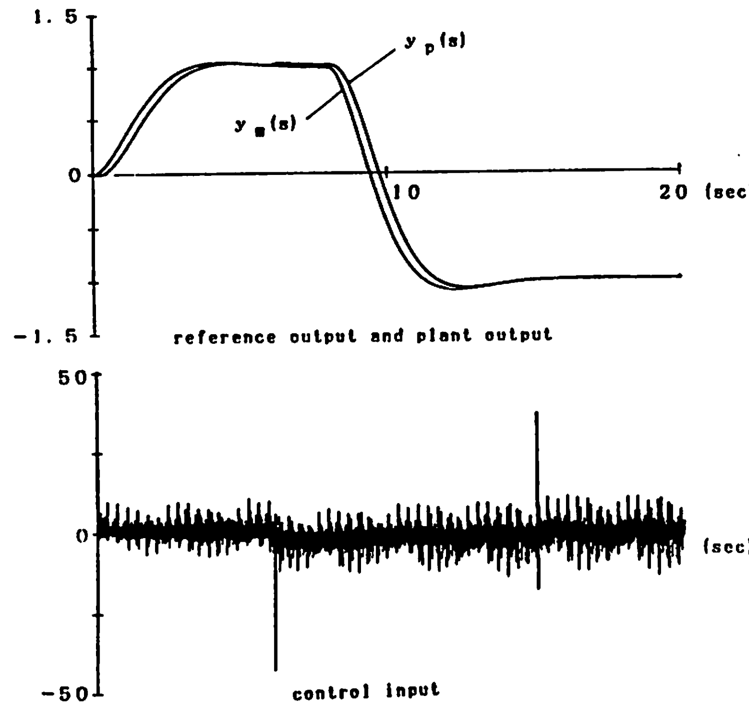


Figure 3.9 Stress test example: the case where all the constraints are taken into consideration.

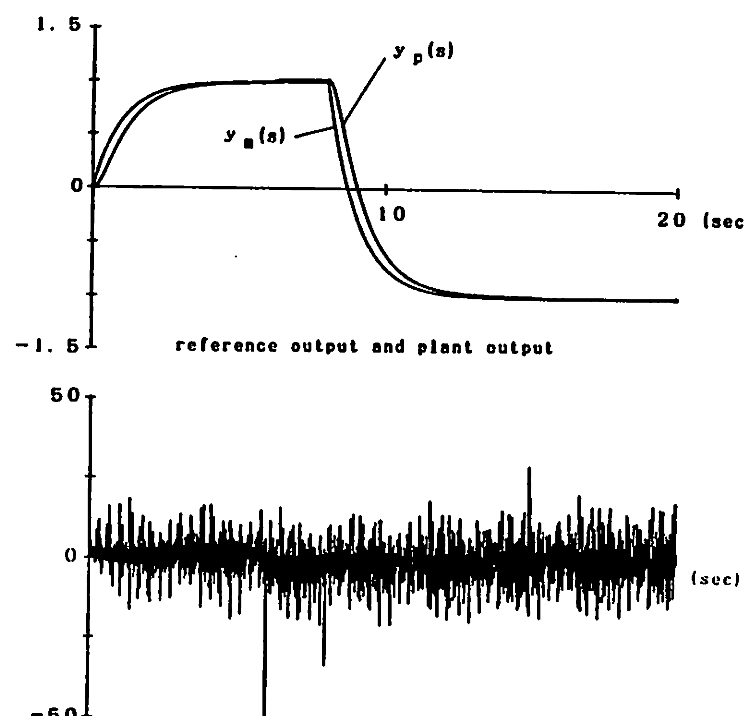


Figure 3.10 Stress test example: first-order reference model.

ii) Simulation results for MIMO plants

The effectiveness of the proposed method for MIMO plants is confirmed by using 2-input and 2-output non-ASPR but minimum-phase plant models.

The plants to be controlled are given as follows.

Case 1:

$$\dot{\boldsymbol{x}}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -2 & 3 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & -2 & 3 \end{bmatrix} \boldsymbol{x}(t) + \begin{bmatrix} 0 & 0 \\ 2 & 2 \\ 0 & 0 \\ 1 & 3 \end{bmatrix} \boldsymbol{u}(t) + \boldsymbol{g}(t) \tag{3.2.62a}$$

$$\boldsymbol{y}(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \boldsymbol{x}(t) \tag{3.2.62b}$$

$$\boldsymbol{g}(t) = \begin{bmatrix} 2\sin(2\pi t/5) \\ \cos(2\pi t/7) \\ \sin(2\pi t/10) \\ 2\cos(2\pi t/5) \end{bmatrix} \tag{3.2.62c}$$

Case 2:

$$\dot{\boldsymbol{x}}(t) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix} \boldsymbol{x}(t) + \begin{bmatrix} 1 & 1 \\ -1 & 0 \\ -1 & 0 \\ 0 & 1 \\ -1 & -15 \end{bmatrix} \boldsymbol{u}(t) + \boldsymbol{g}(t) \tag{3.2.63a}$$

$$\boldsymbol{y}(t) = \begin{bmatrix} 1 & 2 & -1 & -1 & 0 \\ 0 & 1 & 0 & -15 & -1 \end{bmatrix} \boldsymbol{x}(t) \tag{3.2.63b}$$

$$\boldsymbol{g}(t) = \begin{bmatrix} 2\sin(2\pi t/5) \\ \cos(2\pi t/7) \\ \sin(2\pi t/10) \\ 2\cos(2\pi t/7) \\ 2\sin(2\pi/3) \end{bmatrix} \tag{3.2.63c}$$

The transfer function matrices of these plants are given as

Case 1:

$$G(s) = \frac{1}{(s^2-3s+3)(s^2-3s+1)} \begin{bmatrix} 2s^2-6s+5 & 2s^2-6s+7 \\ s^2-3s+4 & 3s^2-9s+8 \end{bmatrix} \tag{3.2.64}$$

Case 2:

$$G(s) = \begin{bmatrix} \dfrac{2}{(s-1)(s-2)(s-3)} & \dfrac{-3}{(s-1)(s-4)} \\ \dfrac{3}{(s-2)(s-5)} & \dfrac{15}{(s-4)(s-5)} \end{bmatrix} \tag{3.2.65}$$

In Case 1, the relative degree d between pole and zero polynomials is 4 and the sum of the relative degrees of diagonal elements d_0 is 4 ($\gamma_{11} = \gamma_{22} = 2$), and, in Case 2, $d = 4$ and $d_0 = 5$ ($\gamma_{11} = 3, \gamma_{22} = 2$).

The reference model that the above plants are required to follow is chosen to be:

$$G_m(s) = \text{diag}[1/(s+1), 1/(s+1)] \tag{3.2.66a}$$

$$\boldsymbol{u}_m(t) = [u_{m1}(t), u_{m2}(t)]^T \tag{3.2.66b}$$

$u_{m1}(t)$: a rectangular wave of amplitude 1

$u_{m2}(t)$: a rectangular wave of amplitude 2

Both the plants are non-ASPR. So, according to the design method proposed in previous sections, we have to construct the parallel feedforward compensators as follows.

Case 1:

$$F(s) = \text{diag}[0.08/(s+5), 0.08/(s+5)] \tag{3.2.67}$$

Case 2:

$$F(s) = F_1(s) + F_2(s) \tag{3.2.68a}$$

$$F_1(s) = \text{diag}[0.1/(s+20)^2, 0.01/(s+20)] \tag{3.2.68b}$$

$$F_2(s) = \text{diag}[0.01/(s+20), 0] \tag{3.2.68c}$$

The design parameters of the adaptive law (3.2.58) are given as

$$\Gamma_{Ia} = \text{diag}[10^8 I_2, 10^3 I_4], \; \Gamma_{Pa} = \text{diag}[10^6 I_2, 10^2 I_4]$$

$$\sigma_{a1} = 0.01, \; \sigma_{a2} = 0.05, \; K(0) = 0$$

Simulation results are shown in Figures 3.11–3.14. Figures 3.11 and 3.13 are the simulation results for Cases 1 and 2, respectively, when disturbance $\boldsymbol{g}(t)$ is set to zero, and Figures 3.12 and 3.14 are the results under the effect of disturbances. In both cases, good tracking performances have been obtained.

Next, we assume that the relative degrees γ_{ii} of diagonal elements of plants are unknown but the upper bounds γ_{ii}^* ($\gamma_{11}^* = \gamma_{22}^* = 3$ for Case 1 and $\gamma_{11}^* = 4, \gamma_{22}^* = 3$ for Case 2) are known. In this case, the parallel feedforward compensators are chosen as follows.

Case 1:

$$F(s) = F_1(s) + F_2(s) \tag{3.2.69a}$$

$$F_1(s) = \text{diag}[0.8/(s+5)^2, 0.8/(s+5)^2] \tag{3.2.69b}$$

$$F_2(s) = \text{diag}[0.08/(s+5), 0.08/(s+5)] \tag{3.2.69c}$$

Case 2:

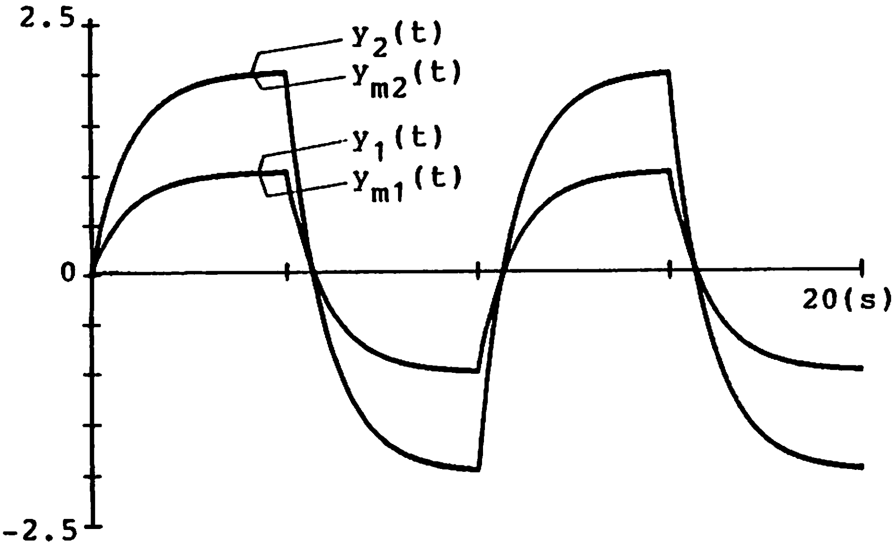
$$F(s) = F_1(s) + F_2(s) + F_3(s) \tag{3.2.70a}$$

$$F_1(s) = \text{diag}[1/(s+20)^3, 0.1/(s+20)^2] \tag{3.2.70b}$$

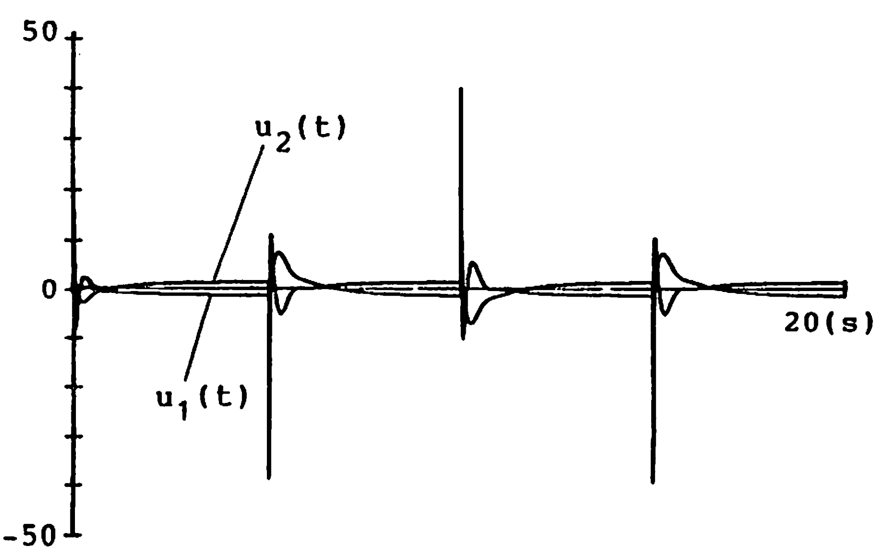
$$F_2(s) = \text{diag}[0.1/(s+20)^2, 0.01/(s+20)] \tag{3.2.70c}$$

$$F_2(s) = \text{diag}[0.01/(s+20), 0] \tag{3.2.70d}$$

Figures 3.15 and 3.16 show the simulation results for Cases 1 and 2 without disturbances, respectively. As compared with the results given in Figures 3.11 and 3.13, off-sets of tracking errors are made to appear by use of an extra compensator, which must be added because the true values of γ_{ii} are unknown.

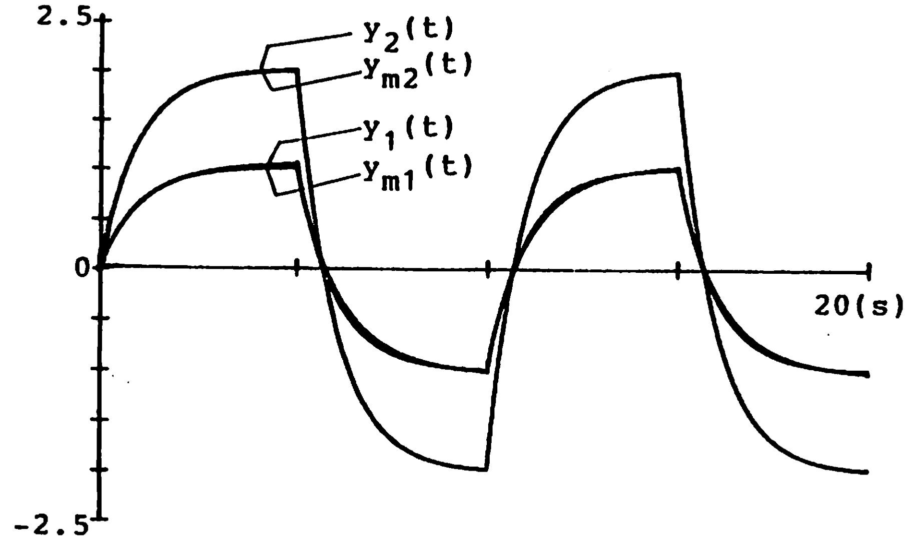


(a) plant outputs and model outputs

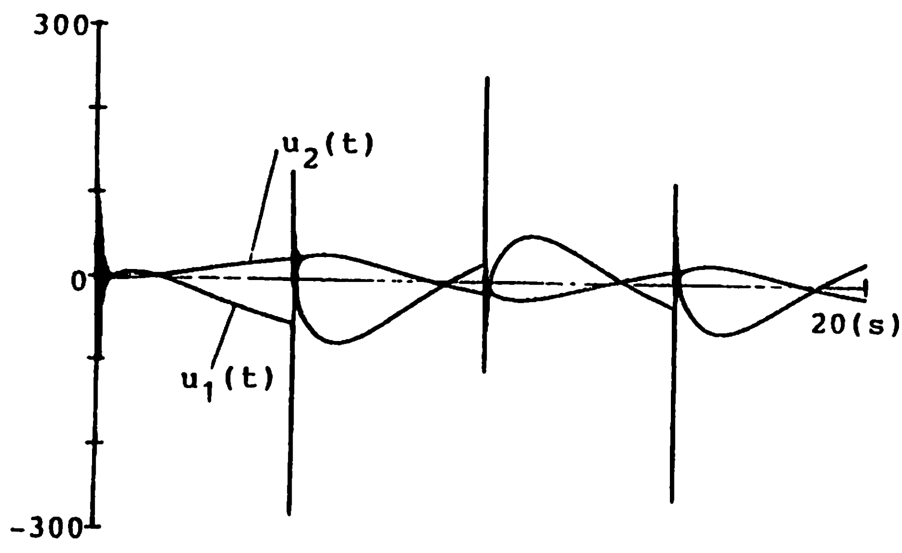


(b) control inputs

Figure 3.11 Simulation results for Case 1 without disturbance (relative degree γ_{ii} is exactly known)

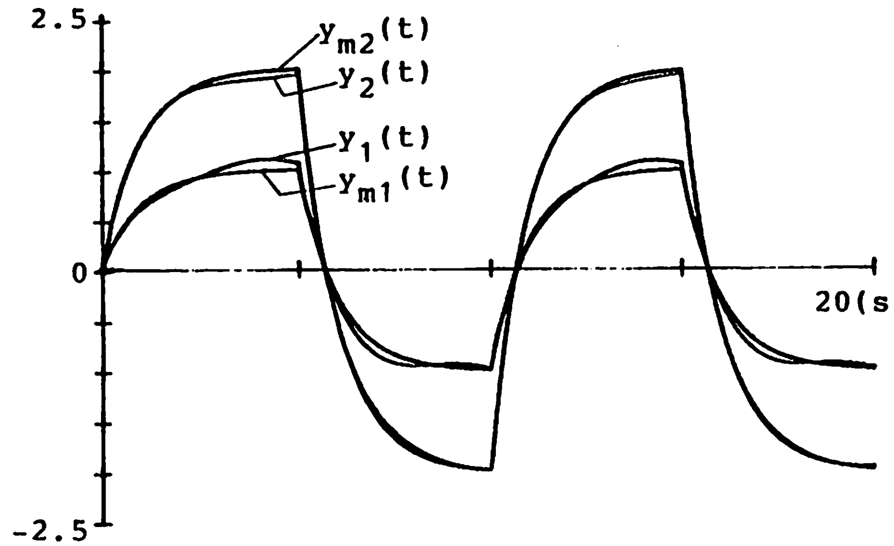


(a) plant outputs and model outputs

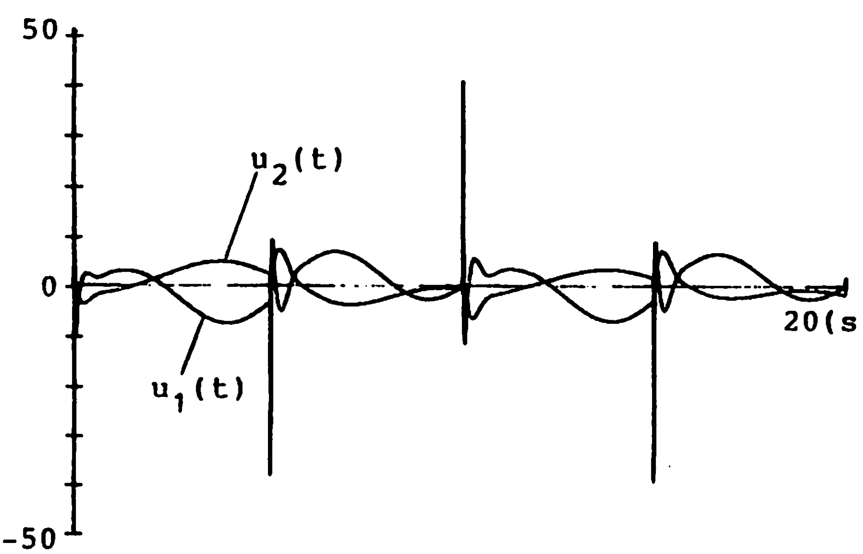


(b) control inputs

Figure 3.12 Simulation results for Case 2 without disturbance (relative degree γ_{ii} is exactly known)

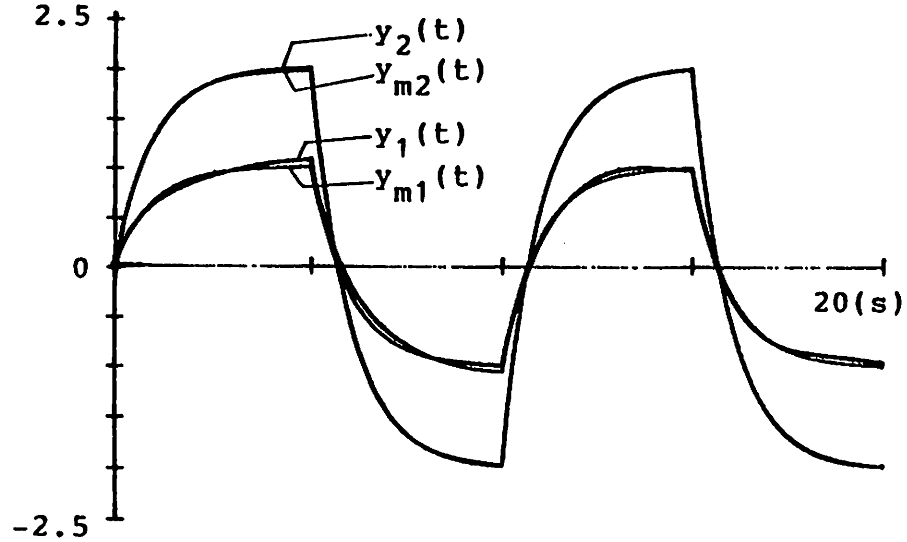


(a) plant outputs and model outputs

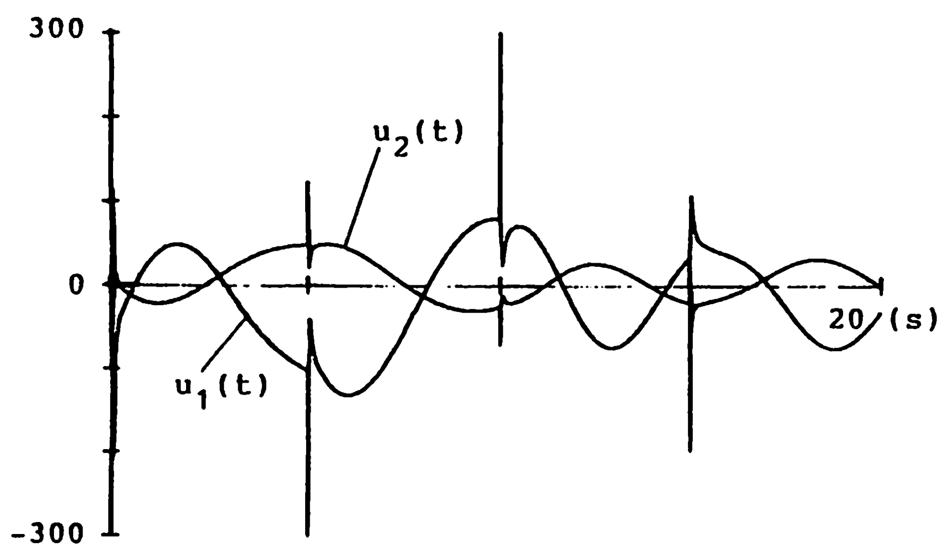


(b) control inputs

Figure 3.13 Simulation results for Case 1 with disturbance (relative degree γ_{ii} is exactly known)

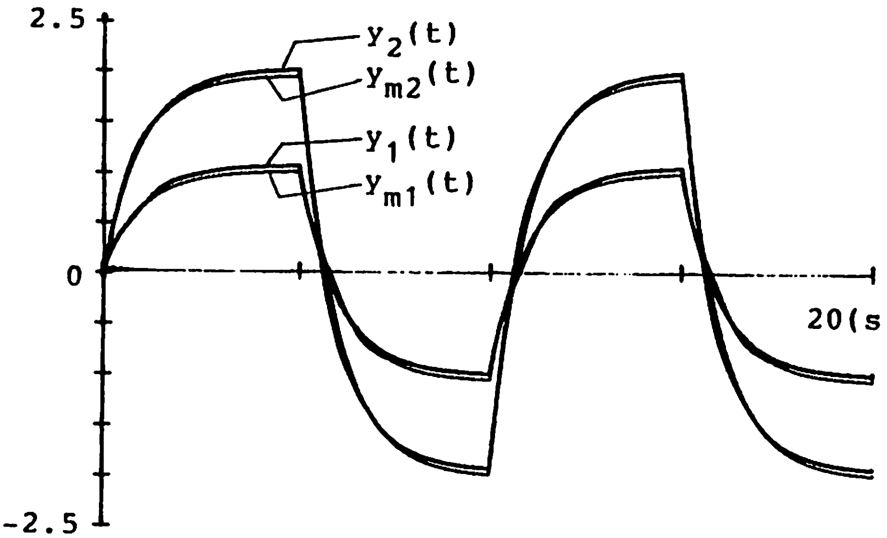


(a) plant outputs and model outputs

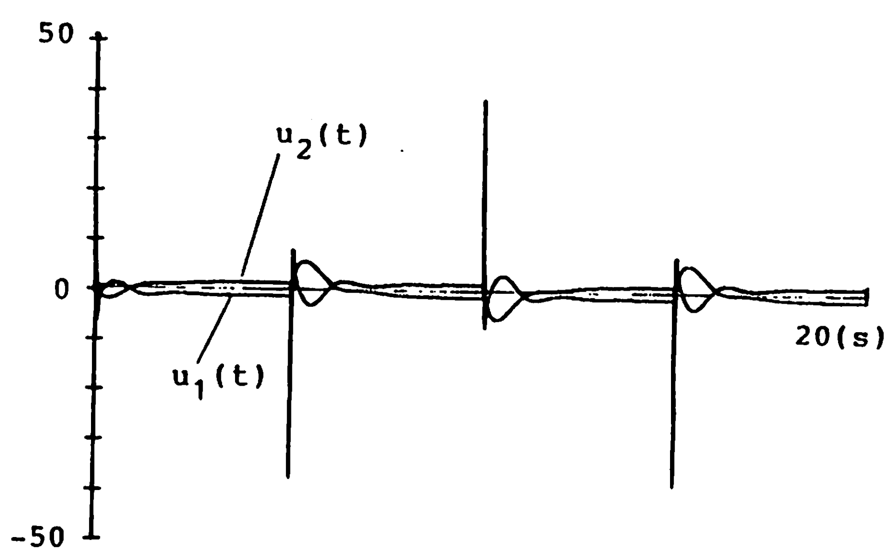


(b) control inputs

Figure 3.14 Simulation results for Case 2 with disturbance (relative degree γ_{ii} is exactly known)

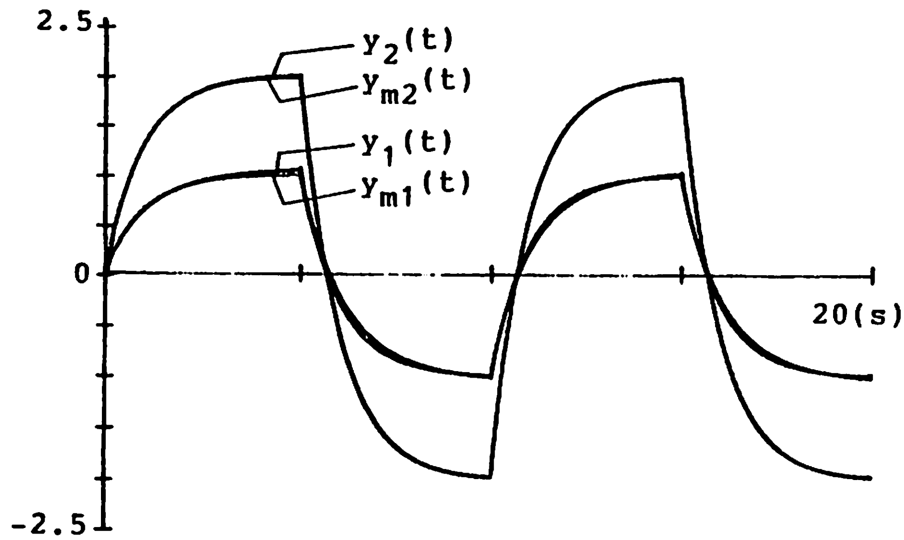


(a) plant outputs and model outputs

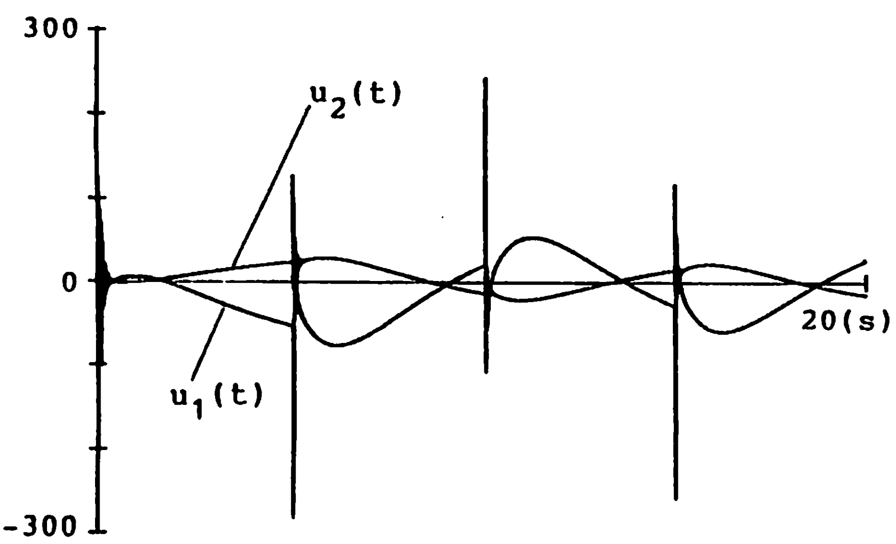


(b) control inputs

Figure 3.15 Simulation results for Case 1 without disturbance (upper bound γ_{ii}^* of relative degree γ_{ii} is known)



(a) plant outputs and model outputs



(b) control inputs

Figure 3.16 Simulation results for Case 2 without disturbance (upper bound γ_{ii}^* of relative degree γ_{ii} is known)

3.3 Compensation for Plants with Unmodelled Dynamics

As shown in the preceding section, if the plant is minimum phase then we can systematically design the PFC which makes non-ASPR plants ASPR. However, there are many plants for which it is not clear whether they are minimum phase or not. In fact, the plant may be non-minimum phase even if the modelled plant is minimum phase because of the existence of unmodelled dynamics.

In this section, we deal with the following SISO plants with unmodelled dynamics.

$$y(t) = G(s)[u(t)] \tag{3.3.1a}$$

$$G(s) = G_0(s)(1 + \Delta(s)) \tag{3.3.1b}$$

where $G_0(s)$ denotes the modelled part (dominant plant) of the plant and $\Delta(s)$ denotes the multiplicative uncertainty (unmodelled dynamics) of the plant. The notation $G(s)[u(t)]$ denotes the output of a system at time t with transfer function $G(s)$ and input $u(t)$. New design schemes of compensators, which make non-ASPR plants with unmodelled dynamics to be virtually ASPR, are suggested.

3.3.1 Almost Strictly Positive Real Condition for Plants with Unmodelled Dynamics

In preparation for investigating the design scheme of compensators, we first discuss the ASPR condition for plants with unmodelled dynamics.

With regard to the ASPR-ness of the plant with unmodelled dynamics, we give the following lemma.

Lemma 3.4: *Suppose that $\Delta(s) \in RH_\infty$ and $\|\Delta(s)\|_\infty < 1$. Then, the system $(1 + \Delta(s))$ is SPR and proper.*

Proof: From the assumptions on $\Delta(s)$, it is apparent that $(1 + \Delta(s))$ is stable and proper.

Let us denote

$$\Delta(j\omega) = a(\omega) + jb(\omega) \tag{3.3.2}$$

Since $\| \Delta(s) \|_\infty < 1$, for all ω, we have

$$\sqrt{a(\omega)^2 + b(\omega)^2} < 1 \tag{3.3.3}$$

It follows from (3.3.3) that $| a(\omega) | < 1$ for all ω. Thus, we have

$$\mathrm{Re}[1 + \Delta(j\omega)] = 1 + a(\omega) > 0 \tag{3.3.4}$$

and the desired result is obtained. □

From the above-mentioned lemma, it is clear that the overall plant $G(s)$ is ASPR under the following assumptions.

Assumption 3.3 :

(1) The modelled plant $G_0(s)$ *a) is minimum phase, b) has relative degree of* 1 *and c) the sign of the high frequency gain is positive*

(2) $\Delta(s) \in RH_\infty$ *and* $\| \Delta(s) \|_\infty < 1$

Assumption 3.3 is a sufficient condition for the plant to be ASPR. That is, in practice there exist ASPR plants even if Assumption 3.3, especially Assumption 3.3(2), does not hold. However, it may be difficult to judge whether the plant with uncertainty $\| \Delta(s) \|_\infty \geq 1$ is ASPR or not.

3.3.2 Basic Design Strategy of Parallel Feedforward Compensator

Let us consider the introduction of the PFC $F(s)$ to the plant (3.3.1). The resulting augmented plant is expressed as follows:

$$\begin{aligned} G_a(s) &= G(s) + F(s) \\ &= (G_0(s) + F(s))(1 + \frac{\Delta(s)}{1 + H(s)}) \end{aligned} \tag{3.3.5}$$

$$H(s) = G_0(s)^{-1} F(s) \tag{3.3.6}$$

Now, consider

$$G_{a0}(s) = G_0(s) + F(s) \tag{3.3.7}$$

to be the augmented modelled plant (modelled plant for the augmented plant), and consider

$$\Delta_a(s) = \frac{\Delta(s)}{1 + H(s)} \tag{3.3.8}$$

to be the new unmodelled dynamics. From Assumption 3.3, a guiding principle of the PFC design is given as follows:

Guiding principle of designing $F(s)$:

(1) $F(s)$ is stable.

(2) the augmented modelled plant $G_{a0}(s)$ is ASPR

(3) $\Delta_a(s) \in RH_\infty$ and $\| \Delta_a(s) \|_\infty < 1$

If we can design the PFC so as to satisfy the above-mentioned conditions, then the augmented plant becomes ASPR even if the original plant with unmodelled dynamics is non-minimum phase.

3.3.3 Practical Design of Parallel Feedforward Compensator

In practice, we cannot utilize $\Delta(s)$ to design the PFC because $\Delta(s)$ is unknown. With this in mind, here we show a practical design scheme of the PFC.

Now we impose the following assumptions on the plant (3.3.1).

Assumption 3.4:

(1) Assumption 3.3(1) holds ($G_0(s)$ is ASPR).

(2) $\Delta(s) \in RH_\infty$ and there exists a known rational function $r(s) \in RH_\infty$ such that

$$| \Delta(j\omega) | \leq | r(j\omega) | , \ \forall \omega \tag{3.3.9}$$

Under this assumption, we have the following theorem with regard to the PFC design.

Theorem 3.3 : *Under Assumption 3.4, the augmented plant $G_a(s)$ with the PFC:$F(s)$ designed according to the following design condition satisfies Assumption 3.3.*

Design condition of $F(s)$:

(1) $F(s)$ is stable and strictly proper

(2) i)$(1 + H(s))$ is proper, inversely stable and its high frequency gain is positive.

ii)

$$\| \frac{1}{1 + H(s)} r(s) \|_\infty < 1 \tag{3.3.10}$$

where

$$H(s) = G_0(s)^{-1} F(s) \tag{3.3.11}$$

Proof : Implementing $F(s)$ on the plant, the augmented plant $G_a(s)$ can be expressed as

$$\begin{aligned} G_a(s) &= G(s) + F(s) \\ &= (G_0(s) + F(s))(1 + \frac{\Delta(s)}{1 + H(s)}) \\ &= G_0(s)(1 + H(s))(1 + \frac{\Delta(s)}{1 + H(s)}) \end{aligned} \tag{3.3.12}$$

Here

$$G_{a0}(s) = G_0(s)(1 + H(s)) \tag{3.3.13}$$

is the augmented modelled plant and

$$\Delta_a(s) = \frac{1}{1 + H(s)}\Delta(s) \tag{3.3.14}$$

is the augmented unmodelled dynamics. From the design condition (2) i) of $F(s)$ and Assumption 3.4(1), it is apparent that the augmented modelled plant $G_{a0}(s)$ satisfies Assumption 3.3(1). Next, since $\mid \Delta(j\omega) \mid \leq \mid r(j\omega) \mid$, it follows that

$$\mid \Delta_a(j\omega) \mid \leq \mid r_a(j\omega) \mid, \ \forall\omega \tag{3.3.15}$$

where

$$r_a(s) = \frac{1}{1 + H(s)} r(s) \tag{3.3.16}$$

Further, from the design condition (2)ii), we have

$$\| r_a(s) \|_\infty < 1 \tag{3.3.17}$$

Thus, Assumption 3.3(2) is also satisfied. □

3.3.4 Use of a Pre-compensator

Assumption 3.4 is also not necessarily satisfied on many practical plants. We therefore show that Assumption 3.4 can be relaxed by using a pre-compensator.

Let us introduce the pre-compensator $Q(s)$ as shown in Figure 3.17. Defining

$$Q(s) = \frac{q_1(s)}{q_2(s)}$$

the augmented plant $\bar{G}_a(s)$ can be represented as follows.

$$\begin{aligned} \bar{G}_a(s) &= G(s)Q(s) \\ &= G_0(s)q_1(s)(\frac{1}{q_2(s)} + \frac{\Delta(s)}{q_2(s)}) \\ &= \bar{G}_{a0}(s)(1 + \bar{\Delta}_a(s)) \end{aligned} \tag{3.3.18}$$

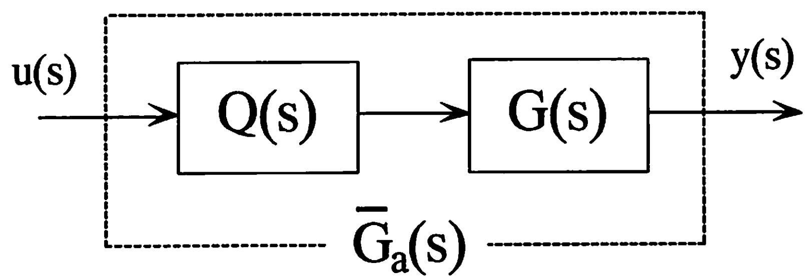


Figure 3.17 Augmented plant with pre-compensator

where

$$\bar{G}_{a0}(s) = G_0(s)q_1(s) \tag{3.3.19a}$$

$$\bar{\Delta}_a(s) = \frac{1}{q_2(s)} + \frac{\Delta(s)}{q_2(s)} - 1 \tag{3.3.19b}$$

Then we have the following theorem.

Theorem 3.4 : *We impose the following assumptions on the plant* (3.3.1).

Assumption 3.5:

(1) The modelled plant $G_0(s)$ is minimum phase and its relative degree is γ (≥ 2)

(2) $\Delta(s)$ is stable and there exists known rational function $r(s)$ satisfying the following relations

$$\frac{1}{q_0(s)} r(s) \in RH_\infty$$

and

$$|\, \Delta(j\omega) \,| \leq |\, r(j\omega) \,|$$

for any Hurwitz polynomial $g_0(s)$ of order n_q.

Under Assumption 3.5, *we give the pre-compensator as follows.*

$$Q(s) = \frac{q_1(s)}{q_2(s)}$$

$$q_1(s): \quad (\gamma-1)th \ order \ stable \ polynomial$$
$$q_2(s): \quad max\{(\gamma-1)\ ,\ n_q\}th \ order \ stable \ polynomial$$

Then the augmented plant

$$\bar{G}_a(s) = G(s)Q(s) = \bar{G}_{a0}(s)(1+\bar{\Delta}_a(s))$$

satisfies Assumption 3.4.

Proof : From(3.3.19a), $\bar{G}_{a0}(s) = G_0(s)q_1(s)$. Since $G_0(s)$ is minimum phase and has relative degree γ and $q_1(s)$ is a $(\gamma-1)$th order stable polynomial, it is apparent that $\bar{G}_{a0}(s)$ satisfies Assumption 3.4(1). Next, using (3.3.19b), we have

$$\begin{aligned} |\ \bar{\Delta}_a(j\omega)\ | &\leq \left|\ \frac{1}{q_2(j\omega)} - 1\ \right| + \left|\ \frac{\Delta(j\omega)}{q_2(j\omega)}\ \right| \\ &\leq \left|\ \frac{1}{q_2(j\omega)} - 1\ \right| + \left|\ \frac{r(j\omega)}{q_2(j\omega)}\ \right| \end{aligned} \tag{3.3.20}$$

Since $q_2(s)$ is $\max\{(\gamma-1), n_q\}$th order stable polynomial, it follows from Assumption 3.5(2) that there exists $\bar{r}_a(s) \in RH_\infty$ such that

$$\left|\ \frac{1}{q_2(j\omega)} - 1\ \right| + \left|\ \frac{r(j\omega)}{q_2(j\omega)}\ \right| \leq |\ \bar{r}_a(j\omega)\ | \tag{3.3.21}$$

for all ω. Hence, Assumption 3.4(2) also holds. □

The above theorem shows a possibility that we can construct the augmented plant for plants with minimum phase modelled part by implementing the pre-compensator. If $\| \bar{r}_a(s) \|_\infty < 1$ holds for $\bar{r}_a(s)$, satisfying relation (3.3.21), then we can directly apply the SAC method to the augmented plant $\bar{G}_a(s)$ with pre-compensator. In this case, we have no need to implement the augmented reference model which will be discussed in the following subsection. On the other hand, in the case of $\| \bar{r}_a(s) \|_\infty \geq 1$, we have to introduce the PFC in addition to pre-compensator.

3.3.5 Improvement of Tracking Performance

As derived in Theorem 3.4, if the plant has minimum phase modelled part and stable unmodelled dynamics, then we can design a new controlled plant which satisfies Assumption 3.4 by using a pre-compensator. Further, implementing the PFC to the plant which satisfies Assumption 3.4, we have the augmented plant satisfying Assumption 3.3 as shown in Theorem 3.3. As discussed in Bar-Kana (1987), Bar-Kana and Kaufman (1988) and in the preceding sections however, if the influence of the PFC $F(s)$ is *large*, then the exact output tracking between the original plant and the original reference model is not attained even if it is attained for the augmented plant with the PFC. This is summarized as follows.

Let us consider the reference model represented by

$$y_m(t) = G_m(s)[u_m(t)] \tag{3.3.22}$$

The actual control objective is to achieve

$$e_y(t) = G(s)[u(t)] - G_m(s)[u_m(t)] \simeq 0 \tag{3.3.23}$$

However, denoting the output error signal between the augmented plant and the reference model as

$$e_{ay}(t) = y_a(t) - y_m(t) \quad , \quad y_a(t) = G_a(s)[u(t)] \tag{3.3.24}$$

the actual tracking error is expressed from (3.2.1) and (3.3.22) as

$$e_y(t) = -F(s)[u(t)] + e_{ay}(t) \tag{3.3.25}$$

Thus, when $e_{ay}(t) \simeq 0$ is achieved, the actual tracking error substantially depends on the "order" of the signal $F(s)[u(t)]$. That is, if the influence of $F(s)$ is *large*, then the actual control objective is not attained even if $e_{ay}(t) \simeq 0$ is achieved.

From this point of view, in the following, we propose a new construction of the SAC system with the PFC.

Recall that we can construct an augmented plant with the PFC which satisfies Assumption 3.3 by considering $G_{a0}(s) = G_0(s)(1 + H(s))$ as the new modelled plant. Thus, the modelled plant of the augmented plant with PFC is expressed by $G_0(s)(1 + H(s))$. Taking this into account, we now introduce the following new reference model which the augmented plant is required to follow:

$$y_{am}(t) = G_m(s)(1 + H(s))[u_m(t)] \tag{3.3.26}$$

In this case, denoting the output tracking error signal between the augmented plant output and the output of the augmented reference model as

$$\bar{e}_{ay}(t) = y_a(t) - y_{am}(t) \tag{3.3.27}$$

we have from (3.3.12),(3.3.26) and (3.3.27) that

$$\begin{aligned} &G_0(s)(1 + H(s))(1 + \frac{1}{1 + H(s)}\Delta(s))[u(t)] \\ &= G_m(s)(1 + H(s))[u_m(t)] + \bar{e}_{ay}(t) \end{aligned} \tag{3.3.28}$$

Thus, we get

$$G_0(s)(1 + \frac{1}{1+H(s)}\Delta(s))[u(t)]$$
$$= G_m(s)[u_m(t)] + \frac{1}{1+H(s)}[\bar{e}_{ay}(t)] \qquad (3.3.29)$$

Considering that

$$e_y(t) = G_0(s)(1+\Delta(s))[u(t)] - G_m(s)[u_m(t)] \qquad (3.3.30)$$

it follows that the actual plant can be expressed as

$$e_y(t) = F(s)\frac{1}{1+H(s)}\Delta(s)[u(t)] + \frac{1}{1+H(s)}[\bar{e}_{ay}(t)] \qquad (3.3.31)$$

Thus, if $\bar{e}_{ay}(t) \simeq 0$ is achieved for the augmented system, then the actual tracking error depends on the "order" of the signal $\frac{F(s)}{1+H(s)}\Delta(s)\ [u(t)]$.

From the above-mentioned fact, taking into account that we give the PFC so as to satisfy

$$\mid \frac{1}{1+H(j\omega)}\Delta(j\omega) \mid < 1\ ,\ \forall\omega$$

we can conclude that the use of the augmented reference model on the augmented plant makes the influence of the parallel feedforward compensator smaller than the use of the original reference model. Further, if

$$\mid \frac{1}{1+H(j\omega)}\Delta(j\omega) \mid \ll 1$$

holds for any frequency range $\omega_0 \leq \omega \leq \omega_1$, then the influence of the parallel feedforward compensator may be neglected on the frequency range $\omega_0 \leq \omega \leq \omega_1$.

As discussed above, introducing the augmented reference model, we can construct a new control system which is less influenced from the PFC. However, in the SAC system, increase in the order of the reference model leads increase in the order of the adaptive controller. Therefore, in practice, instead of introducing the augmented reference model, we give the augmented reference input:

$$u_{am}(t) = (1 + H(s))[u_m(t)] \qquad (3.3.32)$$

Thus the new reference model is given by

$$y_{am}(t) = G_m(s)[u_{am}(t)] \qquad (3.3.33)$$

with the original reference model transfer function and the augmented reference input $u_{am}(t)$ as new reference input.

Remark 3.4: It should be noted that the augmented reference input $u_{am}(t)$ has to satisfy Assumption 2.2(4) in order to construct the above-mentioned SAC system. It is clear that $u_{am}(t)$ satisfies Assumption 2.2(4) if $(1 + H(s))$ is stable and proper and Assumption 2.2(4) holds on the original reference input $u_m(t)$.

Remark 3.5: In the case where the augmented reference modelis used, the SAC controller has the same form as given in (3.2.57) and (3.2.58), only replacing $z_a(t)$ with $\overline{z}_a(t) = [\overline{e}_{ay}(t), x_m(t)^T, u_{am}(t)]^T$.

3.3.6 Numerical Simulations

In this subsection, the effectiveness of the SAC design method for plants with unmodelled dynamics is confirmed through numerical simulations for two types of plants; case 1: non-minimum phase plant with non-ASPR modelled plant and case 2: time delay system with unmodelled dynamics. We use the following reference model in all simulations.

$$G_m(s) = \frac{1}{s+1} \tag{3.3.34}$$

$u_m(t)$: rectangular wave with amplitude 1

It is noted that the main simulation results shown below are the results implementing the augmented reference model.

Case 1:*(Application to the non-minimum phase plant with non-ASPR modelled plant)*

In this simulation, we use the simulation example based on Tao and Ioannou (1991). The plant to be controlled is given as follows.

$$G(s) = G_0(s)(1 + \Delta(s)) \tag{3.3.35}$$

where

$$G_0(s) = \frac{1}{(s^2 - s - 3)}, \ \Delta(s) = \mu\frac{-0.1s^2 - 4s + 2}{s+3} \tag{3.3.36}$$

The total plant is therefore expressed as

$$G(s) = \frac{-0.1\mu s^2 + (1 - 4\mu)s + 3 + 2\mu}{(s^2 - s - 3)(s + 3)} \tag{3.3.37}$$

Thus, this plant is non-minimum phase for all $\mu > 0$. We give $\mu = 0.03$ here. It is assumed that $G_0(s)$ is known and $\Delta(s)$ is unknown but a rational function $r(s)$ such that $| \Delta(j\omega) | \leq | r(j\omega) |$ is known. $r(s)$ is set to be

$$r(s) = \frac{0.02s^2 + 0.1s + 0.04}{s + 1.8} \tag{3.3.38}$$

as illustrated in Figure 3.18. Since the modelled plant $G_0(s)$ is non-ASPR and its relative degree is two, we give the pre-compensator as

$$Q(s) = \frac{s+2}{0.1s+1} \tag{3.3.39}$$

and we set

$$\overline{r}_a(s) = \frac{0.28s + 0.023}{0.2s + 0.8} \tag{3.3.40}$$

so as to satisfy (3.3.21)(Figure 3.19). Further, since $\| \overline{r}_a(s) \|_\infty > 1$, we design the PFC as follows.

$$F(s) = \frac{s+3}{s^2 + 5s + 15} \tag{3.3.41}$$

The design parameters in (3.2.58) are given as

$$\Gamma_{Ia} = \text{diag}[10^7, 10, 10],\ \Gamma_{Pa} = \text{diag}[10^5, 10, 10],\ \sigma_{a1} = 0,\ \sigma_{a2} = 0.03$$

Figures 3.20 and 3.21 show simulation results. In spite of the fact that the plant was non-minimum phase, good tracking performance was obtained. On the contrary, in the case where the augmented reference model was not used, as shown in Figure 3.22, the control performance became worse.

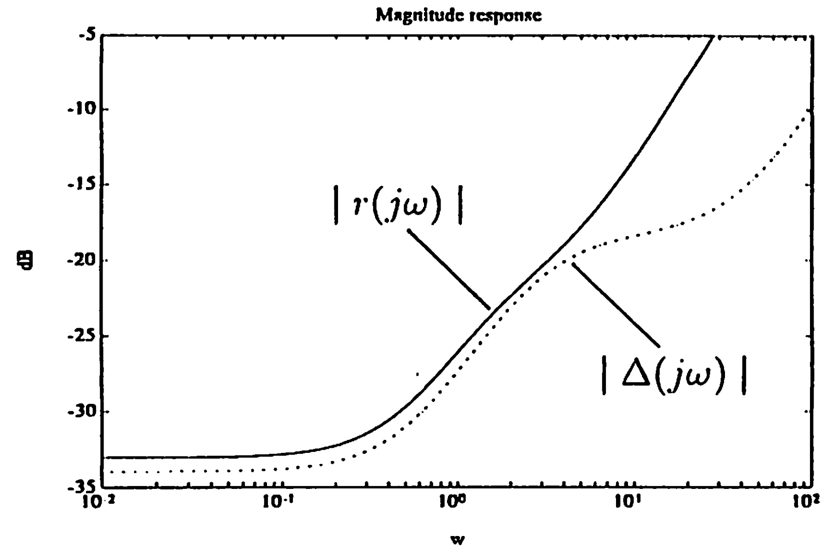


Figure 3.18 Gain diagram of $r(s)$; Case 1.

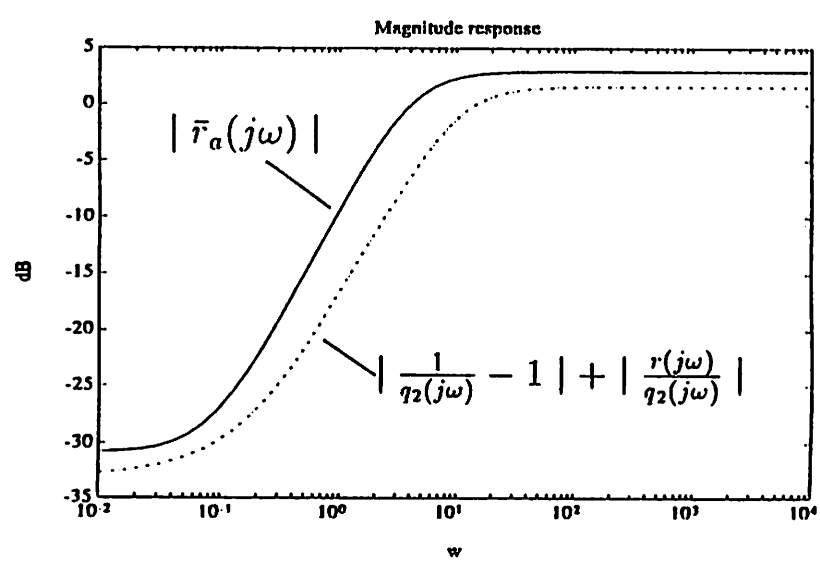
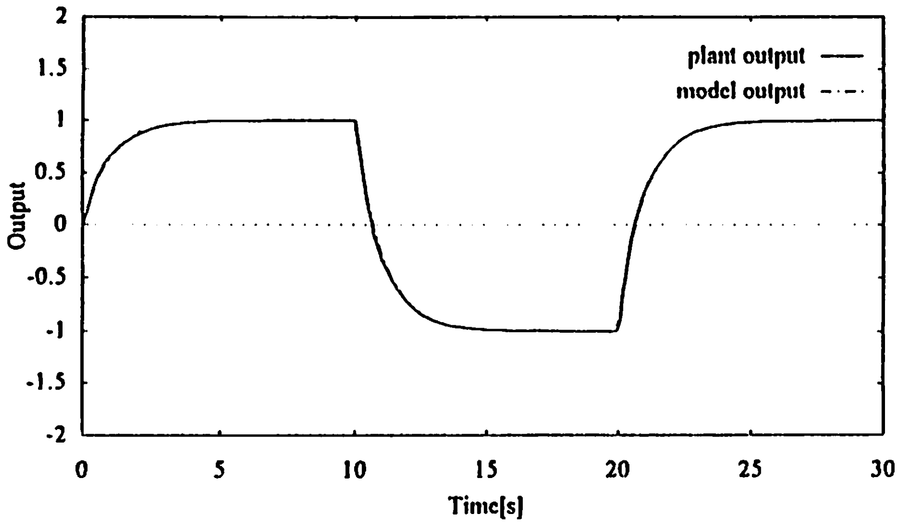
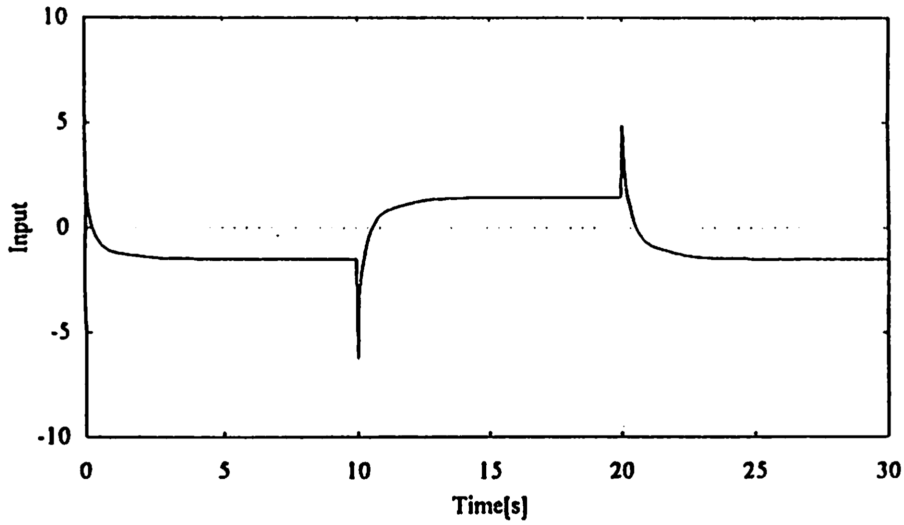


Figure 3.19 Gain diagram of $\bar{r}_a(s)$; Case 1.

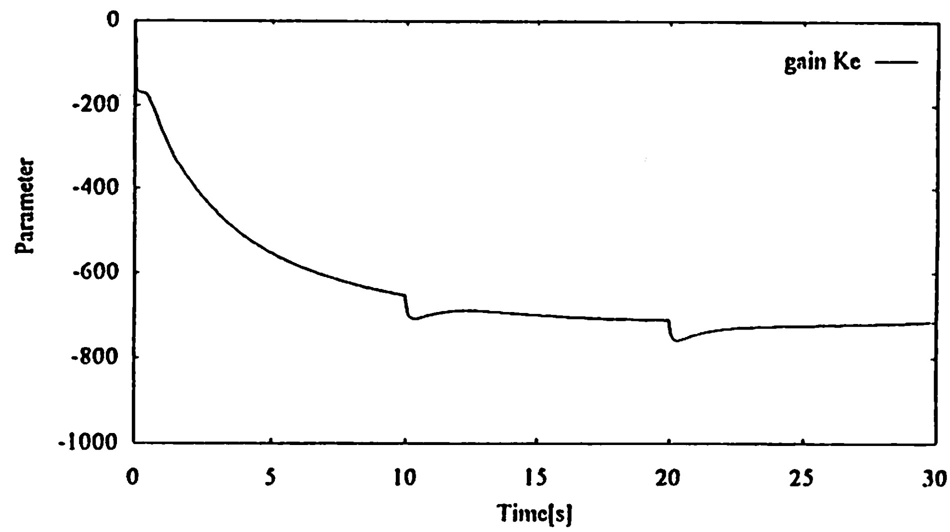


(a) plant output and model output

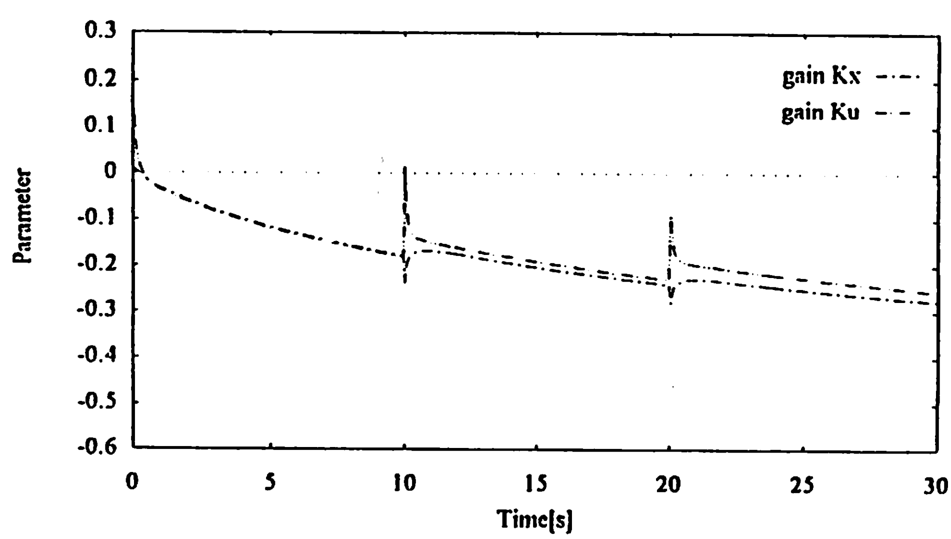


(b) control input

Figure 3.20 Simulation results for Case 1.

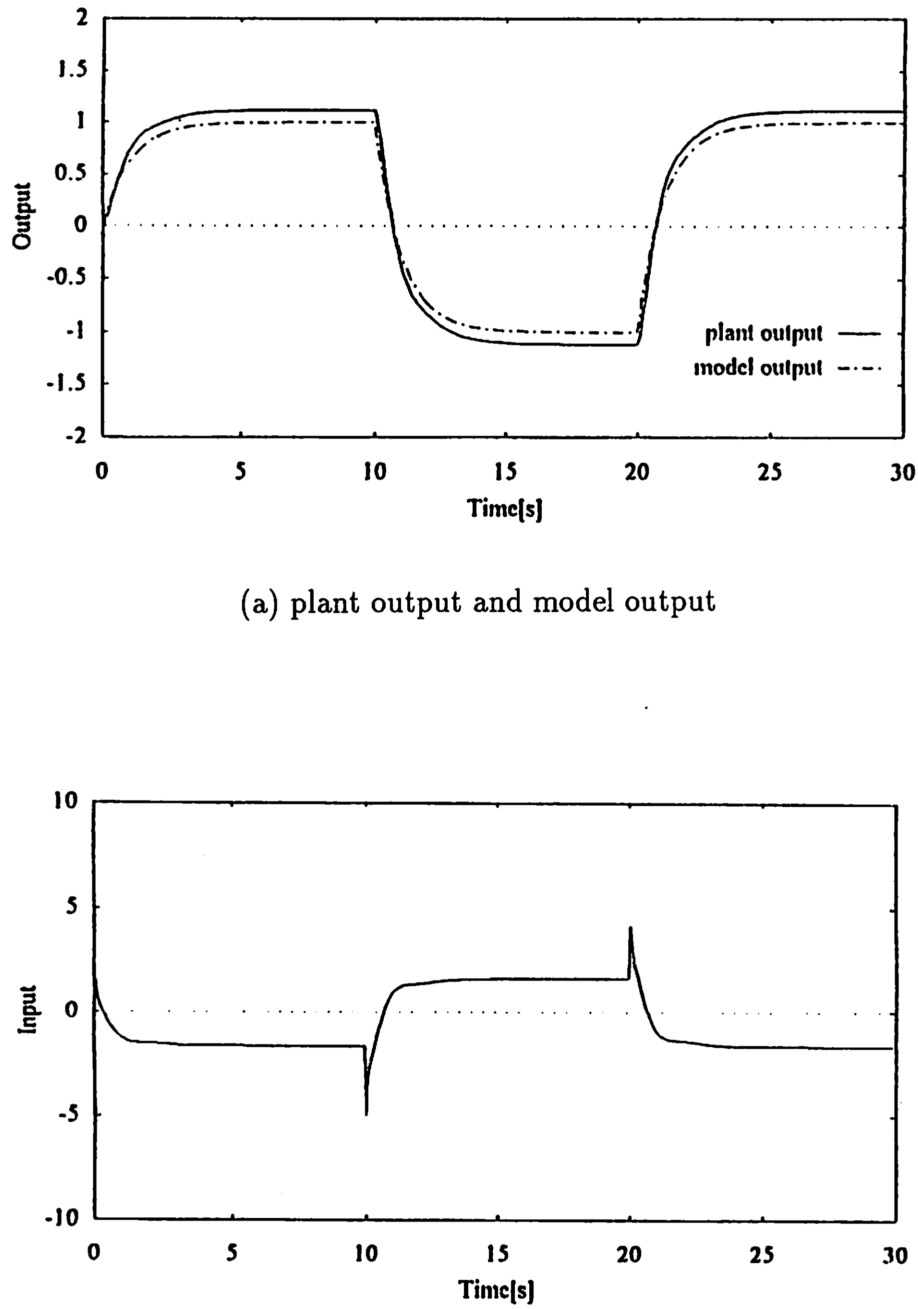


(a) estimated parameter; $k_e(t)$



(b) estimated parameters; $\boldsymbol{k}_x(t)$ and $k_u(t)$

Figure 3.21 Estimated parameters of Case 1.



(a) plant output and model output

(b) control input

Figure 3.22 Simulation results for Case 1; no use of augmented reference model.

Case 2: *(Application to time delay system)*

Consider the following plant with time delay τ_d.

$$G(s) = G_0(s)(1+\Delta(s))e^{-\tau_d s}, \quad \tau_d = 0.1 \tag{3.3.42}$$
$$G_0(s) = \frac{s+2}{s^2+18s-6}$$
$$\Delta(s) = \frac{-s+0.2}{s+1}$$

We suppose that $G_0(s)$ is known but $\Delta(s)$ and the time delay τ_d is unknown. Further, suppose that the rational function:

$$r(s) = \frac{1.12s+0.11}{1s+0.5} \tag{3.3.43}$$

such that

$$| e^{-\tau_d \omega j} + \Delta(j\omega)e^{-\tau_d \omega j} - 1 | \leq | r(j\omega) | \tag{3.3.44}$$

is known (Figure 3.23). From $\|r(s)\|_\infty > 1$, the PFC is set to be

$$F(s) = \frac{s+2}{s^2+100s+150} \tag{3.3.45}$$

The design parameters in (3.2.58) are given as

$$\Gamma_{Ia} = \text{diag}[10^6, 10, 10], \ \Gamma_{Pa} = \text{diag}[10^2, 10, 10], \ \sigma_{a1} = 0, \ \sigma_{a2} = 0.3$$

Figures 3.24 and 3.25 show the simulation results. We can obtain good tracking performance in spite of the presence of time delay by using a reasonable PFC. In this simulation, Considering that the time delay part can be approximated with a stable rational transfer function, for that approximated system, we can guarantee the stability. In fact, for example, using the Padé approximation, we can approximate the time delay part with a stable proper rational transfer function. However, as shown in this simulation, in practice, it is not so important and not necessary to find out the rational function with which the plant uncertainties can be exactly approximated. We are required to have only the information about $r(s)$ which satisfies relation (4.13).

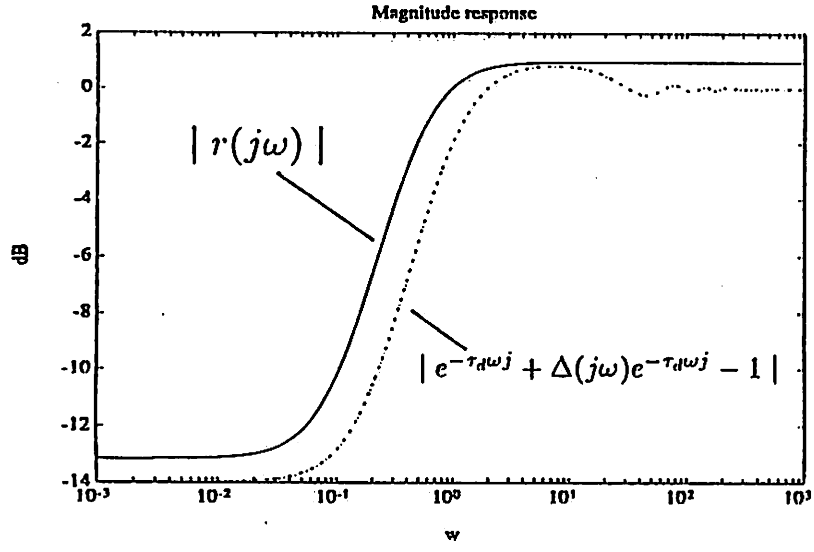
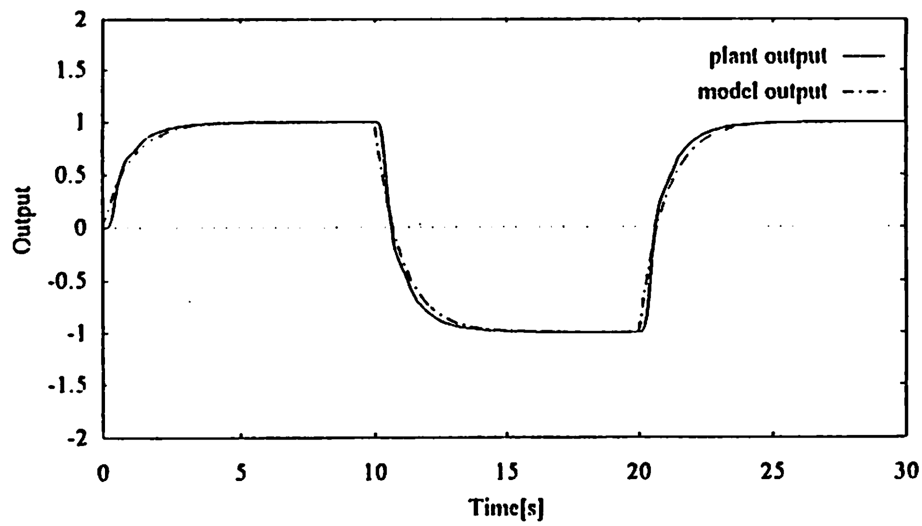
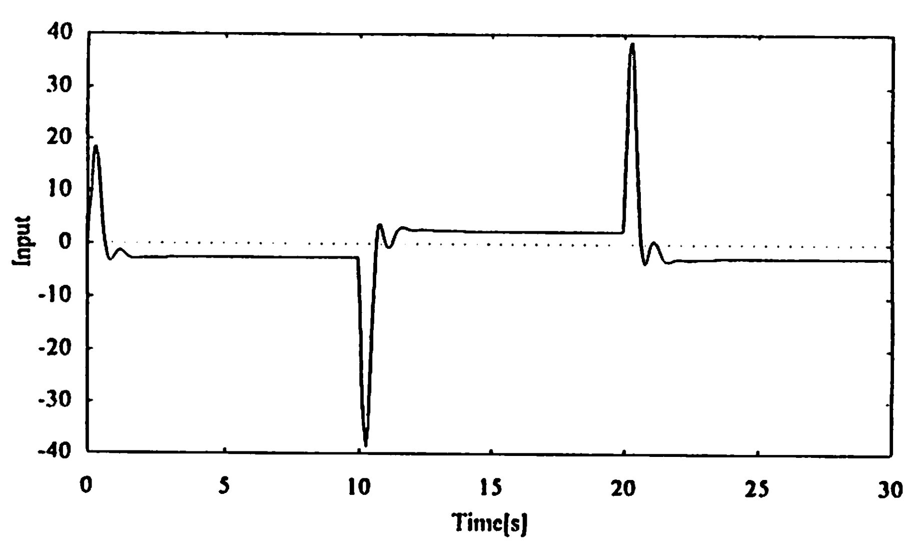


Figure 3.23 Gain diagram of $r(s)$; Case 2.

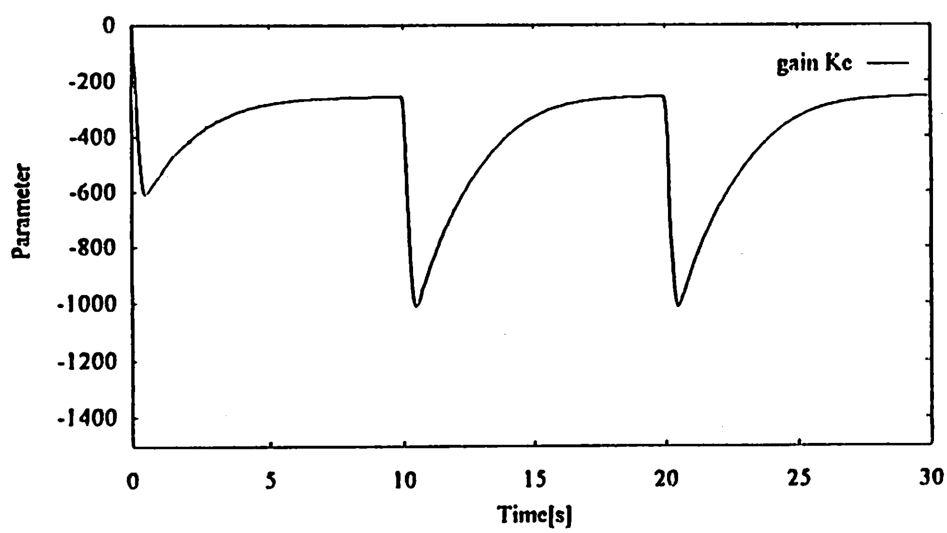


(a) plant output and model output

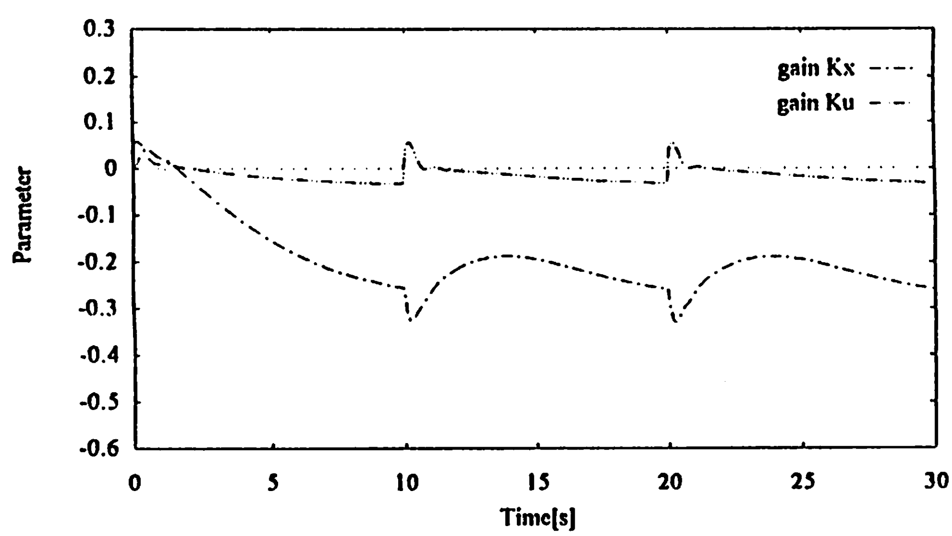


(b) control input

Figure 3.24 Simulation results for Case 2.



(a) estimated parameter; $k_e(t)$



(b) estimated parameters; $\boldsymbol{k}_x(t)$ and $k_u(t)$

Figure 3.25 Estimated parameters of Case 2.

3.4 Conclusions

The SAC method is based on the ASPR-ness of the controlled plant. This is a weak point of the method in the practical application because most real plants do not satisfy this ASPR condition. In this chapter, it was proved that the introduction of compensators makes it possible to apply SAC algorithms to general non-ASPR plants subject to assumptions which are necessary for the construction of regular robust adaptive control systems and/or robust control systems. First of all, the use of the PFC was considered as a practical SAC design option. A systematic and concrete design approach of the PFC which makes non-ASPR plants virtually ASPR was proposed under the assumption that the plant is minimum phase. Secondly, the ASPR condition for the plant with unmodelled dynamics was derived and, using this condition, the design scheme of the PFC was shown. Furthermore, it was shown that the use of a pre-compensator expanded the applicable class of the SAC. The effectiveness of the proposed methods was confirmed through several types of numerical simulations.

3.5 Appendix

Appendix 3.A

Let $\phi_1(s), \phi_2(s), \ldots, \phi_m(s)$ be the columns of $\Phi(s)$. Then, using the unit vectors $e_1, e_2, \ldots, e_m$, from (3.2.27), we have

$$\begin{aligned}\Phi_a(s) = [&\rho_1 f_1(s)p(s)e_1 + \phi_1(s)p_f(s), \rho_2 f_2(s)p(s)e_2 + \phi_2(s)p_f(s), \\ &\ldots, \rho_m f_m(s)p(s)e_m + \phi_m(s)p_f(s)] \qquad (3.A.1)\end{aligned}$$

It follows that the determinant of $\Phi_a(s)$ can be expressed as follows by using the basic transformation of a determinant:

$$\begin{aligned}\det\Phi_a(s) = &\det[\rho_1 f_1(s)p(s)e_1, \rho_2 f_2(s)p(s)e_2 + \phi_2(s)p_f(s), \\ &\ldots, \rho_m f_m(s)p(s)e_m + \phi_m(s)p_f(s)] \\ &\det[\phi_1(s)p_f(s), \rho_2 f_2(s)p(s)e_2 + \phi_2(s)p_f(s), \\ &\ldots, \rho_m f_m(s)p(s)e_m + \phi_m(s)p_f(s)] \qquad (3.A.2)\end{aligned}$$

Repeating this transformation 2^m times for all columns, we can obtain (3.2.28).

Appendix 3.B

From (3.2.29), we get

$$\deg q_h(s) = \deg\left\{\sum_{1\le i_1<\ldots<i_h\le m}\left(\prod_{\substack{j=1\\ j\ne i_1,\ldots,i_h}}^{m}\rho_j f_j(s)\right)\Phi[i_1,\ldots,i_h]\right\}.$$

$$-\deg p_f(s)^{m-h-1} - \deg p(s)^{h-1} \tag{3.B.1}$$

It follows that

$$\begin{aligned}
&\deg q_h(s) - \deg q_{h-1}(s) \\
&= \deg\left\{ \sum_{1\le i_1<\ldots<i_h\le m} \left(\prod_{\substack{j=1 \\ j\neq i_1,\ldots,i_h}}^{m} \rho_j f_j(s) \right) \Phi[i_1,\ldots,i_h] \right\} \\
&\quad -\deg\left\{ \sum_{1\le i_1<\ldots<i_{h-1}\le m} \left(\prod_{\substack{j=1 \\ j\neq i_1,\ldots,i_{h-1}}}^{m} \rho_j f_j(s) \right) \Phi[i_1,\ldots,i_{h-1}] \right\} \\
&\quad +n_f - n
\end{aligned} \tag{3.B.2}$$

Since

$$\begin{aligned}
&\deg\left\{ \sum_{1\le i_1<\ldots<i_h\le m} \left(\prod_{\substack{j=1 \\ j\neq i_1,\ldots,i_h}}^{m} \rho_j f_j(s) \right) \Phi[i_1,\ldots,i_h] \right\} \\
&= \max_{1\le i_1<\ldots<i_h\le m} \left[\deg\left\{ \left(\prod_{\substack{j=1 \\ j\neq i_1,\ldots,i_h}}^{m} \rho_j f_j(s) \right) \Phi[i_1,\ldots,i_h] \right\} \right]
\end{aligned} \tag{3.B.3}$$

$$\begin{aligned}
&\deg\left\{ \sum_{1\le i_1<\ldots<i_{h-1}\le m} \left(\prod_{\substack{j=1 \\ j\neq i_1,\ldots,i_{h-1}}}^{m} \rho_j f_j(s) \right) \Phi[i_1,\ldots,i_{h-1}] \right\} \\
&= \max_{1\le i_1<\ldots<i_{h-1}\le m} \left[\deg\left\{ \left(\prod_{\substack{j=1 \\ j\neq i_1,\ldots,i_{h-1}}}^{m} \rho_j f_j(s) \right) \Phi[i_1,\ldots,i_{h-1}] \right\} \right]
\end{aligned} \tag{3.B.4}$$

and relation (3.2.34) holds for all indices $\{i_1,\ldots,i_h\} \subset M$, we have

$$\begin{aligned}
&\max_{1\le i_1<\ldots<i_h\le m} \left[\deg\left\{ \left(\prod_{\substack{j=1 \\ j\neq i_1,\ldots,i_h}}^{m} \rho_j f_j(s) \right) \Phi[i_1,\ldots,i_h] \right\} \right] \\
&\ge \max \left[\deg\left\{ \left(\prod_{\substack{j=1 \\ j\neq i_1,\ldots,i_h}}^{m} \rho_j f_j(s) \right) \Phi[i_1,\ldots,i_{s-1},i_{s+1},\ldots,i_h] \right\} + n - \gamma_{i_s i_s} \right]
\end{aligned} \tag{3.B.5}$$

If we set

$$\begin{aligned}
&\max_{1\le i_1<\ldots<i_{h-1}\le m}\left[\deg\left\{\left(\prod_{\substack{j=1\\ j\ne i_1,\ldots,i_{h-1}}}^{m}\rho_j f_j(s)\right)\Phi[i_1,\ldots,i_{h-1}]\right\}\right]\\
&= \deg\left\{\left(\prod_{\substack{j=1\\ j\ne k_1,\ldots,k_{r-1},k_{r+1},\ldots,k_h}}^{m}\rho_j f_j(s)\right)\Phi[k_1,\ldots,k_{r-1},k_{r+1},\ldots,k_h]\right\}
\end{aligned} \tag{3.B.6}$$

then we obtain from (3.B.5) and (3.B.6)

$$\begin{aligned}
&(3.B.3)-(3.B.4)\\
&\ge \max\left[\deg\left\{\left(\prod_{\substack{j=1\\ j\ne i_1,\ldots,i_h}}^{m}\rho_j f_j(s)\right)\Phi[i_1,\ldots,i_{s-1},i_{s+1},\ldots,i_h]\right\}+n-\gamma_{i_s i_s}\right]\\
&\quad -\deg\left\{\left(\prod_{\substack{j=1\\ j\ne k_1,\ldots,k_{r-1},k_{r+1},\ldots,k_h}}^{m}\rho_j f_j(s)\right)\Phi[k_1,\ldots,k_{r-1},k_{r+1},\ldots,k_h]\right\}\\
&\ge \deg\left\{\left(\prod_{\substack{j=1\\ j\ne k_1,\ldots,k_h}}^{m}\rho_j f_j(s)\right)\Phi[k_1,\ldots,k_{r-1},k_{r+1},\ldots,k_h]\right\}+n-\gamma_{k_r k_r}\\
&\quad -\deg\left\{\left(\prod_{\substack{j=1\\ j\ne k_1,\ldots,k_{r-1},k_{r+1},\ldots,k_h}}^{m}\rho_j f_j(s)\right)\Phi[k_1,\ldots,k_{r-1},k_{r+1},\ldots,k_h]\right\}\\
&= n-\gamma_{k_r k_r}-(n_f-\gamma_{k_r k_r}+1)=n-n_f+1
\end{aligned} \tag{3.B.7}$$

Therefore, from (3.B.2), it follows that

$$\deg q_h(s)-\deg q_{h-1}(s)\ge -1 \tag{3.B.8}$$

and the desired result is obtained.

4 Robust Simple Adaptive Control

4.1 Introduction

As shown in the preceding chapter, SAC is robust with regard to disturbances, parasitics and so on. Such properties have also been confirmed through several types of numerical simulations and practical experiments by many researchers (Bar-Kana 1987b, Bar-Kana and Kaufman 1988, Ih *et al.* 1987, Bar-Kana *et al.* 1983, Meldrum and Balas 1986, Kaufman *et al.* 1984 and Kawasaki *et al.* 1994). This robust performance of the SAC controller is due to the SAC's ability to make a high gain adaptive feedback control system subject to the ASPR-ness of the controlled plant. However, in the case where *large* external disturbances and/or state-dependent disturbances are present, of course, the control performance might become worse.

In this chapter, the suppression of disturbances in the SAC system is discussed and it will be shown that the use of an additional robust adaptive controller in the SAC algorithm significantly imposes the robustness of the control system when external disturbances exist.

4.2 Robust Simple Adaptive Control for Single-Input Single-Output Plants

In this section, we will discuss the design problem of the SAC system with an additional adaptive controller for reducing the influence of disturbances.

4.2.1 Problem Setup

Consider the following SISO plant with external disturbance.

$$\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + \boldsymbol{b}u(t) + \boldsymbol{b}_1 g(t, \boldsymbol{x}(t)) \qquad (4.2.1a)$$

$$y(t) = \boldsymbol{c}^T \boldsymbol{x}(t) \qquad (4.2.1b)$$

where $g(t, \boldsymbol{x}(t))$ is an unknown disturbance which depends on a measurable state vector. The asymptotically stable reference model which the plant output is required to follow is given as follows:

$$\dot{\boldsymbol{x}}_m(t) = A_m \boldsymbol{x}_m(t) + \boldsymbol{b}_m u_m(t) \tag{4.2.2a}$$

$$y_m(t) = \boldsymbol{c}_m^T \boldsymbol{x}_m(t) \tag{4.2.2b}$$

Here we impose the following assumptions on the plant (4.2.1) and reference model (4.2.2).

Assumption 4.1:

(1) The linear part of the plant (4.2.1) *and the reference model* (4.2.2) *satisfy Assumption* 2.2.

(2) There exist positive but unknown constants ρ_0 *and* ρ_1 *and measurable state vector* $\overline{\boldsymbol{y}}(t)$ *such that*

$$|g(t, \boldsymbol{x}(t))| \leq \rho_0 + \rho_1 \|\overline{\boldsymbol{y}}(t)\| \tag{4.2.3}$$

$$\overline{\boldsymbol{y}}(t) = H\boldsymbol{x}(t) \tag{4.2.4}$$

The objective is to design a controller so as to be able to adaptively reduce the influence of disturbances.

4.2.2 Control Algorithm

The SAC controller with the robust adaptive control term is given as follows:

$$u(t) = \boldsymbol{k}(t)^T \boldsymbol{z}(t) + u_r(t) \tag{4.2.5}$$

where the first term of the right hand side is a regular SAC input and the second term $u_r(t)$ is a robust adaptive control term.

The parameter vector $\boldsymbol{k}(t)$ is adjusted by

$$\begin{cases} \boldsymbol{k}(t) = \boldsymbol{k}_I(t) + \boldsymbol{k}_P(t) \\ \dot{\boldsymbol{k}}_I(t) = -\Gamma_I \boldsymbol{z}(t) e_y(t) - \sigma_I(t) \boldsymbol{k}_I(t) \\ \boldsymbol{k}_P(t) = -\Gamma_P \boldsymbol{z}(t) e_y(t) \\ \sigma_I(t) = \sigma_1 \frac{e_y(t)^2}{1+e_y(t)^2} + \sigma_2 \end{cases} \tag{4.2.6}$$

where

$$\Gamma_I = \Gamma_I^T > 0,\ \Gamma_P = \Gamma_P^T > 0,\ \sigma_1, \sigma_2 > 0$$

The above parameter adjusting law has been defined by (2.3.11) as the form given in the MIMO case. Further, we give the robust adaptive control input $u_r(t)$ as follows:

$$u_r(t) = \begin{cases} -\boldsymbol{\beta}(t)^T \boldsymbol{z}_\beta(t) \mathrm{sgn} e_y(t), & \text{if} |\boldsymbol{\beta}(t)^T \boldsymbol{z}_\beta(t) e_y(t)| > \varepsilon \\ -\{\boldsymbol{\beta}(t)^T \boldsymbol{z}_\beta(t)\}^2 e_y(t)/\varepsilon, & \text{if} |\boldsymbol{\beta}(t)^T \boldsymbol{z}_\beta(t) e_y(t)| \le \varepsilon \end{cases} \tag{4.2.7}$$

where

$$\boldsymbol{\beta}(t) = [\beta_0(t), \beta_1(t)]^T,\ \ \boldsymbol{z}_\beta(t) = [1, \|\overline{\boldsymbol{y}}(t)\|]^T,\ \ \varepsilon > 0$$

and the parameter vector $\boldsymbol{\beta}(t)$ is adjusted by the following parameter adjusting law.

$$\begin{cases} \boldsymbol{\beta}(t) = \boldsymbol{\beta}_I(t) + \boldsymbol{\beta}_P(t) \\ \dot{\boldsymbol{\beta}}_I(t) = \Gamma_{\beta I} \boldsymbol{z}_\beta(t) |e_y(t)| - \sigma_\beta(t) \boldsymbol{\beta}_I(t) \\ \boldsymbol{\beta}_P(t) = \Gamma_{\beta P} \boldsymbol{z}_\beta(t) |e_y(t)| \\ \sigma_\beta(t) = \sigma_{\beta 1} \frac{e_y(t)^2}{1+e_y(t)^2} + \sigma_{\beta 2} \end{cases} \tag{4.2.8}$$

where

$$\Gamma_{\beta I} = \Gamma_{\beta I}^T > 0,\ \Gamma_{\beta P} = \Gamma_{\beta P}^T > 0,\ \sigma_{\beta 1}, \sigma_{\beta 2} > 0$$

4.2.3 Stability of the Control System

The stability analysis of the SAC system with the robust adaptive controller follows the stability analysis of the regular SAC system discussed in subsection 2.3.3.

We have the following error system between the ideal system (2.3.12) with $g(t, \boldsymbol{x}(t)) \equiv 0$ and the plant (4.2.1) with control input (4.2.5).

$$\dot{\boldsymbol{e}}_x(t) = A_c \boldsymbol{e}_x(t) + \boldsymbol{b} u_c(t) + \boldsymbol{b}_1 g(t, \boldsymbol{x}(t)) \tag{4.2.9a}$$

$$e_y(t) = \boldsymbol{c}^T \boldsymbol{e}_x(t) \tag{4.2.9b}$$

where

$$A_c = A + \boldsymbol{k}^* \boldsymbol{b} \boldsymbol{c}^T$$
$$u_c(t) = (\boldsymbol{k}(t) - \boldsymbol{k}^*)^T \boldsymbol{z}(t) - S_{23}(t) + u_r(t)$$

According to Assumption 4.1(1), the linear part of (4.2.9) is SPR. It follows from Kalman-Yakubovich lemma that there exist positive symmetric matrices P and Q satisfying:

$$\begin{aligned} A_c^T P + P A_c &= -Q \\ P\boldsymbol{b} &= \boldsymbol{c} \end{aligned} \tag{4.2.10}$$

Set a positive definite function as

$$V(t) = \boldsymbol{e}_x(t)^T P \boldsymbol{e}_x(t) + \sum_{i=1}^{2} \boldsymbol{\zeta}_i(t)^T \Gamma_{iI}^{-1} \boldsymbol{\zeta}_i(t) \tag{4.2.11}$$

where

$$\begin{aligned} \boldsymbol{\zeta}_1(t) &= \boldsymbol{k}_I(t) - \boldsymbol{k}^*, \quad \boldsymbol{\zeta}_2(t) = \boldsymbol{\beta}_I(t) - \boldsymbol{\beta}^* \\ \boldsymbol{\beta}^* &= [\rho_0 + S_{23}^*, \rho_1]^T, \quad S_{23}^* = \max_t |S_{23}(t)| \\ \Gamma_{1I} &= \Gamma_I, \quad \Gamma_{2I} = \Gamma_{\beta I} \end{aligned}$$

Differentiating (4.2.11) along the trajectory (4.2.9) and taking into account relation (4.2.10) yields

$$\begin{aligned} \frac{dV(t)}{dt} &= -\boldsymbol{e}_x(t)^T Q \boldsymbol{e}_x(t) + 2u_c(t)e_y(t) + 2g(t, \boldsymbol{x}(t))e_y(t) \\ &\quad + 2g(t, \boldsymbol{x}(t))(\boldsymbol{b}_1 - \boldsymbol{b})^T P \boldsymbol{e}_x(t) + 2\sum_{i=1}^{2} \boldsymbol{\zeta}_i(t)^T \Gamma_{iI}^{-1} \dot{\boldsymbol{\zeta}}_i(t) \end{aligned} \tag{4.2.12}$$

From the parameter adjusting laws (4.2.6) and (4.2.8), we have

$$\begin{aligned} &\sum_{i=1}^{2} \boldsymbol{\zeta}_i(t)^T \Gamma_{iI}^{-1} \dot{\boldsymbol{\zeta}}_i(t) \\ &\quad \le -\sum_{i=1}^{2} \sigma_i(t) \boldsymbol{\zeta}_i(t)^T \Gamma_{iI}^{-1} \boldsymbol{\zeta}_i(t) - \sigma_1(t) \boldsymbol{\zeta}_1(t)^T \Gamma_{1I}^{-1} \boldsymbol{k}^* \\ &\qquad -\sigma_2(t) \boldsymbol{\zeta}_2(t)^T \Gamma_{2I}^{-1} \boldsymbol{\beta}^* - [\boldsymbol{k}(t) - \boldsymbol{k}^*]^T \boldsymbol{z}(t) e_y(t) \\ &\qquad + [\boldsymbol{\beta}(t) - \boldsymbol{\beta}^*]^T \boldsymbol{z}_\beta(t) |e_y(t)| \end{aligned} \tag{4.2.13}$$

where $\sigma_1(t) = \sigma_I(t)$ and $\sigma_2(t) = \sigma_\beta(t)$. From (4.2.12) and (4.2.13), we obtain

$$\begin{aligned} \frac{dV(t)}{dt} &\le -\boldsymbol{e}_x(t)^T Q \boldsymbol{e}_x(t) - 2\sum_{i=1}^{2} \sigma_i(t) \boldsymbol{\zeta}_i(t)^T \Gamma_{iI}^{-1} \boldsymbol{\zeta}_i(t) + 2u_r(t)e_y(t) \\ &\quad - 2\sigma_1(t) \boldsymbol{\zeta}_1(t)^T \Gamma_{1I}^{-1} \boldsymbol{k}^* - 2\sigma_2(t) \boldsymbol{\zeta}_2(t)^T \Gamma_{2I}^{-1} \boldsymbol{\beta}^* \\ &\quad + 2\boldsymbol{\beta}^{*T} \boldsymbol{z}_\beta(t) |e_y(t)| + 2[\boldsymbol{\beta}(t) - \boldsymbol{\beta}^*]^T \boldsymbol{z}_\beta(t) |e_y(t)| \\ &\quad + 2g(t, \boldsymbol{x}(t))(\boldsymbol{b}_1 - \boldsymbol{b})^T P \boldsymbol{e}_x(t) \end{aligned} \tag{4.2.14}$$

From (4.2.7), if $|\boldsymbol{\beta}(t)^T \boldsymbol{z}_\beta(t) e_y(t)| > \varepsilon$, then, $u_r(t) = -\boldsymbol{\beta}(t)^T \boldsymbol{z}_\beta(t) \mathrm{sgn} e_y(t)$. Substituting this into (4.2.14) leads to

$$\frac{dV(t)}{dt} \le -e_x(t)^T Q e_x(t) - 2\sum_{i=1}^{2} \sigma_i(t)\zeta_i(t)^T \Gamma_{iI}^{-1}\zeta_i(t)$$
$$-2\sigma_1(t)\zeta_1(t)^T\Gamma_{1I}^{-1}k^* - 2\sigma_2(t)\zeta_2(t)^T\Gamma_{2I}^{-1}\beta^*$$
$$+2g(t,x(t))(b_1 - b)^T P e_x(t)$$
$$= V_1(t) \tag{4.2.15}$$

If $|\beta(t)^T z_\beta(t) e_y(t)| \le \varepsilon$, then

$$\frac{dV(t)}{dt} \le V_1(t) + 2\varepsilon \tag{4.2.16}$$

holds. Moreover, from (4.2.4) we have

$$\|\overline{y}(t)\| \le \|H\|\|e_x(t)\| + \|Hx^*(t)\| \le \rho_3\|e_x(t)\| + \rho_4 \tag{4.2.17}$$

where $\rho_3 = \|H\|$ and $\rho_4 = \max_t \|Hx^*(t)\|$. It follows from Assumption 4.1(b) and (4.2.17) that

$$g(t,x(t))(b_1 - b)^T P e_x(t) \le |g(t,x(t))|\|b_1 - b\|\lambda_{max}[P]\|e_x(t)\|$$
$$\le \lambda_{max}[P]\|b_1 - b\|(\rho_0 + \rho_1\|\overline{y}(t)\|)\|e_x(t)\|$$
$$\le \rho_A\|e_x(t)\|^2 + \rho_B\|e_x(t)\| \tag{4.2.18}$$

where $\rho_A = \lambda_{max}[P]\|b_1 - b\|\rho_1\rho_3$ and $\rho_B = \lambda_{max}[P]\|b_1 - b\|(\rho_0 + \rho_1\rho_4)$. Hence, from (4.2.16) and (4.2.18) we can obtain

$$\frac{dV(t)}{dt} \le -(\lambda_{min}[Q] - 2\rho_A)\|e_x(t)\|^2 + 2\rho_B\|e_x(t)\|$$
$$-2\sum_{i=1}^{2}\sigma_i(t)\lambda_{min}[\Gamma_{iI}]\|\zeta_i(t)\|^2 + 2\sigma_1(t)\lambda_{max}[\Gamma_{1I}]\|k^*\|\|\zeta_1(t)\|$$
$$+2\sigma_2(t)\lambda_{max}[\Gamma_{2I}]\|\beta^*\|\|\zeta_2(t)\| + 2\varepsilon \tag{4.2.19}$$

Since $\sigma_i(t) > 0$, if $\lambda_{min}[Q] - 2\rho_A > 0$ holds, then $dV(t)/dt$ becomes negative definite for large values of $e_x(t)$ and $\zeta_i(t), i = 1,2$. That is, $e_x(t)$ and $\zeta_i(t), i = 1,2$ are uniformly ultimately bounded (Corless and Leitman 1981 and Chen 1989). It can easily be verified from this conclusion that all the signals in the closed-loop system are also uniformly ultimately bounded.

From the above-mentioned fact, we have the following theorem with regard to the stability of the robust SAC system.

Theorem 4.1: *Suppose that Assumption* 4.1 *is satisfied. Further suppose that there exists a positive scalar* ε_1 *such that* $\|b_1 - b\| \le \varepsilon_1 < \lambda_{min}[Q]/(2\rho_1\|H\|\lambda_{max}[P])$. *Then, all the signals in the control system* (4.2.1) *with the control input* (4.2.5) *are uniformly ultimately bounded.*

4.3 Robust Simple Adaptive Control for Multi-Input Multi-Output Plants

In the preceding section, we have shown the design scheme of a robust SAC system for SISO plants with a sort of state-dependent disturbance. In this section, the method will be expanded to the one for MIMO plants.

4.3.1 Problem Setup

Consider the following MIMO plant with a state-dependent disturbance.

$$\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + B\boldsymbol{u}(t) + B_1\boldsymbol{g}(t, \overline{\boldsymbol{y}}) \tag{4.3.1a}$$

$$\boldsymbol{y}(t) = C\boldsymbol{x}(t) \tag{4.3.1b}$$

$$\overline{\boldsymbol{y}}(t) = H\boldsymbol{x}(t) \tag{4.3.1c}$$

where $\boldsymbol{x} \in R^n$, $\boldsymbol{y} \in R^m$ and $\boldsymbol{u} \in R^m$ denote state vector, output vector and control input vector, respectively. $\boldsymbol{g}(t, \overline{\boldsymbol{y}}) \in R^m$ is an unknown disturbance which depends on the measurable state vector $\overline{\boldsymbol{y}}(t)$.

Further consider the following asymptotically stable reference model.

$$\dot{\boldsymbol{x}}_m(t) = A_m\boldsymbol{x}_m(t) + B_m\boldsymbol{u}_m(t) \tag{4.3.2a}$$

$$\boldsymbol{y}_m(t) = C_m\boldsymbol{x}_m(t) \tag{4.3.2b}$$

The control objective is to have the plant output $\boldsymbol{y}(t)$ track the reference model output $\boldsymbol{y}_m(t)$.

Here we make the following assumptions on the plant (4.3.1) and reference model (4.3.2).

Assumption 4.2:

(1) The linear part of the plant (4.3.1) *and the reference model* (4.3.2) *satisfy Assumption* 2.2.

(2) Denoting

$$\boldsymbol{g}(t, \overline{\boldsymbol{y}}) = [g_1(t, \overline{\boldsymbol{y}}), \cdots, g_m(t, \overline{\boldsymbol{y}})]$$

there exist positive unknown constants ρ_{0i} and ρ_{1i} such that

$$|g_i(t, \overline{\boldsymbol{y}})| \leq \rho_{0i} + \rho_{1i}\|\overline{\boldsymbol{y}}(t)\|, \quad i = 1, \ldots, m \tag{4.3.3}$$

4.3.2 Control Algorithm

The SAC controller with the robust adaptive control term for MIMO plants is given as follows:

$$\boldsymbol{u}(t) = K(t)\boldsymbol{z}(t) + \boldsymbol{u}_R(t) \tag{4.3.4}$$

The parameter matrix $K(t)$ is adjusted by the following parameter adjusting law which was given in (2.3.11).

$$\begin{cases} K(t) = K_I(t) + K_P(t) \\ \dot{K}_I(t) = -\boldsymbol{e}_y(t)\boldsymbol{z}(t)^T\Gamma_I - \sigma_I(t)K_I(t) \\ K_P(t) = -\boldsymbol{e}_y(t)\boldsymbol{z}(t)^T\Gamma_P \\ \sigma_I(t) = \sigma_1\frac{\boldsymbol{e}_y(t)^T\boldsymbol{e}_y(t)}{1+\boldsymbol{e}_y(t)^T\boldsymbol{e}_y(t)} + \sigma_2 \end{cases} \tag{4.3.5}$$

where

$$\Gamma_I = \Gamma_I^T > 0,\ \Gamma_P = \Gamma_P^T > 0,\ \sigma_1, \sigma_2 > 0$$

Further, denoting $\boldsymbol{u}_R(t) = [u_{R1}(t), \cdots, u_{Rm}(t)]^T$ and $\boldsymbol{e}_y(t) = \boldsymbol{y}(t) - \boldsymbol{y}_m(t) = [e_{y1}(t), \cdots, e_{y2}(t)]^T$, the robust adaptive control input $\boldsymbol{u}_R(t)$ is given as

$$u_{Ri}(t) = \begin{cases} -\boldsymbol{\beta}_i(t)^T\boldsymbol{z}_\beta(t)\mathrm{sgn}e_{yi}(t), & \mathrm{if}|\boldsymbol{\beta}_i(t)^T\boldsymbol{z}_\beta(t)e_{yi}(t)| > \varepsilon_i \\ -\{\boldsymbol{\beta}_i(t)^T\boldsymbol{z}_\beta(t)\}^2 e_{yi}(t)/\varepsilon_i, & \mathrm{if}|\boldsymbol{\beta}_i(t)^T\boldsymbol{z}_\beta(t)e_{yi}(t)| \le \varepsilon_i \end{cases} \tag{4.3.6}$$

where

$$\boldsymbol{\beta}_i(t) = [\beta_{0i}(t), \beta_{1i}(t)]^T,\quad \boldsymbol{z}_\beta(t) = [1, \|\overline{\boldsymbol{y}}(t)\|]^T,\quad \varepsilon_i > 0$$

The parameter vectors $\boldsymbol{\beta}_i(t), i = 1, \ldots, m$ are adaptively adjusted by the following parameter adjusting laws.

$$\begin{cases} \boldsymbol{\beta}_i(t) = \boldsymbol{\beta}_{Ii}(t) + \boldsymbol{\beta}_{Pi}(t) \\ \dot{\boldsymbol{\beta}}_{Ii}(t) = \Gamma_{\beta Ii}\boldsymbol{z}_\beta(t)|e_{yi}(t)| - \sigma_{\beta i}(t)\boldsymbol{\beta}_{Ii}(t) \\ \boldsymbol{\beta}_{Pi}(t) = \Gamma_{\beta Pi}\boldsymbol{z}_\beta(t)|e_{yi}(t)| \\ \sigma_{\beta i}(t) = \sigma_{\beta 1i}\frac{e_{yi}(t)^2}{1+e_{yi}(t)^2} + \sigma_{\beta 2i} \end{cases} \tag{4.3.7}$$

where

$$\Gamma_{\beta Ii} = \Gamma_{\beta Ii}^T > 0,\ \Gamma_{\beta Pi} = \Gamma_{\beta Pi}^T > 0,\ \sigma_{\beta 1i}, \sigma_{\beta 2i} > 0$$

4.3.3 Stability of the Control System

The stability of the control system can be verified in the same manner as in the SISO case. From (2.3.12) and (4.3.1), we have the following error system between the ideal system with $\boldsymbol{g}(t, \overline{\boldsymbol{y}}) \equiv 0$ and the controlled plant with control input (4.3.4).

$$\dot{\boldsymbol{e}}_x(t) = A_c \boldsymbol{e}_x(t) + B\boldsymbol{u}_c(t) + B_1 \boldsymbol{g}(t, \overline{\boldsymbol{y}}) \tag{4.3.8a}$$

$$\boldsymbol{e}_y(t) = C\boldsymbol{e}_x(t) \tag{4.3.8b}$$

where

$$\begin{aligned} A_c &= A + BK^*C \\ \boldsymbol{u}_c(t) &= (\Delta K(t) - K^*)^T \boldsymbol{z}(t) - S_{23}(t) + \boldsymbol{u}_R(t) \\ \Delta K(t) &= K(t) - K^* \end{aligned}$$

According to Assumption 4.1(1), the linear part of (4.3.7) is SPR. It follows from Kalman-Yakubovich lemma that there exist positive symmetric matrices P and Q satisfying:

$$\begin{aligned} A_c^T P + PA_c &= -Q \\ PB &= C^T \end{aligned} \tag{4.3.9}$$

Set a positive definite function

$$V(t) = \boldsymbol{e}_x(t)^T P \boldsymbol{e}_x(t) + \mathrm{tr}\{\Delta K_I(t) \Gamma_I^{-1} \Delta K_I(t)^T\} + \sum_{i=1}^{m} \boldsymbol{\zeta}_{\beta Ii}(t)^T \Gamma_{\beta Ii}^{-1} \boldsymbol{\zeta}_{\beta Ii}(t) \tag{4.3.10}$$

where

$$\begin{aligned} \Delta K_I(t) &= K_I(t) - K^*, \ \boldsymbol{\zeta}_{\beta Ii}(t) = \boldsymbol{\beta}_{Ii}(t) - \boldsymbol{\beta}_i^* \\ \boldsymbol{\beta}_i^* &= [\rho_{0i} + s_i^*, \rho_{1i}]^T, \ s_i^* = \max_t |s_i(t)| \\ S_{23}(t) &= [s_1(t), \cdots, s_m(t)]^T \end{aligned}$$

From (4.3.8)~(4.3.10), we have

$$\begin{aligned} \frac{dV(t)}{dt} = & -\boldsymbol{e}_x(t)^T Q \boldsymbol{e}_x(t) + 2\boldsymbol{u}_c(t)^T \boldsymbol{e}_y(t) + 2\boldsymbol{g}(t, \overline{\boldsymbol{y}})^T \boldsymbol{e}_y(t) \\ & + 2\boldsymbol{g}(t, \overline{\boldsymbol{y}})^T (B_1 - B)^T P \boldsymbol{e}_x(t) \\ & + \mathrm{tr}\left\{\Delta \dot{K}_I(t) \Gamma_I^{-1} \Delta K_I(t)^T + \Delta K_I(t) \Gamma_I^{-1} \Delta \dot{K}_I(t)^T\right\} \\ & + 2\sum_{i=1}^{m} \boldsymbol{\zeta}_{\beta Ii}(t)^T \Gamma_{\beta Ii}^{-1} \dot{\boldsymbol{\zeta}}_{\beta Ii}(t) \end{aligned} \tag{4.3.11}$$

It follows from (4.3.5) that

$$\begin{aligned} & \mathrm{tr}\left\{\Delta \dot{K}_I(t) \Gamma_I^{-1} \Delta K_I(t)^T + \Delta K_I(t) \Gamma_I^{-1} \Delta \dot{K}_I(t)^T\right\} \\ & \leq -2\boldsymbol{e}_y(t)^T \Delta K(t) \boldsymbol{z}(t) - 2\sigma_I(t) \mathrm{tr}\left\{\Delta K_I(t) \Gamma_I^{-1} \Delta K_I(t)^T\right\} \\ & \quad -2\sigma_I(t) \mathrm{tr}\left\{\Delta K_I(t) \Gamma_I^{-1} K^{*T}\right\} \end{aligned} \tag{4.3.12}$$

and from (4.3.7) that

$$\begin{aligned}
&\sum_{i=1}^{m} \zeta_{\beta Ii}(t)^T \Gamma_{\beta Ii}^{-1} \dot{\zeta}_{\beta Ii}(t) \\
&\quad \le \; -\sum_{i=1}^{m} \sigma_{\beta i}(t) \boldsymbol{\zeta}_{\beta Ii}(t)^T \Gamma_{\beta Ii}^{-1} \zeta_{\beta Ii}(t) - \sum_{i=1}^{m} \sigma_{\beta i}(t) \zeta_{\beta i}(t)^T \Gamma_{\beta Ii}^{-1} \boldsymbol{\beta}_i^* \\
&\qquad + \sum_{i=1}^{m} [\boldsymbol{\beta}_i(t) - \boldsymbol{\beta}_i^*]^T \boldsymbol{z}_\beta(t) |e_{yi}(t)| \qquad (4.3.13)
\end{aligned}$$

Then we have from (4.3.11)~(4.3.13) that

$$\begin{aligned}
\frac{dV(t)}{dt} \le\; & -\boldsymbol{e}_x(t)^T Q \boldsymbol{e}_x(t) + 2\boldsymbol{u}_R(t)^T \boldsymbol{e}_y(t) - 2\sigma_I(t) \mathrm{tr}\left\{\Delta K_I(t) \Gamma_I^{-1} \Delta K_I(t)^T\right\} \\
& -2\sigma_I(t) \mathrm{tr}\left\{\Delta K_I(t) \Gamma_I^{-1} K^{*T}\right\} \\
& -2\sum_{i=1}^{m} \sigma_{\beta i}(t) \boldsymbol{\zeta}_{\beta Ii}(t)^T \Gamma_{\beta Ii}^{-1} \boldsymbol{\zeta}_{\beta i}(t) \\
& -2\sum_{i=1}^{m} \sigma_{\beta i}(t) \boldsymbol{\zeta}_{\beta Ii}(t)^T \Gamma_{\beta Ii}^{-1} \boldsymbol{\beta}_i^* \\
& +2\sum_{i=1}^{m} \boldsymbol{\beta}_i^{*T} \boldsymbol{z}_\beta(t) |e_{yi}(t)| + 2\sum_{i=1}^{m} [\boldsymbol{\beta}_i(t) - \boldsymbol{\beta}_i^*]^T \boldsymbol{z}_\beta(t) |e_{yi}(t)| \\
& +2\boldsymbol{g}(t, \overline{\boldsymbol{y}})^T (B_1 - B)^T P \boldsymbol{e}_x(t) \qquad (4.3.14)
\end{aligned}$$

From (4.3.7), if $|\boldsymbol{\beta}_i(t)^T \boldsymbol{z}_\beta(t) e_{yi}(t)| > \varepsilon_i$ for all i, then we obtain

$$\begin{aligned}
\frac{dV(t)}{dt} \le\; & -\boldsymbol{e}_x(t)^T Q \boldsymbol{e}_x(t) - 2\sigma_I(t) \mathrm{tr}\left\{\Delta K_I(t) \Gamma_I^{-1} \Delta K_I(t)^T\right\} \\
& -2\sigma_I(t) \mathrm{tr}\left\{\Delta K_I(t) \Gamma_I^{-1} K^{*T}\right\} \\
& -2\sum_{i=1}^{m} \sigma_{\beta i}(t) \boldsymbol{\zeta}_{\beta Ii}(t)^T \Gamma_{\beta Ii}^{-1} \boldsymbol{\zeta}_{\beta i}(t) \\
& -2\sum_{i=1}^{m} \sigma_{\beta i}(t) \boldsymbol{\zeta}_{\beta Ii}(t)^T \Gamma_{\beta Ii}^{-1} \boldsymbol{\beta}_i^* \\
& +2\boldsymbol{g}(t, \overline{\boldsymbol{y}})^T (B_1 - B)^T P \boldsymbol{e}_x(t) = V_1(t) \qquad (4.3.15)
\end{aligned}$$

and if $|\boldsymbol{\beta}_i(t)^T \boldsymbol{z}_\beta(t) e_{yi}(t)| > \varepsilon_i$ for $i \in L$ and $|\boldsymbol{\beta}_i(t)^T \boldsymbol{z}_\beta e_{yi}(t)| \le \varepsilon_i$ for $i \in N$, where $L + N \subset M = \{1, 2, \ldots, m\}$, then we have

$$\frac{dV(t)}{dt} \le V_1(t) + 2\sum_{i \in N} \varepsilon_i \qquad (4.3.16)$$

Further, if $|\boldsymbol{\beta}_i(t)^T \boldsymbol{z}_\beta(t) e_{yi}(t)| \le \varepsilon_i$ for all i, then

$$\frac{dV(t)}{dt} \le V_1(t) + 2\sum_{i=1}^{m} \varepsilon_i \tag{4.3.17}$$

holds. Moreover, from (4.3.1c) we have

$$\|\overline{\boldsymbol{y}}(t)\| \le \|H\|\|\boldsymbol{e}_x(t)\| + \|H\boldsymbol{x}^*(t)\| \le \rho_3\|\boldsymbol{e}_x(t)\| + \rho_4 \tag{4.3.18}$$

where $\rho_3 = \|H\|$ and $\rho_4 = \max_t \|H\boldsymbol{x}^*(t)\|$. It follows from Assumption 4.2(b) and (4.3.18) that

$$\begin{aligned}
& g(t, \overline{\boldsymbol{y}})^T (B_1 - B)^T P \boldsymbol{e}_x(t) \\
& \le \|g(t, \overline{\boldsymbol{y}\boldsymbol{y}})\|\|B_1 - B\|\lambda_{max}[P]\|\boldsymbol{e}_x(t)\| \\
& \le \lambda_{max}[P]\|B_1 - B\| \left\{ \sum_{i=1}^{m} (\rho_{0i} + \rho_{1i}\|\overline{\boldsymbol{y}}(t)\|) \right\} \|\boldsymbol{e}_x(t)\| \\
& \le \rho_A \|\boldsymbol{e}_x(t)\|^2 + \rho_B \|\boldsymbol{e}_x(t)\|
\end{aligned} \tag{4.3.19}$$

where

$$\begin{aligned}
\rho_A &= \lambda_{max}[P]\|B_1 - B\|\rho_3 \sum_{i=1}^{m} \rho_{1i} \\
\rho_B &= \lambda_{max}[P]\|B_1 - B\| \left(\sum_{i=1}^{m} \rho_{0i} + \rho_4 \sum_{i=1}^{m} \rho_{1i} \right)
\end{aligned}$$

Finally, from (4.3.17) and (4.3.19) we can obtain

$$\begin{aligned}
\frac{dV(t)}{dt} \le & -(\lambda_{min}[Q] - 2\rho_A)\|\boldsymbol{e}_x(t)\|^2 + 2\rho_B\|\boldsymbol{e}_x(t)\| \\
& -2\sigma_I(t)\lambda_{min}[\Gamma_I^{-1}] \left\{ \sum_{i=1}^{m} \|\Delta \boldsymbol{k}_{Ii}(t)\|^2 \right\} \\
& +2\sigma_I(t)\lambda_{max}[\Gamma_I^{-1}] \left\{ \sum_{i=1}^{m} \|\boldsymbol{k}_i^*\|\|\Delta \boldsymbol{k}_{Ii}(t)\| \right\} \\
& -2\sum_{i=1}^{m} \sigma_{\beta i}(t)\lambda_{min}[\Gamma_{\beta Ii}^{-1}]\|\zeta_{\beta Ii}(t)\|^2 \zeta_{\beta i}(t) \\
& +2\sum_{i=1}^{m} \lambda_{max}[\Gamma_{\beta Ii}^{-1}]\sigma_{\beta i}(t)\|\boldsymbol{\beta}_i^*\|\|\zeta_{\beta Ii}(t)\| \\
& +\sum_{i=1}^{m} \varepsilon_i
\end{aligned} \tag{4.3.20}$$

where vectors $\Delta \boldsymbol{k}_{Ii}(t)$ and $\boldsymbol{k}_i^*$ denote the ith row of matrices $\Delta K_I(t)$ and K^*, respectively. Thus, if $\lambda_{min}[Q] - 2\rho_A > 0$ holds, then all the signals in the control system are uniformly ultimately bounded.

From the above conclusion, we have the following theorem with regard to the stability of the MIMO robust SAC system.

Theorem 4.2: *Suppose that Assumption* 4.2 *holds. Further suppose that the following relation is satisfied.*

$$\frac{\lambda_{min}[Q]}{2\lambda_{max}[P]\left(\sum_{i=1}^{m}\rho_{1i}\right)\|H\|} > \|B_1 - B\|$$

Then, all the signals in the control system (4.3.1) *with the control input* (4.3.4) *are uniformly ultimately bounded.*

4.4 Numerical Simulations

In this section the effectiveness of the proposed methods with robust adaptive control term is confirmed through numerical simulations.

4.4.1 Simulation Results for Single-Input Single-Output Plants

The simulations were generated using a second-order non-ASPR plant with a state dependent disturbance, described by

$$y(t) = G(s)[u(t) + g(t,y)], \quad G(s) = \frac{30}{s^2+50} \tag{4.4.1a}$$

$$g(t,y) = 20y(t)\sin 10\pi t + 10\sin\{5y(t)\} + 5\sin 20\pi t + 1 \tag{4.4.1b}$$

and a first-order model:

$$y_m(t) = G_m(s)[u_m(t)], \quad G_m(s) = \frac{2}{s+2} \tag{4.4.2}$$

In (4.4.1a), G(s) is non-ASPR with relative degree of 2. Therefore we used the following PFC.

$$F(s) = \frac{0.01}{s+10} \tag{4.4.3}$$

Design parameters of adaptive control laws (4.2.6) $\sim$ (4.2.8) were set to be

$$\Gamma_I = \Gamma_P = \Gamma_{\beta I} = \Gamma_{\beta_P} = 10^3 I_3, \quad \sigma_1 = \sigma_{\beta 1} = 0.01$$
$$\sigma_2 = \sigma_{\beta 2} = 0.001, \quad \varepsilon = 0.01, \boldsymbol{k}(0) = 0, \quad \boldsymbol{\beta}(0) = 0$$

The simulation result using a regular SAC algorithm is shown in Figure 4.1. In this case, as shown in Figure 4.2, the introduction of the robust adaptive control term $u_r(t)$ significantly improved the tracking performance.

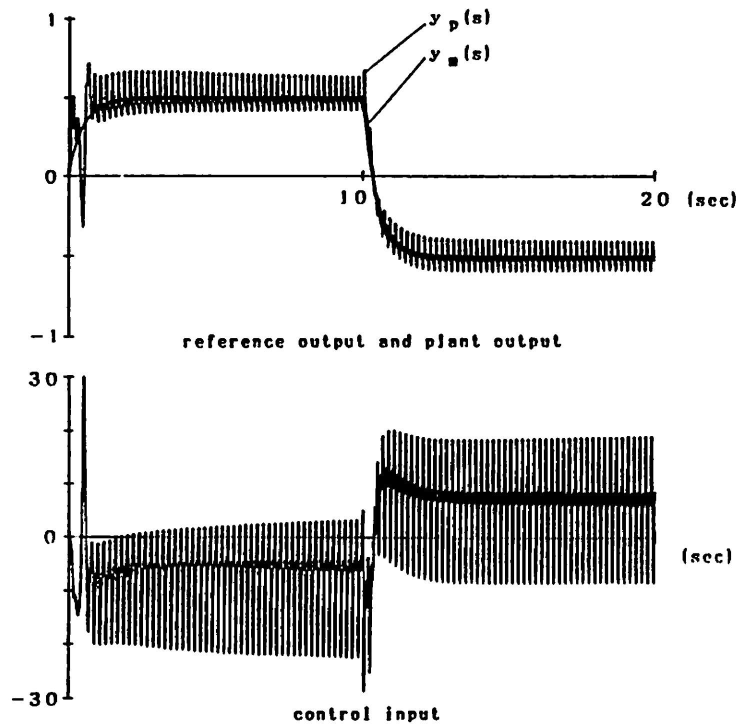


Figure 4.1 Simulation results: the use of the SAC algorithm.

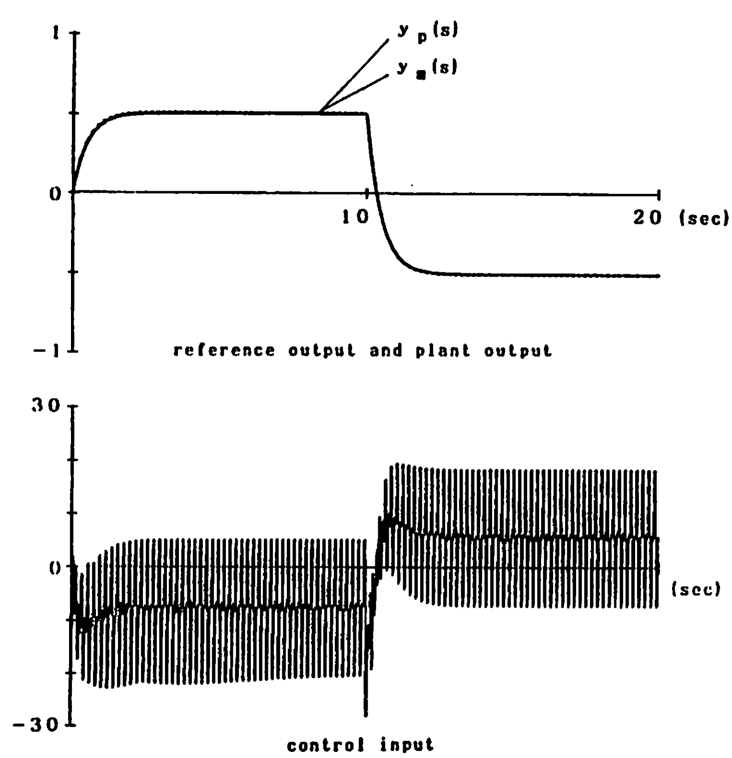


Figure 4.2 Simulation results: the use of the robust SAC algorithm.

4.4.2 Simulation Results for Multi-Input Multi-Output plants

Let us consider the following 4th order 2 input-output ASPR plant.

$$\dot{\boldsymbol{x}}(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \boldsymbol{x}(t) + \begin{bmatrix} -4 & -1 \\ 1 & 0 \\ 4 & -8 \\ 0 & 1 \end{bmatrix} (\boldsymbol{u}(t) + \overline{\boldsymbol{y}}(t) \sin \pi t) \tag{4.4.4a}$$

$$\boldsymbol{y}(t) = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 1 & -3 & 1 & 10 \end{bmatrix} \boldsymbol{x}(t) \tag{4.4.4b}$$

$$\overline{\boldsymbol{y}}(t) = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \boldsymbol{x}(t) \tag{4.4.4c}$$

The reference model which the plant is required to follow was given by

$$\boldsymbol{y}_m(t) = G_m(t)[\boldsymbol{u}_m(t)] \tag{4.4.5}$$

$$G_m(s) = \text{diag}\left[\frac{1}{s+1}, \ \frac{1}{s+1}\right]$$

$\boldsymbol{u}_m(t) = [u_{m1}(t), \ u_{m2}(t)]$

$u_{m1}(t)$: a rectangular wave of amplitude 1

$u_{m2}(t)$: a rectangular wave of amplitude 2

Further, the design parameters of the adaptive adjusting laws (4.3.5) $\sim$ (4.3.7) were set to be

$\Gamma_I = \text{diag}[10^5 I_2, \ 10^3 I_4], \ \ \Gamma_P = \text{diag}[10^4 I_2, \ 10^2 I_4]$

$\Gamma_{\beta Ii} = \text{diag}[450, \ 450], \ \ \Gamma_{\beta Pi} = \text{diag}[20, \ 20], \ i = 1, 2$

$\sigma_1 = 0.1, \ \sigma_2 = 0.05, \ \sigma_{\beta 1i} = 2.0, \ \sigma_{\beta 2i} = 0.8, \ \varepsilon_i = 0.1, \ i = 1, 2$

Simulation results are shown in Figures 4.3 and 4.4. Figure 4.3 is the result with the regular SAC algorithm and Figure 4.4 is the result with the robust SAC algorithm. It is apparent that the use of robust SAC algorithm effectively reduced the influence of the disturbance.

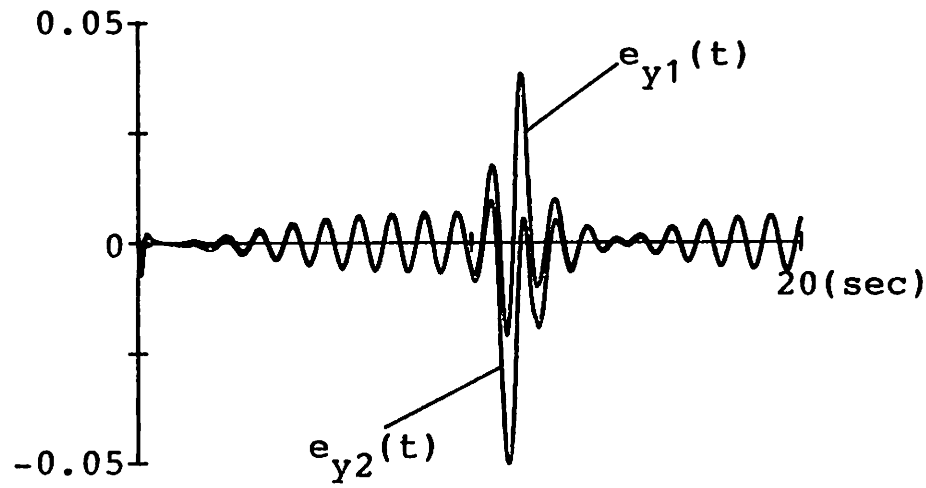


Figure 4.3 Tracking errors with the SAC algorithm.

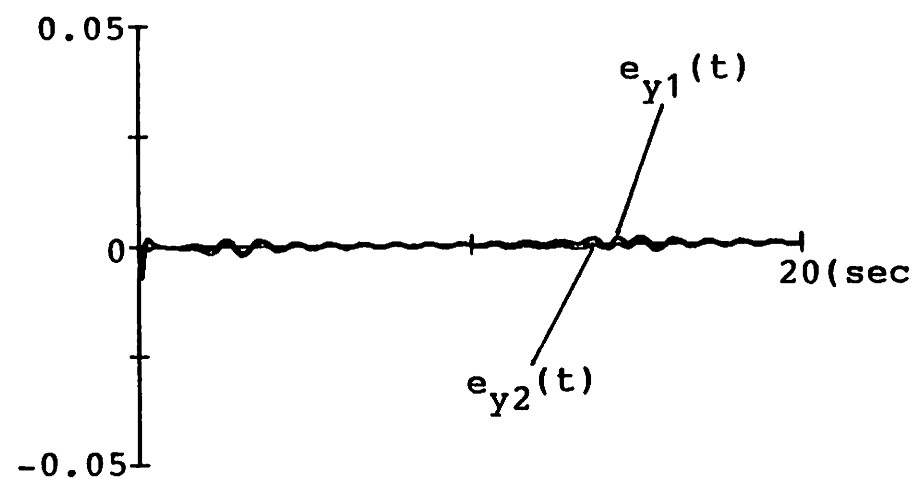


Figure 4.4 Tracking errors with the robust SAC algorithm.

4.5 Conclusions

The SAC has a great robustness with regard to disturbances and parasitics as shown in the preceding section. However, in the case where *large* external disturbances and/or state-dependent disturbances are present, the influence of disturbances is evident.

In this chapter, a robust SAC algorithm with robust adaptive control term which is aimed at positively eliminating the influence of disturbances was considered for both SISO and MIMO plants with state-dependent disturbances. If the order of the disturbance can be evaluated by a measurable state vector, then the influence of the disturbance can be reduced effectively by using the proposed method with estimated values of the upper bounds of disturbances. The effectiveness of the proposed method was confirmed through numerical simulations for both SISO and MIMO plants.

5 Decentralized Simple Adaptive Control

5.1 Introduction

As we have shown in the previous sections, the SAC scheme can easily be applied to MIMO plants. However, controlling the large-scale system might be difficult due to an interconnection of subsystems with unknown parameters, nonlinearities and disturbances. Decentralized control methods (Sandell *et al.* 1978, Huseyin *et al.* 1982, Ikeda *et al.* 1983, Chen *et al.* 1988, 1991), especially decentralized adaptive control methods, are an effective way to handle the large-scale system.

The problem of designing decentralized adaptive control systems for large-scale systems has been considered by many researchers in the last few years, including Hmamed and Radouane (1983), Ioannou (1986) and Gavel and Siljak (1989). Hmamed and Radouane have discussed adaptive feedback stabilization of large-scale interconnected systems. Ioannou developed decentralized adaptive control systems for a class of large-scale systems formed from arbitrary interconnections of subsystems with unknown parameters, nonlinearities and bounded disturbances and proved the stability of the control system by using the *M-matrix* condition. Gavel and Siljak have proposed a decentralized adaptive control method provided that certain structural constraints called the *range condition* are satisfied. The satisfaction of either the M-matrix condition or the range condition seems to play a fundamental role when we consider the stability of the decentralized adaptive control system.

In this chapter, the decentralized SAC method will be considered. It is verified that if *(i)* each subsystem satisfies either the M-matrix condition or the range condition and *(ii)* the fundamental linear part of each subsystem satisfies regular design conditions of the SAC, then the design procedure of the SAC can be applied to each subsystem. Further, it is shown that the implementation of the robust SAC controller significantly improves the control performance of decentralized SAC systems when the interconnection outputs among each subsystem are available as measurement signals.

Yousef *et al.* (1989) discussed a similar decentralized adaptive control method which will be presented in this section. However, their approach is valid only for

ASPR plants.

5.2 Decentralized Simple Adaptive Control

We first consider the case where the large-scale system can be divided into SISO subsystems with arbitrary interconnections from each subsystem.

5.2.1 Problem Setup

Consider an interconnected LTI system composed of N subsystems described by

$$S_i: \quad \dot{\boldsymbol{x}}_i(t) = A_i\boldsymbol{x}_i(t) + \boldsymbol{b}_i u_i(t) + P_i\boldsymbol{v}_i(t) + \boldsymbol{g}_i(t) \tag{5.2.1a}$$

$$y_i(t) = \boldsymbol{c}_i^T\boldsymbol{x}_i(t) \tag{5.2.1b}$$

$$\boldsymbol{w}_i(t) = Q_i\boldsymbol{x}_i(t), \quad i \in N, \; N = \{1, \ldots, N\} \tag{5.2.1c}$$

where $\boldsymbol{x}_i \in R^{n_i}$, $u_i \in R^1$ and $y_i \in R^1$ are the state, the control input and the output of the subsystem S_i, respectively, and $\boldsymbol{v}_i \in R^{m_i}$, $\boldsymbol{w}_i \in R^{l_i}$ are the interconnection input and output vectors associated with S_i, respectively. Furthermore, $\boldsymbol{g}_i \in R^{n_i}$ is a bounded disturbance such that $\|\boldsymbol{g}_i(t)\| \leq g_i^*$, where g_i^* is an unknown but positive constant. $A_i, \boldsymbol{b}_i, \boldsymbol{c}_i, P_i$ and Q_i are matrices and vectors which have appropriate dimensions. It is assumed that the pair $(A_i, \boldsymbol{b}_i)$ is controllable and the pair $(A_i, \boldsymbol{c}_i)$ is observable. Further, we assume that interconnection inputs $\boldsymbol{v}_i$ satisfy the following relations.

$$\boldsymbol{v}_i(t) = \boldsymbol{f}_i(t, \boldsymbol{w}) \tag{5.2.2a}$$

$$\boldsymbol{w}(t) = [\boldsymbol{w}_1(t)^T, \cdots, \boldsymbol{w}_N(t)^T]^T$$

$$\|\boldsymbol{f}_i(t, \boldsymbol{w})\| \leq \sum_{j=1}^{N} \xi_{ij}\|\boldsymbol{w}_j(t)\| \tag{5.2.2b}$$

where ξ_{ij} are unknown but positive constants. The overall system S, which is composed of subsystems S_i, can then be expressed as

$$S: \quad \dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + B\boldsymbol{u}(t) + P\boldsymbol{v}(t) + \boldsymbol{g}(t) \tag{5.2.3a}$$

$$\boldsymbol{y}(t) = C\boldsymbol{x}(t) \tag{5.2.3b}$$

$$\boldsymbol{w}(t) = Q\boldsymbol{x}(t) \tag{5.2.3c}$$

where $\boldsymbol{x} = [\boldsymbol{x}_1^T, \cdots, \boldsymbol{x}_N^T]^T$, $\boldsymbol{y} = [y_1, \cdots, y_N]^T$, $\boldsymbol{u} = [u_1, \cdots, u_N]^T$, $\boldsymbol{v} = [\boldsymbol{v}_1^T, \cdots, \boldsymbol{v}_N^T]^T$ and $\boldsymbol{g} = [\boldsymbol{g}_1^T, \cdots, \boldsymbol{g}_N^T]^T$ and $A = \text{diag}[A_i]$, $B = \text{diag}[\boldsymbol{b}_i]$, $C = \text{diag}[\boldsymbol{c}_i^T]$, $P = \text{diag}[P_i]$

and $Q = \text{diag}[Q_i]$ are the constant block diagonal matrices with appropriate dimensions.

The stable reference models S_{M_i} for each subsystem S_i are given by

$$S_{Mi}: \quad \dot{\boldsymbol{x}}_{mi}(t) = A_{mi}\boldsymbol{x}_{mi}(t) + \boldsymbol{b}_{mi}u_{mi}(t) \tag{5.2.4a}$$

$$y_{mi}(t) = \boldsymbol{c}_{mi}^T\boldsymbol{x}_{mi}(t) \tag{5.2.4b}$$

where $\boldsymbol{x}_{mi} \in R^{n_{mi}}$, $u_{mi} \in R^1$ and $y_{mi} \in R^1$.

The control objective is to have the output of each subsystem S_i track the output of each reference model S_{Mi}.

5.2.2 Basic Design of Control System

Suppose that the subsystem S_i and the reference model S_{Mi} satisfy the following assumptions:

Assumption 5.1:

(1) The linear part of each subsystem $S_i, i \in N$ is ASPR. That is, there exists a constant gain k_{ei}^* such that the transfer function:

$$G_{ci}(s) = \boldsymbol{c}^T(sI - A_{ci})^{-1}\boldsymbol{b}_i \tag{5.2.5}$$

is SPR, where $A_{ci} = A_i + k_{ei}^*\boldsymbol{b}_i\boldsymbol{c}_i^T$.

(2) The matrix $P_i, i \in L, L \subset N$ can be factored as

$$P_i = \boldsymbol{b}_i\boldsymbol{p}_i^T, \quad i \in L \tag{5.2.6}$$

for some constant vector $\boldsymbol{p}_i \in R^{m_i}$. That is, subsystems $S_i, i \in L$ belong to a stabilizable class in the decentralized system whose coupling parameters are within the range of the control input.

(3) Each subsystem S_i and its reference model S_{Mi} satisfy Assumption 2.2 (3), (4). That is, Broussard's model output following condition is satisfied for each subsystem.

Under the above assumptions, the local control laws for each subsystem are given by the form of the SISO SAC as follows:

$$u_i(t) = \boldsymbol{k}_i(t)^T\boldsymbol{z}_i(t) \tag{5.2.7}$$

$$z_i(t) = [e_{yi}(t), x_{mi}(t)^T, u_{mi}(t)]^T$$
$$e_{yi}(t) = y_i(t) - y_{mi}(t)$$
$$k_i(t) = [k_{ei}(t), k_{xi}(t)^T, k_{ui}(t)]^T$$

The parameter vectors $k_i(t)$ are adjusted by

$$\begin{cases} k_i(t) = k_{Ii}(t) + k_{Pi}(t) \\ \dot{k}_{Ii}(t) = -\Gamma_{Ii} z_i(t) e_{yi}(t) - \sigma_i(t) k_{Ii}(t) \\ k_{Pi}(t) = -\Gamma_{Pi} z_i(t) e_{yi}(t) \\ \sigma_i(t) = \sigma_{1i} \frac{e_{yi}(t)^2}{1+e_{yi}(t)^2} + \sigma_{2i} \end{cases} \tag{5.2.8}$$

where

$$\Gamma_{Ii} = \Gamma_{Ii}^T > 0, \ \Gamma_{Pi} = \Gamma_{Pi}^T > 0, \ \sigma_{1i}, \sigma_{2i} > 0$$

5.2.3 Stability of the Control System

Suppose that disturbances and the inter-connections do not exist in the plant (5.2.1). Then, under Assumption 5.1 (3), the perfect output following is attained for each subsystem. That is, we have

$$\dot{x}_i^*(t) = A_i x_i^*(t) + b_i u_i^*(t) \tag{5.2.9a}$$
$$y_i^*(t) = y_{mi}(t) = c_i^T x_i^*(t) \tag{5.2.9b}$$
$$u_i^*(t) = S_{1i} x_{mi}(t) + S_{2i} u_{mi}(t) + S_{3i}(t), \quad i \in N \tag{5.2.9c}$$

where x_i^*, y_i^* and u_i^* are the optimal state, the optimal output and the optimal control input, respectively. $S_{ji}, j = 1, 2$ are the appropriate dimensional vector and scalar, respectively, and $S_{3i}(t)$ is a bounded scalar function (See Section 2.2 or 2.3). Define error signals between the real states and the ideal states by

$$\dot{e}_{xi}(t) = x_i(t) - x_i^*(t) \tag{5.2.10a}$$
$$e_{yi}(t) = y_i(t) - y_i^*(t) = y_i(t) - y_{mi}(t) \tag{5.2.10b}$$

From (5.2.1) and (5.2.9), we have the following error system:

$$\dot{e}_{xi}(t) = A_{ci} e_{xi}(t) + b_i \zeta_i(t) z_i(t) - b_i S_{3i} + P_i v_i(t) + g_i(t) \tag{5.2.11a}$$
$$e_{yi}(t) = c_i^T e_{xi}(t) \tag{5.2.11b}$$

where

$$\boldsymbol{\zeta}_i(t) = \boldsymbol{k}_i(t) - \boldsymbol{k}_i^*, \quad \boldsymbol{k}_i^* = [k_{ei}^*, S_{1i}, S_{2i}]^T \tag{5.2.12}$$

Here, from Assumption 5.1(1), there exist positive definite matrices H_i and G_i satisfying the Kalman-Yakubovich Lemma:

$$\begin{aligned} A_{ci}^T H_i + H_i A_{ci} &= -G_i \\ H_i \boldsymbol{b}_i &= \boldsymbol{c}_i \end{aligned} \tag{5.2.13}$$

Let us consider the following positive definite function:

$$V(t) = \sum_{i=1}^{N} V_i(t) \tag{5.2.14a}$$

$$V_i(t) = V_{1i}(t) + V_{2i}(t) \tag{5.2.14b}$$

$$V_{1i}(t) = \boldsymbol{e}_{xi}(t)^T H_i \boldsymbol{e}_{xi}(t), \quad i \in N \tag{5.2.14c}$$

$$V_{2i}(t) = \left\{ \begin{array}{ll} \boldsymbol{\zeta}_{Ii}(t)^T \Gamma_{Ii}^{-1} \boldsymbol{\zeta}_{Ii}(t), & i \in K = N - L \\ (\boldsymbol{\zeta}_{Ii}(t) + \rho\gamma)^T \Gamma_{Ii}^{-1} (\boldsymbol{\zeta}_{Ii}(t) + \rho\gamma), & i \in L \end{array} \right. \tag{5.2.14d}$$

$$\boldsymbol{\zeta}_{Ii}(t) = \boldsymbol{k}_{Ii}(t) - \boldsymbol{k}_i^*, \ \gamma = [1, 0, \cdots, 0]^T, \ \rho > 0$$

From (5.2.11) and (5.2.13), we have

$$\begin{aligned} \frac{dV_{1i}(t)}{dt} = & -\boldsymbol{e}_{xi}(t)^T G_i \boldsymbol{e}_{xi}(t) + 2\boldsymbol{\zeta}_i(t)^T \boldsymbol{z}_i(t) e_{yi}(t) \\ & +2\boldsymbol{v}_i(t) P_i^T H_i \boldsymbol{e}_{xi}(t) \\ & +2(-\boldsymbol{b}_i S_{3i}(t) + \boldsymbol{g}_i(t))^T H_i \boldsymbol{e}_{xi}(t) \end{aligned} \tag{5.2.15}$$

Furthermore, from the parameter adjusting law (5.2.8), if $i \in K$, then we obtain

$$\begin{aligned} \frac{V_{2i}(t)}{dt} \le & -2\sigma_{2i} \boldsymbol{\zeta}_{Ii}(t)^T \Gamma_{Ii}^{-1} \boldsymbol{\zeta}_{Ii}(t) - 2\sigma_{2i} \boldsymbol{\zeta}_{Ii}(t) \Gamma_{Ii}^{-1} \boldsymbol{k}_i^* \\ & -2\boldsymbol{\zeta}_i(t)^T \boldsymbol{z}_i(t) e_{yi}(t) \end{aligned} \tag{5.2.16}$$

and if $i \in L$, we obtain

$$\begin{aligned} \frac{V_{2i}(t)}{dt} \le & -2\sigma_{2i} \boldsymbol{\zeta}_{Ii}(t)^T \Gamma_{Ii}^{-1} \boldsymbol{\zeta}_{Ii}(t) - 2\sigma_{2i} \boldsymbol{\zeta}_{Ii}(t) \Gamma_{Ii}^{-1} \boldsymbol{k}_i^* \\ & -2\boldsymbol{\zeta}_i(t)^T \boldsymbol{z}_i(t) e_{yi}(t) - 2\rho_{yi}(t)^2 \\ & -2\sigma_{2i} \rho \gamma^T \Gamma_{Ii}^{-1} \boldsymbol{\zeta}_{Ii}(t) - 2\sigma_{2i} \rho \gamma^T \Gamma_{Ii}^{-1} \boldsymbol{k}_i^* \end{aligned}$$

$$
\begin{aligned}
&\leq -2\boldsymbol{\zeta}_i(t)^T \boldsymbol{z}_i(t) e_{yi}(t) - 2\rho_{yi}(t)^2 \\
&\quad -\sigma_{2i}(\boldsymbol{\zeta}_{Ii}(t) + \rho\gamma)^T \gamma_{Ii}^{-1}(\boldsymbol{\zeta}_{Ii}(t) + \rho\gamma) \\
&\quad +\sigma_{2i}(\boldsymbol{k}_i^*(t) - \rho\gamma)^T \gamma_{Ii}^{-1}(\boldsymbol{k}_i^*(t) - \rho\gamma)
\end{aligned} \tag{5.2.17}
$$

Here, it follows from (5.2.1c) that

$$
\|\boldsymbol{w}_j(t)\| \leq \|Q_j\|\|\boldsymbol{e}_{xj}(t)\| + \|Q_j\|\|\boldsymbol{x}_j^*\| \tag{5.2.18}
$$

and from (5.2.2) that

$$
\begin{aligned}
&|\boldsymbol{v}_i(t)^T P_i^T H_i \boldsymbol{e}_{xi}(t)| \\
&\leq \sum_{j=1}^{N} (\|P_i\|\|H_i\|\xi_{ij}\|Q_j\|\|\boldsymbol{e}_{xj}(t)\|)\|\boldsymbol{e}_{xi}\| \\
&\quad + \sum_{j=1}^{N} (\|P_i\|\|H_i\|\xi_{ij}\|Q_j\|\|\boldsymbol{x}_j^*(t)\|)\|\boldsymbol{e}_{xi}\|
\end{aligned} \tag{5.2.19}
$$

Thus, for $i \in K$, we have

$$
\begin{aligned}
\frac{dV_i(t)}{dt} &\leq -\lambda_{min}[G_i]\|\boldsymbol{e}_{xi}(t)\|^2 + 2\sum_{j=1}^{N} R_{ij}\|\boldsymbol{e}_{xj}(t)\|\|\boldsymbol{e}_{xi}(t)\| \\
&\quad +2\sum_{j=1}^{N} R_{ij}\|\boldsymbol{x}_j^*(t)\|\|\boldsymbol{e}_{xi}(t)\| \\
&\quad +2\|\boldsymbol{b}_i S_{3i}(t) - \boldsymbol{g}_i(t)\|\|Hi\|\|\boldsymbol{e}_{xi}(t)\| \\
&\quad -2\sigma_{2i}\lambda_{min}[\Gamma_{Ii}^{-1}]\|\boldsymbol{\zeta}_{Ii}(t)\|^2 \\
&\quad +2\sigma_{2i}\lambda_{max}[\Gamma_{Ii}^{-1}]\|\boldsymbol{k}_i^*\|\|\boldsymbol{\zeta}_{Ii}(t)\|
\end{aligned} \tag{5.2.20}
$$

where

$$
R_{ij} = \|P_i\|\|H_i\|\xi_{ij}\|Q_i\|\ ,\quad i \in k,\ j \in N \tag{5.2.21}
$$

Further, since it follows from (5.2.2) and (5.2.18) that

$$
\begin{aligned}
&\boldsymbol{v}_i(t)^T \boldsymbol{p}_i \boldsymbol{p}_i^T \boldsymbol{v}_i(t) \\
&\leq \lambda_{max}[\boldsymbol{p}_i \boldsymbol{p}_i^T]\boldsymbol{v}_i(t)^T \boldsymbol{v}_i(t) \\
&\leq \lambda max[\boldsymbol{p}_i \boldsymbol{p}_i^T]\lambda_{max}[\boldsymbol{\xi}_i \boldsymbol{\xi}_i^T] \\
&\qquad \times \sum_{j=1}^{N} \|Q_j\|^2(\|\boldsymbol{e}_{xi}(t)\|^2 + 2\|\boldsymbol{x}_j^*(t)\|\|\boldsymbol{e}_{xi}(t)\| + \|\boldsymbol{x}_j^*(t)\|^2)
\end{aligned} \tag{5.2.22}
$$

where $\boldsymbol{\xi}_i = [\xi_{i1}, \xi_{i2}, \ldots, \xi_{iN}]^T$, we obtain

$$
\begin{aligned}
&2\boldsymbol{v}_i(t)^T\boldsymbol{p}_i e_{yi}(t)\\
&= -2\left(\sqrt{\rho}e_{yi}(t) - \frac{1}{2}(\sqrt{\rho})^{-1}\boldsymbol{v}_i(t)^T\boldsymbol{p}_i\right)^2 + \frac{1}{2}\rho^{-1}\boldsymbol{v}_i(t)^T\boldsymbol{p}_i\boldsymbol{p}_i^T\boldsymbol{v}_i(t)\\
&\leq \frac{1}{2}\rho^{-1}\lambda_{pi}\lambda_{\xi i}\sum_{j=1}^{N}\|Q_j\|^2(\|\boldsymbol{e}_{xi}(t)\|^2 + 2\|\boldsymbol{x}_j^*(t)\|\|\boldsymbol{e}_{xi}(t)\| + \|\boldsymbol{x}_j^*(t)\|^2)
\end{aligned} \tag{5.2.23}
$$

where $\lambda_{pi} = \lambda_{max}[\boldsymbol{p}_i\boldsymbol{p}_i^T]$ and $\lambda_{\xi i} = \lambda_{max}[\boldsymbol{\xi}_i\boldsymbol{\xi}_i^T]$. Thus, from Assumption 5.1(2) and (5.2.23), for $i \in L$, we have

$$
\begin{aligned}
\frac{dV_i(t)}{dt} \leq & -\lambda_{min}[G_i]\|\boldsymbol{e}_{xi}(t)\|^2 + \frac{1}{2}\rho^{-1}\lambda_{pi}\lambda_{\xi i}\sum_{j=1}^{N}\|Q_j\|^2\|\boldsymbol{e}_{xj}(t)\|^2\\
& + 2\|\boldsymbol{b}_i S_{3i}(t) - \boldsymbol{g}_i(t)\|\|Hi\|\|\boldsymbol{e}_{xi}(t)\|\\
& + \rho^{-1}\lambda_{pi}\lambda_{\xi i}\sum_{j=1}^{N}\|Q_j\|^2\|\boldsymbol{x}_j^*(t)\|\|\boldsymbol{e}_{xj}(t)\|\\
& + \frac{1}{2}\rho^{-1}\lambda_{pi}\lambda_{\xi i}\sum_{j=1}^{N}\|Q_j\|^2\|\boldsymbol{x}_j^*(t)\|^2\\
& - \sigma_{2i}\lambda_{min}[\Gamma_{Ii}^{-1}]\|\boldsymbol{\zeta}_{Ii}(t) + \rho\gamma\|^2\\
& + \sigma_{2i}\lambda_{max}[\Gamma_{Ii}^{-1}]\|\boldsymbol{k}_i^* - \rho\gamma\|
\end{aligned} \tag{5.2.24}
$$

From (5.2.20) and (5.2.24), we can obtain

$$
\begin{aligned}
\frac{dV(t)}{dt} &= \sum_{i=1}^{N}\frac{dV_i(t)}{dt}\\
&\leq -\sum_{i=1}^{N}\lambda_{min}[G_i]\|\boldsymbol{e}_{xi}(t)\|^2\\
&\quad + 2\sum_{i\in K}\sum_{j=1}^{N}R_{ij}\|\boldsymbol{e}_{xi}(t)\|\|\boldsymbol{e}_{xj}(t)\|\\
&\quad + 2\sum_{i\in K}\sum_{j=1}^{N}R_{ij}\|\boldsymbol{x}_j^*(t)\|\|\boldsymbol{e}_{xi}(t)\|\\
&\quad + 2\sum_{i=1}^{N}\|\boldsymbol{b}_i S_{3i}(t) - \boldsymbol{g}_i(t)\|\|H_i\|\|\boldsymbol{e}_{xi}(t)\|\\
&\quad + \frac{1}{2}\sum_{i\in L}\rho^{-1}\lambda_{pi}\lambda_{\xi i}\sum_{j=1}^{N}\|Q_j\|^2\|\boldsymbol{e}_{xj}(t)\|^2\\
&\quad + \sum_{i\in L}\rho^{-1}\lambda_{pi}\lambda_{\xi i}\sum_{j=1}^{N}\|Q_j\|^2\|\boldsymbol{x}_j^*(t)\|\|\boldsymbol{e}_{xj}(t)\|
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{2}\sum_{i\in L}\rho^{-1}\lambda_{pi}\lambda_{\xi i}\sum_{j=1}^{N}\|Q_j\|^2\|\boldsymbol{x}_j^*(t)\|^2 \\
&-2\sum_{i\in K}\sigma_{2i}\lambda_{min}[\Gamma_{Ii}^{-1}]\|\boldsymbol{\zeta}_{Ii}(t)\|^2 \\
&-\sum_{i\in L}\sigma_{2i}\lambda_{min}[\Gamma_{Ii}^{-1}]\|\boldsymbol{\zeta}_{Ii}(t)+\rho\gamma\|^2 \\
&+2\sum_{i\in K}\sigma_{2i}\lambda_{max}[\Gamma_{Ii}^{-1}]\|\boldsymbol{k}_i^*\|\|\boldsymbol{\zeta}_{Ii}(t)\| \\
&+\sum_{i\in L}\sigma_{2i}\lambda_{max}[\Gamma_{Ii}^{-1}]\|\boldsymbol{k}_i^*-\rho\gamma\|^2
\end{aligned}
\qquad (5.2.25)
$$

Let us define the matrix $M=[m_{ij}]$ as follows:

$$
m_{ij}=\begin{cases}\lambda_{min}[G_i]-2R_{ij}, & i=j\in N\\ -(R_{ij}+R_{ji}), & i\neq j,\ i,j\in N\end{cases}
\qquad (5.2.26)
$$

where

$$
\begin{array}{ll}
R_{ij}=\|P_i\|\|H_i\|\xi_{ij}\|Q_j\|, & i\in K,\ j\in N\\
R_{ij}=0, & i\in L,\ j\in N
\end{array}
\qquad (5.2.27)
$$

Then, supposing that $M>0$ and taking into account that there exist constants such that $\|\boldsymbol{x}_i^*(t)\|\le x_i^*$, $|S_{3i}|\le S_{3i}^*$ and $\|\boldsymbol{g}_i(t)\|\le g_i^*$, we have from (5.2.25) that

$$
\begin{aligned}
\frac{dV(t)}{dt}\le\ &-\lambda_{min}[M]\sum_{i=1}^{N}\|\boldsymbol{e}_{xi}(t)\|^2 \\
&+\rho^{-1}\left(\sum_{j\in L}\frac{1}{2}\lambda_{pj}\lambda_{\xi j}\right)\max_{i\in N}\|Q_i\|^2\sum_{i=1}^{N}\|\boldsymbol{e}_{xi}(t)\|^2 \\
&+\sum_{j=1}^{N}R_{Bi}\|\boldsymbol{e}_{xi}(t)\| \\
&-2\sum_{i\in K}\sigma_{2i}\lambda_{min}[\Gamma_{Ii}^{-1}]\|\boldsymbol{\zeta}_{Ii}(t)\|^2 \\
&-\sum_{i\in L}\sigma_{2i}\lambda_{min}[\Gamma_{Ii}^{-1}]\|\boldsymbol{\zeta}_{Ii}(t)+\rho\gamma\|^2 \\
&+2\sum_{i\in K}\sigma_{2i}\lambda_{max}[\Gamma_{Ii}^{-1}]\|\boldsymbol{k}_i^*\|\|\boldsymbol{\zeta}_{Ii}(t)\| \\
&+g
\end{aligned}
\qquad (5.2.28)
$$

where

$$R_{Bi} = \begin{cases} 2\sum_{j=1}^{N} R_{ij}\boldsymbol{x}_j^* + \sum_{j\in L} \rho^{-1}\lambda_{pj}\lambda_{\xi j}\|Q_i\|^2\boldsymbol{x}_j^* \\ \quad +2(\|\boldsymbol{b}_i\|S_{3i}^* + g_i^*)\|H_i\|, \quad i \in K \\ \sum_{j\in L} \rho^{-1}\lambda_{pj}\lambda_{\xi j}\|Q_i\|^2\boldsymbol{x}_j^* \\ \quad +2(\|\boldsymbol{b}_i\|S_{3i}^* + g_i^*)\|H_i\|, \quad i \in L \end{cases} \tag{5.2.29}$$

and

$$\begin{aligned} g = &+\frac{1}{2}\rho^{-1}\sum_{i\in L}\lambda_{pi}\lambda_{\xi i}\sum_{j=1}^{N}\|Q_j\|^2\boldsymbol{x}_j^{*2} \\ &+\sum_{i\in L}\sigma_{2i}\lambda_{max}[\Gamma_{Ii}^{-1}]\|\boldsymbol{k}_i^* - \rho\gamma\|^2 \end{aligned} \tag{5.2.30}$$

Taking the parameter ρ in (5.2.28) as

$$\rho > \sum_{j\in L}\frac{1}{2}\lambda_{pj}\lambda_{\xi j}\max_{i\in N}\|Q_i\|^2/\lambda_{min}[M], \tag{5.2.31}$$

we can finally obtain

$$\begin{aligned} \frac{dV(t)}{dt} \le &-\lambda_m\sum_{i=1}^{N}\|\boldsymbol{e}_{xi}(t)\|^2 \\ &+\sum_{j=1}^{N}R_{Bi}\|\boldsymbol{e}_{xi}(t)\| \\ &-2\sum_{i\in K}\sigma_{2i}\lambda_{min}[\Gamma_{Ii}^{-1}]\|\boldsymbol{\zeta}_{Ii}(t)\|^2 \\ &-\sum_{i\in L}\sigma_{2i}\lambda_{min}[\Gamma_{Ii}^{-1}]\|\boldsymbol{\zeta}_{Ii}(t) + \rho\gamma\|^2 \\ &+2\sum_{i\in K}\sigma_{2i}\lambda_{max}[\Gamma_{Ii}^{-1}]\|\boldsymbol{k}_i^*\|\|\boldsymbol{\zeta}_{Ii}(t)\| \\ &+g \end{aligned} \tag{5.2.32}$$

where

$$\lambda_m = \lambda_{min}[M] - \rho^{-1}\left(\sum_{j\in L}\frac{1}{2}\lambda_{pj}\lambda_{\xi j}\right)\max_{i\in N}\|Q_i\|^2 > 0 \tag{5.2.33}$$

From the above inequality, we can conclude that dV/dt becomes negative definite for large values of $\|\boldsymbol{e}_{xi}(t)\|$ and $\|\boldsymbol{\zeta}_{Ii}(t)\|$, $i \in N$. Thus, it is apparent that $\boldsymbol{e}_{xi}(t)$ and $\boldsymbol{\zeta}_{Ii}(t)$ are uniformly ultimately bounded, and it can easily be verified from this result that all the signals in the control system are also uniformly ultimately bounded.

The above conclusion leads to the following theorem with regard to the basic stability of the decentralized SAC system.

Theorem 5.1: *Suppose that Assumption* 5.1 *holds and the matrix* $M = [m_{ij}]$ *defined by* (5.2.26) *and* (5.2.27) *is positive definite. Then, the use of the local inputs* (5.2.7) *guarantees the uniform ultimate boundedness of all the signals in the control system.*

Remark 5.1: A sufficient condition for the M-matrix defined by (5.2.26) and (5.2.27) to be positive definite is given by

$$\begin{cases} \lambda_{min}[G_i] > \sum_{j\in N} R_{ij} + \sum_{j\in K} R_{ji}, & i \in K \\ \lambda_{min}[G_i] > \sum_{j\in K} R_{ji}, & i \in L \end{cases} \tag{5.2.34}$$

(Ioanou 1986).

5.2.4 Control System with the Parallel Feedforward Compensator

In Theorem 5.1, we assume that all the subsystems are ASPR. However, as mentioned in the preceding section, the ASPR condition may be a severe restriction. As a countermeasure to this problem, we will introduce PFCs for non-ASPR subsystems.

Let the linear part $G_i(s)$ of subsystem S_i, $i \in J$, $J \subset N$ be non-ASPR. Consider the augmented transfer function with a PFC:

$$G_{ai}(s) = G_i(s) + F_i(s) \tag{5.2.35}$$

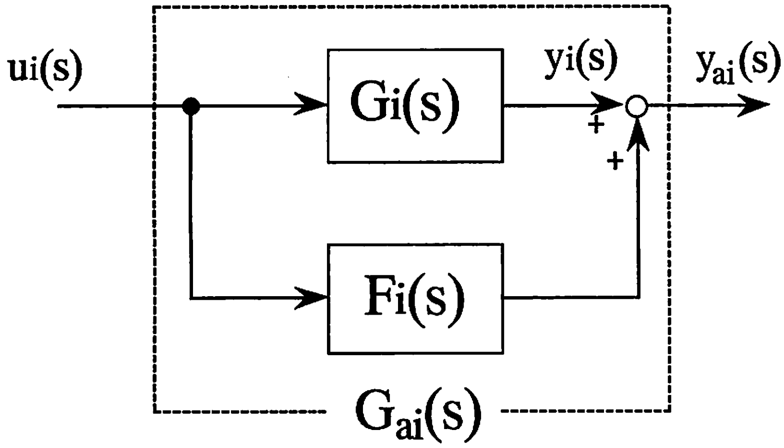


Figure 5.1 Augmented plant with PFC

as shown in Figure 5.1. $F_i(s)$ is chosen so that the resulting augmented transfer function $G_{ai}(s)$ is ASPR. Suppose that the above stated PFC $F_i(s)$ is given in the form of the state equation:

$$\dot{\boldsymbol{x}}_{fi}(t) = A_{fi}\boldsymbol{x}_{fi}(t) + \boldsymbol{b}_{fi}u_i(t) \tag{5.2.36a}$$

$$y_{fi}(t) = \boldsymbol{c}_{fi}^T\boldsymbol{x}_{fi}(t) \tag{5.2.36b}$$

Then the resulting augmented ASPR subsystem is expressed by

$$\overline{S}_i: \quad \dot{\overline{\boldsymbol{x}}}_i(t) = \overline{A}_i\overline{\boldsymbol{x}}_i(t) + \overline{\boldsymbol{b}}_iu_i(t) + \overline{P}_i\boldsymbol{v}_i(t) + \overline{\boldsymbol{g}}_i(t) \tag{5.2.37a}$$

$$\overline{y}_i(t) = y_i(t) + y_{fi}(t) + \overline{\boldsymbol{c}}_i^T\overline{\boldsymbol{x}}_i(t), \quad i \in J \tag{5.2.37b}$$

where

$$\begin{aligned}
\overline{\boldsymbol{x}}_i(t) &= [\boldsymbol{x}_i(t)^T, \boldsymbol{x}_{fi}(t)^T]^T, \ \overline{\boldsymbol{g}}_i(t) = [\boldsymbol{g}_i(t)^T, 0]^T, \\
\overline{A}_i &= diag[A_i, A_{fi}], \ \ \overline{\boldsymbol{b}}_i = [\boldsymbol{b}_i^T, \boldsymbol{b}_{fi}^T]^T, \\
\overline{\boldsymbol{c}}_i &= [\boldsymbol{c}_i^T, \boldsymbol{c}_{fi}^T]^T, \ \ \overline{P}_i = [P_i^T, 0]^T.
\end{aligned}$$

It should be noted that the augmented subsystem $\overline{S}_i$ does not satisfy the range condition even if the original subsystem S_i satisfies the range condition because $\overline{P}_i$ is factored as follows:

$$\overline{P}_i = \overline{\boldsymbol{b}}_i\boldsymbol{p}_i^T - \begin{bmatrix} 0 \\ \boldsymbol{b}_{fi} \end{bmatrix} \boldsymbol{p}_i^T. \tag{5.2.38}$$

Then, under Assumption 5.1 with non-ASPR subsystem $S_i, i \in J$, the characteristics of the overall controlled plant with PFCs are as follows:

Subsystem $S_i, i \in L - L \cap J$: ASPR and satisfying the range condition.
Subsystem $\overline{S}_i, i \in L \cap J$: ASPR with PFC (original subsystem S_i satisfies the range condition).
Subsystem $S_i, i \in K - K \cap J$: ASPR and not satisfying the range condition.
Subsystem $\overline{S}_i, i \in K \cap J$: ASPR with PFC (original subsystem S_i does not satisfy the range condition).

Further, for the augmented subsystem $\overline{S}_i, i \in J$, there exists a constant $\overline{k}_i^*$ such that the transfer function:

$$\overline{G}_{ci} = \overline{\boldsymbol{c}}_i(sI - \overline{A}_{ci})^{-1}\overline{\boldsymbol{b}}_i \tag{5.2.39}$$

is ASPR, where $\overline{A}_{ci} = \overline{A}_i = \overline{k}_i^* \overline{b}_i \overline{c}_i^T$, and thus there exist positive definite matrices $\overline{H}_i$ and $\overline{G}_i$ satisfying the Kalman-Yakubovich Lemma:

$$\begin{aligned} \overline{A}_{ci}^T \overline{H}_i + \overline{H}_i \overline{A}_{ci}^T &= -\overline{G}_i \\ \overline{H}_i \overline{b}_i &= \overline{c}_i \end{aligned} \tag{5.2.40}$$

Considering the above mentioned characteristics of the augmented plant with the PFC, we have the following theorem with regard to stability of the control system.

Theorem 5.2: *Apply the control input* (5.2.7) *to the subsystem* S_i *and apply the control input replacing* $e_{yi}(t)$ *with* $\overline{e}_{yi}(t) = \overline{y}_i(t) - y_m(t)$ *to the subsystem* $\overline{S}_i$*. Then, defining a matrix* $M = [m_{ij}]$ *to be:*

$$m_{ij} = \begin{cases} \lambda_{min}[G_i] - 2R_{ij}, & i = j \in N - J \\ \lambda_{min}[\overline{G}_i] - 2R_{ij}, & i = j \in J \\ -(R_{ij} + R_{ji}), & i \neq j,\ i,j \in N \end{cases} \tag{5.2.41}$$

where

$$\begin{aligned} &R_{ij} = \|P_i\| \|H_i\| \|\xi_{ij}\| \|Q_j\|, && i \in K - K \cap J,\ j \in N \\ &R_{ij} = \|P_i\| \|\overline{H}_i\| \|\xi_{ij}\| \|Q_j\|, && i \in K \cap J,\ j \in N \\ &R_{ij} = \|b_{fi} p_i^T\| \|\overline{H}_i\| \|\xi_{ij}\| \|Q_j\|, && i \in L \cap J,\ j \in N \\ &R_{ij} = 0, && i \in L - L \cap J,\ j \in N \end{aligned} \tag{5.2.42}$$

the uniform ultimate boundedness of all the signals in the overall control system is guaranteed provided that the matrix M *is positive definite.*

Proof: Taking into account that the matrix M given by (5.2.41) is positive definite, the proof is executed in the same manner used in the proof of Theorem 5.1.

Remark 5.2: In the case where all the subsystems satisfy the range condition for input part, if non-ASPR subsystems satisfy the range condition $Q_i = q_i c_i$ for output part, then M-matrix condition defined in Theorem 5.2 (matrix M is positive definite) is necessarily satisfied by choosing b_{fi} sufficiently small. As shown in section 3, if the plant is minimum phase, then we can easily design a PFC with small b_f.

5.2.5 Use of the Robust Simple Adaptive Control

In the preceding subsection, the stability of the decentralized SAC scheme for plants with interconnection inputs satisfying inequality (5.2.2b) has been shown under that M-matrix condition. Further, from Theorem 5.1, it is apparent that the decentralized SAC system is stable provided that all the subsystems satisfy the range condition given by (5.2.6) even if the controlled plant does not satisfy M-matrix condition. However, the influence of interconnections between each subsystem must remain. Here, we introduce a robust SAC scheme given in chapter 4 to alleviate the influence of interconnections for the case where interconnection inputs are available for measurement signal.

Make the following assumptions:

Assumption 5.2:

(1) The plant (5.2.1) *and the reference model* (5.2.2) *satisfy Assumption* 5.1 (1) *and* (3).

(2) matrices $P_i, i \in N$ in (5.2.1) *can be factored as*

$$P_i = \boldsymbol{b}_i \boldsymbol{p}_i^T, \quad i \in N \tag{5.2.43}$$

for some constant vector $\boldsymbol{p}_i \in R^{m_i}$. That is, all subsystems $S_i, i \in N$ satisfy the range condition.

(3) All interconnection outputs $\boldsymbol{w}_i(t), i \in N$ is measurable.

Then, for each subsystem $S_i, i \in N$, we give the control input with a robust adaptive control term as follows:

$$u_i(t) = u_{si}(t) + u_{ri}(t) \tag{5.2.44}$$

where $u_{si}(t)$ is the original SAC input given in (5.2.7) and $u_{ri}(t)$ is the robust adaptive control term. By considering interconnections to be disturbances and $\boldsymbol{w}_i(t)$ to be measurable signals with regard to disturbances, the robust control term $u_{ri}(t)$ is given as follows:

$$u_{ri}(t) = \begin{cases} -\boldsymbol{\beta}_i(t)^T \boldsymbol{z}_{\beta i}(t) \mathrm{sgn} e_{yi}(t), & \text{if} |\boldsymbol{\beta}_i(t)^T \boldsymbol{z}_{\beta i}(t) e_{yi}(t)| > \varepsilon_i, \ \varepsilon_i > 0 \\ -\{\boldsymbol{\beta}_i(t)^T \boldsymbol{z}_{\beta i}(t)\}^2 e_{yi}(t)/\varepsilon_i, & \text{if} |\boldsymbol{\beta}_i(t)^T \boldsymbol{z}_{\beta i}(t) e_{yi}(t)| \le \varepsilon_i \end{cases} \tag{5.2.45}$$

where

$$\boldsymbol{\beta}(t) = [\beta_{i0}(t), \beta_{i1}(t), \cdots, \beta_{iN}(t)]^T, \quad \boldsymbol{z}_{\beta i}(t) = [1, \|\boldsymbol{w}_1(t)\|, \cdots, \|\boldsymbol{w}_N(t)\|]^T$$

and the parameter vectors $\boldsymbol{\beta}_i(t)$ are adjusted by the following parameter adjusting law:

$$\begin{cases} \boldsymbol{\beta}_i(t) = \boldsymbol{\beta}_{Ii}(t) + \boldsymbol{\beta}_{Pi}(t) \\ \dot{\boldsymbol{\beta}}_{Ii}(t) = \Gamma_{\beta Ii}\boldsymbol{z}_{\beta i}(t)|e_{yi}(t)| - \sigma_{\beta i}\boldsymbol{\beta}_{Ii}(t) \\ \boldsymbol{\beta}_{Pi}(t) = \Gamma_{\beta Pi}\boldsymbol{z}_{\beta i}(t)|e_{yi}(t)| \end{cases} \tag{5.2.46}$$

where

$$\Gamma_{\beta Ii} = \Gamma_{\beta Ii}^T > 0,\ \Gamma_{\beta Pi} = \Gamma_{\beta Pi}^T > 0,\ \sigma_{\beta i} > 0$$

Then we have the following theorem concerning stability.

Theorem 5.3: *Under Assumption* 5.2, *all the signals in the control system with the control input* (5.2.44) *are uniformly ultimately bounded.*

Proof: Considering the robust adaptive control term $u_{ri}(t)$ in the error system derived in (5.2.11), we obtain the error system

$$\begin{aligned} \dot{\boldsymbol{e}}_{xi}(t) &= A_{ci}\boldsymbol{e}_{xi}(t) + \boldsymbol{b}_i\boldsymbol{\zeta}_i(t)\boldsymbol{z}_i(t) + \boldsymbol{b}_i u_{ri}(t) \\ &\qquad -\boldsymbol{b}_i S_{3i} + P_i\boldsymbol{v}_i(t) + \boldsymbol{g}_i(t) \end{aligned} \tag{5.2.47a}$$

$$e_{yi}(t) = \boldsymbol{c}_i^T\boldsymbol{e}_{xi}(t) \tag{5.2.47b}$$

Now, let us consider the following positive definite function:

$$V(t) = \sum_{i=1}^{N} V_i(t) \tag{5.2.48a}$$

$$V_i(t) = \boldsymbol{e}_{xi}(t)^T H_i\boldsymbol{e}_{xi}(t) + \boldsymbol{\zeta}_{Ii}(t)^T\Gamma_{Ii}^{-1}\boldsymbol{\zeta}_{Ii}(t) + \boldsymbol{\zeta}_{\beta Ii}(t)^T\Gamma_{\beta Ii}^{-1}\boldsymbol{\zeta}_{\beta Ii}(t) \tag{5.2.48b}$$

$$\boldsymbol{\zeta}_{\beta Ii}(t) = \boldsymbol{\beta}_{Ii}(t) - \boldsymbol{\beta}_i^*,\ \boldsymbol{\beta}_i^* = [S_{3i}^*, \rho_{i1}, \cdots, \rho_{iN}]^T,\ \rho_{ij} = \|\boldsymbol{p}_i\|\xi_{ij}$$

Taking into account that it follows from (5.2.2) that

$$|\boldsymbol{v}_i(t)\boldsymbol{p}_i| \leq \sum_{j=1}^{N}\|\boldsymbol{p}_i\|\xi_{ij}\|\boldsymbol{w}_j(t)\| = \sum_{j=1}^{N}\rho_{ij}\|\boldsymbol{w}_j(t)\|, \tag{5.2.49}$$

from (5.2.8), (5.2.13), (5.2.43) and (5.2.46), we have

$$\begin{aligned} \frac{dV_i(t)}{dt} &= -\boldsymbol{e}_{xi}(t)^T G_i\boldsymbol{e}_{xi}(t) + 2u_{ri}(t)e_{yi}(t) \\ &\quad +2\boldsymbol{\beta}_i^{*T}\boldsymbol{z}_{\beta i}(t)|e_{yi}(t)| \\ &\quad -2\sigma_{2i}\boldsymbol{\zeta}_{Ii}(t)^T\Gamma_{Ii}^{-1}\boldsymbol{\zeta}_{Ii}(t) - 2\sigma_{2i}\boldsymbol{\zeta}_{Ii}(t)\Gamma_{Ii}^{-1}\boldsymbol{k}_i^* \end{aligned}$$

$$-2\sigma_{\beta i}\boldsymbol{\zeta}_{\beta Ii}(t)^T\Gamma_{\beta Ii}^{-1}\boldsymbol{\zeta}_{\beta Ii}(t) - 2\sigma_{\beta i}\boldsymbol{\zeta}_{\beta Ii}(t)\Gamma_{\beta Ii}^{-1}\boldsymbol{\beta}_i^*$$
$$+2[\boldsymbol{\beta}_i(t) - \boldsymbol{\beta}_i^*]^T\boldsymbol{\zeta}_{\beta i}(t)|e_{yi}(t)|$$
$$+2\boldsymbol{g}_i(t)^T H_i \boldsymbol{e}_{xi}(t). \tag{5.2.50}$$

Finally, using the same manner shown in Chapter 4 (stability analysis of the robust SAC), we obtain

$$\begin{aligned}\frac{dV(t)}{dt} \leq &-\sum_{i=1}^{N}\lambda_{min}[G_i]\|\boldsymbol{e}_{xi}(t)\|^2 + 2\sum_{i=1}^{N}\boldsymbol{g}_i^*\|H_i\|\|\boldsymbol{e}_{xi}(t)\| \\ &-2\sum_{i=1}^{N}\sigma_{2i}\lambda_{min}[\Gamma_{Ii}^{-1}]\|\boldsymbol{\zeta}_{Ii}(t)\|^2 \\ &-2\sum_{i=1}^{N}\sigma_{\beta i}\lambda_{min}[\Gamma_{\beta Ii}^{-1}]\|\boldsymbol{\zeta}_{\beta Ii}(t)\|^2 \\ &+2\sum_{i=1}\sigma_{2i}\lambda_{max}[\Gamma_{Ii}^{-1}]\|\boldsymbol{k}_i^*\|\|\boldsymbol{\zeta}_{Ii}(t)\| \\ &+2\sum_{i=1}\sigma_{\beta i}\lambda_{max}[\Gamma_{\beta Ii}^{-1}]\|\boldsymbol{\beta}_i^*\|\|\boldsymbol{\zeta}_{\beta Ii}(t)\| \\ &+2\sum_{i=1}^{N}\varepsilon_i\end{aligned} \tag{5.2.51}$$

Then, it can be concluded that all the signals in the control system are uniformly ultimately bounded.

Remark 5.3: If some subsystems are not ASPR in Theorem 5.3, then we have to introduce the PFCs to non-ASPR subsystems. In addition, the satisfaction of M-matrix condition as in Theorem 5.2 is necessary to guarantee the stability. However, as mentioned in Remark 5.2, M-matrix condition holds with small $\boldsymbol{b}_{fi}$, and as shown in Section 4, the influence of interconnections can be alleviated if $\boldsymbol{b}_{fi}$ is *small.*

5.3 Multivariable Decentralized Simple Adaptive Control

5.3.1 Problem Setup

In the case where the large-scale system is divided into MIMO subsystems, each subsystem S_i will be expressed as:

$$S_i: \quad \dot{\boldsymbol{x}}_i(t) = A_i\boldsymbol{x}_i(t) + B_i\boldsymbol{u}_i(t) + P_i\boldsymbol{v}_i(t) \tag{5.3.1a}$$
$$\boldsymbol{y}_i(t) = C_i\boldsymbol{x}_i(t) \tag{5.3.1b}$$
$$\boldsymbol{v}_i = \boldsymbol{f}_i(t, \boldsymbol{w}), \ \boldsymbol{w}_i(t) = Q_i\boldsymbol{x}_i(t), \quad i \in N, \ N = \{1, \ldots, N\} \tag{5.3.1c}$$

Subsystem S_i is an m_i-output/m_i-input system. Where $\boldsymbol{x}_i \in R^{n_i}$, $u_i \in R^{mi}$ and $y_i \in R^{mi}$ are the state, the control input and the output of the subsystem S_i, respectively, and $\boldsymbol{v}_i \in R^{q_i}$, $\boldsymbol{w}_i \in R^{l_i}$ are the interconnection input and output vectors associated with S_i, respectively. Further, we define $\boldsymbol{w} = [\boldsymbol{w}_1^T, \cdots, \boldsymbol{w}_N^T]^T$.

The reference models S_{Mi} which the outputs of each subsystem S_i are required to follow are given as follows:

$$S_{Mi}: \quad \dot{\boldsymbol{x}}_{mi}(t) = A_{mi}\boldsymbol{x}_{mi}(t) + B_{mi}\boldsymbol{u}_{mi}(t) \tag{5.3.2a}$$

$$\boldsymbol{y}_{mi}(t) = C_{mi}\boldsymbol{x}_{mi}(t) \tag{5.3.2b}$$

where $\boldsymbol{x}_{mi} \in R^{n_{mi}}$, $\boldsymbol{u}_{mi} \in R^{mi}$ and $\boldsymbol{y}_{mi} \in R^{mi}$.

The following assumptions are made on the overall system with subsystems S_i and each reference model S_{Mi}.

Assumption 5.3:

(1) The linear part of each subsystem $S_i, i \in N$ is ASPR.

(2) The matrix $P_i, i \in L, L \subset N$ can be factored as

$$P_i = B_i P_{0i}, \quad i \in L \tag{5.3.3}$$

for some constant matrix $P_{0i} \in R^{m_i \times q_i}$. That is, subsystems $S_i, i \in L$ belong to a stabilizable class in the decentralized system whose coupling parameters are within the range of the control input.

(3) Each subsystem S_i and its reference model S_{Mi} satisfy Assumption 2.2 (3), (4). *That is, Broussard's model output following condition is satisfied for each subsystem.*

(4) Denoting the j-th derivative of $\boldsymbol{u}_{mi}(t)$ as $\boldsymbol{u}_{mi}^{(j)}(t)$, $\boldsymbol{u}_{mi}^{(j)}(t), j = 0, \ldots, m_i$ is uniformly bounded.

(5) There exists a constant $\xi_{ij} > 0$ such that

$$\|\boldsymbol{f}(t, \boldsymbol{w})\| \leq \sum_{j \in N} \xi_{ij} \|\boldsymbol{w}_j\|. \tag{5.3.4}$$

In the following subsection, applications of SAC methods to each subsystem are considered and we analyze the stability of the overall control system.

5.3.2 Design of Control System and Its Stability

First of all, let's consider the use of general SAC algorithms. Thus, the controller for each subsystem is given by

$$u_i(t) = K_i(t)z_i(t) \tag{5.3.5}$$

$$\begin{aligned} z_i(t) &= [e_{yi}(t)^T, x_{mi}(t)^T, u_{mi}(t)^T]^T \\ e_{yi}(t) &= y_i(t) - y_{mi}(t) \\ K_i(t) &= [K_{ei}(t), K_{xi}(t)^T, K_{ui}(t)] \end{aligned}$$

$$\left\{ \begin{aligned} K_i(t) &= K_{Ii}(t) + K_{Pi}(t) \\ \dot{K}_{Ii}(t) &= -e_{yi} z_i(t)^T \Gamma_{Ii} - \sigma_i K_{Ii}(t) \\ K_{Pi}(t) &= -e_{yi} z_i(t)^T \Gamma_{Pi} \end{aligned} \right. \tag{5.3.6}$$

$$\Gamma_{Ii} = \Gamma_{Ii}^T > 0,\ \Gamma_{Pi} = \Gamma_{Pi}^T > 0,\ \sigma_i$$

The following theorem gives the stability result.

Theorem 5.3: *Under Assumption* 5.3, *all the signals in the overall control system with inputs* (5.3.5) *are uniformly ultimately bounded provided that the matrix* $M = [m_{ij}]$ *defined by*

$$m_{ij} = \left\{ \begin{array}{ll} \lambda_{min}[G_i] - 2R_{ij}, & i = j \in N \\ -(R_{ij} + R_{ji}), & i \neq j,\ i, j \in N \end{array} \right. \tag{5.3.7}$$

$$\begin{array}{ll} R_{ij} = \|P_i\| \|H_i\| \|\xi_{ij}\| \|Q_j\|, & i \in K - N - L,\ j \in N \\ R_{ij} = 0, & i \in L,\ j \in N \end{array} \tag{5.3.8}$$

is positive definite matrix. Where, H_i *and* G_i *are positive definite matrices which satisfy the Kalman-Yakubovich Lemma:*

$$\begin{aligned} A_{ci}^T H_i + H_i A_{ci} &= -G_i \\ B_i^T H_i &= C_i \end{aligned} \tag{5.3.9}$$

where

$$A_{ci} = A_i + B_i K_{ei}^* C_i,$$
K_{ei}^* : *A gain matrix for making the plant SPR*

Proof: Denote the ideal state vector and control input in the case where the perfect output following is achieved for each subsystem without interconnections from other subsystems as $x_i^*(t)$ and $u_i^*(t)$, respectively. Since the ideal input $u_i^*(t)$ is given by

$$u_i^*(t) = S_{1i} x_{mi}(t) + S_{2i} u_{mi}(t) + S_{3i}(t) \tag{5.3.10}$$

$S_{3i}(t)$: a bounded vector function of t

under Assumption 5.3(3) and (4), we have the following error system:

$$\dot{e}_{xi}(t) = A_{ci}e_{xi}(t) + B_i(\Delta K_i(t)z_i(t) - S_{3i}(t)) + P_i v_i(t) \quad (5.3.11a)$$
$$e_{yi}(t) = C_i e_{xi}(t) \quad (5.3.11b)$$

where

$$e_{xi}(t) = x_i(t) - x_i^*(t) \quad (5.3.12)$$
$$\Delta K_i(t) = K_i(t) - K_i^*, \quad K_i^* = [K_{ei}^*, S_{1i}, S_{2i}]^T \quad (5.3.13)$$

Then, choosing the positive definite function as

$$V(t) = \sum_{i=1}^{N} V_i(t) \quad (5.3.14a)$$
$$V_i(t) = V_{1i}(t) + V_{2i}(t) \quad (5.3.14b)$$
$$V_{1i}(t) = e_{xi}(t)^T H_i e_{xi}(t), \quad i \in N \quad (5.3.14c)$$
$$V_{2i}(t) = \begin{cases} \mathrm{tr}\{\Delta K_{Ii}(t)\Gamma_{Ii}^{-1}\Delta K_{Ii}^T(t)\}, & i \in K = N - L \\ \mathrm{tr}\{(\Delta K_{Ii}(t) + \rho\Gamma_{0i})\Gamma_{Ii}^{-1}(\Delta K_{Ii}(t) + \rho\Gamma_{0i})^T\}, & i \in L \end{cases} \quad (5.3.14d)$$
$$\Delta K_{Ii}(t) = K_{Ii}(t) - K_i^*, \; \Gamma_{0i} = [I_{mi}, 0, \cdots, 0]^T, \; \rho > 0.$$

the boundedness of all the signals in the overall control system can be easily shown using the same manner as in the case where each subsystem is expressed to be an SISO system (Subsection 5.2.3).

As shown in Subsection 5.2.5, we can introduce the robust SAC scheme to alleviate the influence of interconnections for plants with subsystems satisfying the range condition if interconnection inputs are available.

Assumption 5.4:

(1) All subsystems $S_i, i \in N$ satisfy the range condition. That is, matrices $P_i, i \in N$ can be factored as

$$P_i = B_i P_{0i}, \quad i \in N \quad (5.3.15)$$

for some constant matrix $P_{0i} \in R^{m_i \times q_i}$.

(2) All interconnection inputs $w_i(t), i \in N$, are measurable.

(3) Let be

$$P_{0i}\boldsymbol{f}_i(t,\boldsymbol{w}) = [f_{i1}, f_{i2}, \cdots, f_{i\ mi}]^T \tag{5.3.16}$$

Then, there exist constants ξ_{ijk} *such that*

$$|f_{ij}(t,\boldsymbol{w})| \le \sum_{k\in N} \xi_{ijk}\|\boldsymbol{w}_k\|. \tag{5.3.17}$$

Under the above assumption, the control input with a robust adaptive control term is given as follows:

$$\bar{\boldsymbol{u}}_i(t) = \boldsymbol{u}_i(t) + \boldsymbol{u}_{Ri}(t) \tag{5.3.18}$$

where the robust control input $u_{Ri}(t)$ is given by

$$\boldsymbol{u}_{Ri}(t) = [u_{Ri1}(t), u_{Ri2}(t), \cdots, u_{Rim_i}(t)]^T \tag{5.3.19}$$

$$u_{Rij}(t) = \begin{cases} -\boldsymbol{\beta}_{ij}(t)^T \boldsymbol{z}_\beta(t)\mathrm{sgn} e_{yij}(t), & \mathrm{if}|\boldsymbol{\beta}_{ij}(t)^T \boldsymbol{z}_\beta(t) e_{yij}(t)| > \varepsilon_{ij},\ \varepsilon_{ij} > 0 \\ -\{\boldsymbol{\beta}_{ij}(t)^T \boldsymbol{z}_\beta(t)\}^2 e_{yij}(t)/\varepsilon_{ij}, & \mathrm{if}|\boldsymbol{\beta}_{ij}(t)^T \boldsymbol{z}_\beta(t) e_{yij}(t)| \le \varepsilon_{ij} \end{cases} \tag{5.3.20}$$

where

$$\boldsymbol{e}_{yi}(t) = [e_{i1}(t), e_{i2}(t), \cdots, e_{im_i}(t)]^T, \quad \boldsymbol{z}_\beta(t) = [1, \|\boldsymbol{w}_1(t)\|, \cdots, \|\boldsymbol{w}_N(t)\|]^T$$

and the parameter vectors $\boldsymbol{\beta}_{ij}(t) \in R^{N+1}$ are adjusted by the following parameter adjusting law.

$$\begin{cases} \boldsymbol{\beta}_{ij}(t) = \boldsymbol{\beta}_{Iij}(t) + \boldsymbol{\beta}_{Pij}(t) \\ \dot{\boldsymbol{\beta}}_{Iij}(t) = \Gamma_{\beta Iij} \boldsymbol{z}_\beta(t)|e_{yij}(t)| - \sigma_{\beta ij}\boldsymbol{\beta}_{Iij}(t) \\ \boldsymbol{\beta}_{Pij}(t) = \Gamma_{\beta Pij} \boldsymbol{z}_\beta(t)|e_{yij}(t)| \end{cases} \tag{5.3.21}$$

where

$$\Gamma_{\beta Iij} = \Gamma_{\beta Iij}^T > 0,\ \Gamma_{\beta Pij} = \Gamma_{\beta Pij}^T > 0,\ \sigma_{\beta ij} > 0$$

The stability in this case can also be shown in the same manner as the proof of Theorem 5.3.

5.4 Numerical Simulations

Here, the effectiveness of proposed decentralized SAC method is confirmed through numerical simulations for the example given by Gavel and Siljak (1989). The problem statement in this simulation is as follows:

Consider two inverted pendulums which are connected to each other by a spring (see Figure 5.2). Denoting

$$\boldsymbol{x}_1(t) = [\phi_1(t), \dot{\phi}_1(t)],\ \boldsymbol{x}_2(t) = [\phi_2(t), \dot{\phi}_2(t)],$$

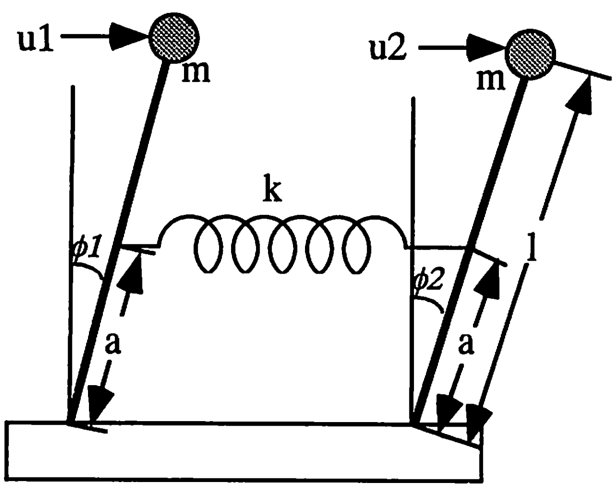


Figure 5.2 Inverted pendulums

we have the following state space representation for the inverted pendulum.

$$\dot{x}_1(t) = \begin{bmatrix} 0 & 1 \\ \frac{g}{l} & 0 \end{bmatrix} x_1(t) + \begin{bmatrix} 0 \\ \frac{1}{ml^2} \end{bmatrix} u_1(t) + \begin{bmatrix} 0 & 0 \\ -\frac{ka^2}{ml^2} & 0 \end{bmatrix} x_1(t)$$

$$+ \begin{bmatrix} 0 & 0 \\ \frac{ka^2}{ml^2} & 0 \end{bmatrix} x_2(t) \tag{5.4.1a}$$

$$\dot{x}_2(t) = \begin{bmatrix} 0 & 1 \\ \frac{g}{l} & 0 \end{bmatrix} x_2(t) + \begin{bmatrix} 0 \\ \frac{1}{ml^2} \end{bmatrix} u_2(t) + \begin{bmatrix} 0 & 0 \\ -\frac{ka^2}{ml^2} & 0 \end{bmatrix} x_2(t)$$

$$+ \begin{bmatrix} 0 & 0 \\ \frac{ka^2}{ml^2} & 0 \end{bmatrix} x_1(t) \tag{5.4.1b}$$

Here, we set $g/l = 1$, $1/ml^2 = 1$ and $k/m = 2$. The uncertainty of the interconnections appears from the position of the spring as $a(t)/l \in [0,1]$. In this case, the corresponding system parameters to (5.2.1) are

$$A_i = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \; b_i = P_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \; c_i^T = Q_i = [1, 0], i = 1, 2$$

and the interconnection input is

$$v(t) = f(t, w) = 2\frac{a(t)}{l^2}\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} w(t)$$

The reference models which the outputs of each subsystem should folllow are given as follows:

$$\dot{x}_{mi}(t) = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} x_{mi}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_{mi}(t) \qquad (5.4.2a)$$

$$y_{mi}(t) = [1, 0]x_{mi}(t), \quad i = 1, 2 \qquad (5.4.2b)$$

$$u_{m1}(t) = \sin 20t + \sin 5t + \sin t \qquad (5.4.2c)$$

$$u_{m2}(t) = \sin 10t + \sin 2t + \sin 0.5t \qquad (5.4.2d)$$

Because each subsystem $S_i(A_i, b_i, c_i), i = 1, 2$, of the plant (5.4.1) is not ASPR, we implement PFCs to each subsystem. PFCs were designed according to the design scheme given in Section 3.2. as follows:

$$F_i(s) = \frac{b_i}{s + a_i}, \; a_i = 2.0, b_i = 0.05, \; i = 1, 2. \qquad (5.4.3)$$

Further, we set the design parameters in (5.2.8) as

$$\Gamma_{Ii} = \mathrm{diag}[10^5 10^2 10^2 10^2], \; \Gamma_{Pi} = \mathrm{diag}[10^4 101010],$$
$$\sigma_{1i} = 0.1, \; \sigma 2i = 0.05, \quad i = 1, 2$$

and the design parameters in (5.2.46) as

$$\Gamma_{\beta Ii} = 10^3 I_3, \quad \Gamma_{\beta Pi} = 10^2 I_3$$
$$\sigma_{\beta 1i} = 0.1, \; \sigma_{\beta 2i} = 0.05, \; \varepsilon = 0.01, \; i = 1, 2$$

Figures 5.3 $\sim$ 5.6 illustrate simulation results. Figure 5.3 is a result which shows the comparison between the proposed method (decentralized SAC) and the method given by Gavel and Siljak (1989). The tracking error amplitude with decentralized SAC is reduced by 10 percent compared with Gavel's method. Figure 5.4 shows the original tracking error between the original plant and reference model outputs and the augmented tracking error between the augmented plant with PFCs (5.4.3) and the reference model outputs. In addition, the results with the robust decentralized SAC are shown in Figure 5.5. We can see from these two figures that although the robust decentralized SAC significantly affects augmented systems, there is no improvement for the original system due to bias affects from PFCs. As a countermeasure to this problem, we redesigned parameters for PFCs by $b_i = 0.005, i = 1, 2$. That is, we employed the PFC with lower gain. The result with robust decentralized SAC is shown in Figure 5.6. It is seen that the tracking performance was sigificantly improved.

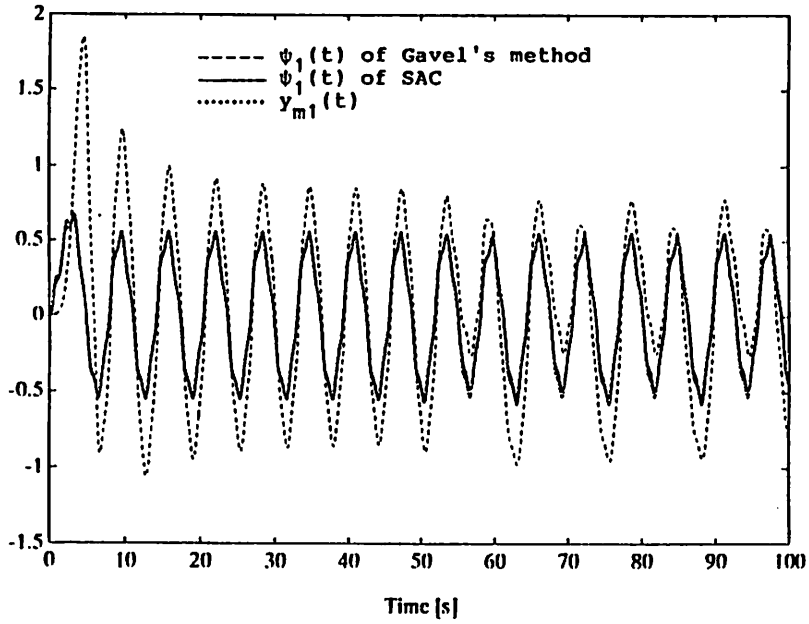


Figure 5.3 Simulation results compared with SAC method and Gavel's method

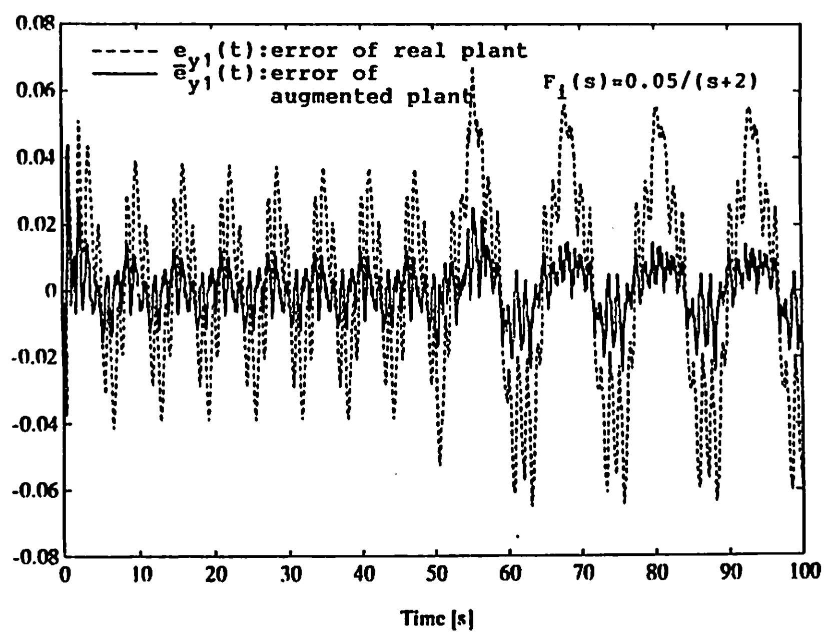


Figure 5.4 Tracking errors of the plant and the augmented plant outputs: The use of SAC

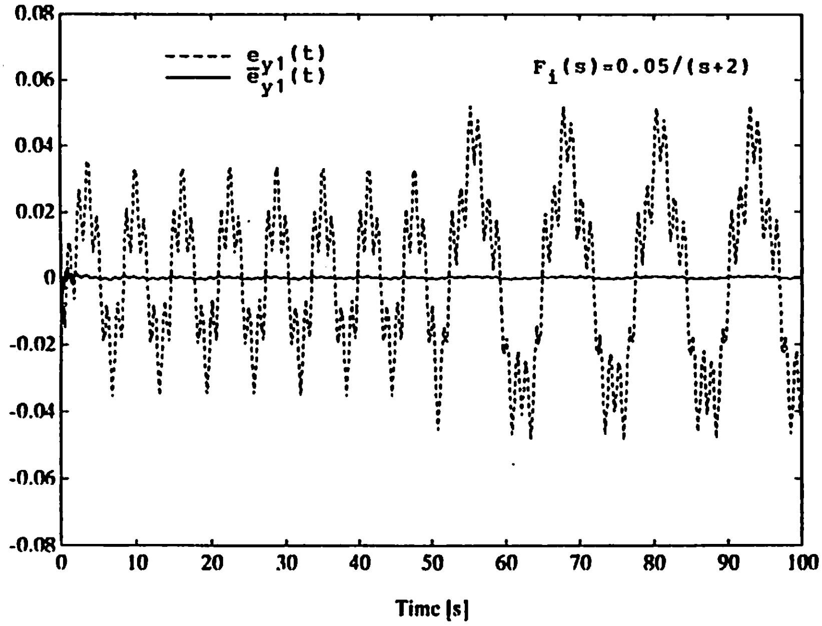


Figure 5.5 Tracking errors of the plant and the augmented plant outputs: The use of robust SAC (first case)

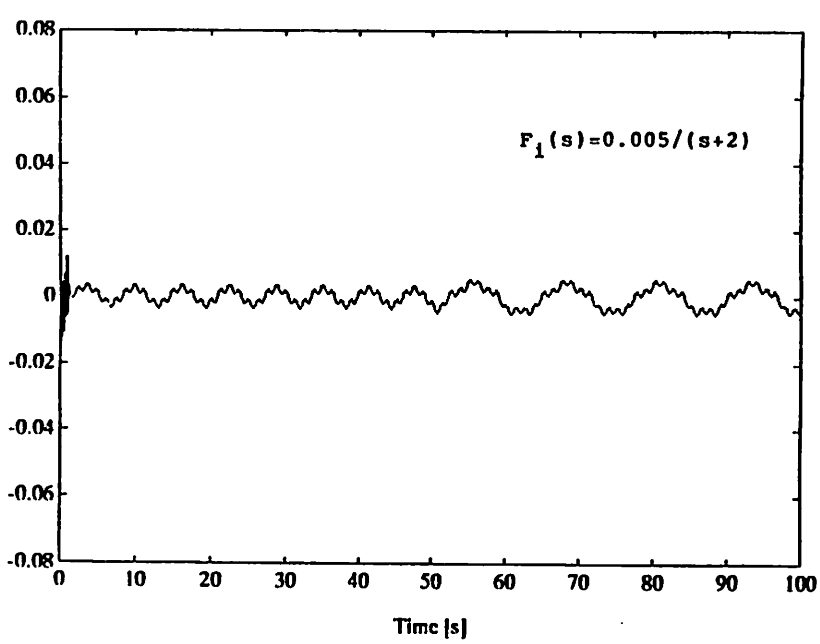


Figure 5.6 Tracking error of the plant output: The use of robust SAC (second case)

5.5 Conclusions

In this chapter, we considered the applications of SAC method and robust SAC method to decentralized control systems. The stability of the obtained decentralized SAC system is guaranteed by the satisfaction of M-matrix condition or the range condition for each subsystem. The effectiveness of the proposed methods were confirmed through a numerical simulation following the example given by Gavel and Siljak (1989). The proposed decentralized SAC scheme has significant robustness with regard to uncertain interconnections in large-scale systems similar to the results obtained for disturbances in the preceding sections.

6 Simple Adaptive Control with Derivative Control Term

6.1 Introduction

We can apply the SAC method to the ASPR augmented plant with a PFC even if the original plant is not ASPR. In this case, if we design a PFC which has a much smaller gain than that of the original plant, the control objective for the original plant can be approximately attained by applying the SAC method to the augmented plant instead of the original plant. However, as shown by Bar-Kana (1991), choosing a smaller gain PFC leads to the increase of the feedback gain which is adaptively adjusted in the augmented control system because the leading coefficient of the augmented plant depends on the leading coefficient of the PFC. In this situation, the amplitude of the control input often increases at the transient state. In this chapter, it is shown that the above mentioned problem is improved by adding a derivative action term to the original SAC input.

6.2 Simple Adaptive Control with Derivative Control Term

6.2.1 Control Algorithm

Consider the following n order m-input/output plant:

$$\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + B\boldsymbol{u}(t) \tag{6.2.1a}$$
$$\boldsymbol{y}(t) = C\boldsymbol{x}(t) \tag{6.2.1b}$$

and n_m order m-input/output reference model:

$$\dot{\boldsymbol{x}}_m(t) = A_m\boldsymbol{x}_m(t) + B_m\boldsymbol{u}_m(t) \tag{6.2.2a}$$
$$\boldsymbol{y}_m(t) = C_m\boldsymbol{x}_m(t) \tag{6.2.2b}$$

Suppose that the plant (6.2.1) and the reference model (6.2.2) satisfy Assumption 2.2. That is, for these plant and reference model, we can design a SAC system. Under this condition, the modified SAC input with derivative term is given as follows:

$$\boldsymbol{u}(t) = K(t)\boldsymbol{z}(t) + K_v\dot{\boldsymbol{e}}_y(t) \tag{6.2.3}$$

where

$$
\begin{aligned}
z(t) &= [e_y(t)^T, x_m(t)^T, u_m(t)^T]^T \\
e_y(t) &= y(t) - y^*(t) = y(t) - y_m(t) \\
K(t) &= [K_e(t), K_x(t), K_u(t)], K_v = K_v^T < 0
\end{aligned}
$$

and the gain matrix $K(t)$ is adaptively adjusted by the following parameter adjusting law:

$$
\begin{cases}
K(t) = K_I(t) + K_P(t) \\
\dot{K}_I(t) = -e_y(t)z(t)^T\Gamma_I - \sigma_I(t)K_I(t) \\
K_P(t) = -e_y(t)z(t)^T\Gamma_P \\
\sigma_I(t) = \sigma_1\frac{e_y(t)^Te_y(t)}{1+e_y(t)^Te_y(t)} + \sigma_2
\end{cases}
\tag{6.2.4}
$$

$$
\Gamma_I = \Gamma_I^T > 0,\ \Gamma_P = \Gamma_P^T > 0,\ \sigma_1, \sigma_2 > 0
$$

6.2.2 Stability of the Control System

The stability is analyzed only for the case where the reference input is constant. It will similarly be shown for the case where the reference input is any function of t satisfying Assumption 2.2(4).

Suppose that the perfect model output following: $e_y(t) \equiv 0$ is attained. Denoting the ideal state, input and output as $x^*(t), u^*(t)$ and $y^*(t)$, respectively, we have

$$\dot{x}^*(t) = Ax^*(t) + Bu^*(t) \tag{6.2.5a}$$

$$y^*(t) = Cx^*(t) = y_m(t) \tag{6.2.5b}$$

$$u^*(t) = S_{21}x_m(t) + S_{22}u_m(t) \tag{6.2.5c}$$

Now, defining $e_x(t) = x(t) - x^*(t)$ and $e_y(t) = y(t) - y^*(t) = y(t) - y_m(t)$, we have from (6.2.1), (6.2.3) and (6.2.5) that

$$(I - BK_vC)\dot{e}_x(t) = A_ce_x(t) + B\Delta u(t) \tag{6.2.6a}$$

$$e_y(t) = Ce_x(t) \tag{6.2.6b}$$

$$\Delta u(t) = \Delta K(t)z(t) \tag{6.2.6c}$$

where

$$
\begin{aligned}
A_c &= A + BK_e^*C \ (K_e^* : \text{a gain which makes the closed loop ASPR}) \\
\Delta K(t) &= K(t) - K^*,\ K^* = [K_e^*, S_{21}, S_{22}]^T
\end{aligned}
$$

Here, from the ASPR-ness of the plant (6.2.1), there exist positive definite matrices P and Q such that

$$A_c^T P + PA_c = -Q,\ B^T P = C \tag{6.2.7}$$

Since $K_v < 0$, it follows from (6.2.7) that

$$\begin{aligned}\det(I - BK_vC) &= \det K_v \det(K_v^{-1} - CB) \\ &= \det K_v \det(K_v^{-1} - B^T CB) \neq 0\end{aligned} \tag{6.2.8}$$

Thus, we obtain the following error system from (6.2.6) and(6.2.8).

$$\dot{e}_x(t) = \bar{A}_c e_x(t) + \bar{B}\Delta u(t) \tag{6.2.9a}$$

$$e_y(t) = Ce_x(t) \tag{6.2.9b}$$

where

$$\bar{A}_c = (I - BK_vC)^{-1}A_c,\ \bar{B} = (I - BK_vC)^{-1}B$$

Further setting $P_1 = P(I - BK_vC)$, since $K_v = K_v^T < 0$ and from (6.2.7), it follows that $P_1 = P_1^T > 0$. From this result and (6.2.7), we have

$$\bar{A}_c^T P_1 + P_1\bar{A}_c = -Q,\ \bar{B}^T P_1 = C \tag{6.2.10}$$

This implies that the Kalman-Yakubovich Lemma holds in error system (6.2.9).

Now, consider the following positive definite function:

$$V(t) = e_x(t)^T P_1 e_x(t) + \mathrm{tr}\{\Delta K_I(t)\Gamma_I^{-1}\Delta K_I(t)^T\} \tag{6.2.11}$$

where

$$\Delta K_I(t) = K_I(t) - K^*$$

The ultimate uniform boundedness of all the signals in the control system can easily be proved by using the same manner shown in Section 2.3. Finally, we have the following theorem concerning the stability of the control system.

Theorem 6.1: *All the signals in the control system with control input* (6.2.3) *are ultimately uniformly bounded under Assumption* 2.2.

6.3 Numerical Simulations

We set the following 2-input/output non-ASPR plant as a controlled plant.

$$G(s) = \begin{bmatrix} \frac{2}{s^2} & \frac{2}{s^2} \\ \frac{1}{s^2} & \frac{3}{s^2} \end{bmatrix} \tag{6.3.1}$$

The reference model is given by

$$G_m(s) = \text{diag}\left[\frac{3}{(s+3)}, \frac{3}{(s+4)}\right] \tag{6.3.2}$$

$$\boldsymbol{u}_m(t) = [0.3 + 1.85\sin 10t, 0.3 + 1.85\cos 10t]^T$$

Since the plant (6.3.1) is not ASPR, we design the PFC as follows:

$$F(s) = \text{diag}\left[\frac{\beta}{(s+1)}, \frac{\beta}{(s+1)}\right], \ \beta > 0 \tag{6.3.3}$$

For any $\beta > 0$, the augmented plant $G_a(s) = G(s) + F(s)$ is ASPR.

Figure 6.1 shows the relation between β and the upper bound ω_0 of ω such that $20|\log|g_{11}(j\omega)/g_{a11}(j\omega)|| \leq 3$ for (1, 1) elements of $G(s)$ and $G_a(s)$. It is seen that a smaller β makes the upper bound ω_0 bigger. This means that the smaller β expands an applicable range of the SAC method in the sense that the frequency range which satisfies $|g_{11}(j\omega)| \simeq |g_{a11}(j\omega)|$ becomes wider. Contrarily, as shown in Table 6.1, the adaptively adjusted gain $K_e(t)$ strikingly increases for a smaller β. However, as shown in Figure 6.2, the maximum value of $K_e(t)$ is suppressed and reduced by adding a derivative control term to the original SAC input. Figure 6.3 and 6.4 show simulation result with $K_v = 0$ and $K_v = -5.0I$, respectively, for $\beta = 0.005$ in (6.3.3). It is apparent that we can obtain better control performance by using the modified SAC input with derivative term than using the original SAC input.

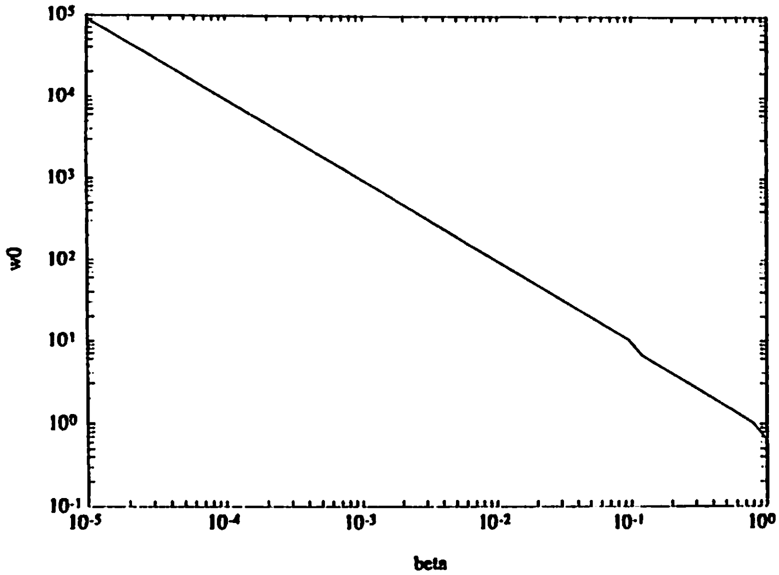


Figure 6.1 Cut off frequency ω_0 vs. feedforward compensator gain β

Table 6.1 Adaptive gain $K_e(t)$ for feedforward compensator gain β

β	0.1	0.05	0.01	0.005
$\|K_e(t=200)\|$	58.9	137.5	262.5	592.8
$\max \|K_e(t)\|$	88.5	637.9	833.3	2139.1

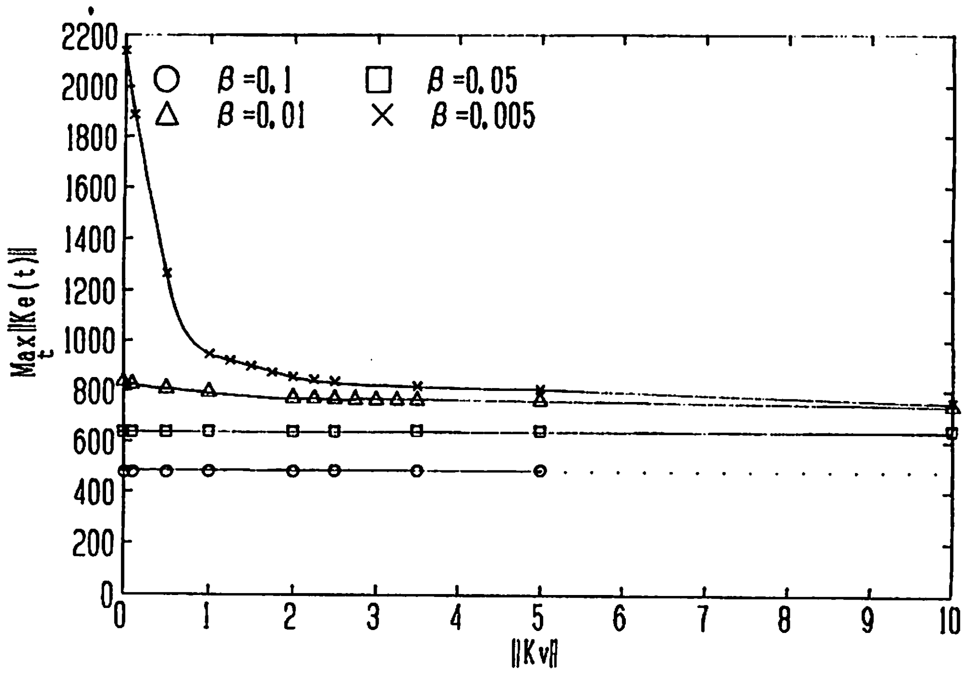
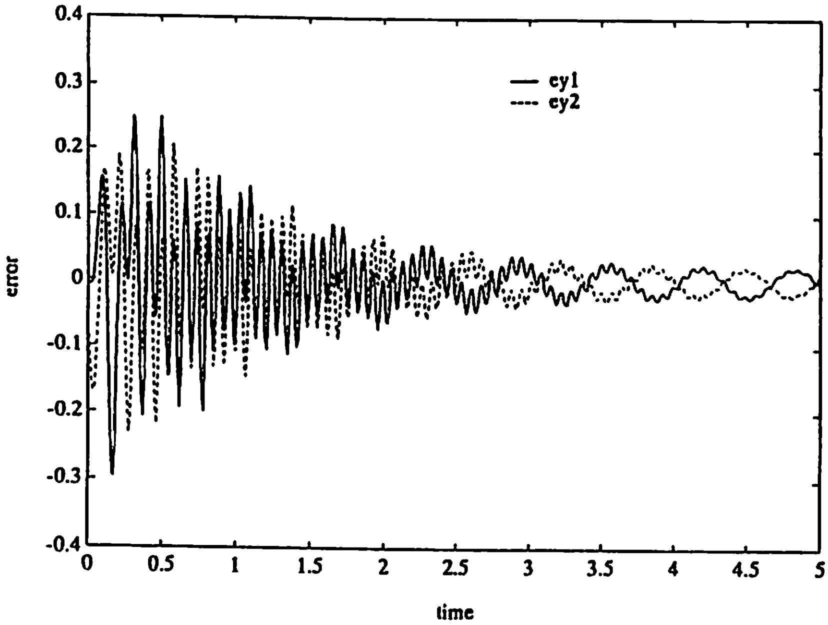
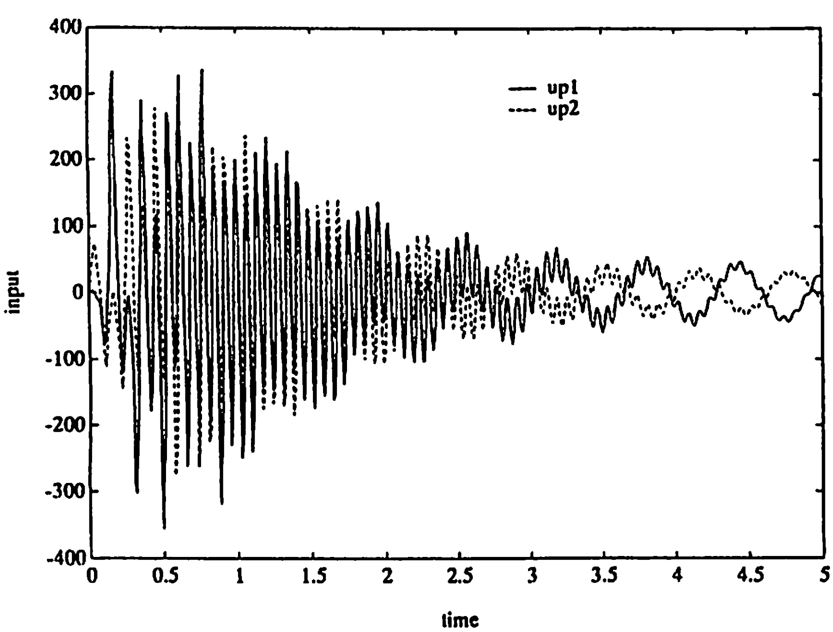


Figure 6.2 Maximum adaptive gain $K_e(t)$ for derivative feedback gain K_v and feedforward compensator gain β

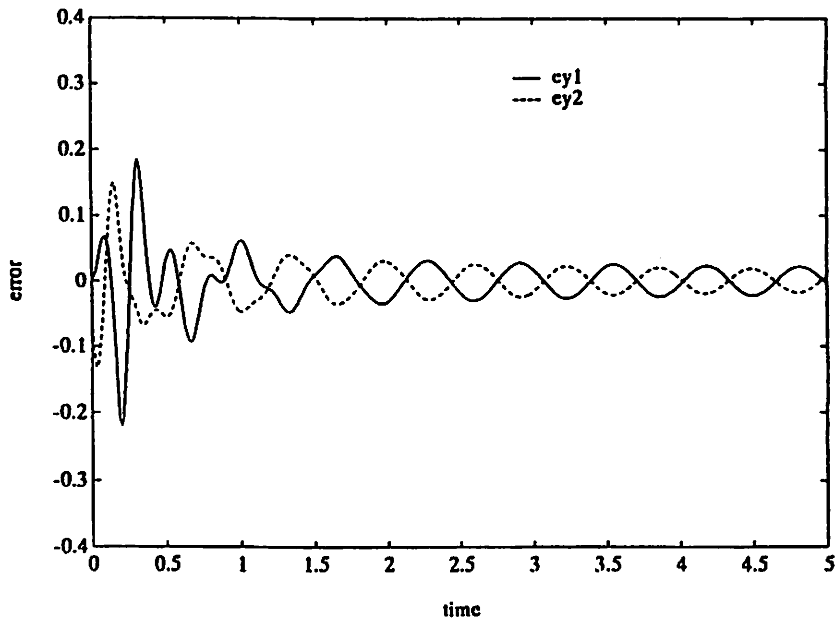


(a) Tracking errors

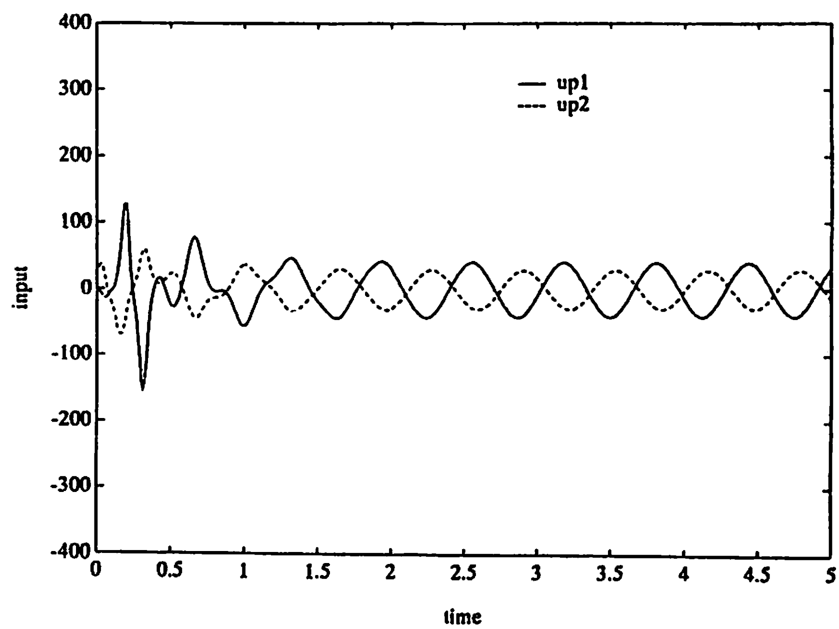


(b) Control inputs

Figure 6.3 Simulation results for original SAC



(a) Tracking errors



(b) Control inputs

Figure 6.4 Simulation results for SAC with derivative control term

6.4 Conclusions

In this chapter, a modified SAC algorithm with derivative control term was proposed. The SAC is generally applied to an augmented plant with PFC in the case where the original plant is not ASPR. In this case, to obtain a better tracking performance for the original plant output, we have to design a PFC with smaller gain. Contrarily, setting the PFC gain small leads to an increase in the adaptive feedback gain so that the tracking performance in the transient state might become worse. The proposed modified SAC method significantly affects this situation.

7 Conclusions

Adaptive control is a direct aggregation of a control methodology with some form recursive system identification. It adjusts controller parameters automatically so as to obtain good control performance during the whole operation in the presence of uncertainties and parameter changes. However, in general, assumptions on the adaptive controls to guarantee the stability of the control system are seldom valied on practical plants. With this point in mind, the robustness of the adaptive control system was discussed during the late 1980's. Many modified adaptive control schemes aimed at improving the robustness of the control systems were proposed. Unfortunately, most of these modifications caused complications of the adaptive controller structure. For practicing engineers, the simplicity of controller structure is extremely fascinating because they can easily implement the control methods. With these points as background, a new strategy to direct model reference adaptive control, so-called *Simple Adaptive Control* (SAC), was proposed. The method makes it possible to construct the adaptive control system regardless of the plant order. The basic idea of this adaptive method is to ensure the stability of the control system by using the output feedback under the *almost strictly positive real* (ASPR) condition on the plant and to attain the model output following by forward compensation based on the *Command Genetator Tracker* (CGT) theory. The SAC is robust with regard to disturbances, unmodelled dynamics, and non-linearities. These robust performances have been confirmed through several numerical simulations and practical experiments. However, there were some severe constraints to implement the method, such as the requirement of the ASPR-ness of the plant. Thus, somewhat serious problems have remained with regard to the applicability of the SAC to the wider class of the controlled plant.

In this research, problems for design of SAC systems were considered to expand the applicable class of the controlled plant of the SAC schemes.

In Chapter 2, a basic concept of SAC for ASPR plants was reviewed for the sake of brevity of discussions in the following chapters. The CGT theory and the ASPR-ness of the plant were discussed. A basic algorithm of the SAC and stability of the control system were also given in this chapter.

To expand the applicability of the SAC to plants not satisfying ASPR condition, design schemes of compensator (which make non-ASPR plants ASPR in the sense that the resulting augmented plant with compensators is ASPR) were presented in Chapter 3. Systematic design schemes for a parallel feedforward compensator (PFC) were given in this chapter for both single-input/single-output and multi-input/multi-output minimum phase plants with unknown orders but known relative degrees. Robust design schemes of compensators (PFC and pre-compensator) using frequency domain analysis were also presented for plants with multiplicative plant uncertainties which might be non-minimum phase in this chapter.

Chapter 4 presented a robust SAC algorithm for plants with state-dependent disturbances. SAC is generally robust with regard to disturbances. This robustness of the SAC is due to the SAC's ability to make a high gain adaptive output feedback control system subject to the ASPR-ness of the controlled plant. However, in the case where *large* external disturbances and/or state-dependent disturbances are present, of course, the control performance might become worse. By adding a robust adaptive control term to the original SAC algorithm, the control performance of the SAC system can be significantly improved.

Controlling the large-scale system is difficult due to unknown interconnection of each subsystem. The decentralized adaptive control methods are an effective way to handle the large-scale system. In Chapter 5, decentralized SAC schemes for large-scale systems with unknown interconnections were presented. The stability conditions of the control system corresponding to *M-matrix* condition and *renge* condition were clarified.. It was also shown that the use of robust SAC scheme given in Chapter 4 in decentralized SAC algorithm was effective in eliminating the affects of interconnections.

In Chapter 6, a modified SAC algorithm with a derivative control term aimed at robust performance in transient state was presented. The SAC is generally applied to an augmented plant with PFCs. In the case, to obtain a better tracking performance for the original plant output, we have to design a PFC with smaller gain. Contrarily, setting the PFC gain small leads to an increase in the adaptive feedback gain so that the tracking performance in the transient state sometimes becomes worse. The proposed method significantly affected this situation.

In each chapter, the effectiveness of proposals were confirmed through several types of numerical simulations.

The proposals in this thesis expand the applicability of the SAC scheme to a wider class of controlled plants including both minimum and non-minimum phase non-ASPR plants with unmodelled dynamics, large-scale systems, plants with unknown disturbances, and so on. The obtained schemes for SAC system design will be very useful and powerful for practical plants with several uncertainties.

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