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On the complete Kählerity of complex spaces

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Abstract

We prove that on every reduced Stein space there exists a complete Kähler metric with a globally defined real-analytic potential function. We also give two results which are generalizations of a theorem of Yasuoka on the complete Kähler exhaustion.

0. Introduction

Grauert [6] proved that on every Stein manifold there exists a complete Kähler metric with a globally defined real-analytic potential function. A complex manifold on which there exists a complete Kähler metric is not necessarily Stein.

In this paper we first give a definition of the completeness of the Kähler metric on a reduced complex space and prove that on every reduced Stein space there exists a complete Kähler metric with a globally defined real-analytic potential function.

Let X be a reduced complex space and $\{D_i\}_{i=1}^{\infty}$ a sequence of complete Kähler open sets of X such that $D_i \subset\subset D_{i+1}$ for every $i \geq 1$. Yasuoka [17] proved that the union $D := \bigcup_{i=1}^{\infty} D_i$ is locally Stein at every $p \in \partial D$ if X is a complex manifold. Therefore D is Stein if X is a Stein manifold by Docquier-Grauert [4]. The second author [9,10] generalized this result for unramified regions over a Stein manifold or over a complex projective space $\mathbf{P}^n$.

We consider the normal complex space X which satisfies the condition that for every $p \in S(X)$ there exist a neighborhood U of p, a complex manifold M and an open and finite holomorphic surjection $\varphi : M \to U$. We give two results on the local Steinness of D which are generalizations of the theorem of Yasuoka [17].

The results of this paper were announced in [2,3] in somewhat restricted forms.

1. Definitions

Throughout this paper all complex spaces are supposed to be second countable. We denote by $S(X)$ the singular locus of a complex space X.

Let X be a reduced complex space and D an open set of X. D is said to be *locally Stein* at $p \in \partial D$

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if there exists a neighborhood U of p such that $U \cap D$ is Stein. Let A be a thin set in X. A point $p \in \partial D \cap A$ is said to be *removable* along A if there exists a neighborhood U of p such that $U - A \subset D$.

We denote by d_g the distance function induced by the Hermitian metric g on a (not necessarily connected) complex manifold. $d_g(x,y) = +\infty$ if there exists no path joining x and y. We denote by $l_g(\gamma)$ the length of the piecewise smooth path γ measured by g. We denote by Ω_g the Kähler form associated to g.

Let X be a reduced complex space. A Hermitian metric g on $X-S(X)$ is called a *Kähler metric* on X if for every $p \in X$ there exist a neighborhood U of p and a C^∞ strictly plurisubharmonic function $\lambda : U \to \mathrm{R}$ such that $\sqrt{-1}\,\partial\bar{\partial}\lambda = \Omega_g$ on $U-S(X)$ (Grauert [7], p.346). Such λ is called a (local) *potential function*. A Kähler metric g on X is said to be *complete* if every Cauchy sequence of points in $X-S(X)$ with respect to the distance d_g converges in X. X is said to be *complete Kähler* if there exists a complete Kähler metric on X.

2. Complete Kähler metric on a Stein space

We prove the following theorem which is a generalization of Satz 7 of Grauert [6].

THEOREM 1. *On every reduced Stein space X there exists a complete Kähler metric g with a globally defined real-analytic potential function.*

PROOF. There exists a real-analytic strictly plurisubharmonic exhaustion function $\psi : X \to \mathrm{R}$ by Narasimhan [14]. Let $\lambda := e^{\psi} : X \to \mathrm{R}$. Let g be the Hermitian metric on $X-S(X)$ defined by the equation that $\sqrt{-1}\,\partial\bar{\partial}\lambda = \Omega_g$ on $X-S(X)$. Then g is a Kähler metric on X. By the similar calculation of the proof of Proposition 16.5 of [13] we have the inequality that $g_{\gamma(t)}(\gamma'(t),\gamma'(t))^{1/2} \geq (e^{\psi(\gamma(t))}/2)^{1/2} \cdot d\psi(\gamma(t))/dt$ for every piecewise smooth path $\gamma : [a,b] \to X-S(X)$. From this we obtain the inequality that $d_g(p,q) \geq \sqrt{2}\,(e^{\psi(q)/2} - e^{\psi(p)/2})$ for every $p,q \in X-S(X)$. Let $\{p_\nu\}_{\nu=1}^{\infty}$ be an arbitrary Cauchy sequence in $X-S(X)$ with respect to d_g. Assume that the sequence $\{\psi(p_\nu)\}_{\nu=1}^{\infty}$ is not bounded above. Then $\sup_{\nu \geq 1} d_g(p_1, p_\nu) \geq \sup_{\nu \geq 1} \sqrt{2}\,(e^{\psi(p_\nu)/2} - e^{\psi(p_1)/2}) = +\infty$. It is a contradiction. Therefore there exists $M > 0$ such that $\psi(p_\nu) < M$ for every $\nu \geq 1$. Since $\{\psi < M\} \subset\subset X$, there exist a subsequence $\{p_{l_\nu}\}_{\nu=1}^{\infty}$ of $\{p_\nu\}_{\nu=1}^{\infty}$ and $p_0 \in X$ such that $\lim_{\nu\to\infty} p_{l_\nu} = p_0$ in X. There exist a neighborhood V of p_0, an open set D of some C^N, a holomorphic embedding $\varphi : V \to D$ and a C^∞ strictly plurisubharmonic function $\hat{\lambda} : D \to \mathrm{R}$ such that $\lambda = \hat{\lambda} \circ \varphi$ on V. Let $\hat{g}$ be the Kähler metric on D defined by the equation that $\sqrt{-1}\,\partial\bar{\partial}\hat{\lambda} = \Omega_{\hat{g}}$ on D. It holds that $g = \varphi^*\hat{g}$ on $V-S(X)$. Let $B_{\hat{g}}(\varphi(p_0), \varepsilon) := \{x \in D \mid d_{\hat{g}}(\varphi(p_0), x) < \varepsilon\}$ for every $\varepsilon > 0$. There exists $\varepsilon_0 > 0$ such that $B_{\hat{g}}(\varphi(p_0), \varepsilon_0) \subset\subset D$. Take an arbitrary $\varepsilon \in (0, \varepsilon_0)$. There exists $N_0 \in \mathrm{N}$ such that $p_{l_\nu} \in \varphi^{-1}(B_{\hat{g}}(\varphi(p_0), \varepsilon/2))$ for every $\nu \geq N_0$ and that $d_g(p_\mu, p_\nu) < \varepsilon/2$ for every $\mu, \nu \geq N_0$. Take an arbitrary $\nu \geq N_0$. Since $d_g(p_{l_\nu}, p_\nu) < \varepsilon/2$, there exists a piecewise smooth path $\gamma : [a,b] \to X-S(X)$ such that $\gamma(a) = p_{l_\nu}$, $\gamma(b) = p_\nu$ and $l_g(\gamma) < \varepsilon/2$. Assume that $c := \sup\{s \in [a,b] \mid \gamma([a,s]) \subset \varphi^{-1}(B_{\hat{g}}(\varphi(p_0), \varepsilon))\} < b$. Since $\gamma(c) \in V - \varphi^{-1}(B_{\hat{g}}(\varphi(p_0), \varepsilon))$, $d_{\hat{g}}(\varphi(\gamma(c)), \varphi(p_0)) \geq \varepsilon$. $l_g(\gamma) \geq l_g(\gamma \mid_{[a,c]}) = l_{\hat{g}}(\varphi \circ (\gamma \mid_{[a,c]})) \geq d_{\hat{g}}(\varphi(p_{l_\nu}), \varphi(\gamma(c))) \geq d_{\hat{g}}(\varphi(p_0), \varphi(\gamma(c))) - d_{\hat{g}}(\varphi(p_0), \varphi(p_{l_\nu})) > \varepsilon - \varepsilon/2 = \varepsilon/2$. It is a contradiction. Therefore $c = b$. It follows that $d_{\hat{g}}(\varphi(p_\nu), \varphi(p_0)) = \lim_{t\to b-0} d_{\hat{g}}(\varphi(\gamma(t)), \varphi(p_0)) \leq \varepsilon$. Therefore $\lim_{\nu\to\infty} \varphi(p_\nu) = \varphi(p_0)$ in D. Since $\varphi : V \to \varphi(V)$ is homeomorphic, it holds that $\lim_{\nu\to\infty} p_\nu = p_0$ in X. Thus we proved the completeness of g. □

3. Lemmas

LEMMA 1. *Let X be a normal complex space and A a thin set in X of order 2. Then every locally Stein open set D of X has no boundary point removable along A.*

PROOF. Suppose that there exists $p \in \partial D \cap A$ removable along A. Then there exists a neighborhood U of p such that $U-A \subset D$ and that $U \cap D$ is Stein. There exists a sequence $\{p_\nu\}_{\nu=1}^{\infty} \subset U-A$ such that $\lim_{\nu\to\infty} p_\nu = p$. There exists $f \in \mathcal{O}(U \cap D)$ such that $\lim_{\nu\to\infty} |f(p_\nu)| = +\infty$. Since U is normal, there exists $\tilde{f} \in \mathcal{O}(U)$ such that $\tilde{f}=f$ on $U-A$. $|\tilde{f}(p)| = \lim_{\nu\to\infty} |\tilde{f}(p_\nu)| = +\infty$. It is a contradiction. □

The following lemma is an improvement of Lemma 3 of the first author's [1] .

LEMMA 2. *Let X be a normal complex space such that for every $p \in S(X)$ there exist a neighborhood U of p, a complex manifold M and an open and finite holomorphic surjection $\varphi : M \to U$. Let D be an open set of X. Assume that D is locally Stein at every $p \in \partial D - S(X)$ and that D has no boundary point removable along $S(X)$. Then D is locally Stein at every $p \in \partial D$.*

PROOF. Let $p \in \partial D \cap S(X)$. There exist a neighborhood U of p, a complex manifold M and an open and finite holomorphic surjection $\varphi : M \to U$. We may assume that U is connected and Stein. Then M is Stein by 73.1 of [11] ,p.313. $\tilde{S} := \varphi^{-1}(S(U))$ is a positive codimensional analytic set of M. Let V be a Stein neighborhood of p such that $V \subset\subset U$. V has no boundary point removable along $S(X)$ by Lemma 1. We can verify that $W := V \cap D$ is locally Stein at every $q \in \partial W - S(U)$ and has no boundary point removable along $S(U)$. Let $\tilde{W} := \varphi^{-1}(W)$. Take an arbitrary $r \in \partial \tilde{W} - \tilde{S}$. Since $\varphi(r) \in \partial W - S(U)$, there exists a neighborhood E of $\varphi(r)$ in U such that $E \cap W$ is Stein. $\varphi^{-1}(E) \cap \tilde{W} = \varphi^{-1}(E \cap W)$ is Stein by 73.1 of [11] . Hence $\tilde{W}$ is locally Stein at every $r \in \partial \tilde{W} - \tilde{S}$. We can also verify that $\tilde{W}$ has no boundary point removable along $\tilde{S}$. $\tilde{W}$ is locally Stein at every $r \in \partial \tilde{W}$ by Lemma of Ueda [16] , p.564, which is originally due to Grauert-Remmert [8] . $\tilde{W}$ is Stein by Docquier-Grauert [4] . Therefore W is Stein by 73. 1 of [11] . Thus we proved that D is locally Stein at every $p \in \partial D \cap S(X)$. □

LEMMA 3. *Let X be a reduced complex space. Let D_1 and D_2 be the open sets of X. Assume that for each $i=1,2$ there exists a Kähler metric g_i on $D_i - S(D_i)$ such that every Cauchy sequence of points in $D_i - S(D_i)$ with respect to the distance d_{g_i} converges in D_i. Then there exists a Kähler metric g on $D_1 \cap D_2$ such that every Cauchy sequence of points in $D_1 \cap D_2 - S(D_1 \cap D_2)$ with respect to the distance d_g converges in $D_1 \cap D_2$.*

PROOF. $g := g_1 + g_2$ is a Kähler metric on $D_1 \cap D_2 - S(D_1 \cap D_2)$. Since it holds that $d_{g_i} \leq d_g$ on $D_1 \cap D_2 - S(D_1 \cap D_2)$ for every $i = 1,2$, every Cauchy sequence in $D_1 \cap D_2 - S(D_1 \cap D_2)$ with respect to the distance d_g converges in $D_1 \cap D_2$. □

LEMMA 4. *Let X be a reduced complex space. Let $\{D_i\}_{i=1}^{\infty}$ be a sequence of open sets of X such that $D_i \subset\subset D_{i+1}$ for every $i \geq 1$. Assume that for every $i \geq 1$ there exists a Kähler metric g_i on $D_i - S(D_i)$ such that every Cauchy sequence of points in $D_i - S(D_i)$ with respect to the distance d_{g_i} converges in D_i. Then*

$D:=\cup_{i=1}^{\infty}D_i$ is locally Stein at every $p\in \partial D-S(X)$.

Proof. Let $p\in \partial D-S(X)$. Let U be a Stein neighborhood of p such that $U\cap S(X)=\emptyset$. There exists a sequence $\{U_i\}_{i=1}^{\infty}$ of Stein open sets such that $U_i\subset\subset U_{i-1}$ for every $i\geq 1$ and that $U=\cup_{i=1}^{\infty}U_i$. By Theorem 1 or by Satz 7 of Grauert [6] there exists a complete Kähler metric on U_i for every $i\geq 1$. By Lemma 3 there exists a complete Kähler metric on $D_i\cap U_i$ for every $i\geq 1$. Since it holds that $D_i\cap U_i \subset\subset D_{i-1}\cap U_{i-1}$ for every $i\geq 1$, $D\cap U=\cup_{i=1}^{\infty}(D_i\cap U_i)$ is Stein by Yasuoka [17] or by the second author's [9]. □

Lemma 5. *Let X be a normal complex space. Assume that for every $p\in S(X)$ there exist a neighborhood U of p, a complex manifold M and an open and finite holomorphic surjection $\varphi: M\to U$. Let $\{D_i\}_{i=1}^{\infty}$ be a sequence of complete Kähler open sets of X such that $D_i\subset\subset D_{i+1}$ for every $i\geq 1$. Then $D:=\cup_{i=1}^{\infty}D_i$ has no boundary point removable along $S(X)$.*

Proof. Suppose that there exists $p\in \partial D\cap S(X)$ removable along $S(X)$. There exist a neighborhood U of p, a complex manifold M and an open and finite holomorphic surjection $\varphi: M\to U$. We may assume that U is Stein and that $U-S(X)\subset D$. $T:=U-D$. The set $\varphi^{-1}(T)(\neq\emptyset)$ is thin in M of order 2. M is Stein by 73.1 of [11], p.313. There exists a sequence $\{U_i\}_{i=1}^{\infty}$ of Stein open sets such that $U_i\subset\subset U_{i+1}$ for every $i\geq 1$ and that $U=\cup_{i=1}^{\infty}U_i$. Take an arbitrary $i\geq 1$. There exists a complete Kähler metric $\mathfrak{g}$ on D_i. There exist an open covering $\{V_\alpha\}$ of $D_i\cap U_i$ and C^∞ strictly plurisubharmonic functions $\lambda_\alpha: V_\alpha\to \mathbf{R}$ such that $\sqrt{-1}\,\partial\bar\partial\lambda_\alpha=\Omega_{\mathfrak{g}}$ on $V_\alpha-S(D_i\cap U_i)$ for every α. We can define the real (1,1)-form ω on $\varphi^{-1}(D_i\cap U_i)$ by the equations that $\omega=\sqrt{-1}\,\partial\bar\partial(\lambda_\alpha\circ\varphi)$ on $\varphi^{-1}(V_\alpha)$. Let g be the positive semi-definite Hermitian form on $\varphi^{-1}(D_i\cap U_i)$ corresponding to ω. It holds that $g=\varphi^*\mathfrak{g}$ on $\varphi^{-1}(D_i\cap U_i)-A$, where $A:=\varphi^{-1}(S(D_i\cap U_i))$. Since $\varphi^{-1}(U_i)$ is Stein by 73.1 of [11], p.313, there exists a complete Kähler metric h on $\varphi^{-1}(U_i)$ by Theorem 1 or by Satz 7 of Grauert [6]. $f:=g+h$ is a Kähler metric on $\varphi^{-1}(D_i\cap U_i)$. Take any $x,y\in \varphi^{-1}(D_i\cap U_i)-A$ such that $d_f(x,y)<+\infty$. Let $\epsilon>0$. We can prove the existence of such a piecewise smooth path $\gamma:[a,b]\to\varphi^{-1}(D_i\cap U_i)-A$ that $\gamma(a)=x$, $\gamma(b)=y$ and $l_f(\gamma)<d_f(x,y)+\epsilon$. Since $g_{\gamma(t)}(\gamma'(t),\gamma'(t))^{1/2}+h_{\gamma(t)}(\gamma'(t),\gamma'(t))^{1/2}\leq\sqrt{2}f_{\gamma(t)}(\gamma'(t),\gamma'(t))^{1/2}$, we have that $d_{\mathfrak{g}}(\varphi(x),\varphi(y))+d_h(x,y)\leq l_{\mathfrak{g}}(\varphi\circ\gamma)+l_h(\gamma)\leq\sqrt{2}\,l_f(\gamma)\leq\sqrt{2}\,d_f(x,y)+\sqrt{2}\,\epsilon$. By letting $\epsilon\to+0$ we obtain that $d_{\mathfrak{g}}(\varphi(x),\varphi(y))+d_h(x,y)\leq\sqrt{2}\,d_f(x,y)$. Using this inequality we can prove the completeness of the distance d_f. Thus we proved that $\varphi^{-1}(D_i\cap U_i)$ is complete Kähler for every $i\geq 1$. Since it holds that $\varphi^{-1}(D_i\cap U_i)\subset\subset\varphi^{-1}(D_{i+1}\cap U_{i+1})$ for every $i\geq 1$, the union $\cup_{i=1}^{\infty}\varphi^{-1}(D_i\cap U_i)=\varphi^{-1}(D\cap U)=M-\varphi^{-1}(T)$ is Stein by Yasuoka [17] or by the second author's [9]. It contradicts Lemma 1. □

Lemma 6. *Let X be a normal complex space. Assume that $S(X)$ is discrete and that for every $p\in S(X)$ there exist a neighborhood U of p, a complex manifold M and an open and finite holomorphic surjection $\varphi: M\to U$. Let $\{D_i\}_{i=1}^{\infty}$ be a sequence of open sets of X such that $D_i\subset\subset D_{i+1}$ for every $i\geq 1$. Assume that for every $i\geq 1$ there exists a Kähler metric $\mathfrak{g}_i$ on $D_i-S(D_i)$ such that every Cauchy sequence of points in $D_i-S(D_i)$ with respect to the distance $d_{\mathfrak{g}_i}$ converges in D_i. Then $D:=\cup_{i=1}^{\infty}D_i$ has no boundary point removable along $S(X)$.*

Proof. Suppose that there exists $p\in \partial D\cap S(X)$ removable along $S(X)$. There exist a neighborhood U of p, a complex manifold M and an open and finite holomorphic surjection $\varphi: M\to U$. We may assume that U is Stein, that $U \cap S(X) = \{p\}$ and that $U - S(X) \subset D$. Then M is Stein by 73.1 of [11]. There exists a sequence $\{U_i\}_{i=1}^{\infty}$ of Stein open sets such that $U_i \subset\subset U_{i-1}$ for every $i \geq 1$ and that $U = \cup_{i=1}^{\infty} U_i$. Take an arbitrary $i \geq 1$. There exists a Kähler metric $\mathfrak{g}_i$ on $D_i - S(X)$ such that every Cauchy sequence in $D_i - S(X)$ with respect to the distance $d_{\mathfrak{g}_i}$ converges in D_i. Since $\varphi^{-1}(U_i)$ is Stein by 73.1 of [11], there exists a complete Kähler metric h_i on $\varphi^{-1}(U_i)$ by Theorem 1 or by Satz 7 of Grauert [6]. $g_i := \varphi^* \mathfrak{g}_i + h_i$ is a Kähler metric on $\varphi^{-1}(D_i \cap U_i)$. Since it holds that $d_{\mathfrak{g}_i}(\varphi(x), \varphi(y)) + d_{h_i}(x,y) \leq \sqrt{2}\, d_{g_i}(x,y)$ for any $x, y \in \varphi^{-1}(D_i \cap U_i)$, every Cauchy sequence in $\varphi^{-1}(D_i \cap U_i)$ with respect to the distance d_{g_i} converges in $\varphi^{-1}(D_i \cap U_i)$. Therefore g_i is a complete Kähler metric on $\varphi^{-1}(D_i \cap U_i)$. Since it holds that $\varphi^{-1}(D_i \cap U_i) \subset\subset \varphi^{-1}(D_{i+1} \cap U_{i+1})$ for every $i \geq 1$, the union $\cup_{i=1}^{\infty} \varphi^{-1}(D_i \cap U_i) = \varphi^{-1}(D \cap U) = M - \varphi^{-1}(p)$ is Stein by Yasuoka [17] or by the second author's [9]. Since dim M = dim $U \geq 2$ and $\varphi^{-1}(p)$ is finite, it contradicts Lemma 1. □

Lemma 7. *Let M and X be reduced complex spaces and $\varphi : M \to X$ a finite holomorphic surjection. Let D be an open set of X. If D is locally Stein at every $p \in \partial D$, then $\varphi^{-1}(D)$ is locally Stein at every $q \in \partial \varphi^{-1}(D)$.*

Proof. Take an arbitrary $q \in \partial \varphi^{-1}(D)$. Since $\varphi(q) \in \partial D$, there exist a neighborhood U of $\varphi(q)$ such that $U \cap D$ is Stein. $\varphi^{-1}(U)$ is a neighborhood of q. $\varphi^{-1}(U) \cap \varphi^{-1}(D) = \varphi^{-1}(U \cap D)$ is Stein by 73.1 of [11], p.313. □

4. Complete Kähler exhaustion

We give two theorems which are generalizations of Theorem 3 of Yasuoka [17].

Theorem 2. *Let X be a normal complex space such that for every $p \in S(X)$ there exist a neighborhood U of p, a complex manifold M and an open and finite holomorphic surjection $\varphi : M \to U$. Let $\{D_i\}_{i=1}^{\infty}$ be a sequence of complete Kähler open sets of X such that $D_i \subset\subset D_{i+1}$ for every $i \geq 1$. Then the union $D := \cup_{i=1}^{\infty} D_i$ is locally Stein at every $p \in \partial D$.*

Proof. By Lemmas 2,4 and 5. □

Corollary. *Let X be a weighted projective space. Let $\{D_i\}_{i=1}^{\infty}$ be a sequence of complete Kähler open sets of X such that $D_i \subset\subset D_{i+1}$ for every $i \geq 1$. Let $D := \cup_{i=1}^{\infty} D_i$. Then D is a Stein open set of X or $D = X$.*

Proof. There exist an integer n and a finite subgroup G of Aut (P^n) such that X is biholomorphic to the quotient complex space P^n / G by [11], p.208. X is normal by 72.5 of [11], p.312. The natural projection $\varphi : \mathrm{P}^n \to X$ is finite, open and surjective. D is locally Stein at every $p \in \partial D$ by Theorem 2. $\tilde{D} := \varphi^{-1}(D)$ is locally Stein by Lemma 7. Therefore $\tilde{D}$ is Stein or $\tilde{D} = \mathrm{P}^n$ by the theorem of Fujita-Takeuchi-Kieselman on Levi's problem [5,15,12]. In case that $\tilde{D}$ is Stein, D is Stein by 73.1 of [11], p.313. □

THEOREM 3. *Let X be a normal complex space such that $S(X)$ is discrete and that for every $p \in S(X)$ there exist a neighborhood U of p, a complex manifold M and an open and finite holomorphic surjection $\varphi : M \to U$. Let $\{D_i\}_{i=1}^{\infty}$ be a sequence of open sets of X such that $D_i \subset\subset D_{i+1}$ for every $i \geq 1$. Assume that for every $i \geq 1$ there exists a Kähler metric g_i on $D_i - S(D_i)$ such that every Cauchy sequence of points in $D_i - S(D_i)$ with respect to the distance d_{g_i} converges in D_i. Then the union $D := \bigcup_{i=1}^{\infty} D_i$ is locally Stein at every $p \in \partial D$.*

PROOF. By Lemmas 2,4 and 6.□

References

[1] M.Abe, *Domains in a two dimensional Stein space with quotient singularities*, The Bulletin of Oshima National College of Maritime Technology 21(1988),103-110.

[2] M.Abe, *A generalization of Yasuoka's theorem on the complete Kähler exhaustion*, Proceedings of the Second Korean-Japanese Colloquium on Finite of Infinite Dimensional Complex Analysis, Fukuoka, Japan, 27-29 July 1994(eds. Kajiwara, J.,Kazama, H. and Shon, K. H.), Kyushu University Co-operation, Fukuoka, 1994, pp.1-5.

[3] M. Abe and H. Hamada, *On the open set exhausted by complete Kähler open sets in a Stein space with isolated singularities*, Proceedings of the Second Korean -Japanese Colloquium on Finite or Infinite Dimensional Complex Analysis, Fukuoka, Japan, 27-29 July 1994 (eds. Kajiwara, J., Kazama, H. and Shon, K. H.), Kyushu University Co-operation, Fukuoka, 1994, pp. 7-13.

[4] F. Docquier and H. Grauert, *Levisches Problem und Rungescher Satz für Teilgebiete Steinscher Mannigfaltigkeiten*, Math. Ann.140 (1960),94-123.

[5] R.Fujita, *Domaines sans point critique intérieur sur l'espace projectif complexe*, J. Math. Soc. Japan 15(1963), 443-473.

[6] H.Grauert, *Charakterisierung der Holomorphiegebiete durch die vollständige Kählersche Metrik*, Math. Ann. 131(1956), 38-75.

[7] H. Grauert, *Über Modifikationen und exzeptionelle analytische Mengen*, Math. Ann. 146 (1962), 331-368.

[8] H. Grauert and R.Remmert, *Konvexität in der komplexen Analysis. Nicht-holomorphkonvexe Holomorphiegebiete und Anwendungen auf die Abbildungstheorie*, Comm. Math. Helv. 31 (1956), 152-183.

[9] H. Hamada, *A domain exhausted by complete Kähler domains over a Stein manifold*, Mem. Fac. Sci. Kyushu Univ. 39 (1985),105-111.

[10] H. Hamada, *A domain exhausted by complete Kähler domains over* $\mathbf{P}^n$, Math. Rep. College Gen. Educ. Kyushu Univ. 19(1993), 26-30.

[11] L. Kaup and B. Kaup, *Holomorphic functions of several variables*, Walter de Gruyter, Berlin-New York, 1983.

[12] C. O. Kieselman, *On entire functions of exponential type and indicators of analytic functionals*, Acta Math. 117 (1967), 1-35.

[13] S. Nakano, *Theory of functions of several variables* (in Japanese), Asakura Shoten, Tokyo, 1981.

[14] R. Narasimhan, *The Levi problem for complex spaces*, Math. Ann. 142 (1961), 355-365.

[15] A. Takeuchi, *Domaines pseudoconvexes infinis et la mètrique riemannienne dans un espace projectif*, J. Math. Soc. Japan 16 (1964), 159-181.

[16] T. Ueda, *Domains of holomorphy in Segre cones*, Publ. Res. Inst. Math. Sci. 22 (1986), 561-569.

[17] T. Yasuoka, *A domain exhausted by complete Kähler domains*, Kobe J. Math. 2 (1985), 57-64.