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A Study of a Class of Simple Salesforce Compensation Plans

Amiya K. Basu
Kalyanaram Gurumurthy

College of Commerce and Business Administration
Bureau of Economic and Business Research
University of Illinois Urbana-Champaign

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Amiya K. Basu, Professor
Department of Business Administration

Kalyanaram Gurumurthy
University of Texas at Dallas

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Abstract

This paper addresses the problem of finding the optimal salesforce compensation plan in an uncertain selling environment. A procedure is developed to determine the optimal compensation plan of the form $(A + Bx)^{\alpha}$ where x is the sales level. This class includes the commonly used linear compensation plan which consists of a salary and a straight commission, and also the optimal agency theoretic compensation plan in the cases studied. A comparative statics analysis is performed using numerical techniques to study how the linear and the agency theoretic compensation plans would be affected by changes in the selling environment, and the results are compared with earlier findings from the theoretical salesforce compensation literature. Also, the relative performance of the linear and the agency theoretic compensation plans is investigated. The simpler, linear plan is found to perform almost as well as the more complex agency theoretic plan in a wide range of situations.

1. INTRODUCTION & OBJECTIVES

Selling activities constitute a major expense of running a business. In 1981, U.S. companies spent about $150 billion on personal selling, far exceeding the $61 billinon spent on advertising that year (Kotler, 1986, p.499). Not surprisingly, salesforce management and salesforce compensation have received considerable attention from researchers. The existing research literature on salesforce compensation can be broadly classified as follows : (1) description of industry practice (e.g. Stanton & Buskirk, 1974; Steinbrink, 1978), (2) normative research using quantitative models (discussed next), and (3) assessing the external validity of normative models (e.g. John & Weitz, 1988 & 1989; Coughlan & Sen, 1988; Coughlan & Narasimhan, 1989). The present research belongs to the second category identified above.

Normative research on the determination of an optimal salesforce compensation plan started with the seminal work of Farley (1964) which addressed the problem of finding optimal commission rates for a salesperson selling multiple products. Tapiero & Farley (1975), Weinberg (1975, 1978), Albers (1980) and Srinivasan (1981) relaxed the assumptions originally made by Farley and extended the scope of the research. The salient features of this stream of research are the following : (1) the selling environment is deterministic, i.e. a given amount of sales effort always generates the same sales volume,[1] (2) the salesperson is responsible for a portfolio of products, and (3) the employing firm compensates the salesperson on a pure commission basis. The focus of the research was on finding the optimal commission rates and identifying when the rates should vary from product to product.

In a major departure from previous research, Basu, Lal, Srinivasan and Staelin (1985), henceforth called BLSS, applied the paradigm of agency theory, first developed in mathematical economics, to the problem of finding optimal salesforce compensation plans. The model used by BLSS deviated from the research stream outlined in the previous paragraph in the following ways : (1) the selling environment is uncertain, the sales generated being a random variable whose distribution depends on the salesperson's effort, (2) the salesperson is risk averse, and (3) the function relating the salesperson's earnings to sales generated is not restricted a priori, in contrast to previous research which used pure commission plans exclusively. The BLSS model considered the case where one salesperson sells a single product in a single time period, and the focus was on finding the shape of the optimal compensation plan, and analysing how it depends on parameters of the selling environment. For example, BLSS demonstrated that as the environment becomes more uncertain, the optimal compensation plan should have a larger ratio of salary to total compensation. Also, BLSS showed that the optimal compensation plan obtained from agency theory could be described as

[1] Weinberg (1975) discussed an uncertain selling environment. However, since the salesperson was considered to be risk neutral, uncertainty did not materially affect the solutions.

one consisting of a fixed salary and a sliding commission rate, and it could approximate several compensation plans commonly used in practice. This added strength to the claim that model assumptions were realistic.

The work by BLSS was the first of several applications of agency theory to the problem of finding optimal salesforce compensation plans. Lal (1986) studied the impact of delegating pricing responsibilities to the salesforce. Lal & Staelin (1986) and Rao (1988) analyzed the problem of designing compensation plans for a heterogeneous salesforce. These works are structurally similar to BLSS in that the firm and the salesperson(s) play a leader-follower game in **one period:** the firm offers the salesperson a compensation plan, and the salesperson responds by selecting **a level of effort for the period** under consideration.

The BLSS model of salesforce compensation, in spite of the elegance of the approach, yields compensation plans which are considerably more complex than plans used in practice. For example, in 1984, 29.8% of U.S. firms used salary and commission while 33.6% used salary plus bonuses to compensate their salesforces (Wilson & Bennet, 1986). While these plans can be approximated by those derived from agency theory, the departure can be significant over the range of possible sales volume. This difference can be attributed to the possibility that the agency theoretic plans are too complex for a firm to implement. Lal & Srinivasan (1988), henceforth called LS, provided a different explanation for this phenomenon. LS applied results derived by Holmstrom & Milgrom (1987) to finding optimal salesforce compensation plans in a dynamic environment where the time period for which the firm declares a compensation plan corresponds to a large number of periods for the salesperson for which he/she chooses level of effort and gets paid. The salesperson's choice of effort in response to the compensation plan declared by the firm evolves over these smaller periods, and he/she can engage in 'creative accounting', claiming compensation for sales in a period at a later time. This allows the salesperson to exploit nonlinearities in the compensation plan to the detriment of the profitability of the firm. In this situation, LS obtained the significant result that the optimal salesforce compensation plan here would be linear, i.e. consist of a salary plus a straight commission on sales.

In addition to incorporating a dynamic perspective, the LS model differed from the BLSS approach in the following significant details :

(1) The LS model used a constant absolute risk aversion utility function for the salesperson rather than the square-root utility function used by BLSS.

(2) The salesperson's utility function is multiplicatively separable in earnings and effort. In contrast, the BLSS model assumed additive separability.

(3) The sales generated follows a Weiner process in the LS model,leading to normally distributed sales. This is a realistic model of a selling environment where the salesperson calls on a large number of small accounts. In contrast, the BLSS model used the gamma and the

binomial distributions to model sales, and is better suited to describe the case with fewer, larger accounts.

In spite of the differences in model development, the LS and the BLSS model obtained very similar implications for how the compensation plan would be affected by changes in parameters of the selling environment.

The present research adopts the BLSS framework and assumes (unlike the LS model) that the time period is identical for the firm and the salesperson. The compensation plan is restricted to be of the form $(A + Bx)^\alpha$ where x is the sales level. This class of compensation plans includes, in addition to the linear plan, the agency theoretic compensation plan in a wide range of situations.

Like BLSS, the present research considers the case of one salesperson selling a single product in one time period. The objectives of the research are as follows :

(1) Develop a procedure for determining the optimal compensation plan of the form $(A + Bx)^\alpha$. This is done here for the case where the salesperson has a power utility function for earnings, using a combination of analytical and numerical techniques.

(2) Compare the firm's expected profits under the agency theoretic compensation plan with profits from a linear compensation plan. If the linear plan is found to perform almost as well as the agency theoretic plan, that will demonstrate that even under the assumptions of the BLSS model the managerial practice of using linear compensation plans is close to optimal, and will complement the findings of Lal & Srinivasan (1988).

(3) The final objective of the study is to determine the robustness of the findings of the BLSS study. For example, this study aims to investigate whether the result derived by BLSS that with an increase in uncertainty, the compensation plan should have a higher ratio of salary to total income, would hold for a linear compensation plan, or even for the agency theoretic plan if the salesperson's utility function for income is not restricted to be a square-root function (as assumed by BLSS). A numerical 'experiment' is conducted to perform comparative statics analysis for the linear and the agency theoretic compensation plans in order to address these questions.

The paper is organized as follows. Section 2 presents the notations used throughout the paper and lists the basic assumptions of the model development (additional, more specific assumptions are made in later sections). The BLSS model and its relation to the present work are discussed briefly. Section 3 formulates the problem of finding an optimal compensation plan of the form $(A + Bx)^\alpha$, and discusses how this problem can be solved using a combination of analytical and numerical techniques. Section 4 presents numerical evidence comparing performances of the linear and agency theoretic compensation plans, and investigates how these plans are affected by changes in parameters of the selling environment such as uncertainty. Section 5 summarizes the findings of the paper.

For clarity of exposition, the proofs and much of the mathematical development used in the paper have been placed in the appendices. Appendix A contains the proofs to the propositions developed in the paper while Appendices B & C derive mathetical results used in Appendix A.

2. ASSUMPTIONS, NOTATIONS AND MODEL DEVELOPMENT

In this section we develop the notations used in the rest of the paper, and present the major assumptions of the model development. The assumptions made and their presentation closely follow the BLSS paper and details are omitted here unless there is a difference between the approaches.

Notations :

x = level of sales.

t = time (effort) spent by the salesperson in the period. The firm cannot observe t directly.

$f(x|t)$ = probability density function of x given t.

$F(x|t)$ = cumulative probability distribution of x given t.

$f_t(x|t)$ and $F_t(x|t)$ represent $\frac{d}{dt}f(x|t)$ and $\frac{d}{dt}F(x|t)$, respectively. To simplify notations, we will sometimes used f, f_t, F and F_t to represent $f(x|t)$, $f_t(x|t)$, $F(x|t)$ and $F_t(x|t)$, respectively.

$g(t) = E(x|t)$, and we will usually denote this by g.

Assumptions :

(a) The salesperson's utility function for earnings s and effort t is the additively separable function, $U(s) - V(t)$.

(b) $U(s) \geq 0$, $U'(s) > 0$, $U''(s) < 0$.

(c) $V(t) \geq 0$, $V'(t) > 0$, $V''(t) > 0$.

We will sometimes denote $V(t)$ by V, etc.

(d) The firm's objective is to maximize expected profit in the single time period under consideration.

(e) c, the marginal cost of production and distribution expressed as a fraction of price, is constant.

(f) The sales-effort response function is equally well known to the salesperson and the sales manager (acting on behalf of the firm).

(g) The sales manager acting on behalf of the firm knows the salesperson's utility function.

(h) x depends on the effort devoted by the salesperson, the marketing mix of the firm, and the uncertain marketing environment.

(i) $f(x|t)$ satisfies the condition, $\dfrac{f_t(x|t)}{f(x|t)} = K_0(x - g(t))$, where K_0 is a positive constant.

Assumption (i) is more restrictive than its counterpart in the BLSS approach regarding $F_t(x|t)$ (assumption (i), BLSS). However, it is satisfied by the gamma and the binomial density functions used by BLSS. (An important consequence of assumption (i) is, $\frac{f_t}{f}$ is strictly increasing in x. This property has been extensively used to derive the proofs in this paper. Appendix C lists the specific results used in this study.)

As in the BLSS model, the firm and the salesperson here are assumed to participate in a leader-follower game where the firm declares a compensation plan $s(x)$ relating the salesperson's earnings to the sales level. The salesperson responds by choosing the level of effort t which maximizes his/her expected utility, i.e. the salesperson selects

$$t = \operatorname{argmax} \int U(s(x)) f(x|t) dx - V(t). \tag{1}$$

The salesperson will agree to be employed by the firm if and only if he/she can derive an expected utility of at least m. As BLSS (and Holmstrom, 1979), we make the assumption that salesperson's choice of t is uniquely determined by the first order condition for (1), i.e.

(j) given $s(x)$, the salesperson's choice of t can be uniquley determined by solving

$$\int U(s(x))\, f_t(x|t)\, dx = V'(t). \tag{2}$$

Model Development. Using the approach developed by Grossman and Hart (1983), the firm's problem addressed by BLSS can be expressed in two stages as follows :[2]

Problem P1.

Stage 1. For each level of effort t, find the compensation plan $s(x)$ which will induce the salesperson to devote t at the least expected cost to the firm, i.e.

$$\min_{s(x)} \int s(x) f(x|t) dx, \tag{3}$$

such that

$$\int U(s(x)) f(x|t) dx \geq m, \tag{4}$$

and

$$\int U(s(x)) f_t(x|t) dx = V'(t). \tag{5}$$

[2]This approach is presented in more detail in Mathematical Appendices B & C to BLSS.

Let $s_t(x)$ denote the compensation plan obtained here.
Compute the expected profit of the firm, $\pi(t)$, corresponding to $s_t(x)$, i.e.

$$\pi(t) \quad = \quad \int [(1-c)x - s_t(x)] f(x|t) dx \; = \; (1-c)g(t) \; - \int s_t(x) f(x|t) dx. \tag{6}$$

Stage 2. Choose t to maximize $\pi(t)$. The corresponding compensation plan, denoted by $s^*(x)$, will be optimal for the firm.

Assuming that a Lagrangean solution exists to the problem defined by (3) - (5) and that it is globally optimal, BLSS demonstrated that $s_t(x)$,the optimal compensation plan corresponding to t, is given by

$$\frac{1}{U'(s)} \quad = \quad \lambda + \mu \frac{f_t(x|t)}{f(x|t)}, \tag{7}$$

where $\lambda \geq 0$ and μ are Lagrangean multipliers for constraints (4) and (5) respectively. λ and μ depend on t. As a consequence of assumption (i), equation(7) can be restated as

$$\frac{1}{U'(s)} \quad = \quad a + bx, \tag{8}$$

where a and b do not depend on x. Equation (8) defines the shape of the optimal compensation plan for a given t, which is the same for all values of t including the optimal t (a and b depend on t.).

As shown by BLSS, the nature of the optimal compensation plan for three commonly used utility functions for income can now be expressed as follows:
(1) If the salesperson's utility for earnings is the power function

$$U(s) \quad = \quad \frac{1}{\delta} s^{\delta}, \quad 0 < \delta < 1, \tag{9}$$

then

$$s^*(x) \quad = \quad [A + Bx]^{1/(1-\delta)}. \tag{10}$$

(2) If the utility function is logarithmic, i.e.

$$U(s) \quad = \quad \ln(s), \tag{11}$$

then

$$s^*(x) \quad = \quad A + Bx. \tag{12}$$

(3) If the salesperson has constant absolute risk aversion for income, i.e. the utility function can be expressed as

$$U(s) \quad = \quad -e^{-s}, \tag{13}$$

then

$$s^*(x) \quad = \quad \ln[A + Bx]. \tag{14}$$

It is interseting to note that these three utility functions are all concave and difficult to distinguish empirically. However, they give rise to compensation plans ranging from strictly concave to strictly convex. The wide acceptance of the linear compensation plan in industry practice may conceivably be due to the firm's inability to identify the salesperson's true utility function and the consequent decision to use the intermediate, linear compensation plan.

Another possible explanation for the use of the linear compensation plan is the bounded rationality of the salesperson. For example, let us consider the power utility function with $\delta = 2/3$. $s^*(x)$ will have the form $(A + Bx)^3$, i.e. large, positive values of x will have a large impact on the salesperson's computation of expected utility. If the salesperson with bounded rationality truncates the probability distribution of x while assessing the employment contract, he/she will underestimate expected utility for the employment. The firm would have to counter this by offering greater compensation for sales achieved in order to make the compensation plan acceptable to the salesperson. The linear compensation plan relies less on extreme values of x and is likely to be more robust.

In the present research, we ignore the possible explanations for adopting the linear compensation plan outlined above, and use the BLSS framework which favors the agency theoretic compensation plan. The objective here is to establish that even in this situation, the linear compensation plan would perform reasonably well.

3. MODEL DEVELOPMENT WITH THE COMPENSATION PLAN $(A + Bx)^\alpha$

3.1 Problem Formulation. In the rest of the paper, the following assumption is made about the salesperson's utility function :
(k) The salesperson's utility function for income, $U(s)$ is the power utility function given by equation(9). If $s < 0$, then $U(s) = -\infty$.

As a consequence of assumption (k), any compensation plan which involves a possibility of monetary loss for the salesperson is infeasible.

We will now develop a procedure for finding the best compensation plan from the class of plans of the form

$$s(x) \quad = \quad (A + Bx)^\alpha, \quad \text{where} \quad 1 \leq \alpha \leq \frac{1}{1-\delta}. \tag{15}$$

We will use (A, B) to represent the compensation plan given by (15). Following BLSS, we will call A the salary parameter of the compensation plan, and B the commission rate parameter.

If $\alpha = 1$, $s(x)$ is the linear compensation plan. As shown in the previous section, the Lagrangean solution to the agency theoretic problem, if it exists, corresponds to $\alpha = \dfrac{1}{1-\delta}$. As $\delta \to 0$, the power utility function given by equation(9) converges to the logarithmic function given by equation(11), and $s^*(x)$ becomes linear. The study of the class of compensation plans given by equation(15) allows us to examine the relative performances of the linear and the agency theoretic compensation plans as δ deviates from 0.

Let

$$Z = \int s(x) f\, dx = \int (A + Bx)^{\alpha} f\, dx, \quad \text{the salesperson's expected earnings,} \tag{16}$$

$$E_1 = E[U(s(x))|t] = \frac{1}{\delta} \int (A + Bx)^{\alpha\delta} f\, dx, \tag{17}$$

$$E_2 = \frac{d}{dt} E[U(s(x))|t] = \frac{1}{\delta} \int (A + Bx)^{\alpha\delta} f_t dx. \tag{18}$$

Then, restricting our attention to $s(x)$ of the form given by equation(15), the firm's problem can be expressed in two stages as follows :

Problem P2.

Stage 1. For each t, choose (A, B) which will induce effort t from the salesperson at the least expected cost to the firm, i.e.

$$\min_{A,B} Z \tag{19}$$

subject to

$$E_1 \geq m + V, \tag{20}$$

$$E_2 = V'. \tag{21}$$

It can be easily shown that an optimal solution (A, B) exists to the problem defined by (19) - (21). Let $Z(t)$ denote the minimum Z obtained here.

Compute the firm's expected profit $\pi(t)$ corresponding to this compensation plan :

$$\pi(t) = (1 - c)g(t) - Z(t). \tag{22}$$

Stage 2. Select t to maximize $\pi(t)$.

Stage 1 and stage 2 of P2 are discussed in sections 3.2 and 3.3, respectively.

3.2 Finding Optimal (A, B) For A Given t. The firm can induce the salesperson to devote zero effort at the least cost if the compensation plan pays $U^{-1}(m)$ for all values of x. In what follows, we will implicitly assume that inducing $t = 0$ is not optimal for the firm and will only consider $t > 0$.

We now make the following additional assumptions regarding the probability density function $f(x|t)$:

(l) $f(x|t)$ is defined on $0 < x < \infty$, and it is a strictly positive continuous function of x for $0 < x < \infty$.

(m) If $A \geq 0$, $B \geq 0$, and at least one of A and B is strictly positive, then $E\{x^a(A+Bx)^b|t\} < \infty$ for any $a \geq 0$, and $b > -1$.

For example, the gamma function used by BLSS has property (m) for $q \geq 1$.

We now state two propositions about properties of the optimal solution to the stage 1 problem defined by (19) - (21).

Proposition 1. A feasible compensation plan must have $A \geq 0$ and $B > 0$.

Assumptions (k) & (l) jointly dictate the nonnegativity of A and B while $B > 0$ is necessary to induce the salesperson to provide a positive effort.

Proposition 2. A locally optimal solution to the problem defined by (19) - (21) must satisfy at least one of the following two conditions : (i)$A = 0$. (ii) $E_1 = m + V$.

This result follows from the fact that it is possible to reduce A and change B simultaneously such that equation(21) is satisfied, thereby inducing the salesperson to provide effort t. This process reduces both Z and E_1, and can be carried on indefinitely unless one of the two conditions of proposition 2 is satisfied.

To summarize, a locally optimal solution (A, B) to the firm's problem defined by equations (19)-(21) will satisfy the following conditions :

$$A \geq 0, \quad B > 0, \tag{23}$$

$$E_2 = \frac{1}{\delta}\int (A + Bx)^{\alpha\delta} f_t dx = V', \tag{24}$$

$$E_1 = \frac{1}{\delta}\int (A + Bx)^{\alpha\delta} f dx \geq m + V, \tag{25}$$

and if $A > 0$,

$$E_1 = \frac{1}{\delta}\int (A + Bx)^{\alpha\delta} f dx = m + V. \tag{26}$$

Proposition 3. If (A, B) satisfies (23)-(26) then (A, B) is unique, and is the globally optimal solution to the firm's problem defined by (19) - (21).

As shown in Appendix A, it is always possible to construct (A, B) which satisfies (23)-(26). A brief outline of the procedure to accomplish that is now presented. Consider the set of compensation plans (A, B) which satisfies $E_1 = K > 0$. This is a nonempty set and any member of this set satisfies the conditions, $0 \leq A \leq AM$ and $0 \leq B \leq BM$, where $E_1(AM, 0) = K$, $E_1(0, BM) = K$. For this set, as A increases, B and E_2 decrease until they both become 0 for the plan $(AM, 0)$. E_2 is largest for the compensation plan $(0, BM)$. Let $E_{2m}(K)$ denote this largest value of E_2. E_{2m} is a strictly increasing function of K.

Let us consider $K = m + V$. The following three cases are possible :
(1) $E_{2m}(m+V) > V'$. Here, the optimal compensation plan will have $A > 0$, and will strictly satisfy the minimum expected utility requirement (20).
(2) $E_{2m}(m+V) = V'$. Here, the optimal compensation plan will have $A = 0$ and will strictly satisfy (20).
(3) $E_{2m}(m+V) < V'$. Here, the optimal compensation plan will have $A = 0$, and we will have $E_1 > m + V$.

Note that in case (1), the optimal compensation plan has a fixed and a variable part. However, in cases (2) & (3), the compensation plan is completely variable. For example, if $\alpha = 1$, the compensation plan in cases (2) & (3) will be a staright commissions plan.

Case (3) is significant because if it holds for $\alpha = \dfrac{1}{1-\delta}$, then constraint (20) will not be strictly satisfied by the optimal compensation plan. Following BLSS, it can be shown that if the Lagrangean solution to the problem defined by (3)-(5) is the gloabally optimal compensation plan, then we must have $\lambda > 0$ and hence constraint (4) (equivalent to constarint (20)) must be strictly satisfied. Clearly, that is not happening here, implying that the Lagrangean procedure cannot be used to give us the optimal solution in this case. We will discuss this further in a more specific context.

If $A > 0$ for the optimal solution, we call the solution 'interior'. Otherwise we call it a 'boundary' solution. We conclude section 3.2 by presenting the following proposition about an interior optimal solution (A, B) to (19)-(21).

Proposition 4. If the optimal (A, B) is an interior solution, then A, B and $Z(t)$ are differentiable functions of t and $\dfrac{dA}{dt} < 0$, $\dfrac{dB}{dt} > 0$, and $\dfrac{dZ}{dt} > 0$.

Proposition 4 implies that the firm should reduce guaranteed compensation and increase rewards for achieving higher sales in order to induce the salesperson to work harder. Also, if for any t the optimal compensation plan is interior, continuity implies that it must be an interior solution for any level of effort lower than t. Conversely, if we have a boundary solution at t, we will have a boundary solution for any level of effort higher than t as well.

3.3 Determination of the optimal sales effort, t^*. In section 3.2 we have discussed how we can construct (A, B) which satisfies conditions (23)- (26) and hence gives us the globally optimal solution to the stage 1 of P2 which considers a specific sales effort t. To solve P2 completely, we need to find the optimal t, i.e. determine

$$t^* \quad = \quad \text{argmax}\,\{(1-c)g(t) \;-\; Z(t)\}\,. \tag{27}$$

Since we could not determine (A, B) in a closed form for an interior optimum, we have to determine the optimal t numerically. We now present a procedure to accomplish that under the following additional assumptions which hold for the numerical study presented in section 4 :

(n) $f(x|t)$ is the gamma function used by BLSS, and given by

$$f(x|t) \quad = \quad \frac{1}{\Gamma(q)}\left(\frac{q}{g(t)}\right)\left(\frac{qx}{g(t)}\right)^{q-1} e^{-qx/g(t)}. \tag{28}$$

For this function, $E(x|t) = g(t)$, and $\sigma^2(x|t) = g^2(t)/q$. A larger q represents a more certain selling environment. To be consistent with assumption (m), we will only consider the case where $q \geq 1$.

(o) $g(t) \;=\; h + kt^{\gamma_1}$, $V(t) \;=\; dt^{\gamma_2}$, where $d > 0$, $h > 0$, $k > 0$, $0 < \gamma_1 < 1$, and $\gamma_2 > 1$.

Assumption (o) is equivalent to assumptions (o) & (p) of BLSS, and implies that $g(t)$ is a strictly increasing, strictly concave and differentiable function of t. Since $V'(0) = 0$ here, we will only consider $t > 0$ to be consistent with assumption (c).

(p) $\delta \;\leq\; \dfrac{\gamma_2}{\gamma_1 + \gamma_2}$, and $\gamma_2 - \gamma_1 \geq 1$.
This is a technical assumption needed to develop the model.

(q) $m > 0$, i.e. the salesperson will not be satisfied with zero pay for zero effort.

Under these conditions, the following two propositions hold:

Proposition 5. There exists $0 < t_m < \infty$ such that the optimal solution to (19)-(21) is interior if $t < t_m$, and boundary if $t \geq t_m$. t_m is strictly increasing in m or α.

Proposition 6. If $t \geq t_m$, $Z(t)$ is a strictly convex function of t.

Proposition 6 implies that $\pi(t) \;=\; (1-c)g(t) - Z(t)$ is strictly concave in t for $t \geq t_m$. Also, combining assumption (o) with the result that $Z(t)$ is a strictly increasing differentiable function of t for an interior optimum (from Proposition 4), we find that $\pi(t)$ is a differentiable function of t for $t < t_m$. The optimal sales effort t^* can now be determined as follows :

Stage 1. Compute t_m.

Stage 2. Use grid search to compute the t which maximizes $\pi(t)$ for $0 < t < t_m$. Denote this by t_1^* and the corresponding expected profit by π_1^*.
[Since $\pi(t)$ is a differentiable function of t, the use of grid search is justified.]

Stage 3. Since $\pi(t)$ is strictly concave in t for $t \geq t_m$, the optimal sales effort, t_2^*, from the set $t_m \leq t \leq \infty$ can be obtained with any desired degree of accuracy using numerical techniques. (Figure 1 presents the flow chart of a simple algorithm which can accomplish that.) Let π_2^* denote the corresponding expected profit of the firm.

Figure 1 about here

Stage 4. Compare π_1^* and π_2^* to determine the globally optimal sales effort t^*.

This concludes our discussion of how to solve problem P2. It should be noted that for $t > t_m$, the Lagrangean procedure cannot be used to solve stage 1 of P1. Thus, in those cases, stage 1 of P1 and stage 1 of P2 using $\alpha = \dfrac{1}{1-\delta}$ may not be exactly comparable. However, if we solve P2 using $\alpha = \dfrac{1}{1-\delta}$ and obtain $t^* < t_m$ i.e. the optimal solution to P2 is interior, it can be easily shown that this solution constitutes a Lagrangean solution to the agency theoretic problem P1. In that case, we will assume that the solution to P2 gives us the optimal solution to P1.

4. NUMERICAL RESULTS.

4.1 Study Design. In this section we study (i) how the nature of the optimal linear compensation plan and the agency theoretic compensation plan depend on parameters of the selling environment, and (ii) the relative profitablity of the linear and agency theoretic compensation plans.

Due to limited computational resources, a numerical 'experiment' was conducted using the following five values of δ : 1/3 , .4, .5, .6, 2/3, i.e. we used a range of $\pm\frac{1}{6}$ around $\delta = .5$ used by BLSS.

For each value of δ, a full factorial design ($2^8 = 256$ units) was used with two levels of each of the following 8 parameters of the selling environment, m, q, γ_1, γ_2, h, k, d and c. The levels of parameter values used are presented in Table 1. For numerical convenience, levels of m and k chosen were not the same across δs. The parameter values were so chosen that in each of the 1280 cases studied, the optimal solution to P2 using $\alpha = \dfrac{1}{1-\delta}$ was interior. (We found that we could always get an interior solution to P2 by using an adequately large m. This procedure is partially justified by the fact that t_m increases with m [Proposition 5].)

Table 1 about here

We would like to stress that apart from making sure that the solution to P2 with $\alpha = \dfrac{1}{1-\delta}$ was interior, i.e. it was also the optimal solution to P1, we did not choose parameter values to favor either the linear or the agency theoretic compensation plan. This, combined with the fact that we analysed a wide spectrum of possibilities, should establish that the results obtained would hold over a large range of cases if not everywhere.

For each of the 1280 cases, P2 was solved using $\alpha = 1$ (linear plan) and $\alpha = \dfrac{1}{1-\delta}$ (agency theoretic plan). Section 4.2 presents comparative statics results for the linear and the agency theoretic plans, and section 4.3 discusses how the two plans performed relative to each other.

4.2 Comparative Statics. Proceeding as BLSS, we studied how the optimal compensations plans were affected by changes in m, q, h, k, and c for each value of δ. For example, to investigate the effect of changes in m, we organized the 256 cases in 128 pairs such that in each pair, the two cases were identical except for the fact that the 'low' level of m was used in one case and the 'high' level of m in the other. The solutions for each pair were compared to determine how they differed on expected profit, π, optimal sales effort, t^*, the salary parameter A, the commission parameter B, the expected income of the salesperson, Z, and salary as a frcation of total expected compensation, A^α/Z. Table 2 presents results of comparative statics for the agency theoretic compensation plan, and Table 3 the results for the linear compensation plan.

Table 2 about here

Table 3 about here

Following BLSS, $\uparrow$ indicates that the quantity in consideration always increased from the low to the high level of the parameter considered, $\downarrow$ indicates a decrease in all cases, and $\updownarrow$ that both increases and decreases were observed. Also, since we used numerical analysis, the change in the quantity considered was sometimes small enough to be due to rounding errors (e.g. a change of 10^{-6} in t^*). We used '0' to represent such cases and also cases where there was no change. Thus, $\uparrow$ 0 means that in the cases studied, the changes in the quantity in consideration were either positive or too small to be measured by the techniques used.

Results for the agency theoretic plan. A comparison of Table 2 with the results obtained by BLSS (Table 3, page 287) shows that the numerical analysis here support the findings of BLSS except for the follwing :

(1) For $\delta = 1/3$, an increase in minimum utility required, m, increases B.

(2) For $\delta = 2/3$, an increase in m increases optimal sales effort t^*.

These two results are similar to the findings of Lal & Srinivasan (1988) (Table 1, page 31) that B and t^* do not depend on m.

Results for the linear plan. The results for the linear compensation plan differed in many ways from BLSS and LS. To be specific, we observed the following:

1. Effect of m. The effects of changes in m on π, A, Z, and A^α/Z generally supported the results of BLSS with the difference that in some cases, the changes in A and A^α/Z were too small to measure with confidence.

Unlike BLSS, an increase in m sometimes increased t^* and always increased the commission rate B.

2. Effect of q. The impact of an incraese in q i.e. greater certainty in the selling environmet is generally similar to BLSS except for the following:
(i) Commission rate B and salesperson's expected income Z may sometimes decrease when the environment becomes more certain.
(ii) The effect of changes in q on A, t^* or A^α/Z was sometimes too small to measure.

3. Effect of base sales level h. The effects of changes in h on π, t^*, B, and Z were consistent with the findings of BLSS. In the cases studied, A and A^α/Z either decreased or changed imperceptibly (BLSS is inconclusive in these cases.).

4. Effect of k. Results are generally consistent with BLSS (the effect on t^* here is sometimes too small to measure).

5. Effect of c. Results are generally consistent with BLSS. The changes in quantities other than π are sometimes too small to measure.

Summarizing, we find that for the agency theoretic compensation plan, the findings support BLSS results in most cases. The results for the linear compensation plan differ more from BLSS, but still the similarity in findings is remarkable. In particular, the BLSS results about the effects of the parameters of the selling environment on the firm's expected profit π held in all the cases studied. The BLSS results about the ratio of salary to total income also were found to hold weakly, i.e. the change was sometimes too small to detect.

4.3 Relative Performances of Linear and Agency Theoretic Plans. For each case studied, we define :
1. π_A = expected profit of the firm from the agency theoretic plan.
2. t_A = optimal selling effort for the agency theoretic plan.
3. π_l = expected profit of the firm from the linear plan.
4. t_l = optimal selling effort for the linear plan.

5. $\pi_0 = -(\delta m)^{1/\delta}$.
This is the firm's expected profit in the worst case where it pays the salesperson a fixed amount (equal to π_0) to satisfy the minimum expected utility requirement, and receives no selling effort in return.
6. $R_1 = \frac{t_l}{t_A}$. This expresses the optimal selling effort under the linear plan as a fraction of the optimal selling effort for the agency theoretic plan.
7. $R_2 = \{\pi_l - \pi_0\}/\{\pi_A - \pi_0\}$. R_2 expresses the fraction of the agency theoretic plan's profit achieved by the linear plan, where both profits are measured from the base π_0.

R_1 and R_2 were computed in each case and used to measure how the linear plan performed compared to the agency theoretic compensation plan. We wanted to determine, for each value of δ selected, how R_1 and R_2 varied from case to case and depended on the model parameters.
For each value of δ, the following regression models were estimated:

$$\text{Model 1.} \quad R_1 = \alpha_0 + \sum_{i=1}^{8} \alpha_i D_i + \sum_{i=2}^{8} \sum_{j<i} \alpha_{ij} D_i D_j + \epsilon_1. \tag{29}$$

$$\text{Model 2.} \quad R_2 = \beta_0 + \sum_{i=1}^{8} \beta_i D_i + \sum_{i=2}^{8} \sum_{j<i} \beta_{ij} D_i D_j + \epsilon_2. \tag{30}$$

The α's and β's are regression parameters. $D_1, \ldots, D_8$ are dummy variables corresponding to m, q, k, γ_1, γ_2, d, h, and c, respectively (The dummy variable is -1 if the parameter is at the low level, and 1 if the parameter is at the high level.).

Results about R_1. Table 4 presents the regression results relating R_1 to the independent variables for each of the five values of δ selected (after eliminating insignificant predictors). In every case, the estimated intercept term $\hat{\alpha}_0$ is the average of R_1 for the sample of size 256. The following patterns emerge from Table 4 :
1. As δ increases, R_1 tends to go down, the decline being very slow for $\delta \leq .5$. Thus, as the salesperson becomes less risk averse, the agency theoretic compensation plan is more effective in inducing higher effort from the salesperson compared to the linear plan.
2. As the environment becomes more certain such that risk aversion of the salesperson plays less of a role in selection of effort level, the agency theoretic plan is once again more effective in inducing higher effort than the linear plan.
3. As m increases, i.e. the cost of inducing any level of effort goes up, R_1 increases.

The above discussions are of course limited by the fact that the analysis has been based on arbitrarily chosen parameter values.

Table 4 about here

Results about R2. The regression results relating R_2 to the predictors are presented in Table 5 (after eliminating insignificant predictors). For each δ, the estimated intercept term $\hat{\beta}_0$ is the average value of R_2 for the sample of size 256.

Table 5 about here

From an inspection of Table 5, it is clear that the linear plan performs almost as well as the agency theoretic plan when $\delta \leq .5$ with the average R_2 exceeding 99% in each case. If δ exceeds .5, the relative performance of the linear plan declines significantly.

Even though the regression results are limited by our arbitrary choice of parameter values, it is interesting to note the high explanatory power of the regression model for the higher values of δ (For the lower values of δ, the variation of R_2 is small.), and the fact that certain results tend to hold over the range of δ's considered. Table 5 shows that the relative profitability of the linear plan declines significantly when δ increases beyond .5, or when the certainty parameter q is high. Noting that $\delta = 1$ signifies risk neutrality, it is clear that when the salesperson is not greatly affected by uncertainty, the agency theoretic plan performs much better than the linear plan.

An inspection of Tables 4 & 5 reveals that when either k or γ_1 is high, the relative performance of the linear plan is adversely affected. Intutitively, in these situations, the output x depends strongly on the selling effort t. The agency theoretic plan, being more flexible than the linear plan, can motivate the salesperson more effectively in these situations. Conversely, when it is costly to induce additional effort (m is high, d is high, γ_2 is high, or q is low), or the revenue is not significantly affected by salesperson's effort (h is relatively high), the relative performance of the linear plan improves.

Comparison with the First Best Solution. In order to explore the effect of uncertainty on performance further, we compared the results from the linear and the agency theoretic compensation plans with the 'first best' solution to the firm's problem of designing an optimal compensation plan.

Here, the first best solution corresponds to the case where the firm can measure selling effort t perfectly and without cost, and thus can 'force' the salesperson to devote any specific amount of effort,[3] i.e. the constraint given by equation(5) signifying the salesperson's freedom to choose effort level can be removed. The first best solution is thus equivalent to solving problem P1 with only constraint 4, and the 'first best' expected profit provides an upper

[3]For example, the firm can obtain effort t from the salesperson using the compensation plan which pays $U^{-1}\{m + V(t)\}$ if effort is t or more, and 0 if effort is lower than t.

bound for the expected profit achievable from the agency theoretic compensation plan (also known as the 'second best solution') which includes the constraint given by equation(5) as well. It is a well known result in agency theory that even when the firm cannot observe t directly, it can achieve the expected profit of the first best solution using a compensation plan based on x if either of the following two conditions is satisfied : (i) the salesperson is risk neutral, or, (ii) the selling environment is deterministic (see Holmstrom 1979, or Shavell 1979 for discussions of the first best solution). The first best solution therefore corresponds to the hypthetical case where uncertainty has no effect on profitability.

Let t_1 and π_1 represent the optimal selling effort and expected profit, respectively, for the first best solution. It can be easily shown that

$$t_1 = \text{argmax}\left\{(1-c)g(t) - U^{-1}(m+V(t))\right\}, \quad \pi_1 = (1-c)g(t_1) - U^{-1}\left(m+V(t_1)\right) \quad (31)$$

In each of the 1280 cases studied, the following additional quantities were computed:

1. t_1.
2. π_1.
3. $R_3 = (t_A - t_l)/(t_1 - t_A)$.
4. $R_4 = (\pi_A - \pi_l)/(\pi_1 - \pi_A)$.

Thus, R_3 and R_4 measure the loss in performance resulting from using the linear plan instead of the agency theoretic plan in terms of the loss in performance resulting from uncertainty in the selling environment.

The following two additional regression models were estimated for each of the five values of δ considered :

$$\text{Model 3.} \quad R_3 \quad = \quad \eta_0 + \sum_{i=1}^{8} \eta_i D_i + \sum_{i=2}^{8} \sum_{j<i} \eta_{ij} D_i D_j \quad + \epsilon_3. \quad (32)$$

$$\text{Model 4.} \quad R_4 \quad = \quad \nu_0 + \sum_{i=1}^{8} \nu_i D_i + \sum_{i=2}^{8} \sum_{j<i} \nu_{ij} D_i D_j \quad + \epsilon_4. \quad (33)$$

The results from the estimation of model 3 and model 4 are presented in Tables 6a and 6b, respectively. (For clarity of exposition, only the estimated coefficeients of $D_1, \ldots, D_8$ are presented in the tables.)

Table 6a about here

Table 6b about here

The findings are consistent with the results presented in Tables 4 & 5 earlier: when the salesperson is more risk averse or the selling environment less certain, the linear plan performs almost as well as the agency theoretic plan. With a reduction in the effect of uncertainty (more certain environment or less risk averse salesperson), the more flexible agency theoretic plan can exploit the opportunities presented by a kinder environment more effectively than the linear compensation plan.

5. CONCLUSION

This paper develops a procedure for finding optimal compensation plans of the form $(A + Bx)^{\alpha}$, and uses it to (a) investigate how the linear compensation plan performs compared to the agency theoretic plan, and (b) determine if the comparative statics results for the optimal agency theoretic compensation plan derived by BLSS in the context of the square-root utility function can be extended to (i) the power utility function in general, and (ii) the linear compensation plan.

It was found that for $\delta \leq .5$, the linear compensation plan was almost as profitable as the agency theoretic plan. This is consistent with the industry practice of using simpler compensation plans. When δ exceeds .5, the relative performance of the linear plan declines significantly. However, in these situations, the salesperson would be less risk averse and hence less affected by variations in income. Thus it is likely that a simple compensation plan consisting of a salary and a bonus if a quota is achieved would perform very well. It appears therefore that there is a simple alternative to the agency theoretic compensation plan in a wide range of cases.

The study also shows that under the linear compensation plan, the salesperson devotes significantly less effort compared to the agency theoretic plan. Thus, if we relax the assumption that the marginal cost of production does not depend on sales level, it is possible that the relative profitability of the linear compensation would go down even more. We leave that study to future research.

The comparative statics results of BLSS were found to be remarkably stable with respect to changes in δ for the agency theoretic plan and most of them were at least approximately true for the linear compensation plan.

In contrast to Lal & Srinivasan (1988), the present study adhered to the more traditional BLSS farmework. Still, it is interesting to note the convergence in results from both approaches, leading to the conclusion that the simple linear plan is often a very efficient way to motivate a salesforce.

APPENDIX A

A.1 Proof of Propositions 1, 2 & 3. These propositions relate to finding an optimal solution to the firm's problem of inducing the salesperson to devote a given selling effort t at the least expected cost, i.e.

$$\min_{A,B} Z \tag{A-1}$$

such that

$$E_1(A,B) \quad \geq \quad m + V(t), \tag{A-2}$$

$$E_2(A,B) \quad = \quad V'(t). \tag{A-3}$$

Proposition 1. A feasible compensation plan to the frim's problem defined by (A-1) - (A-3) must have $A \geq 0$ and $B > 0$.

Proof. Nonnegativity of A & B follows from assumptions (k) & (l). To show that we need $B > 0$, consider any compensation plan (A, B) such that $B \leq 0$. Since $(A + Bx) > 0$ $\forall x \in (0, \infty)$, it can be easily shown that $(A + Bx)^\alpha$ is a decreasing function of x for $0 < x < \infty$.

Since $\frac{f_t}{f}$ is a strictly increasing function of x (assumption (i)), we have, using Theorem MLR2, Appendix C,

$$\int (A + Bx)^\alpha f_t \, dx \quad \leq \quad 0. \tag{A-4}$$

Since $V' > 0$, equation(A-3) cannot be satisfied.

QED

Proposition 2. A locally optimal solution (A, B) to the problem defined by (A-1) - (A-3) must satisfy at least one of the following two conditions :
(i) $A = 0$. (ii) $E_1 = m + V$.

Proof. Let us consider the set of compensation plans $\{A, B\}$ for which E_2 is constant at $V'(t)$. For this set,

$$\delta E_2 = \left.\frac{\partial E_2}{\partial A}\right|_B \delta A + \left.\frac{\partial E_2}{\partial B}\right|_A \delta B = 0 \Longrightarrow \left.\frac{dB}{dA}\right|_{E_2} = -\frac{\left.\frac{\partial E_2}{\partial A}\right|_B}{\left.\frac{\partial E_2}{\partial B}\right|_A}. \tag{A-5}$$

Using property 1 & property 2 from Appendix B, $\frac{dB}{dA}$ is always finite, i.e. B can be expressed as a differentiable function of A.

Now, $$\left.\frac{dE_1}{dA}\right|_{E_2} = \left.\frac{\partial E_1}{\partial A}\right|_B + \left.\frac{\partial E_1}{\partial B}\right|_A \left.\frac{dB}{dA}\right|_{E_2} = \frac{\partial E_1}{\partial A} - \frac{\frac{\partial E_1}{\partial B}\frac{\partial E_2}{\partial A}}{\frac{\partial E_2}{\partial B}} > 0, \tag{A-6}$$
substituting from (A-5) and using property 3, Appendix B.
Similarly,
$$\left.\frac{dZ}{dA}\right|_{E_2} = \left.\frac{\partial Z}{\partial A}\right|_B + \left.\frac{\partial Z}{\partial B}\right|_A \left.\frac{dB}{dA}\right|_{E_2} = \frac{\partial Z}{\partial A} - \frac{\frac{\partial Z}{\partial B}\frac{\partial E_2}{\partial A}}{\frac{\partial E_2}{\partial B}} > 0, \tag{A-7}$$
using property 4, Appendix B.
Therefore, if we keep E_2 constant at V' and reduce A, Z and E_1 are decreased simultaneously. This can be done and Z reduced while satisfying constraints (A-2) & (A-3) unless at least one of the following two conditions hold :
(i) $A = 0$, or (ii) $E_1 = m + V$.

QED

Combining propositions 1 & 2 with conditions (A-2) & (A-3), a locally optimal solution (A, B) to the firm's problem defined by (A-1)-(A-3) will satisfy the following conditions:
$$A \geq 0, \quad B > 0, \tag{A-8}$$
$$\frac{d}{dt}E[U(s(x))|t] = \frac{1}{\delta}\int (A+Bx)^{\alpha\delta} f_t\, dx = V', \tag{A-9}$$
$$E[U(s(x))|t] = \frac{1}{\delta}\int (A+Bx)^{\alpha\delta} f\, dx \geq m + V, \tag{A-10}$$
and, if $A > 0$,
$$\frac{1}{\delta}\int (A+Bx)^{\alpha\delta} f\, dx = m + V. \tag{A-11}$$

Proposition 3. If (A, B) satisfies (A-8) - (A-11), then (A, B) is unique, and is the globally optimal solution to the firm's problem defined by (A-1)-(A-3).

To prove proposition 3, we first prove two lemmas.

Lemma 1. Suppose the compensation plan $(0, B)$ satisfies (A-8)-(A-11). Then, B is uniquely determined.

Proof. For this plan, equation(A-9) becomes
$$\frac{1}{\delta}\int (Bx)^{\alpha\delta} f_t dx = B^{\alpha\delta}\left[\frac{1}{\delta}\int x^{\alpha\delta} f_t(x|t)\, dx\right] = V'$$

$$\Longrightarrow \quad B = \left[\frac{\delta V'}{\int x^{\alpha\delta} f_t(x|t)\, dx}\right]^{\frac{1}{\alpha\delta}}. \tag{A-12}$$
Equation (A-12) uniquely determines B.

Lemma 2. It is **not** possible to have compensation plans (A_1, B_1) and (A_2, B_2) such that they both satisfy (A-8)-(A-11) and $A_1 = 0$, $A_2 > 0$.

Proof. Suppose (A_1, B_1) and $A_2, B_2)$ both satisfy (A-8)-(A-11), and $A_1 = 0$, $A_2 > 0$. Using conditions (A-9)-(A-11), we have,

$$\frac{1}{\delta}\int (A_1 + B_1 x)^{\alpha\delta} f\, dx \quad \geq \quad \frac{1}{\delta}\int (A_2 + B_2 x)^{\alpha\delta} f\, dx \quad = \quad m + V, \tag{A-13}$$

and

$$\frac{1}{\delta}\int (A_1 + B_1 x)^{\alpha\delta} f_t\, dx \quad = \quad \frac{1}{\delta}\int (A_2 + B_2 x)^{\alpha\delta} f_t\, dx \quad = \quad V'. \tag{A-14}$$

From (A-13)-(A-14),

$$\frac{\int (A_1 + B_1 x)^{\alpha\delta} f_t\, dx}{\int (A_1 + B_1 x)^{\alpha\delta} f\, dx} \leq \frac{\int (A_2 + B_2 x)^{\alpha\delta} f_t\, dx}{\int (A_2 + B_2 x)^{\alpha\delta} f\, dx}$$

$$\Longleftrightarrow \int h_1(x)\left(\frac{f_t}{f}\right) dx \quad \leq \quad \int h_2(x)\left(\frac{f_t}{f}\right) dx, \tag{A-15}$$

where

$$h_1(x) = \frac{(A_1 + B_1 x)^{\alpha\delta} f}{\int (A_1 + B_1 x)^{\alpha\delta} f dx}, \quad h_2(x) = \frac{(A_2 + B_2 x)^{\alpha\delta} f}{\int (A_2 + B_2 x)^{\alpha\delta} f dx}. \tag{A-16}$$

Note that for $0 < x < \infty$, $h_1(x) > 0$, $h_2(x) > 0$, and $\int_0^\infty h_1(x)\, dx = \int_0^\infty h_2(x)\, dx = 1$.

Therefore, $h_1(x)$ and $h_2(x)$ can be used as probability density functions defined on $x \in (0, \infty)$.

Also, we can express $\frac{h_1(x)}{h_2(x)}$ as

$$\frac{h_1(x)}{h_2(x)} \quad = \quad C_1 \frac{(A_1 + B_1 x)^{\alpha\delta}}{(A_2 + B_2 x)^{\alpha\delta}} \tag{A-17}$$

where $C_1 > 0$ does not depend on x.

It can be easily shown that here, $\dfrac{d}{dx}\dfrac{h_1(x)}{h_2(x)}$ has the same sign as $(B_1 A_2 - A_1 B_2)$ for all $x \in (0, \infty)$.

Since $B_1 > 0$ (from proposition 1), $A_1 = 0$, and $A_2 > 0$, $B_1 A_2 - A_1 B_2 > 0$, i.e. $\dfrac{h_1(x)}{h_2(x)}$ is a strictly increasing function of x.

Since $\frac{f_t}{f}$ is strictly increasing in x, we have, using Theorem MLR2 (Appendix C),

$$\int h_1(x)\frac{f_t}{f}\, dx \quad > \quad \int h_2(x)\frac{f_t}{f}\, dx, \tag{A-18}$$

which contradicts inequality (A-15), i.e. it is **not** possible to have solutions (A_1, B_1), (A_2, B_2) to the firm's problem given by (A-1)-(A-3) such that $A_1 = 0$, $A_2 > 0$.

QED

Proof of Proposition 3. Suppose compensation plans (A_1, B_1) and (A_2, B_2) both satisfy (A-8)-(A-11). Without loss of generality, let us assume that $A_1 \leq A_2$.

Since $A_1 \geq 0$, $A_2 \geq 0$, the following three mutually exclusive cases collectively exhaust all the possibilities :

Case 1. $A_1 = A_2 = 0$,
Case 2. $A_1 = 0$, $A_2 > 0$.
Case 3. $A_1 > 0$, $A_2 > 0$.

Let us consider these three cases individually.

Case 1. Here, $A_1 = A_2$. Also, using lemma 1, $B_1 = B_2$, i.e. the two compensation plans are identical.
Case 2. From lemma 2, case 2 cannot happen.
Case 3. Suppose (A_1, B_1) and (A_2, B_2), both represent solutions to the problem defined by (16)-(18), and $A_1 > 0$, and $A_2 > 0$.
From equations (A-8)-(A-11), we have,

$$B_1 > 0, \quad B_2 > 0, \tag{A-19}$$

$$\frac{1}{\delta}\int (A_1 + B_1 x)^{\alpha\delta} f\, dx = \frac{1}{\delta}\int (A_2 + B_2 x)^{\alpha\delta} f\, dx = m + V, \tag{A-20}$$

$$\frac{1}{\delta}\int (A_1 + B_1 x)^{\alpha\delta} f_t dx = \frac{1}{\delta}\int (A_2 + B_2 x)^{\alpha\delta} f_t dx = V'. \tag{A-21}$$

Proceeding as in the proof of proposition 4, (A-20) & (A-21) can be combined to give

$$\int h_1(x)\, \frac{f_t}{f}\, dx = \int h_2(x)\, \frac{f_t}{f}\, dx, \tag{A-22}$$

where $h_1(x) = \dfrac{(A_1 + B_1 x)^{\alpha\delta} f}{\int (A_1 + B_1 x)^{\alpha\delta} f\, dx}$ and $h_2(x) = \dfrac{(A_2 + B_2 x)^{\alpha\delta} f}{\int (A_2 + B_2 x)^{\alpha\delta} f\, dx}$ can be used as probability density functions defined on $x \in (0, \infty)$.
Once again, $\dfrac{d}{dx}\dfrac{h_1(x)}{h_2(x)}$ has the same sign as $(B_1 A_2 - A_1 B_2)$ for all $x \in (0, \infty)$.
Therefore, $\frac{h_1(x)}{h_2(x)}$ is strictly increasing, strictly decreasing, or constant for $0 < x < \infty$.
Since $\frac{f_t}{f}$ is a strictly increasing function of x, Theorem MLR2 (Appendix C) implies that equation(A-22) will hold in the present case if and only if $\frac{h_1(x)}{h_2(x)}$ is constant for $0 < x < \infty$, which can be true if and only if

$$B_1 A_2 - A_1 B_2 = 0 \iff \frac{A_1}{A_2} = \frac{B_1}{B_2} \iff A_1 + B_1 x = C_2(A_2 + B_2 x), \tag{A-23}$$

where C_2 is a constant.
Since $V' > 0$, equation (A-21) will be satisfied only if $C_2 = 1$, i.e. $A_1 = A_2$, $B_1 = B_2$.

Summarizing, we find that if (A, B) satisfies conditions (A-8)-(A-11), it will be unique, and hence it will represent the global optimum for the firm's problem defined by (A-1)-(A-3).

QED

A.2. Constructing the optimal soultion for a given t.

We will now demonstrate that it is always possible to construct a compensation plan that satisfies conditions (A-8)-(A-11) and thereby represents the gloabally optimal solution to the firm's problem defined by (A-1)-(A-3).

Let us consider the set of compensation plans (A, B) which satisfy

$$A \geq 0,\ B \geq 0, \quad \text{and} \quad E_1 = \int U(s(x)) f dx = \frac{1}{\delta} \int (A + Bx)^{\alpha\delta} f dx = K > 0 \qquad \text{(A-24)}$$

where K is a given constant.

This set is nonempty since it can be verified by direct substitution that the condition $E_1 = K$ is satisfied by the following two compensation plans :

$$\text{Plan 1.} \quad A = (\delta K)^{\frac{1}{\alpha\delta}} = AM > 0\,(\text{say}), \quad B = 0.$$

$$\text{Plan 2.} \quad A = 0, \quad B = \left[\frac{\delta K}{\int x^{\alpha\delta} f\, dx}\right]^{\frac{1}{\alpha\delta}} = BM > 0\,(\text{say}). \qquad \text{(A-25)}$$

Combining property 1 & property 2, Appendix B, we have $0 < \frac{\partial E_1}{\partial A} < \infty$, $0 < \frac{\partial E_1}{\partial B} < \infty$. This implies that for the set defined above, we must have $0 \leq A \leq AM$ and $0 \leq B \leq BM$. Also, for any $A \in (0, AM)$, $E_1(A, 0) < K$, and $E_1(A, BM) > K$, since $\frac{\partial E_1}{\partial B} > 0$. Therefore, $\exists B \in (0, BM)$ such that $E_1(A, B) = K$. Finally, for this set of compensation plans,

$$\left.\frac{dB}{dA}\right|_{E_1=K} = -\frac{\left.\frac{\partial E_1}{\partial A}\right|_B}{\left.\frac{\partial E_1}{\partial B}\right|_A} < 0, \qquad \text{(A-26)}$$

since $\dfrac{\partial E_1}{\partial A} > 0$, $\dfrac{\partial E_1}{\partial B} > 0$, i.e. for this set, B decreases as A increases.

Now, for this set,

$$\left.\frac{dE_2}{dA}\right|_{E_1=K} = \left.\frac{\partial E_2}{\partial A}\right|_B + \left.\frac{\partial E_2}{\partial B}\right|_A \left.\frac{dB}{dA}\right|_{E_1=K} = \frac{\partial E_2}{\partial A} - \frac{\frac{\partial E_2}{\partial B}\frac{\partial E_1}{\partial A}}{\frac{\partial E_1}{\partial B}} < 0, \qquad \text{(A-27)}$$

using Property 3, Appendix B.

Using Theorem MLR2 (Appendix C), $E_2(AM, 0) = 0$. As A is reduced and B increased to keep E_1 unchanged at K, E_2 increases and attains the largest value at $(0, BM)$, given by

$$E_2(0, BM) = \frac{1}{\delta} \int (BM \cdot x)^{\alpha\delta} f_t\, dx = \frac{K \int x^{\alpha\delta} f_t dx}{\int x^{\alpha\delta} f dx} = E_{2m}(K) \quad (\text{say}), \qquad \text{(A-28)}$$

substituting BM from (A-25). Note that $E_{2m}(K)$ is a strictly increasing function of K.

Let us now consider $K = m + V$, i.e.

$$BM = \left[\frac{\delta\,(m+V)}{\int x^{\alpha\delta} f\,dx}\right]^{\frac{1}{\alpha\delta}}, \qquad E_{2m}(m+V) \;=\; \frac{(m+V)\int x^{\alpha\delta} f_t\,dx}{\int x^{\alpha\delta} f\,dx}. \tag{A-29}$$

The following three cases are possible :

Case 1. $E_{2m} > V'$.

In this case, $E_2(AM, 0) = 0$, and $E_2(0, BM) = E_{2m} > V'$.

From the continuity of E_2 it follows that we can have $0 < A < AM$, $0 < B < BM$ such that $E_2(A,B) = V'$. (A,B) satisfies conditions (A-8)-(A-11) and represents the gloabally optimal solution to (A-1)-(A-3).

Figure 2 presents the flow chart for an algorithm which can determine the optimal (A,B) iteratively with any degree of accuracy desired.

Case 2. $E_{2m} = V'$.

It can be shown by direct substitution that the compensation plan $(0, BM)$ satisfies (A-8)-(A-11). The minimum expected utility constraint of the salesperson is binding here.

Case 3. $E_{2m} < V'$.

Let

$$B = \left[\frac{\delta\,V'}{\int x^{\alpha\delta}\, f_t\,dx}\right]^{\frac{1}{\alpha\delta}}. \tag{A-30}$$

It can be shown by direct substitution that $(0, B)$ satisfies (A-8) - (A-11).

In this case, $E_1 = K = \dfrac{V'\int x^{\alpha\delta} f dx}{\int x^{\alpha\delta} f_t dx} > m + V$, i.e. the salesperson derives a higher expected utility than m from the optimal compensation plan.

Summarizing, we have shown by construction that we can always have a compensation plan (A,B) which satisfies (A-8)-(A-11) and hence gives us the unique globally optimal solution to the problem defined by (A-1)-(A-3). In cases 2 & 3 we have a boundary solution, i.e. the compensation has no guaranteed component (salary). The compensation plan here can be obtained in a closed form. In case 1, the optimal solution is interior, i.e. it consists of a guaranteed income plus a variable component, and it can be estimated iteratively.

A.3 Proof of Proposition 4.

Proposition 4. If the optimal (A,B) is an interior solution, then A, B and $Z(t)$ are differentiable functions of t and $\dfrac{dA}{dt} < 0$, $\dfrac{dB}{dt} > 0$, and $\dfrac{dZ}{dt} > 0$.

Proof. Rewriting (A-8)-(A-11), (A, B) is the unique solution to the system of equations

$$\phi_1(A,B,t) = E_1(A,B) - m - V(t) = 0, \tag{A-31}$$

$$\phi_2(A,B,t) = E_2(A,B) - V'(t) = 0. \tag{A-32}$$

Using the implicit function theorem,

$$\begin{aligned}\begin{pmatrix} \frac{dA}{dt} \\ \frac{dB}{dt} \end{pmatrix} &= \begin{pmatrix} \frac{\partial\phi_1}{\partial A} & \frac{\partial\phi_1}{\partial B} \\ \frac{\partial\phi_2}{\partial A} & \frac{\partial\phi_2}{\partial B} \end{pmatrix}^{-1} \begin{pmatrix} -\frac{\partial\phi_1}{\partial t} \\ -\frac{\partial\phi_2}{\partial t} \end{pmatrix} \\ &= \frac{1}{\frac{\partial E_1}{\partial A}\frac{\partial E_2}{\partial B} - \frac{\partial E_2}{\partial A}\frac{\partial E_1}{\partial B}} \begin{pmatrix} \frac{\partial E_2}{\partial B} & -\frac{\partial E_1}{\partial B} \\ -\frac{\partial E_2}{\partial A} & \frac{\partial E_1}{\partial A} \end{pmatrix} \begin{pmatrix} -\frac{\partial\phi_1}{\partial t} \\ -\frac{\partial\phi_2}{\partial t} \end{pmatrix},\end{aligned} \tag{A-33}$$

since here, $\dfrac{\partial\phi_1}{\partial A} = \dfrac{\partial E_1}{\partial A}$, $\dfrac{\partial\phi_1}{\partial B} = \dfrac{\partial E_1}{\partial B}$, $\dfrac{\partial\phi_2}{\partial A} = \dfrac{\partial E_2}{\partial A}$, and $\dfrac{\partial\phi_2}{\partial B} = \dfrac{\partial E_2}{\partial B}$.

The matrix inversion performed above is always valid since $\dfrac{\partial E_1}{\partial A}\dfrac{\partial E_2}{\partial B} - \dfrac{\partial E_2}{\partial A}\dfrac{\partial E_1}{\partial B} > 0$ (Using Property 3, Appendix B).

It can be easily shown that

$$\phi_2(t) = \frac{\partial\phi_1}{\partial t} = \frac{\partial}{\partial\tau}u(A,B,\tau)\bigg|_{\tau=t}, \quad \text{and} \quad \frac{\partial\phi_2}{\partial t} = \frac{\partial^2\phi_1}{\partial t^2} = \frac{\partial^2}{\partial\tau^2}u(A,B,\tau)\bigg|_{\tau=t}, \tag{A-34}$$

where $u(A, B, \tau)$ is the salesperson's expected utility for devoting τ under compensation plan (A, B).
Since $u(A, B, \tau)$ attains a strict maximum at $t = \tau$, we must have

$$\frac{\partial\phi_1}{\partial t} = 0 \qquad \text{and} \qquad \frac{\partial\phi_2}{\partial t} < 0. \tag{A-35}$$

Combining (A-33) & (A-35) and simplifying,

$$\begin{pmatrix} \frac{dA}{dt} \\ \frac{dB}{dt} \end{pmatrix} = \frac{-\frac{\partial\phi_2}{\partial t}}{\frac{\partial E_1}{\partial A}\frac{\partial E_2}{\partial B} - \frac{\partial E_1}{\partial B}\frac{\partial E_2}{\partial A}} \begin{pmatrix} -\frac{\partial E_1}{\partial B} \\ \frac{\partial E_1}{\partial A} \end{pmatrix}. \tag{A-36}$$

Using (A-35) & (A-36) together with Properties 2 & 3, Appendix B, we have $\dfrac{dA}{dt} < 0$, $\dfrac{dB}{dt} > 0$. Property 2 & Property 3, Appendix B, also ensure that $\dfrac{dA}{dt}$ and $\dfrac{dB}{dt}$ are finite i.e. A and B are differentiable functions of t.

Finally, expressing Z as $Z(A,B,t)$, we have,

$$\frac{dZ}{dt} = \frac{\partial Z}{\partial t} + \frac{\partial Z}{\partial A}\frac{dA}{dt} + \frac{\partial Z}{\partial B}\frac{dB}{dt}. \tag{A-37}$$

Since $A > 0$ and $B > 0$, $(A+Bx)^{\alpha}$ is strictly increasing in x. Using Theorem MLR2 (Appendix C), we have

$$\frac{\partial Z}{\partial t} = \int (A+Bx)^{\alpha} f_t dx > 0. \tag{A-38}$$

Using (A-36) and simplifying,

$$\frac{\partial Z}{\partial A}\frac{dA}{dt} + \frac{\partial Z}{\partial B}\frac{dB}{dt} = -\left(\frac{\partial \phi_2}{\partial t}\right)\left\{\frac{\frac{\partial E_1}{\partial A}\frac{\partial Z}{\partial B} - \frac{\partial E_1}{\partial B}\frac{\partial Z}{\partial A}}{\frac{\partial E_1}{\partial A}\frac{\partial E_2}{\partial B} - \frac{\partial E_1}{\partial B}\frac{\partial E_2}{\partial A}}\right\} > 0, \tag{A-39}$$

using (A-35) together with Properties 3 & 5, Appendix B.

From (A-38) & (A-39), $\frac{dZ}{dt} > 0$. Also, Properties 2, 3 and 5 of Appendix B and assumption (m) jointly imply that $\frac{dZ}{dt}$ is finite i.e. Z is differentiable in t.

QED

A.4 Proofs of Propositions 5 & 6. We will first establish some results which will be used to prove the propositions.

Restating assumptions (n) & (o), we have,

$$f(x|t) = \frac{1}{\Gamma(q)}\left(\frac{q}{g(t)}\right)\left(\frac{qx}{g(t)}\right)^{q-1} e^{-qx/g(t)}, \tag{A-40}$$

$$g(t) = h + kt^{\gamma_1} \quad \text{and} \quad V(t) = dt^{\gamma_2}. \tag{A-41}$$

Result 1. From (A-40), it can be easily shown that if $\beta \geq 0$,

$$\int x^{\beta} f(x|t) dx = \frac{\Gamma(q+\beta)}{\Gamma(q)}\left(\frac{g}{q}\right)^{\beta}. \tag{A-42}$$

Result 2. Differentiating both sides of (A-42) and simplifying, it can be easily shown that

$$\int x^{\beta} f_t dx = \frac{d}{dt}\int x^{\beta} f dx = \frac{\beta g'}{g}\frac{\Gamma(q+\beta)}{\Gamma(q)}\left(\frac{g}{q}\right)^{\beta}. \tag{A-43}$$

Result 3. Combining (A-42) and (A-43),

$$\frac{\int x^\beta f_t dx}{\int x^\beta f dx} = \frac{\beta g'}{g}. \tag{A-44}$$

Result 4. From (A-41), it can be easily shown that

$$\frac{V'g}{g'} = (\frac{\gamma_2 dh}{\gamma_1 k})t^{\gamma_2-\gamma_1} + \frac{\gamma_2}{\gamma_1}V. \tag{A-45}$$

Result 5. Since V is strictly convex in t (assumption c), it follows from (A-45) that if $\gamma_2 \geq \gamma_1 + 1$, then $\frac{V'g}{g'}$ is a strictly convex function of t.

Result 6. $\frac{V'g}{g'}$ is a strictly increasing function of t. This result follows directly from assumption (o).

We will now use the results obtained above to prove propositions 5 & 6.

Proposition 5. There exists $0 < t_m < \infty$ such that the optimal solution to (A-1) - (A-3) is an interior solution if $t < t_m$, and a boundary solution if $t \geq t_m$. t_m is strictly increasing in m or α.

Proof. The optimal solution to the firm's problem defined by (A-1)-(A-3) is interior if and only if

$$E_{2m}(m+V) = \frac{(m+V)\int x^{\alpha\delta} f_t dx}{\int x^{\alpha\delta} f dx} > V'$$

$$\Longleftrightarrow m > (\frac{hd}{\alpha\delta k})t^{\gamma_2-\gamma_1} + (\frac{\gamma_2}{\alpha\delta\gamma_1} - 1)V, \tag{A-46}$$

using (A-44) & (A-45) and simplifying.

Since $\gamma_2 > \gamma_1$ (from assumption (p)), $t^{\gamma_2-\gamma_1}$ is a strictly increasing function of t.
Also, since $\frac{\gamma_2}{\alpha\delta\gamma_1}$ is strictly decreasing in α and $\alpha \leq \frac{1}{1-\delta}$, $\frac{\gamma_2}{\alpha\delta\gamma_1} - 1 \geq 0$ for any $\alpha \in [1, \frac{1}{1-\delta}]$ if

$$\frac{\gamma_2}{(\frac{1}{1-\delta})\delta\gamma_1} - 1 \geq 0 \iff \delta \leq \frac{\gamma_2}{\gamma_1+\gamma_2}. \tag{A-47}$$

Therefore, from assumption (p), the RHS of (A-46) is a strictly increasing function of t. Also, the RHS of (A-46) is 0 if $t = 0$, and it goes to ∞ as $t \to \infty$. Since it is also a continuous function of t, there is a unique solution $0 < t_m < \infty$ to the equation

$$m = \left(\frac{hd}{\alpha\delta k}\right)t^{\gamma_2-\gamma_1} + \left(\frac{\gamma_2}{\alpha\delta\gamma_1} - 1\right)V. \tag{A-48}$$

We have an interior optimum if $t < t_m$ and a boundary optimum if $t \geq t_m$.
If m increases, (A-48) will be satisfied for a larger t. If α increases, the RHS of (A-46) is reduced for any given t, implying that (A-48) will be satisfied for a larger t.

QED

t_m for the linear compensation plan is always lower than t_m for the agency theoretic compensation plan. It is interesting to note that t_m does not depend on q.

Proposition 6. If $t \geq t_m$, $Z(t)$ is a strictly convex function of t.

Proof. From equation(A-30), the optimal compensation plan for $t \geq t_m$ is $(0, B)$, where

$$B = \left[\frac{\delta V'}{\int x^{\alpha\delta} f_t dx}\right]^{\frac{1}{\alpha\delta}} = \left[\frac{\Gamma(q)}{\Gamma(q+\alpha\delta)} \frac{V'g}{\alpha g'}\right]^{\frac{1}{\alpha\delta}} \left(\frac{q}{g}\right), \tag{A-49}$$

using (A-43).

The salesperson's expected income is

$$Z(t) = \int (Bx)^{\alpha} f dx = \left\{\frac{\Gamma(q)}{\alpha\Gamma(q+\alpha\delta)}\right\}^{\frac{1}{\delta}} \left[\frac{\Gamma(q+\alpha)}{\Gamma(q)}\right] \left(\frac{V'g}{g'}\right)^{\frac{1}{\delta}}, \tag{A-50}$$

using (A-42) & (A-49) and simplifying.

Therefore, $Z(t)$ can be expressed as $K(\frac{V'g}{g'})^{\frac{1}{\delta}}$, where K is strictly positive and does not depend on t.

Using results (5) & (6), $\frac{V'g}{g'}$ is a strictly increasing convex function of t. Since $\frac{1}{\delta} > 1$, it follows that $Z(t)$ is strictly convex in t.

QED

APPENDIX B : Properties of Partial Derivatives of Z, E_1 and E_2

As defined in section (3),

$$Z = \int s(x) f\, dx = \int (A + Bx)^{\alpha} f\, dx, \tag{B-1}$$

$$E_1 = \int U(s(x)) f\, dx = \frac{1}{\delta} \int (A + Bx)^{\alpha\delta} f\, dx, \tag{B-2}$$

$$E_2 = \frac{d}{dt} E_1 = \frac{1}{\delta} \int (A + Bx)^{\alpha\delta} f_t\, dx. \tag{B-3}$$

Differentiating under the integral sign, we have,

$$\left.\frac{\partial Z}{\partial A}\right|_B = \alpha \int (A + Bx)^{\alpha - 1} f\, dx, \tag{B-4}$$

$$\left.\frac{\partial Z}{\partial B}\right|_A = \alpha \int x(A + Bx)^{\alpha - 1} f\, dx, \tag{B-5}$$

$$\left.\frac{\partial E_1}{\partial A}\right|_B = \alpha \int (A + Bx)^{\alpha\delta - 1} f\, dx, \tag{B-6}$$

$$\left.\frac{\partial E_1}{\partial B}\right|_A = \alpha \int x(A + Bx)^{\alpha\delta - 1} f\, dx \quad > \quad 0, \tag{B-7}$$

$$\left.\frac{\partial E_2}{\partial A}\right|_B = \alpha \int (A + Bx)^{\alpha\delta - 1} f_t\, dx, \tag{B-8}$$

and

$$\left.\frac{\partial E_2}{\partial B}\right|_A = \alpha \int x(A + Bx)^{\alpha\delta - 1} f_t dx \tag{B-9}$$

In the discussions that follow we will assume that $A \geq 0$, $B \geq 0$, and at least one of A and B is strictly positive, since these conditions will hold in all cases we are interested in.

Property 1. $\frac{\partial Z}{\partial A}$, $\frac{\partial Z}{\partial B}$, $\frac{\partial E_1}{\partial A}$, $\frac{\partial E_1}{\partial B}$, $\frac{\partial E_2}{\partial A}$, and $\frac{\partial E_2}{\partial B}$ are all finite.
This property follows directly from assumption (m).

Property 2. $\frac{\partial Z}{\partial A}$, $\frac{\partial Z}{\partial B}$, $\frac{\partial E_1}{\partial A}$, $\frac{\partial E_1}{\partial B}$, and $\frac{\partial E_2}{\partial B}$ are all strictly positive.

Proof. $f(x|t) = 0$ if $x \leq 0$ (assumption l), and in cases we are interested in, $(A + Bx) > 0$ if $0 < x < \infty$. An inspection of the right hand sides of (B-4), (B-5), (B-6), and (B-7) shows that in each case, the integrand is positive for $x > 0$. Therefore, $\frac{\partial Z}{\partial A}$, $\frac{\partial Z}{\partial B}$, $\frac{\partial E_1}{\partial A}$, and $\frac{\partial E_1}{\partial B}$ are all strictly positive. Also, it can be easily shown that $x(A+Bx)^{\alpha\delta-1}$ is strictly increasing in x for $0 < x < \infty$. Using Theorem MLR2 (Appendix C), we have,

$$\frac{\partial E_2}{\partial B} = \alpha \int x(A+Bx)^{\alpha\delta-1} f_t dx > 0.$$

QED

Property 3. $\dfrac{\partial E_1}{\partial A} - \dfrac{\frac{\partial E_1}{\partial B}\frac{\partial E_2}{\partial A}}{\frac{\partial E_2}{\partial B}} > 0.$

Proof. Combining (B-6), (B-7), (B-8), and (B-9),

$$\frac{\frac{\partial E_2}{\partial B}}{\frac{\partial E_1}{\partial B}} = \frac{\int x(A+Bx)^{\alpha\delta-1} f_t\, dx}{\int x(A+Bx)^{\alpha\delta-1} f\, dx} = \frac{\int \left[x(A+Bx)^{\alpha\delta-1} f\right] \frac{f_t}{f}\, dx}{\int \left[x(A+Bx)^{\alpha\delta-1} f\right] dx} = \int g_1(x) \frac{f_t}{f}\, dx, \quad \text{(B-10)}$$

$$\text{and} \quad \frac{\frac{\partial E_2}{\partial A}}{\frac{\partial E_1}{\partial A}} = \frac{\int (A+Bx)^{\alpha\delta-1} f_t\, dx}{\int (A+Bx)^{\alpha\delta-1} f\, dx} = \int g_2(x) \frac{f_t}{f}\, dx, \quad \text{(B-11)}$$

where

$$g_1(x) = \frac{x(A+Bx)^{\alpha\delta-1} f}{\int x(A+Bx)^{\alpha\delta-1} f\, dx} \quad \text{and} \quad g_2(x) = \frac{(A+Bx)^{\alpha\delta-1} f}{\int (A+Bx)^{\alpha\delta-1} f\, dx}. \quad \text{(B-12)}$$

Since, for $0 < x < \infty$, $g_1(x) > 0$, $g_2(x) > 0$, and $\int_0^\infty g_1(x)dx = \int_0^\infty g_2(x)dx = 1$, $g_1(x)$ and $g_2(x)$ can be used as probability density functions defined for $0 < x < \infty$.
$\dfrac{g_1(x)}{g_2(x)}$ is a strictly increasing function of x, being equal to a positive constant times x. Also, $\dfrac{f_t}{f}$ is a strictly increasing function of x. Therefore, using Theorem MLR1 (Appendix C),

$$\int g_1(x)\frac{f_t}{f} dx > \int g_2(x)\frac{f_t}{f} dx \iff \frac{\frac{\partial E_2}{\partial B}}{\frac{\partial E_1}{\partial B}} > \frac{\frac{\partial E_2}{\partial A}}{\frac{\partial E_1}{\partial A}} \iff \frac{\partial E_1}{\partial A} - \frac{\frac{\partial E_1}{\partial B}\frac{\partial E_2}{\partial A}}{\frac{\partial E_2}{\partial B}} > 0, \quad \text{(B-13)}$$

since $\frac{\partial E_2}{\partial B} > 0$, $\frac{\partial E_1}{\partial A} > 0$, and $\frac{\partial E_1}{\partial B} > 0$ (property 2.).

QED

Property 4. If $\alpha \le 1/(1-\delta)$ and $B > 0$, then $\dfrac{\partial Z}{\partial A} - \dfrac{\frac{\partial Z}{\partial B}\frac{\partial E_2}{\partial A}}{\frac{\partial E_2}{\partial B}} > 0.$

Property 5. $\dfrac{\partial E_1}{\partial A}\dfrac{\partial Z}{\partial B} > \dfrac{\partial E_1}{\partial B}\dfrac{\partial Z}{\partial A}.$

The proofs of properties 4 & 5 are similar to that of property 3, and are omitted.

APPENDIX C : Monotone Likelihood Ratio Theorems

This paper makes an extensive use of probability density functions which satisfy the Monotone Likelihood Ratio (MLR) property. The specific results used in the paper are presented below (without proof) as two theorems. The interested reader may refer to the text by Lehmann (1959, pages 68-75) for a more general discussion of the concept.

In the following, we only consider continuous probability density functions which are strictly positive if $x > 0$ and 0 if $x \leq 0$.

Theorem MLR1 Let $f_1(x)$ and $f_2(x)$ be probability density functions defined on $0 < x < \infty$ such that $\dfrac{f_1(x)}{f_2(x)}$ is a strictly increasing function of x, and let $g(x)$ be a function defined for $0 < x < \infty$.

Then :

1. If $g(x)$ is a (strictly) increasing function of x, then

$$\int_0^\infty f_1(x)\, g(x)\, dx \quad (>) \geq \quad \int_0^\infty f_2(x)\, g(x)\, dx. \tag{C-1}$$

2. If $g(x)$ is a (strictly) decreasing function of x, then

$$\int_0^\infty f_1(x)\, g(x)\, dx \quad (<) \leq \quad \int_0^\infty f_2(x)\, g(x)\, dx. \tag{C-2}$$

3. If $g(x)$ is a constant, then

$$\int_0^\infty f_1(x)\, g(x)\, dx \quad = \quad \int_0^\infty f_2(x)\, g(x)\, dx. \tag{C-3}$$

Theorem MLR2. Let $f(x|t)$ be a probability density function defined on $0 < x < \infty$ such that $\dfrac{f_t}{f}$ is strictly increasing in x, and let $g(x)$ be a function of x defined for $0 < x < \infty$.

Then :

1. If $g(x)$ is (strictly) increasing in x, then

$$\int_0^\infty f_t\, g(x)\, dx \quad (>) \geq \quad 0. \tag{C-4}$$

2. If $g(x)$ is (strictly) decreasing in x, then

$$\int_0^\infty f_t\, g(x)\, dx \quad (<) \leq \quad 0. \tag{C-5}$$

3. If $g(x)$ is constant, then

$$\int_0^\infty f_t\, g(x)\, dx \quad = \quad 0. \tag{C-6}$$

References

Albers, S. (1980), 'Sales Force Control by Compensation Schemes', **Zeitschrift fur Betriebswirtschaft**, 50, 713-36.

Basu, A.K., R.Lal, V.Srinivasan, and R.Staelin (1985), 'Salesforce Compensation Plans : An Agency Theoretic Perspective', **Marketing Science**, 4 (Fall), 267-291.

Coughlan, A.T. and C.Narasimhan (1989), 'An Economic Approach To Salesforce Compensation: Empirical Insights', Working Paper, January.

Coughlan, A.T. and S.Sen (1986), 'Salesforce Compensation: Insights From Management Science', Report No. 86-101 (Cambridge, MA: Marketing Science Institute, March).

Farley, J.U. (1964), 'An Optimal Plan for Salesman's Compensation', **Journal of Marketing Research**, 1 (May), 39-43.

Grossman,S.J. and O.D.Hart (1983), 'An Analysis of The Principal-Agent Problem', **Econometrica**, 51(January), 7-45.

Holmstrom, B. (1979), 'Moral Hazard and Observability', **Bell Journal Of Economics**, 10(Spring), 74-91.

Holmstrom,B. and P.Milgrom (1987), 'Aggregation and Linearity in the Provision of Intertemporal Incentives', **Econometrica**, 55(March), 303-328.

John, G. and B.Weitz (1988), 'Explaining Variation in Sales Compensation Plans: Empirical Evidence for the Basu et. al. Model', Working Paper, July.

John, G. and B.Weitz (1989), 'Salesforce Compensation: An Empirical Investigation of Factors Related to Use of Salary Versus Incentive Compensation', **Journal of Marketing Research**, 26 (February), 1-14.

Kotler, P. (1986), 'Principles of Marketing', 3rd ed., Prentice-Hall.

Lal, R. (1986), 'Delegating Pricing Responsibility To The Salesforce', **Marketing Science**, vol 5 (Spring), 159 - 168.

Lal, R. and V.Srinivasan (1988), 'Salesforce Compensation Plans: A Dynamic Perspective', Graduate School of Business, Stanford University, Research Paper No.999.

Lal, R. and R.Staelin (1986), 'Salesforce Compensation Plans in Environments with Asymmetric Information', **Marketing Science**, 5 (Summer), 179-98.

Lehmann, E.L. (1959), 'Testing Statistical Hypotheses', John Wiley & Sons, Inc.

Rao, R.C. (1988), 'Compensating Heterogeneous Salesforces: Some Explicit Solutions', Working Paper, May.

Shavell, S. (1979), 'Risk sharing and incentives in the principal and agent relationship', **Bell Journal of Economics**, 10(Spring), 55-73.

Srinivasan, V. (1981), 'An Investigation of the Equal Commission Rate Policy for a Multi-product Salesforce', **Management Science**, 27 (July), 731-56.

Stanton, W.J. and R.H.Buskirk (1983), 'Management of the Salesforce', 6th ed., Homewood, IL : Irwin.

Steinbrink, J.P. (1978), 'How to Pay Your Sales Force', **Harvard Business Review**, 56 (July-August), 111-122.

Tapiero, C.S. and J.U.Farley (1975), 'Optimal Control of a Sales-Force Effort in Time', **Management Science**, 21 (May), 976-85.

Weinberg, C.B. (1975), 'An Optimal Compensation Plan for Salesman's Control over Price', **Management Science**, 21 (April), 937-43.

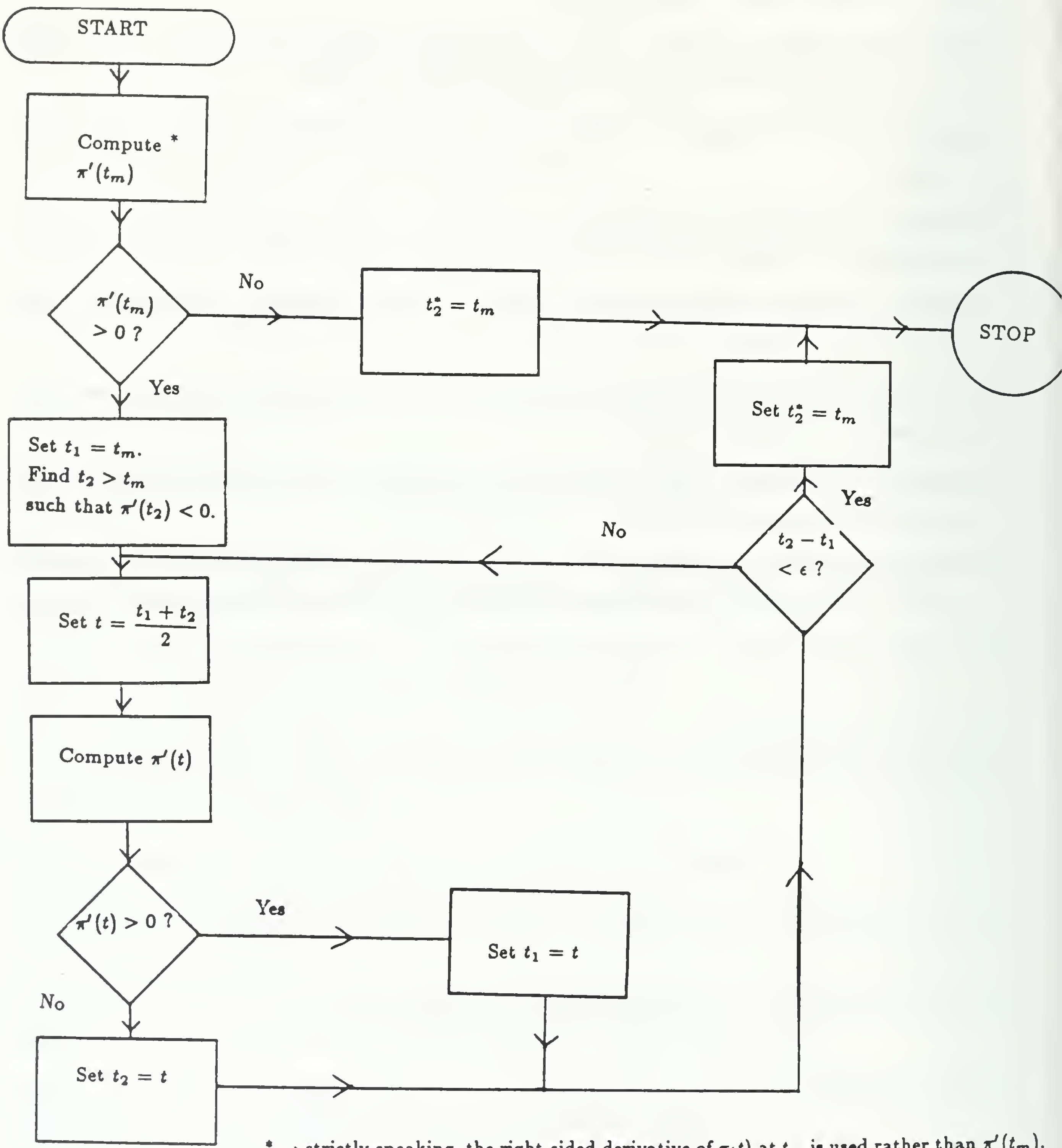
Weinberg, C.B. (1978), 'Jointly Optimal Sales Commissions for Non-income Maximizing Sales Forces', Management Science, 24 (August), 1252-8.

Wilson, D. and P.Bennett (1986), 'Motivating the Field Salesforce Through the Compensation Plan', **Institute for the Study of Business Markets**, Working Series No. 8-1986.

Figure 1

Finding t_2^*, the best t for $t \geq t_m$.

[ϵ is a prespecified small positive number.]



* → strictly speaking, the right-sided derivative of $\pi(t)$ at t_m is used rather than $\pi'(t_m)$.

Figure 2

Finding the interior optimum (A, B) for a given t.

[ϵ_1, ϵ_2 are prespecified positive numbers.

AM and BM satisfy $E_1(AM, 0) = E_1(0, BM) = m + V$.]

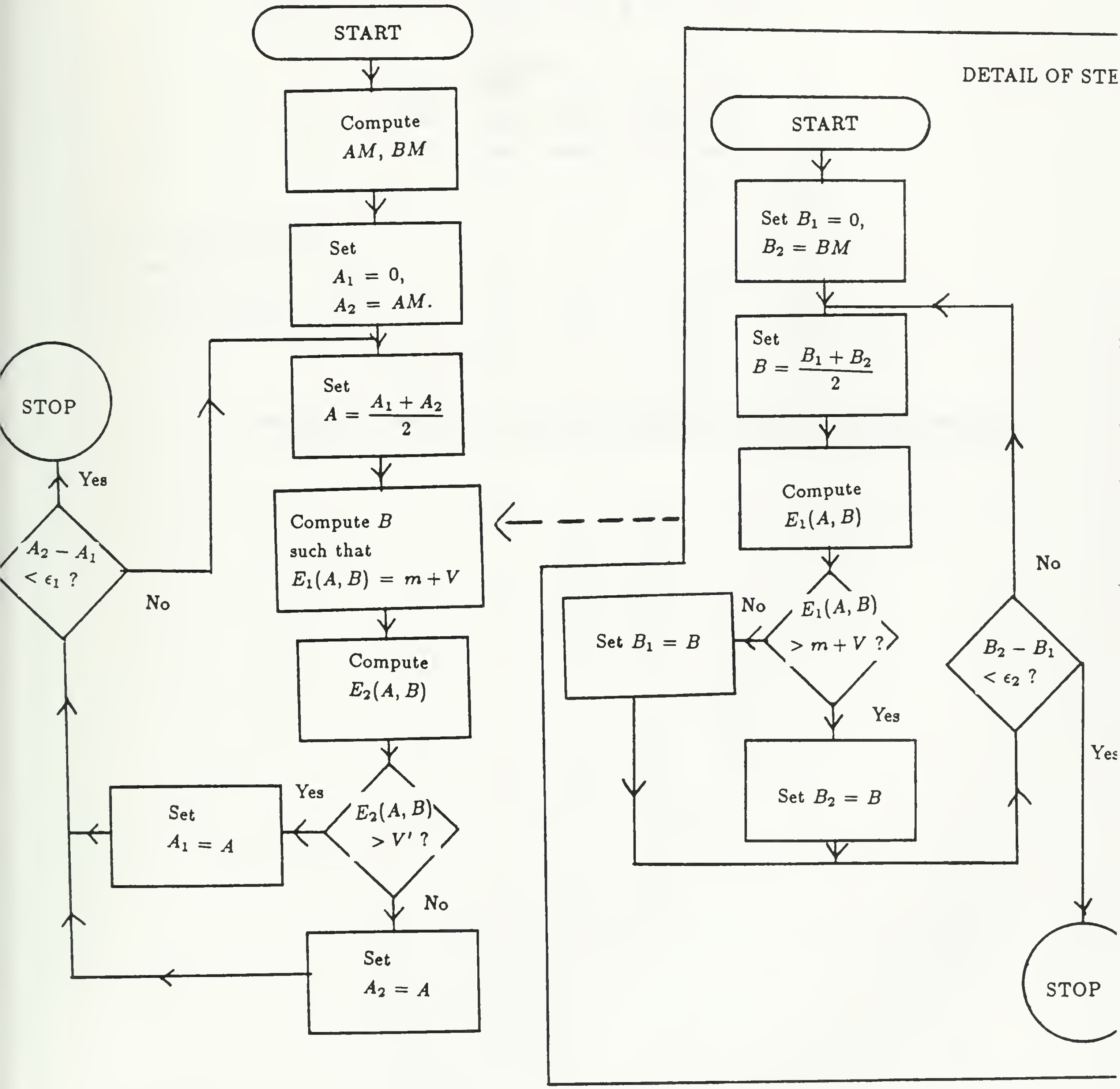


Table 1

Study Design

	$\delta = 1/3$	$\delta = .4$	$\delta = .5$	$\delta = .6$	$\delta = 2/3$
m_1	50	70	130	180	230
m_2	55	80	150	200	250
q_1	2	2	2	2	2
q_2	10	10	10	10	10
h_1	0	0	0	0	0
h_2	4000	4000	4000	4000	4000
k_1	4000	4000	4000	2500	2500
k_2	5000	5000	5000	3000	3000
$(\gamma_1)_1$	.5	.5	.5	.5	.5
$(\gamma_1)_2$	.6	.6	.6	.6	.6
d_1	1	1	1	1	1
d_2	1.5	1.5	1.5	1.5	1.5
$(\gamma_2)_1$	2	2	2	2	2
$(\gamma_2)_2$	2.5	2.5	2.5	2.5	2.5
c_1	0	0	0	0	0
c_2	.2	.2	.2	.2	.2

For every parameter, subscript 1 represents the low level, and subscrip 2 the high level.

Table 2

Summary of Comparative Statics Results For The Agency Theoretic Plan.

	Effect on Optimal					
Effect of	Profit π	Effort t	Salary Parameter A	Commission Rate Parameter B	Expected Income Z	Salary/ Expected Income
m ↑	↓	↕[1,2]	↑	↕[1,3]	↑	↑
q ↑	↑	↑	↓	↑	↑	↓
h ↑	↑	↓	↓	↓	↓	↓
k ↑	↑	↑	↓	↓ 0[4]	↑	↓
c ↑	↓	↓	↑	↓	↓	↑

1 → inconsistent with BLSS.

2 → ↑ observed only for $\delta = 2/3$.

3 → ↑ observed only for $\delta = 1/3$.

4 → 0 observed only for $\delta = 1/3$.

Table 3

Summary of Comparative Statics Results For Linear Plan

	Effect on Optimal					
Effect of	Profit π	Effort t	Salary Parameter A	Commission Rate Parameter B	Expected Income Z	Salary/ Expected Income
$m \uparrow$	$\downarrow$	$\updownarrow^{1}$	$\uparrow 0^{2}$	$\uparrow^{1}$	$\uparrow$	$\uparrow 0^{2}$
$q \uparrow$	$\uparrow$	$\uparrow 0^{2}$	$\downarrow 0^{2}$	$\updownarrow^{1}$	$\updownarrow^{1}$	$\downarrow 0^{2}$
$h \uparrow$	$\uparrow$	$\downarrow$	$\downarrow 0$	$\downarrow$	$\downarrow$	$\downarrow 0$
$k \uparrow$	$\uparrow$	$\uparrow 0^{2}$	$\downarrow 0^{2}$	$\updownarrow^{3}$	$\uparrow 0$	$\downarrow 0$
$c \uparrow$	$\downarrow$	$\downarrow 0^{2}$	$\uparrow 0^{2}$	$\downarrow 0^{2}$	$\downarrow 0^{2}$	$\uparrow 0^{2}$

1 $\rightarrow$ inconsistent with BLSS.

2 $\rightarrow$ similar to BLSS but weaker.

3 $\rightarrow$ $\downarrow$ observed only for $\delta = 1/3$.

Table 4

	(Effect of)	$\delta = 1/3$	$\delta = .4$	$\delta = .5$	$\delta = .6$	$\delta = 2/3$
$\hat{\alpha}_0$		.9926	.9758	.9433	.8607	.7321
$\hat{\alpha}_1$	(m)	.0052	.0144	.0207	.0196	.0156
$\hat{\alpha}_2$	(q)	−.0052	−.0195	−.0430	−.0753	−.0768
$\hat{\alpha}_3$	(k)	−.0033	−.0078	−.0152	−.0199	−.0231
$\hat{\alpha}_4$	(γ_1)	−.0020*	−.0057	−.0132	−.0268	−.0355
$\hat{\alpha}_5$	(γ_2)	.0034	.0090	.0211	.0451	.0674
$\hat{\alpha}_6$	(d)	.0016†	.0038	.0072	.0112	.0128
$\hat{\alpha}_7$	(h)	.0046	.0103	.0181	.0288	.0325
$\hat{\alpha}_8$	(c)	−.0063	−.0169	−.0265	−.0535	−.0559
$\hat{\alpha}_{12}$	$(m * q)$	.0050	.0133	.0162	.0055	0
$\hat{\alpha}_{13}$	$(m * k)$	.0020*	.0034	.0030	0	0
$\hat{\alpha}_{14}$	$(m * \gamma_1)$	.0016†	.0023†	.0027	0	0
$\hat{\alpha}_{15}$	$(m * \gamma_2)$	−.0023	−.0043	−.0053	−.0041	0
$\hat{\alpha}_{16}$	$(m * d)$	0	0	0	0	0
$\hat{\alpha}_{17}$	$(m * h)$	−.0034	−.0039	−.0030	0	0
$\hat{\alpha}_{18}$	$(m * c)$	.0049	.0093	.0053	.0019†	0
$\hat{\alpha}_{23}$	$(q * k)$	−.0029	−.0072	−.0114	−.0053	.0011*
$\hat{\alpha}_{24}$	$(q * \gamma_1)$	−.0020*	−.0051	−.0092	−.0039	.0046
$\hat{\alpha}_{25}$	$(q * \gamma_2)$	.0028	.0079	.0154	.0122	0
$\hat{\alpha}_{26}$	$(q * d)$	0	.0035	.0049	.0028	0
$\hat{\alpha}_{27}$	$(q * h)$	.0043	.0095	.0140	.0079	0
$\hat{\alpha}_{28}$	$(q * c)$	−.0056	−.0156	−.0217	−.0240	−.0081
$\hat{\alpha}_{34}$	$(k * \gamma_1)$	0	0	−.0025	−.0023	0
$\hat{\alpha}_{35}$	$(k * \gamma_2)$	0	.0025*	.0046	.0048	.0018
$\hat{\alpha}_{36}$	$(k * d)$	0	0	0	0	0
$\hat{\alpha}_{37}$	$(k * h)$	.0015†	.0028*	.0026	0	0
$\hat{\alpha}_{38}$	$(k * c)$	−.0028	−.0034	0	.0020†	.0051
$\hat{\alpha}_{45}$	$(\gamma_1 * \gamma_2)$	0	.0021†	.0051	.0093	.0063
$\hat{\alpha}_{46}$	$(\gamma_1 * d)$	0	0	.0020*	.0023*	0
$\hat{\alpha}_{47}$	$(\gamma_1 * h)$	0	0	.0027	.0030	0
$\hat{\alpha}_{48}$	$(\gamma_1 * c)$	−.0020*	−.0028*	−.0024*	−.0021†	.0024
$\hat{\alpha}_{56}$	$(\gamma_2 * d)$	0	0	−.0028	−.0039	−.0024
$\hat{\alpha}_{57}$	$(\gamma_2 * h)$	−.0018*	−.0031	−.0050	−.0066	−.0023
$\hat{\alpha}_{58}$	$(\gamma_2 * c)$	.0028	.0054	.0056	.0078	.0014
$\hat{\alpha}_{67}$	$(d * h)$	0	0	0	0	0
$\hat{\alpha}_{68}$	$(d * c)$	.0015†	0	0	0	−.0029
$\hat{\alpha}_{78}$	$(h * c)$	.0042	.0058	.0042	.0021†	−.0021
Adj R^2		.6153	.8539	.9630	.9781	.9960
$s(R_1)$		.0225	.0490	.0779	.1204	.1309

† → $p < .1$, * → $p < .05$, $p < .01$ otherwise. 0 → insignificant parameter.
$s(R_1)$ is the sample standard deviation of R_1.

Table 5

	(Effect of)	$\delta = 1/3$	$\delta = .4$	$\delta = .5$	$\delta = .6$	$\delta = 2/3$
$\hat{\beta}_0$		.9995	.9980	.9936	.9808	.9481
$\hat{\beta}_1$	(m)	.0003	.0013	.0030	.0038	.0049
$\hat{\beta}_2$	(q)	$-.0001^{\dagger}$	$-.0012$	$-.0042$	$-.0115$	$-.0213$
$\hat{\beta}_3$	(k)	$-.0002$	$-.0008$	$-.0024$	$-.0044$	$-.0086$
$\hat{\beta}_4$	(γ_1)	$-.0002$	$-.0008$	$-.0026$	$-.0071$	$-.0166$
$\hat{\beta}_5$	(γ_2)	.0002	.0009	.0030	.0085	.0200
$\hat{\beta}_6$	(d)	$.0001^{\dagger}$	.0004	.0012	.0026	.0050
$\hat{\beta}_7$	(h)	.0003	.0010	.0026	.0057	.0106
$\hat{\beta}_8$	(c)	$-.0003$	$-.0011$	$-.0025$	$-.0059$	$-.0059$
$\hat{\beta}_{12}$	$(m * q)$	.0002	.0011	.0024	.0021	.0013
$\hat{\beta}_{13}$	$(m * k)$	$.0002^{*}$	.0005	.0009	.0006	.0004
$\hat{\beta}_{14}$	$(m * \gamma_1)$	$.0001^{\dagger}$	.0005	.0010	.0012	.0010
$\hat{\beta}_{15}$	$(m * \gamma_2)$	$-.0002^{*}$	$-.0006$	$-.0012$	$-.0014$	$-.0013$
$\hat{\beta}_{16}$	$(m * d)$	0	$-.0003^{\dagger}$	$-.0004^{\dagger}$	$-.0004^{*}$	$-.0002^{\dagger}$
$\hat{\beta}_{17}$	$(m * h)$	$-.0002$	$-.0006$	$-.0010$	$-.0008$	$-.0004$
$\hat{\beta}_{18}$	$(m * c)$	.0002	.0008	.0009	0	$-.0004$
$\hat{\beta}_{23}$	$(q * k)$	$-.0001^{*}$	$-.0007$	$-.0019$	$-.0024$	$-.0022$
$\hat{\beta}_{24}$	$(q * \gamma_1)$	$-.0001^{\dagger}$	$-.0006$	$-.0019$	$-.0038$	$-.0040$
$\hat{\beta}_{25}$	$(q * \gamma_2)$	$.0001^{\dagger}$	.0007	.0023	.0046	.0049
$\hat{\beta}_{26}$	$(q * d)$	0	$.0004^{*}$	.0009	.0014	.0012
$\hat{\beta}_{27}$	$(q * h)$	.0002	.0008	.0021	.0032	.0027
$\hat{\beta}_{28}$	$(q * c)$	$-.0003$	$-.0011$	$-.0023$	$-.0041$	$-.0021$
$\hat{\beta}_{34}$	$(k * \gamma_1)$	0	$-.0003^{*}$	$-.0009$	$-.0014$	$-.0020$
$\hat{\beta}_{35}$	$(k * \gamma_2)$	0	$.0004^{*}$	.0011	.0017	.0024
$\hat{\beta}_{36}$	$(k * d)$	0	0	0	$.0005^{*}$	.0004
$\hat{\beta}_{37}$	$(k * h)$	$.0001^{*}$	$.0004^{*}$	.0008	.0010	.0009
$\hat{\beta}_{38}$	$(k * c)$	$-.0002^{*}$	$-.0004$	$-.0005^{*}$	0	.0008
$\hat{\beta}_{45}$	$(\gamma_1 * \gamma_2)$	0	$.0004^{*}$	.0013	.0034	.0065
$\hat{\beta}_{46}$	$(\gamma_1 * d)$	0	0	$.0005^{*}$	.0011	.0016
$\hat{\beta}_{47}$	$(\gamma_1 * h)$	$.0001^{\dagger}$	$.0004^{*}$	.0009	.0018	.0024
$\hat{\beta}_{48}$	$(\gamma_1 * c)$	$-.0001^{*}$	$-.0004$	$-.0007$	$-.0011$	$-.0003$
$\hat{\beta}_{56}$	$(\gamma_2 * d)$	0	0	$-.0006$	$-.0013$	$-.0019$
$\hat{\beta}_{57}$	$(\gamma_2 * h)$	$-.0001^{*}$	$-.0004$	$-.0011$	$-.0022$	$-.0030$
$\hat{\beta}_{58}$	$(\gamma_2 * c)$	$.0002^{*}$	.0005	.0009	.0014	.0004
$\hat{\beta}_{67}$	$(d * h)$	0	0	$-.0004^{\dagger}$	$-.0006$	$-.0005$
$\hat{\beta}_{68}$	$(d * c)$	0	0	0	0	$-.0005$
$\hat{\beta}_{78}$	$(h * c)$	.0002	.0006	.0007	$.0004^{*}$	$-.0009$
Adj R^2		.4251	.7113	.8831	.9834	.9977
$s(R_2)$		.0013	.0046	.0109	.0220	.0394

$\dagger \rightarrow p < .1$, $* \rightarrow p < .05$, $p < .01$ otherwise. $0 \rightarrow$ insignificant parameter.
$s(R_2)$ = sample standard deviation of R_2.

Table 6a

[Only the estimates of $\eta_0, \ldots, \eta_8$ are presented.]

	(Effect of)	$\delta = 1/3$	$\delta = .4$	$\delta = .5$	$\delta = .6$	$\delta = 2/3$
$\hat{\eta}_0$		.0321	.1194	.3370	.7883	1.4940
$\hat{\eta}_1$	(m)	−.0231	−.0699	−.1147	−.0854	−.0488
$\hat{\eta}_2$	(q)	.0272	.1094	.3067	.6546	1.0503
$\hat{\eta}_3$	(k)	.0158	.0434	.0947	.1114	.1035
$\hat{\eta}_4$	(γ_1)	.0110	.0389	.1039	.1959	.2801
$\hat{\eta}_5$	(γ_2)	−.0144	−.0432	−.1195	−.2218	−.2988
$\hat{\eta}_6$	(d)	−.0073†	−.0216	−.0430	−.0619	−.0566
$\hat{\eta}_7$	(h)	−.0202	−.0515	−.1012	−.1293	−.1072
$\hat{\eta}_8$	(c)	.0261	.0700	.1017	.0662	−.1115
Adj(R^2)		.6150	.8405	.9448	.9756	.9972
$s(R_3)$		.1027	.2478	.4906	.8089	1.1833

$\dagger \rightarrow p < .1$, $* \rightarrow p < .05$, $p < .01$ otherwise. $0 \rightarrow$ insignificant parameter.
$s(R_3)$ is the sample standard deviation of R_3.

Table 6b

[Only the estimates of $\nu_0, \ldots, \nu_8$ are presented.]

	(Effect of)	$\delta = 1/3$	$\delta = .4$	$\delta = .5$	$\delta = .6$	$\delta = 2/3$
$\hat{\nu}_0$		.0087	.0372	.1279	.4018	.9940
$\hat{\nu}_1$	(m)	−.0052	−.0244	−.0567	−.0653	−.0645
$\hat{\nu}_2$	(q)	.0050	.0300	.1101	.3406	.7589
$\hat{\nu}_3$	(k)	.0036	.0150	.0444	.0769	.1174
$\hat{\nu}_4$	(γ_1)	.0028*	.0130	.0456	.1195	.2311
$\hat{\nu}_5$	(γ_2)	−.0033	−.0155	−.0547	−.1443	−.2810
$\hat{\nu}_6$	(d)	0	−.0076	−.0216	−.0441	−.0660
$\hat{\nu}_7$	(h)	−.0047	−.0181	−.0500	−.0984	−.1399
$\hat{\nu}_8$	(c)	.0054	.0237	.0535	.1109	.0533
Adj(R^2)		.4542	7544	.9097	.9817	.9947
$s(R_4)$		.0263	.0897	.2239	.4933	.9190

$\dagger \rightarrow p < .1$, $* \rightarrow p < .05$, $p < .01$ otherwise. $0 \rightarrow$ insignificant parameter.
$s(R_4)$ is the sample standard deviation of R_4.