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the Tokyo Stock Exchange:  
An Exploratory Study

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College of Commerce and Business Administration

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May 1989

Price and Volume in the Tokyo Stock Exchange:  
An Exploratory Study

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This paper was written when I was visiting the University of Illinois at Urbana-Champaign. I thank Josef Lakonishok for stimulation and inspiration.





## Abstract

In this paper, we consider the unconditional distributions of market return and trading volume in the Tokyo Stock Exchange. We also examine the relationship between price changes and trading volumes. The problems considered are directed by findings in the U.S. market, with a view to ascertain similarities and differences.

Daily market returns exhibit intra-week periodicity. Returns over daily, weekly and monthly intervals are negatively skewed and leptokurtic. While daily returns are slightly positively autocorrelated, the autocorrelations for weekly and monthly returns are statistically insignificant. There is evidence against the stable Paretian hypothesis for market returns; and the empirical results are in support of market returns following a low order mixture of normal distribution. Trading volume is highly positively autocorrelated. This suggests that trading volume is not a good proxy for information. Our results also show that the price-volume relationship is weak and ambiguous.



## 1. Introduction

Due to the steady growth of the Japanese economy and capital market, together with the rapid appreciation of the Japanese yen against the U.S. dollar in the second half of the 1980s, the Tokyo Stock Exchange (TSE) has emerged as the world's largest stock market in terms of aggregate market capitalization. The First Section of the TSE has a total market value of 462.90 trillion yens (\$3.56 trillion) as of the end of 1988, which is 1.45 times the total market value of \$2.46 trillion of the New York Stock Exchange (NYSE). Despite the size and the obvious importance of the Japanese market, there is relatively little academic research in English in this market. The literature in this area, which has just started to grow, has been largely motivated by topics of research interest and controversies in the U.S. market, with an aim to provide insights and additional evidence to resolve these controversies. Issues that have been studied include: the size and periodicity anomalies (Jaffe and Westerfield (1985b), Kato and Schallheim (1985), Kato (1988a, 1988b), Kato, Ziemba and Schwartz (1989) and Ziemba (1989)), the multi-index model and the arbitrage pricing theory (Elton and Gruber (1988) and Hamao (1988)), general market characteristics (Hamao (1989)) and the Japanese market in an international perspective (Hamao, Masulis and Ng (1989), Gultekin and Gultekin (1983), Jaffe and Westerfield (1985a) and Poterba and Summers (1988)).

This paper focuses on the structure of price and volume in the TSE. We consider statistical descriptions of the stock market returns and trading volumes.<sup>1</sup> The relationship between the variability of

returns and trading volumes is also investigated. Our study is exploratory in nature and is directed by findings in the U.S. market, with a view to ascertain similarities and differences. It is of interest as it examines the international robustness and generality of the empirical results in the U.S. market.

Assumptions concerning the statistical distribution of asset returns are of great importance to certain financial models, such as the mean-variance portfolio selection theory and the pricing of derivative securities. The mean-variance theory can be justified by assumptions of risk aversion and normally distributed asset returns. The celebrated Black-Scholes option pricing formula assumes that returns of the asset upon which the contingent claim is based are normally distributed. Thus testing the normality hypothesis has been a topic of much research interest. In one of the early studies in this area, Fama (1965) found that stock returns have higher kurtosis (fatter tails) than one would predict for a normal distribution. A viable alternative using the stable Paretian distribution had been suggested by Mandelbrot (1963), whose work led to further studies by Fama and Roll (1968, 1971). Although the stable Paretian distribution may explain the "fat tail" findings of stock returns, it introduces the problem of infinite variance in the distribution, except for the special case of normality.<sup>2</sup>

Later studies reported evidence that is not consistent with the stable Paretian distribution. Officer (1972) found that the sample standard deviation of daily market returns is a well-behaved measure

of scale. Hsu, Miller and Wichern (1974) showed that the characteristic exponent of the stable Paretian distribution of a sequence of nonoverlapping sums of successive observations increases with the sum size.<sup>3</sup> This finding is inconsistent with the stable Paretian hypothesis. Hsu et al. suggested a time-varying nonhomogeneous distribution as an explanation for the high kurtosis. This suggestion was supported by Boness, Chen and Jatusipitak (1974), who put forward a theory of changes in capital structure as an explanation for the nonstationarity in prices. Blattberg and Gonedes (1974) compared the stable law with the Student's t distribution. Both hypotheses were derived from the framework of subordinated stochastic processes. In particular, if the variance of a normal random variable follows an inverted gamma distribution, then the posterior distribution is a Student's t.<sup>4</sup> The empirical results of Blattberg and Gonedes supported the Student's t distribution in preference to the stable Paretian model.

Recently Kon (1984) considered a discrete mixture of normal distribution (MND) as an explanation for the observed excess kurtosis and positive skewness of stock returns in the U.S. market.<sup>5</sup> He estimated MND models for the daily returns of the 30 stocks of the Dow Jones Industrial Index and three market indices. By fitting MND models with orders up to five, he argued that the MND is considerably more descriptive of the data generating process than the simple normal model and the Student's t distribution. This finding is congruent with a time-varying nonstationary return process. Periodicity anomalies and varying financial and operating leverages were offered as

explanations for the nonstationarity. Additional evidence in support of the MND was provided by Harris (1986) and Hall, Brorsen and Irwin (1989).

In comparison, empirical research in the distribution of trading volume per se has been very scarce--volume has been of interest mainly as a covariate in explaining the distribution of asset return. Clark (1973) assumed that asset return follows a subordinated stochastic process in which the directing process is the cumulative volume. Thus the volume in each nonoverlapping period was assumed to be independently distributed. As a special case, Clark considered the log-normal directing process and provided empirical evidence supporting this assumption. Tauchen and Pitts (1983) suggested a mixing variable model in which price change and volume are simultaneously determined. These two variables are driven by a mixing variable which represents the amount of information reaching the market. As the mixing variable is serially independent, both price change and volume are serially independent. Tauchen and Pitts estimated the parameters of the price and volume processes simultaneously under the assumptions that the mixing variable follows a Poisson as well as a log-normal distribution.

In both the Clark and Tauchen-Pitts models trading volume is serially independent. While the efficient market hypothesis requires stock return to be serially uncorrelated, this restriction would seem to be too stringent when imposed on trading volume. The restriction may be violated when volume is an imperfect proxy for information or when it is not directed by the mixing variable with the given structure.

The earliest empirical examinations of the price-volume relationship were conducted by Granger and Morgenstern (1963) and Godfrey, Granger and Morgenstern (1964). Theoretical models explaining the relationship were suggested by Epps (1975), with further developments in Epps and Epps (1976). Since then, the research output on this topic has become very substantial. The recent survey by Karpoff (1987) provided an excellent review of the literature as well as an extensive bibliography. In particular, he summarized the following stylized facts regarding the price-volume relationship in the U.S. stock market: (i) volume is positively related to the magnitude of the price change, and (ii) volume is positively related to the price change per se.<sup>6</sup>

The plan of the rest of the paper is as follows. In Section 2, we briefly survey some features of the Japanese market. Section 3 describes the data. The market return is examined in Section 4. It is found that the daily return periodicity has become more similar to that in the U.S. market. Excess kurtosis occurs, but it cannot be explained by the periodicity anomaly. Somewhat surprisingly, the return is found to be negatively skewed. Its distribution can be adequately described by a low order MND; and there is evidence against a stable Paretian distribution. In Section 5, we analyze the volume distribution. Significant serial correlation is found in the trading volume, which raises questions about the use of volume as a proxy for information. The price-volume relationship is examined in Section 6. The results suggest that volume is positively related to price change

per se, although the evidence is not very strong. Some concluding comments are given in Section 7.

## 2. The Japanese Market and Its Indicators

There have been a number of publications that describe and explain the Japanese financial markets; a notable comprehensive introduction is the book by Viner (1988). The articles by Hodder and Tschoegl (1985), Hamao (1989) and Kato, Ziemba and Schwartz (1989) are also useful references.

There are eight exchanges in Japan. Listings on the TSE represent more than 95 percent of the total market value. The TSE is divided into two sections: the First Section and the Second Section. The First Section includes the top rated and most actively traded companies and the Second Section, with less severe listing requirements, consists of smaller stocks. As of the end of 1988, the First Section consisted of 1135 stocks and contributed over 95 percent of the total market value of the TSE.

Currently, the TSE trades five days a week, on Monday through Friday. Historically, there was trading on all Saturdays until the end of 1972. Then until July 1983, the exchange was closed on the third Saturday, and later the second Saturday as well from August 1983 to July 1986. From August 1987 to January 1989 there was trading on Saturday in the first and fourth weeks of the month (and on the fifth if there was one).

The most prominent market indicator for the TSE is the Nikkei Stock Average based on 225 issues (N225 hereafter). This index, which



started in 1949, was created by the Dow Jones Company and was then named Nikkei-Dow Average. Following the inauguration of the Tokyo Stock Exchange Average (Topix) in 1969, the Nikkei-Dow Average was discontinued in 1971. In 1975, the Nihon Keizai Shimbun Company (NKS) reinitiated the Nikkei-Dow Average, and renamed it the Nikkei Stock Average in 1985. Apart from the N225, the NKS also provides a Nikkei Stock Average based on 500 representative issues (N500 hereafter), which has been calculated since 1972.

Both N225 and N500 are price-weighted indices. Their computation is similar to that of the Dow Jones Industrial Average. Prices of the component stocks (225 for N225 and 500 for N500) in the index are added and their sum is divided by a divisor which is adjusted when prices of component stocks move due to nonmarket factors, such as stock splits and rights issues. Adjustments are also made when constituents are deleted or added. While there have been few changes in the identity of the 225 issues in N225, the 500 companies in N500 are reselected once a year. On February 28, 1989 the N225 index was 31985.60 and the N500 index was 1870.50.

Topix is a value-weighted index of all stocks listed in the First Section of the TSE. Its methodology is similar to that of the S&P 500. The index was initially set at 100 on July 4, 1968, and its value on February 28, 1989 was 2447.23. Although less popular than the N225, the Topix is considered the most representative indicator of the total market. In particular, the Topix is able to reflect changes in the industrial structure and is less affected by price changes in a handful of smaller companies' stocks trading at high prices.

### 3. The Data

The data used in this study were extracted from the NEEDS (Nikkei Economic Electronic Database System) data base provided by NKS. A description of this data base can be found in Roehl (1985).

Daily, weekly and monthly series for each of the Topix, N225 and N500 indices were extracted. As the weekly and monthly series of the Topix index were given in arithmetic means over the interval (instead of end of the interval), we did not use them in this study. Table 1 summarizes the sampling periods of the data. The market indices were converted to continuously compounded rate of return by calculating the first difference of the logarithm. Trading volume series were also extracted for the same sampling intervals and periods. We considered three measures of volume:

$V1 = \text{number of shares traded (in billions);}$

$V2 = (V1 \times \text{arithmetic stock price average}) / (\text{total market value at end of interval});$

$V3 = (\text{sales value during the interval}) / (\text{total market value at end of interval}).$

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Insert Table 1 About Here  
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V3 measures the sales as a proportion (in percent) of the total market value. This is a better measure for the trading volume than V1, which is positively trended due to the increase in the total number of shares over time. However, as V3 was available only monthly, V2 was calculated as a proxy for V3 for the daily and weekly data. The

correlation coefficients of the three price series and the three volume series are given in Table 1.

The October 1987 crash created some discontinuity jumps in the return series. The Topix index dropped by 15.8 percent on October 20 and recovered by 9.0 percent on October 21.<sup>7</sup> Including these two observations in the data created some anomalies. For example, when the observations were included, the first order autocorrelation coefficient of the return of the N500 index was 0.0037; and the skewness and kurtosis coefficients were, respectively, -3.88 and 73.26. When these observations were excluded, the autocorrelation coefficient increased to 0.1165; and the skewness and kurtosis coefficients became -0.32 and 10.20, respectively. Thus, the extreme return values due to the crash bias the autocorrelation coefficient towards zero, leading to the erroneous conclusion of insignificant autocorrelation, and overstate the higher order moments.

As argued strongly by Greenwald and Stein (1988) in their comments on the Task Force Report, the October 1987 crash should be regarded as a unique event.<sup>8</sup> Thus, we excluded two observations in our data set for all computations involving daily market returns. For weekly data, we also excluded the observation for the week in which the crash occurred.<sup>9</sup> Examining the volume series, we found that there were no abnormalities caused by the crash. Indeed, the trading volumes on October 20 and 21 were below the average over the sampling period. Thus, no observation was excluded for computations that involve only the volume series.

From Table 1, we see that the correlation for the daily data between Topix and N225 is higher than that between Topix and N500, despite the fact that N500 is a bigger subset of Topix than N225.<sup>10</sup> For the monthly volume data, the correlation between V1 and V2 is higher than that between V1 and V3, and V2 and V3. As V2 depends directly on V1, the higher correlation between V1 and V2 is perhaps not unexpected. However, the fact that the correlation between V2 and V3 is virtually the same as that between V1 and V3 implies that V2 is probably not a better proxy for V3 than V1.

#### 4. Price

Table 2 summarizes the first four moments of the return series. To examine the intra-week periodicity, we calculated the summary statistics for different days of the week. Monday was found to have negative return, while returns for all other days were positive, with the return on Tuesday being close to zero. Although trading on Saturday was only for half day, the return was highest among all days of the week. To test the hypotheses: (i) returns are equal Monday through Saturday, and (ii) returns are equal Tuesday through Saturday, we apply the Wald statistics, which are distributed as  $\chi^2$ . At 5 percent significance level, the first hypothesis is rejected while the second one cannot be rejected.

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Insert Table 2 About Here  
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The above findings are different from others in the recent literature. Kato et al. (1989) pointed out that studies in the Japanese

market since 1970 indicate typically small losses on Monday and substantial losses on Tuesday. Compared with results in the U.S. market on the day of the week effects (see, e.g., Dimson (1987)), the shift from the "Tuesday effect" to the "Monday effect" found here is more in line with the periodicity in the U.S. market. However, earlier findings for the Japanese market in the 50s and 60s indicated that the heaviest losses were on Monday (see Kato et al. (1989, Table 6)). Thus, it is unclear whether the present evidence should be regarded as an indication of the narrowing of differences across national markets, or of the period-specific nature of the intra-week periodicity in the TSE.

Return distributions are negatively skewed and leptokurtic.<sup>11</sup> This finding is true for the daily, weekly as well as monthly data; although the excess kurtosis is smaller for returns over longer interval. The importance of skewness in the distribution of asset returns to investors' decisions has been extensively studied (see, e.g., Arditti (1967, 1971) and Francis (1975)). As returns in the equity market are bounded from below but not from above, it is likely that market returns are positively skewed. Studies in the U.S. market (see, e.g., Fielitz and Smith (1972), Fielitz (1976), Simkowitz and Beedles (1980), and Kon (1984)) have ascertained positive skewness in returns, a fact in contrast with the present finding for the TSE.

One explanation that has been postulated for the nonnormality of daily market returns is the intra-week periodicity. Specifically, the observed skewness and fat tail are caused by pooling different normal distributions characterizing different days of the week.<sup>12</sup> If this

hypothesis is correct, we would expect the nonnormality to vanish for returns categorized by day of the week. However, evidence from Table 2 shows that this hypothesis is not supported--the excess kurtosis does not get smaller for returns thus categorized. If nonnormality is caused by the mixing of distributions, the mixing structure is more complex than a simple intra-week partition.

In Table 3, we summarize the results for testing for autocorrelation in the return as well as the variance of the return. For each return series, we calculated the deviation from mean, which is denoted as  $e_t$ . Since it had been established that the mean return for Monday was different from the rest of the week, we used a dummy variable to take account of this difference in the daily return series. For each residual series  $e_t$ , we calculated the first and second order autocorrelation coefficients, denoted as  $r_1$  and  $r_2$ , respectively, and the statistics for two tests for autocorrelation: the runs test and the Box-Pierce portmanteau test. The same statistics were also calculated for  $e_t^2$  as tests for autocorrelation in the variance.<sup>13</sup>

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Insert Table 3 About Here  
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For  $e_t$ ,  $r_1$  is significantly different from zero for the daily data for all indices, although its magnitude is quite small in all cases. Both the runs test and the Box-Pierce test show evidence of serial correlation. As  $r_2$  and higher order autocorrelation coefficients (not reported in the table) are insignificant, the daily return series appear to be adequately described by a first order moving average

process with a small moving average parameter.<sup>14</sup> For data over longer intervals, the residual returns behave like a white noise.

For  $e_t^2$ , there is significant autocorrelation for the daily and weekly data. From results not presented in the table, the autocorrelation coefficients are significantly different from zero even for very high order lags. One possible explanation for the serial correlation in variance is that the complete sampling period may consist of subperiods of homogeneous distributions (Hsu et al. (1974)) determined by varying leverage structures (Boness et al. (1974)). Nonstationarity in the variance is sufficient to explain the leptokurtosis of the return distribution. A statistical model which may capture the structure of the serial correlation in the variance is the autoregressive conditional heteroskedasticity (ARCH) model due to Engle (1982).<sup>15</sup> However, instead of pursuing the serial structure of the variance, we now turn to the problem of modeling the unconditional return distribution.

There has been much empirical research in the unconditional distribution of asset return in the U.S. market. The simple normal distribution, stable Paretian law, Student's t distribution and mixture of normal distribution have been considered, among others.<sup>16</sup> The conclusions that seem to have emerged can be summarized as follows. The simple normal distribution is inadequate to describe the distribution of an asset return as it understates the kurtosis. The stable Paretian hypothesis, which was at one time favored by some researchers, has been found to be less descriptive than the Student's t distribution (Blattberg and Gonedes (1974)). However, as emphasized

by some studies (Fielitz and Smith (1972) and Fielitz (1976)), asset returns are typically skewed--a fact that is inconsistent with the Student's t distribution. A generalized version of the stable Paretian law that allows for skewness has been proposed and estimated (Simkowitz and Beedles (1980) and Akgiray and Lamoureux (1989)). Simkowitz and Beedles suggested that returns might be mixtures of stable distributions. Kon (1984) compared the discrete MND with the Student's t distribution and argued in favor of the former. In view of the above findings and the demonstrated skewness and kurtosis of returns in the TSE, we consider the following unconditional distributions for the TSE market return: the stable Paretian law and the MND.

The full stable Paretian family that allows for asymmetric distributions is characterized by four parameters. There have been some variations in the definitions for the stable family, which at times have led to confusion. We follow the definition proposed by Zolotarev (1957) and adopted by McCulloch (1986), in which the log characteristic function of a stable Paretian distribution has the form

$$\begin{aligned} \psi(t) &= \log E(e^{itx}) \\ &= \begin{cases} it\delta - |ct|^\alpha [1 - i\beta \text{sign}(t) + \tan \frac{\pi\alpha}{2}] & \alpha \neq 1 \\ it\delta - |ct| [1 + i\beta \frac{2}{\pi} \text{sign}(t) \log|t|] & \alpha = 1, \end{cases} \quad (1) \end{aligned}$$

where  $x$  is a stable Paretian variable,  $t$  is the parameter of the characteristic function,  $i^2 = -1$  and  $\alpha$ ,  $\beta$ ,  $\delta$  and  $c$  are, respectively, the characteristic exponent, the skewness parameter, the location parameter and the scale parameter. The normal distribution is a



special member of the stable family with  $\alpha = 2$ , and is the only stable distribution for which the variance exists. When  $\alpha < 2$ , absolute moments of order less than  $\alpha$  exist, while those of order equal to or greater than  $\alpha$  do not.

The density and distribution functions of a stable distribution do not exist in closed form, except for the normal ( $\alpha=2$ ) and Cauchy ( $\alpha=1$ ) cases. However, infinite series expansions for these functions are available.<sup>17</sup> The lack of analytic expression for the density function makes the estimation of the parameters by maximum likelihood method very difficult. Fama and Roll (1968, 1971) suggested a fractile method based on order statistics. This method was improved by McCulloch (1986), who extended the Fama-Roll method to take account of asymmetric distributions and a broader range of  $\alpha$ . The small asymptotic biases in the Fama-Roll estimators were also eliminated. McCulloch's procedure involves the calculations of five sample quantiles, with proper corrections for continuity.<sup>18</sup> The stable distribution parameters are then obtained from tables by simple interpolation. The results reported below were obtained by this method.<sup>19</sup>

An important property of the stable Paretian law is that it is invariant under addition. That is, a sum of independently identically distributed stable variables with characteristic exponent  $\alpha$  is stable with the same exponent. This property has been used by Fama and Roll (1971) as the basis for a test for the hypothesis that a random variable is stable Paretian. Hsu et al. (1974) and Hall et al. (1989) followed Fama and Roll in their studies in the behavior of the U.S stock market and futures market returns. Hsu et al. showed that

randomizing the stock market return series before summing up the adjacent observations increases the power of the test--the shift in the characteristic exponent is more prominently demonstrated for the randomized series.<sup>20</sup> This randomization procedure was adopted by Hall et al. in their study in the futures market. Their results also provided evidence against the stability of the characteristic exponent. In view of these findings, we estimated the stable Paretian distribution for the randomized as well as nonrandomized return series.

We considered two methods of forming sums of returns. First, we arranged a returns series in chronological order and summed up adjacent observations  $k$  terms at a time to form a new series. We defined this method as the chronological sampling scheme, denoted as C. Second, we randomized the returns series and summed up adjacent observations again as above. This method was defined as the randomized sampling scheme, denoted as R. As the summed series had to have sufficient observations for estimation, the value of  $k$  was constrained. For daily data, we considered  $k = 2, 4, 6,$  and  $8$ ; and for weekly data, we considered  $k = 2$ . The stable Paretian distribution was estimated for the original series as well as the summed series. Estimates of  $\alpha$  are presented in Table 4.<sup>21</sup>

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Insert Table 4 About Here  
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The results show very clearly that, for the randomized series, the estimated characteristic exponent approaches two as the sum size increases. The evidence is thus against the stable Paretian distribution.

Similar to findings in the U.S. market, the chronological series fail to detect shifts in  $\alpha$ . This would be the case if shifts in return distributions occur over stretches of homogeneous structures, or if the shifts are serially correlated.

Having rejected the stable Paretian law, we now consider the MND model as a description of the unconditional distribution of the TSE market return. In the MND model, the market return is assumed to be a random drawing from a set of  $m$  normal distributions. Specifically, let  $x_i$  be normally distributed with mean  $\mu_i$  and variance  $\sigma_i^2$ , for  $i = 1, \dots, m$ . Suppose  $\{\lambda_i\}$  represent a discrete probability function defined on  $i = 1, \dots, m$  such that  $1 \geq \lambda_i \geq 0$  and  $\lambda_1 + \dots + \lambda_m = 1$ . Then  $y$  is said to be a mixture of normal random variable if  $y = x_i$  with probability  $\lambda_i$ . Thus, if we denote the density function of  $x_i$  by  $f(x_i | \theta_i)$  where  $\theta_i = (\mu_i, \sigma_i^2)'$ , then the density function of  $y$  is given by

$$g(y|\theta) = \sum_{i=1}^m \lambda_i f(y|\theta_i), \quad (2)$$

where  $\theta = (\mu_1, \dots, \mu_m, \sigma_1^2, \dots, \sigma_m^2, \lambda_1, \dots, \lambda_{m-1})'$  is the vector of parameters with  $3m-1$  elements. Assuming we have a sample with  $n$  independent observations of  $y$  denoted by  $y_1, \dots, y_n$ , we can write down the likelihood function of the sample as

$$\begin{aligned} L(\theta; m) &= \prod_{t=1}^n g(y_t | \theta) \\ &= \prod_{t=1}^n \left[ \sum_{i=1}^m \lambda_i f(y_t | \theta_i) \right]. \end{aligned} \quad (3)$$

Thus, maximum likelihood estimates of  $\theta$  can be obtained by maximizing  $L(\theta;m)$  (or  $\log L(\theta;m)$ ) with respect to  $\theta$  using nonlinear optimization methods.<sup>22</sup>

Kon (1984) estimated the MND model for the 30 component stocks of the Dow Jones Industrial Index and three market indices. He contended that the MND is a better description of the stock market return than the Student's  $t$  distribution. In particular, he found that the market indices follow a MND of order three. As for the 30 stocks, their orders vary from two to four.

In Kon's study, the optimization procedure did not impose any restrictions on  $\lambda_i$ . In some cases, the conclusion about the order of the MND model was weakened due to the failure of the numerical optimization procedure to converge to a well-defined distribution.<sup>23</sup> To overcome this difficulty, we use a reparameterization of the quantities representing the probabilities. Thus, we consider  $m-1$  parameters  $\delta_i$ , for  $i = 1, \dots, m-1$ , and define

$$\lambda_i = \exp(\delta_i) / (1 + \sum_{i=1}^{m-1} \exp(\delta_i)), \quad (4)$$

for  $i = 1, \dots, m-1$  and  $\lambda_m = 1 / (1 + \sum_{i=1}^{m-1} \exp(\delta_i))$ .<sup>24</sup> Although  $\delta_i$  are unrestricted, the transformation satisfies the restrictions  $1 > \lambda_i > 0$  and  $\lambda_1 + \dots + \lambda_m = 1$ . Maximum likelihood estimates of  $\theta$  can be obtained by maximizing the likelihood with respect to  $\theta_1, \dots, \theta_m$  and  $\delta_1, \dots, \delta_{m-1}$  and then recovering  $\lambda_i$  from Equation (4). The estimation results for the Japanese market are summarized in Table 5.

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Insert Table 5 About Here  
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To determine the order of the MND model and test the variability of the means and variances across the set of normal distributions, we applied the following procedure. For each of the return series, we fitted the MND model with increasing order until the increase in the log likelihood was insignificant. The criterion for the stopping point was the likelihood ratio statistic, i.e., twice the increase in the log likelihood, which is distributed as  $\chi_3^2$ . Thus,  $m$  was selected as the smallest integer such that  $2\log(L(\theta;m+1)/L(\theta;m)) < \chi_{3,0.05}^2 = 7.82$ . Of all the series considered, the MND models fitted are of very low order. Specifically, the monthly N225 series is of order one, the weekly N225 series is of order three and all other series are of order two. The likelihood ratio statistics  $L_1 = 2\log(L(\theta;m+1)/L(\theta;m))$  and  $L_2 = 2\log(L(\theta;m)/L(\theta;m-1))$  are presented in Table 5. For the daily data, the sharp drops from  $L_2$  to  $L_1$  are indicative of the unambiguous specification of  $m$  in these cases.<sup>25</sup>

Once  $m$  was determined, we tested the following hypotheses separately: (i)  $\mu_i$  are equal for all  $i$ , and (ii)  $\sigma_i^2$  are equal for all  $i$ .<sup>26</sup> Of all the cases considered, the second hypothesis was always rejected. The first hypothesis was rejected for the daily Topix and N500 series. The estimates reported take account of the accepted restrictions. To determine the goodness of fit of the final model for each series, we computed the implied first four moments of the MND and compared them with the sample quantities given in Table 2. These results are given in the last four columns of Table 5, where the figures in parentheses are the sample quantities. For most cases, the agreement of the two sets of figures is surprisingly good. The only

exceptions come from the daily Topix and N500 series, in which cases imposing equality of means brings about discrepancies in the implied and sample first moments. However, relaxing this restriction results in poorer fit for the skewness and kurtosis.

To sum up, the market return in the TSE exhibits intra-week periodicity. Monday has the lowest return, while returns on all other days of the week are not significantly different from one another. This finding shows a reversal to the periodicity during the 50s and 60s, and coincides with that of the U.S. market. Daily returns are weakly positively autocorrelated and follow a moving average process of order one. Serial correlations for returns over a week and a month are statistically insignificant. Return data are negatively skewed and leptokurtic. These features persist for returns calculated by the day of the week. Thus, nonnormality cannot be explained by intra-week periodicity. Finally, there is evidence against the stable Paretian law as a description of the unconditional return distribution. A better model is the MND, which fits the data adequately with low order mixtures.

## 5. Volume

There have been few studies in the statistical distribution of stock market sales volume per se. Most studies in stock return and volume focus on the distribution of return and use volume as a covariate in the model. However, the statistical distribution of volume may be of interest for two reasons. First, it is often argued that volume is a proxy for information arrival. As information

arrival is random, volume is expected to be serially uncorrelated. Thus, the serial structure of volume may indicate whether volume is a good proxy for information. Second, some models either postulate the price-volume relationship jointly (Tauchen and Pitts (1983)) or explain the return process as a subordinated process driven by a directing process that can be proxied by volume (Clark (1973) and Blattberg and Gonedes (1974)). Knowledge about the marginal distribution of volume can be used to check the implications and assumptions of these models.

Summary statistics of the three volume measures defined in Section 3 are presented in Table 6. Except for V3, of which only monthly data are available, volume statistics were calculated for daily (further categorized by day of the week), weekly and monthly data. The table shows the first four moments and the first order sample autocorrelation coefficients.<sup>27</sup> We observe that volume distributions are positively skewed and slightly leptokurtic. However, as there are large and statistically significant autocorrelations in the volume series, the asymptotic tests for skewness and excess kurtosis conducted in Table 2 are biased and hence are not used here.<sup>28</sup> Point estimates of the mean show that Saturday volume is lower than other days of the week, which is expected because of the shorter trading time on Saturday.

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Insert Table 6 About Here  
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To capture the autocorrelation structures of the volume series, we fitted time series distributed lag regressions for the volume data. Specifically, we considered the following model

$$V_t = \beta_0 + \beta_1 V_{t-1} + \beta_2 V_{t-2} + \beta_3 t^*, \quad (5)$$

where  $V_t$  is the volume and  $t^*$  is a time variable. For each series, we estimated (5) starting with one lag term in  $V_t$  and proceeded to include higher order terms if necessary. To examine the residual autocorrelation, we calculated  $r_1$  and  $h$ , which is Durbin's statistic for testing residual autocorrelation.<sup>29</sup> The lowest order distributed lag regression with insignificant residual autocorrelation was selected. Insignificant regression variables were then discarded and the model was reestimated. Table 7 summarizes the regression results.

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Insert Table 7 About Here  
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For the daily series dummy variables that represent different intercepts for different days of the week were included. These dummies were found to be statistically significantly different from one another. To save space, results for the dummies and intercepts are not reported. The time variable  $t^*$  was scaled in such a way that  $\beta_3$  represents the change in the volume per sampling interval over a period of one year. For example, the first row in the table shows that  $V1$  increases by 0.0377 billion shares per day over a year; and the last row shows that  $V3$  increases by 0.0027 percentage points per month over a year.

It can be seen that volumes follow low order autoregressive processes. Although  $t^*$  is statistically significant for most cases, it is economically insignificant for  $V2$  and  $V3$ . By contrast, the rate of increase in  $V1$  is economically significant. For example, the increase



in V1 per month over a year is 16.2 percent ( $2.474 \div 15.317$ ) of the average monthly volume over the sampling period. However, this increase may be largely due to the increase in the number of shares outstanding. The residual analysis shows that there are significant positive skewness and excess kurtosis. This departure from normality is significant for all cases except for the monthly V3 series.

To sum up, the volume series follow low order autoregressive processes with statistically significant time trends. The increase in the sales ratio over time is economically insignificant; and, in the case of the number of shares traded, the economically significant time effect may be due to the growing number of shares outstanding. The volume distributions are positively skewed and leptokurtic, except for the monthly V3 series, which is not statistically different from a normal distribution. As V3 is likely to be the best measure for volume, this finding is notable.

## 6. Price-Volume Relationship

To find out if there is any empirical regularity in the relationship between price changes and trading volumes in the TSE, we estimated regressions of price changes on trading volumes. Measures of price changes considered were:  $\Delta \log P_t$ ,  $(\Delta \log P_t)^2$  and  $|\Delta \log P_t|$ , where  $P_t$  is one of the three market indices. All three volume measures defined in Section 3 were used as regressors. As the results for different volume measures are qualitatively similar, only findings for the regressions on V2 are reported. The estimated equations and residual diagnostics are summarized in Table 8. For the daily data, we

included a dummy variable to take account of the different Monday return. The dummy takes a value of one for Monday and zero otherwise.

Karpoff's (1987) asserted "stylized fact" for the U.S. market that trading volume has effects on both  $\Delta \log P_t$  and  $|\Delta \log P_t|$  is ambiguous for the TSE. Volume is significant for the daily Topix and N225 indices when the dependent variable is  $\Delta \log P_t$ ; and is also significant for the weekly N225 index for all the three price-change measures considered. Otherwise, the effects of volumes on price changes are insignificant. Thus, the results are not only dependent on the sampling intervals, but also on the price indices. It is perhaps safe to conclude that the relationship between price changes and volumes in the market, if there is any, is weak. In Karpoff's review, most studies in the U.S. market examined individual stocks. The conflicting results for different indices in Table 8 may be due to the aggregation over different stock components. Also, as the volume measures apply to all tradings in the First Section of the TSE, some data incompatibility may occur. In view of these difficulties, it would be interesting to consider the price-volume relationship for individual stocks, which would not be subject to the problems of aggregation and data incompatibility. Future research in this area will be required to ascertain any price-volume regularity.

## 7. Conclusions

We have examined the unconditional distributions of the market returns and trading volumes in the Tokyo Stock Exchange, as well as the relationship between price changes and volumes. Our results have

demonstrated some similarities and differences between the Japanese and U.S. markets.

The intra-week periodicity in daily return in the TSE has shifted from a previous Tuesday loss, as found by many authors, to a Monday loss in the period under this study. Negative skewness and excess kurtosis are found in daily returns; and this evidence of nonnormality persists in data categorized by the day of the week. Daily returns are slightly positively autocorrelated, while returns over a week or a month are not significantly serially correlated. Although the returns are characterized by fat tails, the stable Paretian hypothesis, which was once postulated to account for this problem, is rejected on the basis that the characteristic exponent is unstable under addition. The mixture of normal distribution appears to be a better statistical model. In particular, low order mixtures do sufficiently well to describe the returns data.

We have measured trading volumes by three quantities and have argued that the sales ratio is the best choice. Trading volumes exhibit positive serial correlation and are unlikely to be good proxies for information arrival. The volumes follow low order autoregressive processes with statistically significant time trends, although the trends are only economically significant for volumes measured by sales in numbers of shares.

The price-volume relationship in the TSE is weak and ambiguous. However, the lack of a strong conclusion in this aspect may be due to the use of aggregate market returns and the incompatibility of the volume measures. Further studies using company data are required to ascertain any regularities of this relationship.

Footnotes

<sup>1</sup>In this paper, return is defined as the continuously compounded yield, computed as the first difference in the log price. We shall use the terms price structure and return structure interchangeably.

<sup>2</sup>Normal distribution is a special case of the stable Paretian family. See Mandelbrot (1973) for a discussion of the problem of infinite variance.

<sup>3</sup>Hsu et al. used a randomized sequence to uncover the shift in the characteristic exponent.

<sup>4</sup>See Appendix A of Blattberg and Gonedes (1974) and Raiffa and Schlaifer (1961, § 7.9).

<sup>5</sup>The MND model was earlier suggested by Fama and Roll (1971). A Poisson mixture of normal distribution was considered by Press (1967).

<sup>6</sup>The term "price change" here may be interpreted as price change relative, or continuously compounded rate of return.

<sup>7</sup>As shown in Table 2 below, the average daily return is only 0.11 percent with a standard deviation of 0.99 percent.

<sup>8</sup>Greenwald and Stein (1988, p. 16) concluded that "the stock market crash of October 1987 was fundamentally a unique event, wholly different in character from normal market behavior." If this conclusion is accepted, observations of the crash event should be regarded as outliers.

<sup>9</sup>The drop in this week for the N225 index was 12.37 percent. From Table 2 below, the mean weekly return is 0.49 percent with a standard deviation of 1.80 percent.

<sup>10</sup>It is recalled that N225 and N500 are price-weighted, while Topix is value-weighted.

<sup>11</sup>A distribution is said to be leptokurtic if its kurtosis coefficient is greater than three, which is the kurtosis value for a normal distribution. Tests for symmetry and no excess kurtosis are conducted in Table 2; and significance is indicated by an asterisk. These tests assume independent sample observations. As shown in Table 3 below, the daily return series are slightly positively autocorrelated. This implies that the significance of the tests may be overstated. However, for the kurtosis test, this problem is unlikely to affect the results qualitatively, as the kurtosis coefficients are quite large.

<sup>12</sup>See Kon (1984) for discussions of this hypothesis and other possibilities of nonstationarity in the mean and variance.

<sup>13</sup>The distribution of the Box-Pierce statistic for  $e_t^2$  was established by McLeod and Li (1983).

<sup>14</sup>Hamao et al. (1989) also used a moving average process of order one to correct for residual autocorrelation in their study in international market links.

<sup>15</sup>See Engle and Bollerslev (1986) for a survey of the ARCH literature.

<sup>16</sup>Some authors considered mixed diffusion-jump processes. See Akgiray and Booth (1986) for more details.

<sup>17</sup>Fama and Roll (1968, pp. 818-819) provided expressions for these convergent series.

<sup>18</sup>The quantities required are the 0.05, 0.25, 0.50, 0.75 and 0.95 sample quantiles. These quantiles are different from those used by Fama and Roll. The continuity corrections are critical for obtaining good estimates of  $\beta$ .

<sup>19</sup>Al alternative method is the iterative regression procedure of Koutrouvelis (1980, 1981). See Akgiray and Lamoureux (1989) for a comparison of the two techniques.

<sup>20</sup>The procedure suggested by Fama and Roll is not a significance test in the usual sense. Conclusion is drawn by observing whether there is any "obvious" shift in the estimates of  $\alpha$ . Thus, the terms "power" and "test" used here should not be interpreted strictly statistically.

<sup>21</sup>Due to insufficient observations, monthly data were not considered in this exercise. Although we also estimated  $\beta$ , the results are not reported here, as our main objective is to test for the stability of  $\alpha$  and thus the acceptability of the stable Paretian law. As pointed out by Akgiray and Lamoureux (1989), estimates of  $\beta$  are not robust to small perturbations in the sample and their standard errors are very large. Furthermore, as  $\alpha$  approaches two,  $\beta$  loses its effects on the distribution (see McCulloch (1986, p. 1111)).

<sup>22</sup>The optimization subroutine used in this study is DUMINF of the IMSL library. Standard errors were calculated by numerically differentiating the Hessian matrix using the DFDHES subroutine. As the daily return series are autocorrelated, the standard errors computed may be downward biased. However, in view of the small magnitude of the autocorrelation coefficient, this problem is unlikely to be serious.

<sup>23</sup>The failure resulted from reiterating on values of  $\lambda_i$  straying outside the feasible space. For example, Kon reported that when  $m = 3$ , only 15 of the 30 stocks reached well-defined optimum.

<sup>24</sup>This transformation is a particular case of the choice probability of a multinomial logit model (see, e.g., Amemiya (1985, Chapter 9)). Using this transformation, we did not encounter any nonconvergence for all cases considered.

<sup>25</sup>As is apparent from the  $L_2$  statistic, rejection of  $m = 1$  against  $m = 2$  for the monthly N500 series is only marginally significant at 5 percent. Also, rejection of  $m = 2$  against  $m = 3$  for the weekly N225 series is significant at 5 percent but not at 1 percent.

<sup>26</sup>Note that when  $\gamma_i$  are equal for all  $i$ , the MND is symmetrical.

<sup>27</sup>For volume classified according to day of the week,  $r_1$  is calculated for the subseries. Although there are gaps in the volume subseries,  $r_1$  is still moderately large and statistically significant.

<sup>28</sup>Some correct test results are presented in Table 7 below.

<sup>29</sup>If there is no residual autocorrelation,  $h$  is distributed asymptotically as a standard normal.

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Table 1

Data Description and Correlation Coefficients

Sampling Interval	Sampling Period <sup>a</sup>	Variables	Correlation Coefficient	Number of Observations <sup>b</sup>
Panel A: Prices <sup>c</sup>				
Daily	86/1/4 - 89/2/28	(Topix, N225)	0.9529	861
		(Topix, N500)	0.9124	861
		(N225, N500)	0.9455	861
Weekly	83/7/31 - 89/2/19	(N225, N500)	0.8666	288
Monthly	83/8 - 89/2	(N225, N500)	0.8766	66
Panel B: Volumes <sup>d</sup>				
Daily	86/1/4 - 89/2/28	(V1, V2)	0.9796	864
Weekly	83/7/31 - 89/2/19	(V1, V2)	0.9754	290
Monthly	83/8 - 89/1	(V1, V2)	0.9743	66
		(V1, V3)	0.9241	66
		(V2, V3)	0.9297	66

<sup>a</sup>For weekly data, a week is designated by the date of the first day (Sunday) of the week.

<sup>b</sup>For daily data, the returns on October 20 and 21, 1987 are excluded. For weekly data, the returns over the week beginning October 18, 1987 are excluded.

<sup>c</sup>The correlation coefficients calculated are for the first difference in the log price, i.e., the continuously compounded rate of return. The prices refer to the end-of-interval (day, week or month) records.

<sup>d</sup>V1 = number of shares traded (in billions) during the interval;  
V2 = (V1 x arithmetic stock price average)/(total market value at the end of the interval);  
V3 = (sales value during the interval)/(total market value at the end of the interval).

V3 is the sales as a proportion of the total market value. For daily and weekly data, this series is not available and is approximated by V2. Both V2 and V3 are measured in percent.

Table 2

Summary Statistics of Returns


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Data	Mean <sup>a</sup>	Standard <sup>a</sup> Deviation	Skewness <sup>b</sup> Coefficient	Kurtosis <sup>c</sup> Coefficient	Number of Observations(n)
Panel A: Topix					
All Days	0.106	0.992	-0.065	7.702*	861
Monday	-0.127	0.986	-1.231*	6.964*	153
Tuesday	0.019	0.894	-0.174	6.980*	154
Wednesday	0.171	1.060	0.732*	10.247*	155
Thursday	0.213	0.930	-0.083	4.871*	156
Friday	0.180	1.093	0.168	7.455*	155
Saturday	0.232	0.905	-0.321	4.398*	88

Test mean returns are equal Monday through Saturday,  $\chi^2_5 = 14.514^d$ .

Test mean returns are equal Tuesday through Saturday,  $\chi^2_4 = 4.120^e$ .

## Panel B: N225

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All Days	0.112	0.940	-0.347*	7.480*	861
Monday	-0.159	0.948	-1.317*	7.210*	153
Tuesday	0.027	0.850	-0.563*	6.385*	154
Wednesday	0.210	0.978	0.239	8.509*	155
Thursday	0.215	0.914	-0.102	4.538*	156
Friday	0.180	0.996	-0.207	9.649*	155
Saturday	0.254	0.866	-0.293	3.833	88
Weekly	0.490	1.795	-0.385*	4.092*	288
Monthly	1.890	4.486	-0.282	4.002	66

Test mean returns are equal Monday through Saturday,  $\chi^2_5 = 20.776^d$ .

Test mean returns are equal Tuesday through Saturday,  $\chi^2_4 = 5.001^e$ .

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Table 2 (continued)

Data	Mean <sup>a</sup>	Standard <sup>a</sup> Deviation	Skewness <sup>b</sup> Coefficient	Kurtosis <sup>c</sup> Coefficient	Number of Observations(n)
Panel C: N500					
All Days	0.078	0.845	-0.316*	10.199*	861
Monday	-0.156	0.856	-1.886*	11.666*	153
Tuesday	0.007	0.764	-0.821*	7.585*	154
Wednesday	0.154	0.867	0.821*	12.237*	155
Thursday	0.157	0.840	-0.093	5.034*	156
Friday	0.150	0.908	0.086	12.050*	155
Saturday	0.211	0.740	-0.318	5.519*	88
Weekly	0.361	1.717	-0.309*	3.886*	288
Monthly	1.280	4.453	-0.635*	4.564*	66

Test mean returns are equal Monday through Saturday,  $\chi^2_5 = 18.989^d$ .

Test mean returns are equal Tuesday through Saturday,  $\chi^2_4 = 4.487^e$ .

<sup>a</sup>Mean and standard deviation are given in percent.

<sup>b</sup>Skewness coefficient is the third sample central moment divided by the third power of the sample standard deviation. An asterisk denotes a case where the skewness coefficient differs from zero by more than twice the standard error, which is approximated by  $(6/n)^{1/2}$ .

<sup>c</sup>Kurtosis coefficient is the fourth sample central moment divided by the square of the sample variance. An asterisk denotes a case where the kurtosis coefficient differs from three by more than twice the standard error, which is approximated by  $(24/n)^{1/2}$ .

$$^d \chi^2_{5,0.05} = 11.071.$$

$$^e \chi^2_{4,0.05} = 9.488.$$

Table 3

Autocorrelation in Returns<sup>a</sup>

Sampling Interval	Index	$e_t$				$e_t^2$			
		$r_1^b$	$r_2^b$	Runs Test <sup>c</sup>	Q(24) <sup>d</sup>	$r_1^b$	$r_2^b$	Runs Test <sup>c</sup>	Q(24) <sup>d</sup>
Daily	Topix	0.18 (0.03)	-0.01 (0.03)	-3.82	57.9	0.13 (0.03)	0.23 (0.03)	-5.45	176.3
	N225	0.13 (0.03)	-0.02 (0.03)	-1.91	51.2	0.20 (0.03)	0.31 (0.03)	-6.77	306.0
	N500	0.12 (0.03)	-0.02 (0.03)	-2.33	64.7	0.20 (0.03)	0.34 (0.03)	-5.02	378.1
Weekly	N225	0.01 (0.06)	0.00 (0.06)	-0.05	8.6	0.10 (0.06)	0.27 (0.06)	-0.56	57.5
	N500	-0.01 (0.06)	0.03 (0.06)	-0.10	11.0	0.00 (0.06)	0.08 (0.06)	0.93	40.9
Monthly	N225	-0.04 (0.12)	0.00 (0.12)	0.39	18.8	-0.04 (0.12)	0.13 (0.12)	-1.89	6.3
	N500	0.07 (0.12)	0.06 (0.12)	-0.87	19.7	-0.10 (0.12)	0.00 (0.12)	1.04	9.0

<sup>a</sup>This table summarizes tests for autocorrelation in the residual (deviation from mean) returns, denoted by  $e_t$ . Tests for both  $e_t$  and  $e_t^2$  are presented. For daily data, the differential return on Monday as against the rest of the week are accounted for. The residuals are obtained from the following regressions:

$$\text{Topix: } y_t = 0.157 - 0.284d_t + e_t$$

$$\text{N225: } y_t = 0.170 - 0.330d_t + e_t$$

$$\text{N500: } y_t = 0.129 - 0.285d_t + e_t$$

where  $d_t = 1$  for Monday and zero otherwise.

<sup>b</sup> $r_1$  and  $r_2$  are, respectively, the first and second order sample autocorrelation coefficients. Figures in parentheses are the standard errors.

<sup>c</sup>The runs test statistic is asymptotically distributed as a standard normal if there is no autocorrelation.

<sup>d</sup>Q(24) is the Box-Pierce portmanteau test statistic for autocorrelation, which is asymptotically distributed as a chi-square with 24 degrees of freedom. At 5 percent significance level, the critical value is 36.4.

Table 4

Estimates of the Characteristic Exponent ( $\alpha$ ) of the  
Stable Paretian Distribution for Market Returns<sup>a</sup>

Index	Sampling Scheme <sup>b</sup>	Sum Sizes (k)				
		1	2	4	6	8
Panel A: Daily Data						
Topix	C	1.544	1.584	1.388	1.749	1.586
	R		1.559	1.729	2.000	2.000
N225	C	1.558	1.758	1.470	1.706	1.371
	R		1.538	1.686	1.708	2.000
N500	C	1.606	1.671	1.798	2.000	1.698
	R		1.869	2.000	1.852	1.905
Number of Observations:		861	430	215	143	107
Panel B: Weekly Data						
N225	C	1.471	1.619			
	R		2.000			
N500	C	1.526	1.626			
	R		2.000			
Number of Observations:		288	144			

<sup>a</sup>Estimates were calculated using McCulloch's (1986) method. Due to insufficient observations, monthly data as well as weekly data with sum size greater than two were not considered.

<sup>b</sup>C denotes chronological sampling and R denotes randomized sampling.

Table 5

Estimates of the Mixture of Normal Distribution for Market Returns<sup>a</sup>

Sampling Interval	Index	Estimated Parameters <sup>b</sup>				Likelihood Ratio Tests <sup>c</sup>		Implied Moments <sup>d</sup>				
		$\mu_1$	$\mu_2$	$\sigma_1$	$\sigma_2$	$\lambda_1$	$L_1$	$L_2$	Mean	Standard Deviation	Skewness Coefficient	Kurtosis Coefficient
Daily	Top1x <sup>e</sup>	0.127 (0.028)		0.680 (0.035)	1.885 (0.192)	0.831	5.04	148.32	0.127 (0.106)	0.992 (0.992)	0.0 (-0.065)	7.158 (7.702)
	N225	0.172 (0.030)	-0.195 (0.183)	0.659 (0.035)	1.751 (0.184)	0.837	5.88	138.16	0.112 (0.112)	0.940 (0.940)	-0.481 (-0.347)	6.890 (7.480)
	N500 <sup>e</sup>	0.108 (0.024)		0.607 (0.026)	1.953 (0.242)	0.900	6.72	190.58	0.108 (0.078)	0.845 (0.845)	0.0 (-0.316)	9.301 (10.199)
Weekly	N225 <sup>f</sup>						1.20	9.15	0.490 (0.490)	1.791 (1.795)	-0.443 (-0.385)	4.374 (4.092)
	N500	0.093 (0.270)	0.653 (0.155)	2.129 (0.214)	1.023 (0.206)	0.520	0.28	14.04	0.361 (0.361)	1.714 (1.717)	-0.290 (-0.309)	4.044 (3.886)
Monthly	N225 <sup>g</sup>	1.890 (0.548)		4.452 (0.387)			4.01		1.890 (1.890)	4.452 (4.486)	0.0 (-0.282)	3.0 (4.002)
	N500	-12.526 (1.372)	1.697 (0.474)	1.728 (1.085)	3.756 (0.347)	0.029	2.85	7.84	1.280 (1.280)	4.419 (4.453)	-0.736 (-0.635)	4.610 (4.564)



Table 5 (continued)

<sup>a</sup>The estimates were obtained using maximum likelihood method. Mixture of normal distributions (MND) were estimated with increasing order until there was no significant increase in the log-likelihood, as indicated by the likelihood ratio statistic. Then the accepted MND model was tested for (i) equality of mean, and (ii) equality of variance, separately. The results reported take into account the acceptance (or otherwise) of these hypotheses. Of all models estimated, no nonconvergence was found. Apart from the weekly N225 returns (which follow a third order MND) and the monthly N225 returns (which follow a simple normal distribution), all other returns are described by a second order MND.

<sup>b</sup>Figures in parentheses are standard errors.  $\lambda_1$  is recovered from Equation (4). The mean and standard deviation are given in percent.

<sup>c</sup> $L_1$  is the likelihood ratio statistic of testing against a MND of one order higher.  $L_2$  is the likelihood ratio statistic of testing for the acceptability of a MND of one degree lower. Both statistics are distributed as  $\chi_3^2$ , with critical value at 5 percent being 7.82.

<sup>d</sup>These are the moments implied by the estimated MND. Definitions of the skewness and kurtosis coefficients are given in footnotes b and c of Table 2. Figures in parentheses are the sample results extracted from Table 2.

<sup>e</sup>For these cases, the hypotheses  $H_0: \mu_1 = \mu_2$  was not rejected at 5 percent level. When this constraint was relaxed, closer fit for the mean was obtained, although this was achieved at the expense of poorer fit for the skewness and kurtosis.

<sup>f</sup>The weekly N225 data follow a MND of third order. The parameter estimates and their standard errors are given below:

$\mu_1$	$\mu_2$	$\mu_3$	$\sigma_1$	$\sigma_2$	$\sigma_3$	$\lambda_1$	$\lambda_2$
-5.701	0.437	0.795	0.127	1.964	0.801	0.010	0.674
(0.079)	(0.165)	(0.165)	(0.054)	(0.151)	(0.154)		

Tests of the hypotheses (i)  $\mu_1 = \mu_2 = \mu_3$  and (ii)  $\sigma_1 = \sigma_2 = \sigma_3$  were rejected at 5 percent level.

<sup>g</sup> $L_2$  is not defined in this case.

Table 6

Summary Statistics of Volumes

Data	Mean	Standard Deviation	Skewness Coef- ficient <sup>a</sup>	Kurtosis Coef- ficient <sup>a</sup>	$r_1^b$	Number of Obser- vations
Panel A: V1 <sup>c</sup>						
All Days	0.902	0.482	1.100	4.264	0.693	863
Monday	0.720	0.344	0.627	3.027	0.460	153
Tuesday	0.885	0.459	0.922	3.586	0.509	155
Wednesday	1.048	0.538	0.964	3.455	0.597	156
Thursday	1.006	0.486	0.966	3.840	0.555	156
Friday	1.039	0.503	1.021	4.038	0.479	155
Saturday	0.565	0.268	1.230	5.129	0.264	88
Weekly	3.558	2.200	1.125	3.815	0.797	289
Monthly	15.317	8.329	0.998	3.404	0.733	65
Panel B: V2 <sup>d</sup>						
All Days	0.294	0.150	1.153	4.547	0.674	863
Monday	0.234	0.105	0.556	2.898	0.411	153
Tuesday	0.287	0.142	1.046	4.183	0.462	155
Wednesday	0.341	0.166	1.010	3.804	0.577	156
Thursday	0.328	0.154	1.119	4.331	0.520	156
Friday	0.337	0.153	1.006	3.886	0.438	155
Saturday	0.186	0.082	1.290	5.800	0.201	88
Weekly	1.239	0.647	1.162	4.402	0.741	289
Monthly	5.296	2.351	1.216	4.421	0.649	65
Panel C: V3 <sup>d</sup>						
Monthly	4.746	1.544	0.804	2.804	0.593	65

<sup>a</sup>Definitions of the skewness and kurtosis coefficients are given in footnotes b and c of Table 2. In view of the large and highly significant autocorrelation coefficients, tests for significant difference from the normal quantities based on the (uncorrected) asymptotic standard errors (see Table 2) were not conducted. See Table 7 for the correct test results.

<sup>b</sup> $r_1$  is the sample first order autocorrelation coefficient. All estimates (except for V2, Saturday) are significantly different from zero.

<sup>c</sup>V1 is the sales in billion shares.

<sup>d</sup>V2 and V3 are the sales as a percentage of the total market value. See footnote d of Table 1.

Table 7

Estimates of the Equation<sup>a</sup>

$$V_t = \beta_0 + \beta_1 V_{t-1} + \beta_2 V_{t-2} + \beta_3 t^*$$

Sampling Interval	Data <sup>b</sup>	Estimates <sup>c</sup>			Residual Analysis			
		$\beta_1$	$\beta_2$	$\beta_3$	Skewness Coef- ficient	Kurtosis Coef- ficient	$r_1$	$h^d$
Daily	V1	0.705 (0.023)		0.0377 (0.0130)	1.046 (0.083)	5.920 (0.167)	-0.015	-0.606
	V2	0.709 (0.023)			1.057 (0.083)	6.040 (0.167)	-0.007	-0.293
Weekly	V1	0.667 (0.044)		0.2911 (0.0608)	0.819 (0.144)	5.563 (0.287)	-0.054	-1.376
	V2	0.653 (0.045)		0.0007 (0.0002)	0.699 (0.144)	5.197 (0.287)	-0.055	-1.431
Monthly	V1	0.634 (0.125)	-0.286 (0.127)	2.4794 (0.6496)	0.708 (0.302)	6.091 (0.595)	0.027	1.639
	V2	0.595 (0.126)	-0.268 (0.128)	0.0060 (0.0019)	0.848 (0.302)	6.463 (0.595)	0.016	1.569
	V3	0.454 (0.115)		0.0027 (0.0012)	0.255 (0.302)	3.298 (0.595)	0.049	0.947

<sup>a</sup>Volume variables were regressed on their lagged values and a time trend. The reported equations were selected based on the following criteria: (i) the residuals pass the autocorrelation test, and (ii) all regression parameter estimates are significant.

<sup>b</sup>See footnote d of Table 1 for the definitions.

<sup>c</sup>For daily data five dummy variables representing different mean values were included. All intercepts and dummies were statistically significant. Furthermore, the dummies were statistically different from one another. The results for the intercepts and dummies are not reported. The time variable was rescaled in such a way that  $\beta_3$  presents the change in volume over one year. For example, the daily volume as measured by V1 increases by 0.0377 billion shares per day in one year and the weekly volume as measured by V2 increases by 0.0007 percent per week in one year.

<sup>d</sup> $h$  is Durbin's  $h$ -statistic for testing residual autocorrelation. It is approximately distributed as a standard normal if there is no autocorrelation.

Table 8

Regressions of Return on Volume

Price Index	Dependent Variable	Regression Estimates <sup>a</sup>			Diagnostics <sup>b</sup>	
		Constant	Dummy	Volume	DW	r <sub>1</sub>
Panel A: Daily Data						
Topix	$\Delta \log P_t$	-0.153 (0.078)	-0.211 (0.089)	1.011 (0.226)*	1.628	0.135
	$(\Delta \log P_t)^2$	1.042 (0.205)	-0.026 (0.232)	-0.146 (0.591)	1.764	0.118
	$ \Delta \log P_t $	0.707 (0.057)	-0.031 (0.064)	0.015 (0.163)	1.546	0.227
N225	$\Delta \log P_t$	-0.084 (0.074)	-0.270 (0.084)	0.831 (0.214)*	1.740	0.129
	$(\Delta \log P_t)^2$	1.021 (0.179)	-0.002 (0.203)	-0.426 (0.516)	1.609	0.196
	$ \Delta \log P_t $	0.707 (0.053)	-0.022 (0.060)	-0.082 (0.153)	1.465	0.267
N500	$\Delta \log P_t$	0.033 (0.067)	-0.262 (0.076)	0.313 (0.194)	1.754	0.122
	$(\Delta \log P_t)^2$	0.900 (0.173)	-0.004 (0.195)	-0.615 (0.499)	1.622	0.189
	$ \Delta \log P_t $	0.655 (0.049)	-0.025 (0.055)	-0.200 (0.141)	1.487	0.257
Panel B: Weekly Data						
N225	$\Delta \log P_t$	-0.161 (0.225)		0.526 (0.161)*	2.023	-0.012
	$(\Delta \log P_t)^2$	1.932 (0.701)		1.226 (0.502)*	1.916	0.041
	$ \Delta \log P_t $	0.975 (0.148)		0.368 (0.106)*	1.880	0.059
N500	$\Delta \log P_t$	0.202 (0.219)		0.129 (0.156)	2.027	-0.013
	$(\Delta \log P_t)^2$	3.047 (0.630)		0.018 (0.451)	2.072	-0.037
	$ \Delta \log P_t $	1.227 (0.143)		0.095 (0.103)	2.148	-0.077

Table 8 (continued)

Price Index	Dependent Variable	Regression Estimates <sup>a</sup>			Diagnostics <sup>b</sup>	
		Constant	Dummy	Volume	DW	$r_1$
Panel C: Monthly Data						
N225	$\Delta \log P_t$	0.554 (1.389)		0.254 (0.240)	2.097	-0.051
	$(\Delta \log P_t)^2$	13.917 (11.227)		1.852 (1.940)	2.168	-0.085
	$ \Delta \log P_t $	3.036 (0.937)		0.149 (0.162)	2.229	-0.116
N500	$\Delta \log P_t$	1.732 (1.388)		-0.079 (0.240)	1.841	0.074
	$(\Delta \log P_t)^2$	19.364 (11.015)		0.400 (1.904)	2.281	-0.143
	$ \Delta \log P_t $	3.849 (0.950)		-0.067 (0.164)	2.342	-0.184

<sup>a</sup>V2 was used as the measure of volume in the regressions. In accordance with the results of Table 2, a dummy variable to take account of differential mean return was added in the regression for daily data. The dummy is defined as one for Monday and zero otherwise. Estimates of the volume coefficient that are significant at 5 percent level are marked by an asterisk.

<sup>b</sup>DW is the Durbin-Watson statistic and  $r_1$  is the sample first order autocorrelation coefficient.













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