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Maximizing Net Income Under the Tax Reform Act of 1986

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Abstract

The Tax Reform Act of 1986 requires the property-liability insurance industry to develop a new strategy for maximizing net income after taxes. Statutory income is no longer the basis of insurance taxation, as loss reserves are discounted for tax purposes and part of the unearned premium reserve is included in taxable income. A more inclusive alternative minimum income tax calculation will also apply in many cases. In this paper binomial and trinomial lattice models are used to develop an investment allocation strategy between fully taxable and municipal bonds that maximizes net income under stochastic interest rates and underwriting profits. These models illustrate that the optimal investment allocation can vary depending on whether interest rates and underwriting profits are deterministic or stochastic.

Section I - Introduction

The Tax Reform Act of 1986 (TRA) dramatically revised the tax provisions applicable to the property-liability insurance industry. For the first time, statutory accounting conventions do not serve as the basis for determining taxation and previously tax exempt income is now subject to taxation. The industry will have to develop an entirely new approach to operational planning in order to cope with the new tax legislation. This study will focus on how an insurer can allocate its investment portfolio among fully taxable corporate and U.S. government bonds and only partially taxable municipal bonds in such a way that underwriting income and investment income are combined to produce the highest net after-tax income.

TRA includes four major changes in property-liability insurance taxation and additional changes directed only at special classes of insurers. The first major change is that, starting in 1987, loss reserves are to be discounted using the applicable federal rate on midmaturity (three to nine year) securities based on the five year period prior to the calendar year for which discounting is applied. However, months prior to August, 1986, are not included in determining the discount rate. A "fresh-start" approach applies under which beginning reserves are treated as having been discounted, but the change in accounting profits generated by applying discounting to previously undiscounted loss reserves is not subject to taxation. Insurers can use either industry loss payout patterns calculated by the Treasury Department or company payout patterns. The second major change is that 20 percent of the change in unearned premium reserve each year is included in taxable income and, additionally, one-sixth of 20 percent of the 1986 year end unearned premium reserve is included in taxable income each year from 1987 through 1992. The third major change includes 15 percent of previously tax-exempt interest received on investments made after August 7, 1986, in taxable income. In addition, 15 percent of the dividends that are normally excluded from taxable income (80 percent of dividends from non-affiliated, domestic corporations based on TRA) for securities acquired after August 7, 1986, is also included in taxable income. The fourth major effect for property-liability insurers is a change in the general corporate tax code that includes 50 percent of the difference between "book income" and regular taxable income in the alternative minimum taxable income calculation. In this context book income is the largest pre-tax income value included in financial reports for any purpose, including to shareholders, regulators or creditors. In these reports, property-liability insurers include all regularly tax exempt income in determining pre-tax income. However, by being included in book income, this otherwise tax exempt income may be subject to additional taxation depending on the relationship between the regular income tax and the alternative minimum tax. Effective for tax year 1990 and beyond, 75 percent of the difference between regular taxable income and adjusted current earnings (a term that has not yet been fully defined by the IRS) will be included in the alternative minimum taxable income calculation.

Thus, the property-liability insurance industry now faces a radically different tax regime than it has been accustomed to and must develop new strategies for coping with this environment. This research will propose a method of allocating investments between fully taxable and partially taxable securities in a world of stochastic interest rates and underwriting results that effectively maximizes after-tax income.

Section II - Literature Review

Prior research has demonstrated that net income is maximized when the regular income tax and the alternative minimum tax are equal [Almagro and Ghezzi (1988), Gleeson and Lenrow (1987)]. This occurs because the effective tax rate on municipal bonds acquired after August 7, 1986, goes from 5.1 percent when the insurer is subject to the regular income tax to 11.5 percent (for 1987-1989) or 15.75 percent (for 1990 and beyond) when the insurer is subject to the alternative minimum tax.[1] At the same time, the tax rate on fully taxable investment income drops from 34 percent when the regular income tax applies to 20 percent when the alternative minimum tax applies. Thus, if municipal bonds provide yields of at least 69.55 percent of the yield for equivalent risk fully taxable bonds but less than 90.4 percent (1987-1989) or 94.96 percent (1990 and beyond),[2] which would typically be the case, the insurer increases net after-tax income by shifting investments from fully taxable bonds to municipal bonds as long as the regular income tax rate applies. However, as soon as the alternative minimum tax applies, no further shifting should occur.

One strategy for equalizing the regular tax and the alternative minimum tax is to adjust the investment allocation between fully taxable and partially tax exempt bonds so that the regular tax and the alternative minimum tax will be equal at the end of the tax year. However, the values for various components of income, such as underwriting profit or loss, fully taxable interest, municipal bond interest, dividends and capital gains, are not known until the end of the tax year. These values can be estimated before and during the year, but are subject to random fluctuation. Also, for capital gains, the insurer can determine the timing of the tax liability since capital gains and losses are taxable only when the securities are sold. Adjustments in the investment mix made late in a tax year are more costly to the insurer than adjustments made before the year begins. Transaction costs are associated with buying and selling securities, and a greater dollar value of investments would be involved in the transaction if the allocation were being done late in the year. Thus, the optimal strategy would be to estimate the values of the various components of regular taxable income and alternative minimum taxable income before the year begins, develop an investment allocation that recognizes the stochastic nature of the values and periodically adjust the allocation as experience develops.

One approach to evaluating uncertain outcomes is the use of a lattice, or series of branches, with each node representing a particular event or series of events. An early and famous use of this approach is termed Pascal's Triangle [7], which is illustrated in Figure 1. A earlier discovery of this triangle is attributed to the Chinese mathematician Chia Hsien around 1100 [9]. This lattice indicated the likelihood of obtaining any possible set of outcomes of a binomial series such as achieved from tossing a coin a set number of times. For convenience, the nodes will be labeled in a pattern, with node 1 representing the starting point of the triangle, node 2A representing movement along the first upwards branch and node 2B representing movement along the first downwards branch, node 3A representing two upwards movements, node 3B the center branch after two moves, node 3C representing two downwards movements, and so forth. If the coin, when tossed for the first time, comes up heads, the outcome is represented by node 2A. If it comes up tails, the outcome is represented by 2B. Of the two possible outcomes, the result will be at node 2A one time and node 2B one time, indicating a 1/2 probability of each outcome. The coin is then tossed a second time. If the prior outcome were at node 2A, another head would move it to node 3A and a tail to 3B. If the prior outcome were at 2B, then a head on the second coin toss would move the results to 3B and another tail to 3C. The results move through the lattice as long as additional coin tosses are made. The outcomes and probabilities are illustrated in Figure 1.

Several significant patterns can be illustrated by Pascal's Triangle. One trait, termed path independence, is that the fact that if the current outcome is at any interior node, which could be achieved by more than one pathway through the lattice, the particular path followed to get to that node is irrelevant. It makes no difference whether the first toss was a head and the second a tail or vice versa. The relevant information is that two coin tosses resulted in one head and one tail. Another feature of Pascal's Triangle is that the numerical value of each node, used as the numerator to determine the probability of each outcome, is the sum of the numerical values at each of the nodes that branch into that node. This number represents the number of different pathways that could be followed through the lattice that lead to that node.

The lattice approach was applied to valuing stock options by Cox, Ross and Rubinstein (CRR) (1979). To use this methodology required assuming a binomial model of stock price movements: over a small time interval the stock price could either increase by a certain amount or decline another predetermined amount - remaining at the same level was not allowed. In order to utilize the lattice approach illustrated by Pascal's Triangle, consecutive price movements had to converge after up and down movements. One method to achieve this would have been to use equal dollar value movements, i.e. up one dollar in price or down one dollar. After a large number of moves along a lattice, this assumption would have approximated a normal distribution for the final stock price. One problem with this approach would be the possibility of negative stock prices. An alternative approach to price movements would be to allow the price to move up or down a certain percentage. This approach has the advantage that the stock price would never be negative. Additionally, stock price movements are more conventionally valued in percentage terms. This assumption led to a lognormal distribution for stock prices after a large number of lattice movements, which fits with conventional pricing models. Under this approach the upward move was represented by a value u that was greater than one and the stock price at the upper node was the initial stock price S times u or Su. The downward move would be S divided by u or S/u. The stock price at node 3B would thus be Su/u, or S, regardless of whether the upward move preceded or followed the downward move. Without convergence, the number of nodes at each level of branching would increase exponentially rather than linearly, dramatically increasing the complexity of the model.

One additional feature introduced in the lattice model for stock prices by CRR was the determination of the probabilities of up and down movements based on a risk-neutral world and a no-arbitrage condition. The current stock price would have to be the discounted value of the next level of stock prices weighted by the probabilities of up and down movements. This condition led to a determination of the probabilities and, unlike Pascal's Triangle, the values were not 1/2. The purpose of the CRR lattice was to determine the potential stock price levels at the time of expiration of a particular option and then to work backwards through the lattice to establish the value of the option at each intervening node and, eventually, the initial node which represented the current option value.

Ho and Lee (1986) applied the binomial lattice approach to interest rate levels, rather than stock prices, to value bond options. The Ho and Lee model is similar to the CRR approach, but the nodes represent changes in interest rates.

Boyle (1988) used a trinomial model to value stock options on two underlying securities. The three moves from one level of the lattice to the next involve an upward or downward jump, as included in CRR and Ho and Lee, and also a horizontal move in which the asset's price does not change. By including this additional possibility, Boyle found that option prices converged after far fewer iterations than were needed for the binomial model.

Cummins and Nye (1980) investigated the stochastic characteristics of underwriting profits. Diffusion models for underwriting profits have been utilized by Doherty and Garven (1986) and Cummins (1988). Doherty and Garven determine indicated underwriting profit margins based on the Black-Scholes option pricing model under stochastic rates of return. Security prices are modeled both on a normal and lognormal distribution. Cummins determines pre-assessment guaranty fund premiums based on a diffusion process for insurance profitability that includes a poisson jump process reflecting catastrophes. Neither study directly utilized a lattice framework for underwriting profits.

In this paper the lattice approach will be used to determine the investment strategy for property-liability insurers under the provisions of the Tax Reform Act of 1986. As has previously been shown, an insurer achieves the highest after tax income under typical market conditions if the tax level under the regular tax calculation and the minimum tax calculation are equal. If the level of interest rates and the statutory underwriting profit are known in advance, then an insurer can select the optimal investment allocation to maximize after-tax net income. This research will address the allocation process given stochastic interest rates and underwriting profits.

Section 3 - Research Methodology

The purpose of this research is to devise an investment strategy for property-liability insurers under TRA based on stochastic interest rates and underwriting profits. A lattice approach is used to model interest rate movements and underwriting results. Under the first model, interest rates are stochastic but underwriting profits are fixed. Under the second model, underwriting profits are stochastic but interest rates are fixed. Under the third model, both interest rates and underwriting profits are stochastic.

The lattice approach that has been used to value options on stocks and interest rate securities is used to model the stochastic elements of this determination. A binomial lattice is used to illustrate the potential outcomes when one variable is stochastic.

Several assumptions are made to simplify the presentation of the model. The two investment choices are a fully taxable money market type of investment and a municipal bond type of money market fund that would be partially tax exempt. As both investments are short term, it can be assumed that all municipal bonds would have been purchased after August 7, 1986, and therefore not completely exempt from taxation. The statutory and taxable underwriting profits are assumed to be the same. Based on the revenue offset provision of TRA, 20 percent of the increase in the unearned premium reserve over a calendar year plus, for calendar years 1987 through 1992, one-sixth of 20 percent of the 12/31/86 unearned premium reserve would be included in the taxable underwriting profit but not the statutory underwriting profit. After 1992, the provision for 1986 unearned premium reserve no longer applies. For an insurer with a level premium volume each year, no change in the unearned premium reserve would occur, so the revenue offset provision would be zero. Although most insurers have an increasing premium volume, since the revenue offset amount depends on the assumed growth rate, for simplicity a zero growth rate is assumed to eliminate this term.

The other major difference between statutory and taxable underwriting profits is the effect of discounting the loss reserves. For all years, including 1987, the first year that TRA applied to the insurance industry, the impact of discounting is the difference in discounted loss reserves from the beginning to the end of the year. For an insurer with the same level of loss reserves and payout patterns at the beginning and end of the year, as long as the same interest rate is used to discount the reserves, the change in discounted loss reserves will be the same as the change in undiscounted reserves, zero in both cases. Although a change in interest rates would change this value and current interest rates are assumed to be stochastic, the interest rate used to discount loss reserves is a five year moving average value that is established before the year begins. Thus, that interest rate is deterministic. On the assumption that both premiums and loss reserves are level, the statutory and taxable underwriting profit values are the same for the models. In practice, the taxable underwriting profit will tend to be larger than the statutory value and the difference will be a function of the growth rate of the company and the loss experience.

Section 4 - Model 1: Stochastic Interest Rates

The optimal investment allocation between the fully taxable money market fund and the municipal bond fund for an insurer depends on the interest rate level, the differential between municipal bond interest rates and fully taxable interest rates and the underwriting profit of the insurer. If the interest rates and underwriting performance were known in advance, the insurer could determine the allocation between fully taxable and municipal bonds that would equate the regular tax level with the alternative minimum tax level. In practice, these values are not known, but must be estimated.

In the first model, the underwriting profit is assumed to be known, but the level of interest rates is stochastic. The insurer has a portfolio of $10,000,000 that is to be divided between a fully taxable and a municipal bond money market fund. The municipal bond fund yields 80 percent of the fully taxable money market fund, whatever that yield turns out to be. The best initial estimate of the fully taxable money market fund interest rate over the course of the year is 10 percent. After three months the estimate will be revised to be either 11.11 percent or 9 percent, depending on which of two sets of information is revealed during that quarter. The revised estimate applies to the entire year, not just the remaining three quarters. In the terminology of the lattice literature, the stretch factor, u_1, is 1.1111. By convention, to assure convergence of upward and downward moves, the downward move, d_1, is 1/1.1111 = .9. The probability of an upward move, p_1, is $(1-d_1)/(u_1-d_1)=.4737$. The expected value of the interest rate distribution is, by construction, the same as the initial interest rate estimate:

$$(.4737)(11.11)+(.5263)(9.0)=10.00 \qquad (1)$$

During the second quarter, additional new information is revealed that again raises the expected interest rate level by a multiplicative factor of 1.1111, with a probability of .4737, or lowers it by a factor of .9, with a probability of .5263. Thus, after two quarters the expected interest rate is either 12.35, 10.00 or 8.10 percent. Additional informational releases in the third and fourth quarters continue to increase or decrease the expected interest rate level for the year. The potential outcomes are illustrated by a lattice in Figure 2.

The interest rates for the year are known to be one of five possible rates, $15.24=10.00(u_1)^4$ (node 5A), $12.35=10.00(u_1)^3(d_1)$ (node 5B), $10.00=10.00(u_1)^2(d_1)^2$ (node 5C), $8.10=10.00(u_1)(d_1)^3$ (node 5D) or $6.56=10.00(d_1)^4$ (node 5E). The probabilities of these values, as shown in Figure 2 for each node, are $(p_1)^4$ for node 5A, $4(p_1)^3(1-p_1)$ for 5B, $6(p_1)^2(1-p_1)^2$ for 5C, $4(p_1)(1-p_1)^3$ for 5D and $(1-p_1)^4$ for 5E, based on the number of different pathways that lead to each node and the probabilities of upward and downward moves at each interior node. The number of pathways is the same number derived in Pascal's Triangle. Based on the ultimate values for the interest rates and the probabilities, the expected interest rate at the beginning of the year is 10.00 percent. For each of the potential year end interest rate levels, an optimal investment allocation between fully taxable and municipal bonds can be determined by equating the regular tax level with the alternative minimum tax level. For example, based on a 10.00 percent interest rate level, the optimal percent of investable assets to be in fully taxable investments, F, for the tax rates that will apply for tax years 1990 and beyond is determined by:

$$(10{,}000{,}000)(.10)(F)(.34)+(10{,}000{,}000)(.08)(1-F)(.051) +(250{,}000)(.34) =(10{,}000{,}000)(.10)(F)(.20) +(10{,}000{,}000)(.08)(1-F)(.1575) +(250{,}000)(.20) \qquad (2)$$

$F = .2229$

The left hand side of equation (2) is the amount of taxes owed based on the regular tax calculation. The right hand side of this equation is the amount of taxes owed based on the alternative minimum tax calculation. The first term on the left hand side of equation (2) is the amount of taxes generated by investing in a taxable money market fund; this investment income is taxed at the 34 percent rate. The second term is the amount of taxes generated from investing the remainder of the investable assets (1-F) in a municipal bond money market fund; 15 percent of this investment income is taxed at the 34 percent rate. The third term is the amount of taxes generated by underwriting income, which is taxed at the 34 percent rate. The three terms on the right hand side of equation (2) represent the same calculations, but the tax rates are different, 20 percent versus 34 percent for fully taxable investment income and underwriting income and 15.75 percent versus 5.1 percent for municipal bond income (see footnote 1). The allocation between fully taxable and municipal bond investments is determined to equalize the two possible tax levels by solving for F.

Equation (2) can be expressed algebraically as follows[3]:

$$F = .3783304 - .6216696\ (W/Ar) \qquad (3)$$

where F = percentage of investable assets allocated to a fully taxable money market fund
W = underwriting profit
A = investable assets
r = fully taxable interest rate

The values for F, calculated as described above for the final nodes (level 5), are included in Figure 2. At the end of the year, when interest rates are known, the exact allocation of investable assets to optimize after-tax returns can be determined. However, by then it is too late for the insurer to reallocate assets to meet this optimal level. Therefore, the insurer is forced to determine an asset allocation in advance, based on the known probability distribution of the final interest rates. For example, if, after three-quarters of the year had elapsed and node 4A applied, then the expected fully taxable interest rate level for the year would be 13.72 percent based on the final rate being 15.24 percent with a probability of .4737 and 12.35 percent with a probability of .5263. The optimal allocation value, F, at this node would be the weighted average of the respective final optimal allocation values, or.2638 ((.4737)(.2763)+ (.5263)(.2525)=.2638)). Following this logic through the entire lattice back to node 1, the weighted average of the optimal values of F is .2159 ((.0767)(.2763)+(.2762)(.2525)+(.3729)(.2229) +(.2238)(.1865)+(.0504)(.1415)=.2159). Although the expected interest rate at node 1 is 10 percent, the value for F at node 1 is not the same value as would be optimal if interest rates ended up at 10 percent (node 5C). The optimal allocation under stochastic interest rates is not the same as that when the interest rate is deterministic. In this case, the initial allocation in taxable investments is less than would be indicated if interest rates were known, .2159 versus .2229. This difference occurs because the interest rate is included in the denominator of equation (3), so that even though the probabilities of the various interest rates sum to one, the sum of the quotients is not the same as the expected value.

Section 5 - Model 2: Stochastic Underwriting Profits

In the second model, interest rates are fixed at 10 percent for the fully taxable fund and 8 percent for the municipal bond fund, but the underwriting profit is stochastic. The initial estimate, at node 1, of underwriting profits for the year is 250,000. Although a linear model could be applied to underwriting profits as there is no constraint on underwriting profit remaining positive, for convenience the same model as was used for interest rates will be adopted. This results in, for a large lattice, an approximately lognormal distribution of underwriting profits.

The same pattern utilized for interest rate changes is applied to underwriting profits. After three months, the insurer has new information that indicates that the initial estimate of underwriting profits will either increase by 11.11 percent to 277,778 or decline by 10 percent to 225,000. However, as can be seen from equation (3), an increase in underwriting profits reduces the value of F, whereas an increase in interest rates increased F. To keep the lattice the same direction, the upward move for underwriting profits will be defined as the movement that increases F, even though this is actually a decline in underwriting profits. Thus, the upward stretch factor for underwriting profits, u_2, is .9 and the downward factor, d_2, is 1.1111. The probability of an upward move for underwriting profits, p_2, is $(1-d_2)/(u_2-d_2)=.5263$. The lattice for underwriting profits is illustrated in Figure 3. The values for F at the end of the lattice, nodes 5A-E, are by construction the same as the comparable values for the stochastic interest rate model, but the probabilities are reversed because the probability of an upward move for stochastic underwriting profits is the complement of the probability of an upward move for stochastic interest rates (i.e., $p_2=1-p_1$). When the values of F are calculated by working backwards through the lattice, the weighted average value for the initial point, node 1, is .2229, which is the same as the value for a known underwriting profit margin of 250,000, the initial expected value. This agreement occurs because the underwriting profit is included in the numerator of equation (3), so the expected value and the sum of the possible underwriting profit values weighted by the respective probabilities are equal.

Section 6 - Stochastic Interest Rates and Underwriting Profits

In the third model, both interest rates and underwriting profits are stochastic. After one-quarter of the year has elapsed, the insurer has a revised estimate of the full year's interest rate, which is either 11.11 percent or 9.0 percent, and a revised estimate of the full year's underwriting profit margin, either 225,000 or 277,778. Thus, four possible situations could occur:

1) both the interest rate and the underwriting profit margin could move along the upward path on the lattice

2) the interest rate could move along the upward path and the underwriting profit along the downward path

3) the interest rate could move along the downward path and the underwriting profit along the upward path

4) both the interest rate and the underwriting profit could move along the downward path

This could be visualized by a three dimensional figure with four paths emerging from a node. From each succeeding node four more paths would emerge. To minimize the proliferation of pathways, the interest rate and underwriting profit models were selected to assure that the outcomes of alternatives (2) and (3) described above were equivalent as far as generating the same value for F. Based on equation (3), an increase in the interest rate in the denominator to 11.11 percent is exactly offset by a similar increase in the underwriting profit, which represents downward movement on the underwriting lattice. In addition, a decrease in the interest rate to 9 percent is offset by a similar decrease (an upward move on the underwriting profit lattice) in underwriting profit. Thus, for both of these alternatives, the net effect of the offsetting changes is for the lattice to have one horizontal move along a lattice. The result is a trinomial lattice, similar to that used by Boyle (1988) for evaluating options.

The probability of an upward move in the trinomial lattice is p_1p_2=(.4737)(.5263)=.2493. The probability of a downward move is the same, $(1-p_1)(1-p_2)$=(.5263)(.4737)=.2493, resulting in a symmetrical lattice. The probability of a horizontal move, which can occur if the interest rate moves up or down and the underwriting profit moves the opposite direction along its lattice, is $(p_1)(1-p_2)+(1-p_1)(p_2)$=.5014. The initial upward move along the trinomial lattice results in an interest rate of 11.11 percent (10.00(u_1)) and an underwriting profit of 225,000 (250,000(u_2)). The downward move results in an interest rate of 9.00 percent (10.00(d_1)) and an underwriting profit of 277,778 (250,000(d_2)). The horizontal move results in either an interest rate of 11.11 percent and an underwriting profit of 277,778 or an interest rate of 9.00 percent and an underwriting profit of 225,000. Both of these combinations yield the same optimal investment allocation for the insurer. The subsequent lattice points are determined similarly.

The trinomial lattice has three nodes for the first quarter, five nodes for the second quarter, seven nodes for the third quarter and nine nodes for the year end values. As a result of the selected values for upward and downward moves being approximately .5, the probabilities of the outcomes are approximately equivalent to the values indicated by Pascal's Triangle for twice as many binomial choices. The probabilities of the nine possible nodes after four quarterly moves through the trinomial lattice are almost the same as the probabilities of the nine possible outcomes after eight moves through a binomial lattice (the probabilities of 0 to 8 heads on eight coin tosses). However, for each interior node at the year end position, more than one combination of interest rates and underwriting profit values combine to produce the same optimal investment allocation value F. This situation is analogous to the different ordering of heads and tails in a series of coin tosses that yield the same number of heads. The trinomial lattice and the numerical values for the nodes are displayed in Figure 4. As in model 1, the optimal investment allocation for the beginning of the year, .2198, differs from the value obtained for deterministic values equal to the initial expected values of the parameters, or .2229. Thus, the stochastic nature of the variables affects the optimal allocation of investments.

Section 7 - Extensions

In practice the range of F is limited to 0 to 1, as an insurer can invest no more than all of its investable assets in either fully taxable or only partially taxable municipal bond type investments. However, the values of F determined by equation (3) could exceed 1 (for large negative underwriting profits) or be less than zero (for high underwriting profits and/or low interest rate levels). In this case, the value of F used in the lattice to calculate the optimal investment allocation for prior periods would be limited to the range of zero to one. Thus, the initial or interim F values could diverge significantly from the F value associated with the expected value of the stochastic parameter(s) at that point. If this were the case, the lattice model would approximate the results of CRR.

The effect of limiting the range of F can be illustrated by an example based on Model 2, where only the underwriting profit is stochastic, and the original estimated underwriting profit is $600,000. The five possible final values for the underwriting profit based on an upward stretch factor of .9, the optimal allocation to fully taxable investments based on equation (3) and the probabilities of each outcome would be:

Node	Underwriting Profit	F	Likelihood
5A	393,660	.1336	.0767
5B	486,000	.0762	.2762
5C	600,000	.0053	.3729
5D	740,741	.0000	.2238
5E	914,495	.0000	.0504

The optimal allocation at node 1, calculated recursively, at which the expected underwriting profit is $600,000 would be 3.33 percent. The optimal allocation at year end if the underwriting profit were known to be $600,000 would be only 0.53 percent. This difference occurs only because of the effect of limiting the range of F as the deterministic and stochastic values for unrestricted values of F are the same for underwriting profits.

Several other enhancements could be included in this model to increase its applicability. For insurers, additional income items affect the tax calculation. Some dividend income is fully taxed and the remaining dividend income is treated similarly to municipal bond interest. Capital gains are taxed at the same rate as fully taxable interest, but the insurer has some discretion over when to realize gains for tax purposes. Additionally, the statutory underwriting profit used in the alternative minimum tax calculation will tend to differ from the taxable underwriting profit used for the regular income tax calculation. Also, insurers will often have tax loss carryforwards that can be applied to current tax liabilities that will affect the optimal investment allocation. These and other practical considerations will increase the complexity of determining the optimal investment allocation, but the same lattice based approach could be followed.

A major simplifying assumption in these models was that insurers invested only in money market funds. This allowed the yield curve to be represented by a single value. Redington (1952) assumed a similar yield curve, but by allowing long term bond investments he violated the no arbitrage constraint. Adding more realism to the investment choices will require more complex term structure models, such as proposed by Cox, Ingersoll and Ross (1985), Ho and Lee (1986) or Vasicek (1977). With a bond portfolio that ranges over both short and long term issues, interest rate changes will generate unrealized capital gains or losses that will have additional tax consequences.

Finally, the number of levels lattice can be increased to represent shorter time periods between reallocation of investments. Monthly decisions would be represented by twelve levels in the lattice, and weekly decisions by 52 levels. For options, current lattice type models have such short time intervals that supercomputers are necessary to complete the iterative calculations.

Conclusion

Tax planning for insurers under TRA will require the use of new tools and techniques. This paper suggests one such approach. The lattice technique previously applied for valuing options can be used to establish a dynamic tax planning strategy for property-liability insurers. This research illustrates that under stochastic interest rates and underwriting profit margins, the optimal investment allocation during the year differs from the level that would prevail under deterministic values. The lattice approach allows for an investment strategy that changes as new information is revealed. Use of this technique should allow insurers to achieve a higher net income than less responsive investment strategies.

Footnotes

1. For municipal bond interest, 15 percent of this investment income is taxable at the maximum corporate tax rate of 34 percent (.15 x .34 = .051) when the insurer is subject to the regular income tax. For taxable years 1987 through 1989, when the insurer is subject to the alternative minimum tax, 15 percent of this investment income is taxed at the alternative minimum tax rate of 20 percent and one-half of the remaining 85 percent is taxed at 20 percent (.15 x .20 + (.5 x .85 x .20) = .115). For taxable years 1990 and beyond, when the insurer is subject to the alternative minimum tax, 15 percent of municipal bond income is taxed at the 20 percent rate and three-quarters of the remaining 85 percent is taxed at the 20 percent rate (.15 x .20 + (.75 x .85 x .20) = .1575).

2. For investments acquired after August 7, 1986, when the regular tax rate applies, a municipal bond would provide a higher after-tax yield than a fully taxable bond if the ratio of the municipal bond yield divided by an equivalent risk fully taxable bond were at least 69.55 percent. The after-tax income on a fully taxable bond would be 66 percent of its interest rate (1-.34=.66). The after-tax income on a municipal bond would be 94.9 percent of its interest rate (1-.051=.949). The breakeven ratio is .66/.949=.6955. When the alternative minimum tax calculation applies, the breakeven ratio is 90.4 percent for 1987-1989 ((1-.20)/(1-.115)=.904) or 94.96 percent for 1990 and beyond ((1-.20)/(1-.1575)=.9496).

3. Equation (3) results from the TRA tax rates applicable to 1990 and later and the assumption that municipal bonds yield 80 percent of fully taxable bonds. If instead this ratio is denoted as a variable m, then the value of F would be:

$$F = (.1065m/(.14+.1065m)) - (.14/(.14+.1065m))(W/Ar)$$

References

[1] Almagro, Manuel and Ghezzi, Thomas L., "Federal Income Taxes - Provisions Affecting Property/Casualty Insurers," Proceedings of the Casualty Actuarial Society, forthcoming.

[2] Boyle, Phelim P., "A Lattice Framework for Option Pricing with Two State Variables," Journal of Financial and Quantitative Analysis, Vol. 23 (1988), pp. 1-12.

[3] Cox, J. C., Ingersoll, J. and Ross, Stephen A., "A Theory of the Term Structure of Interest Rates," Econometrica, Vol. 53 (1985), pp. 385-407.

[4] Cox, J. C., Ross, S. A. and Rubinstein, M., "Option Pricing: A Simplified Approach," Journal of Financial Economics, Vol. 7 (1979), pp. 229-263.

[5] Cummins, J. David, "Risk-Based Premiums For Insurance Guaranty Funds," Journal of Finance, Vol. 43 (1988), pp. 823-839.

[6] Cummins, J. David and Nye, David J., "The Stochastic Characteristics of Property-Liability Insurance Company Underwriting Profits," Journal of Risk and Insurance, Vol. 47 (1980), pp. 61- .

[7] David, F., Games, Gods and Gambling (New York: Hafner Press, 1962), Chapter 9.

[8] Doherty, Neil A. and Garven, James R., "Price Regulation in Property-Liability Insurance: A Contingent Claims Approach," Journal of Finance, Vol. 41 (1986), pp. 1031-1050.

[9] Edward, A. W. F., Pascal's Arithmetical Triangles (New York: Oxford University Press, 1987), pp. 50-55.

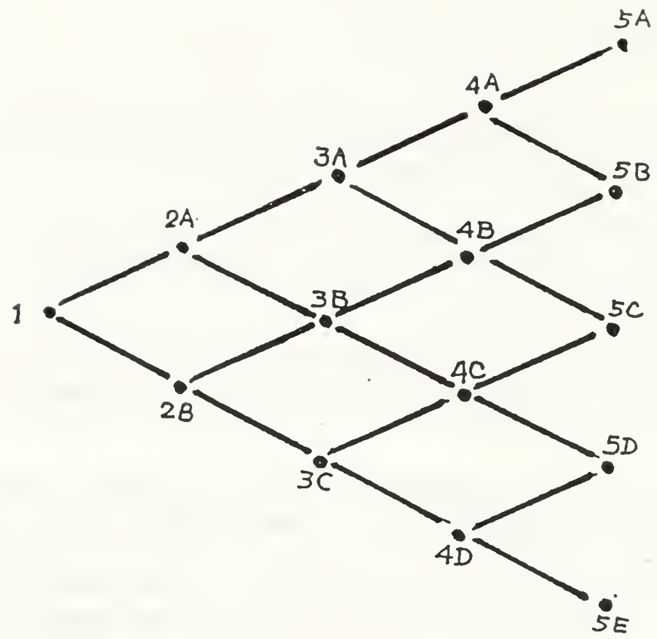
[10] Gleeson, Owen M. and Lenrow, Gerald I., "An Analysis of the Impact of the Tax Reform Act on the Property/Casualty Industry," Casualty Actuarial Society Discussion Paper Program, 1987, pp. 119-189.

[11] Ho, Thomas S. Y. and Lee, Sang-Bin, "Term Structure Movements and Pricing Interest Rate Contingent Claims," Journal of Finance, Vol. 41 (1986), pp. 1011-1029.

[12] Redington, F. M., "Review of the Principles of Life Office Valuations," Journal of the Institute of Actuaries, Vol. 78 (1952), pp. 286-315.

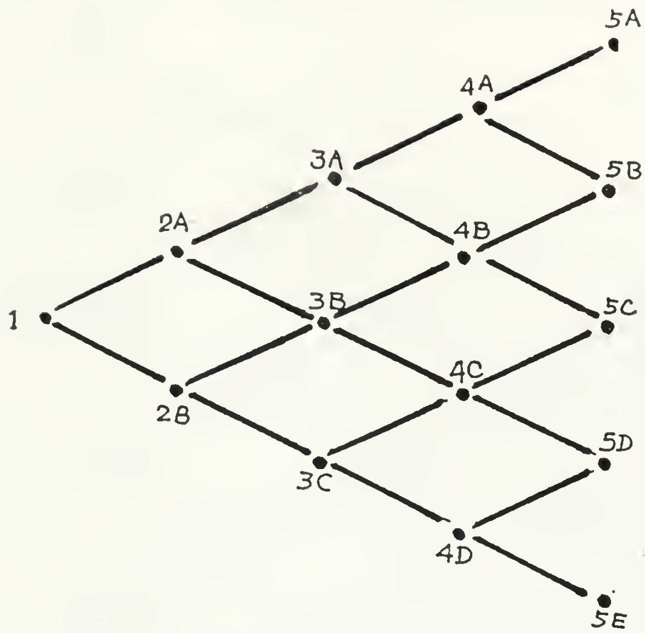
[13] Vasicek, Oldrich, "An Equilibrium Characterization of the Term Structure," Journal of Financial Economics, Vol. 5 (1977), pp. 177-188.

FIGURE 1
PASCAL´S TRIANGLE



NODE	OUTCOME	LIKELIHOOD
1	---	---
2A	H	1/2
2B	T	1/2
3A	2H	1/4
3B	1H, 1T	2/4
3C	2T	1/4
4A	3H	1/8
4B	2H, 1T	3/8
4C	1H, 2T	3/8
4D	3T	1/8
5A	4H	1/16
5B	3H, 1T	4/16
5C	2H, 2T	6/16
5D	1H, 3T	4/16
5E	4T	1/16

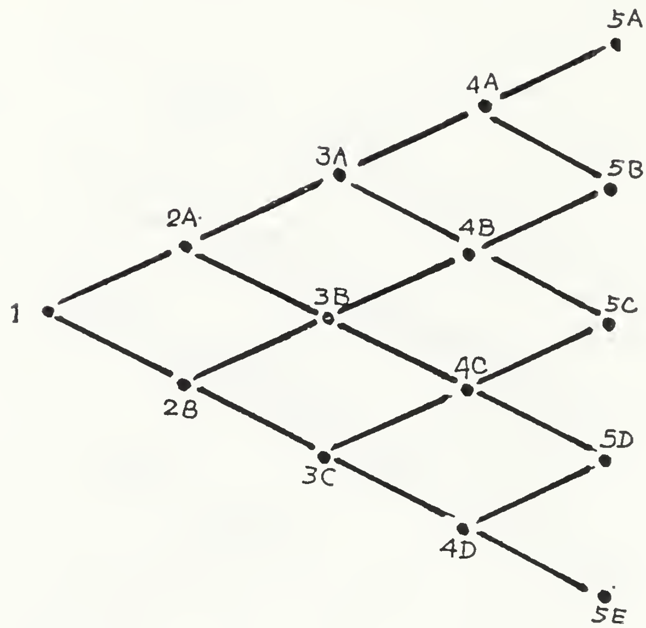
FIGURE 2
STOCHASTIC INTEREST RATE



NODE	INTEREST RATE%	F*	LIKELIHOOD
1	10.00%	.2159	----
2A	11.11	.2337	.4737
2B	9.00	.1998	.5263
3A	12.35	.2496	.2244
3B	10.00	.2194	.4986
3C	8.10	.1822	.2770
4A	13.72	.2638	.1063
4B	11.11	.2369	.3543
4C	9.00	.2037	.3936
4D	7.29	.1628	.1458
5A	15.24	.2764	.0504
5B	12.35	.2524	.2238
5C	10.00	.2229	.3729
5D	8.10	.1865	.2762
5E	6.56	.1414	.0767

* Fully Taxable Investment Allocation

FIGURE 3
STOCHASTIC UNDERWRITING PROFITS



NODE	UNDERWRITING PROFIT	F	LIKELIHOOD
1	250,000	.2229	---
2A	225,000	.2384	.5263
2B	277,778	.2057	.4737
3A	202,500	.2524	.2770
3B	250,000	.2229	.4986
3C	308,642	.1865	.2244
4A	182,250	.2650	.1458
4B	225,000	.2384	.3936
4C	277,778	.2057	.3543
4D	342,936	.1651	.1063
5A	164,025	.2764	.0767
5B	202,500	.2524	.2762
5C	250,000	.2229	.3729
5D	308,642	.1865	.2238
5E	381,039	.1414	.0504

FIGURE 4
STOCHASTIC INTEREST RATES AND UNDERWRITING PROFITS

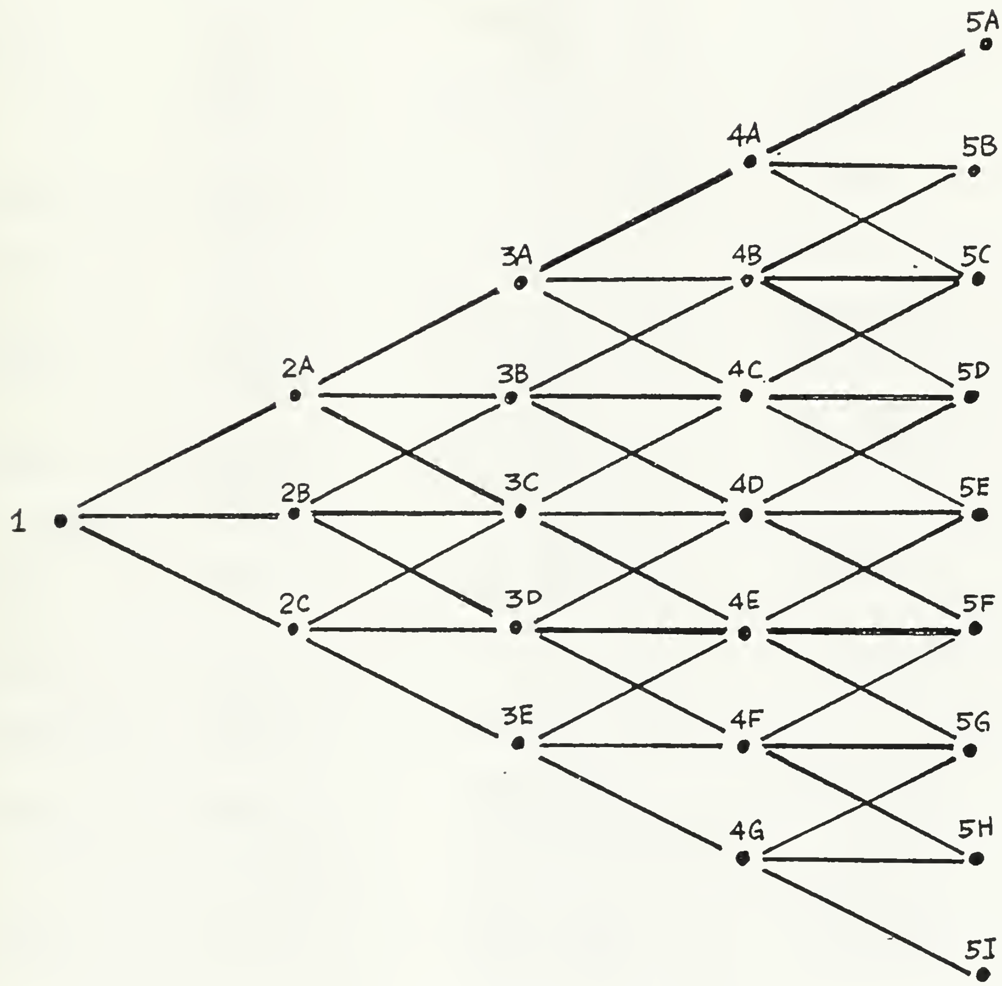


FIGURE 4 (CONTINUED)

NODE	INTEREST RATE%	UNDERWRITING PROFIT	F	LIKELIHOOD
1	10.00	250,000	.2198	---
2A	11.11	225,000	.2496	.2493
2B	{11.11 9.00	277,778 225,000}	.2219	.5014
2C	9.00	277,778	.1859	.2493
3A	12.35	202,500	.2741	.0622
3B	{12.35 10.00	250,000 202,500}	.2501	.2500
3C	{12.35 10.00 8.10	308,642 250,000 202,500}	.2241	.3757
3D	{10.00 8.10	308,642 250,000}	.1893	.2500
3E	8.10	308,642	.1409	.0622
4A	13.72	182,250	.2948	.0155
4B	{13.72 11.11	225,000 182,250}	.2752	.0935
4C	{13.72 11.11 9.00	277,778 225,000 182,250}	.2510	.2345
4D	{13.72 11.11 9.00 7.29	342,936 277,778 225,000 182,250}	.2259	.3130
4E	{11.11 9.00 7.29	342,936 277,778 225,000}	.1939	.2345

FIGURE 4 (CONTINUED)

4F	9.00 7.29	342,936 277,778	.1436	.0935
4G	7.29	342,936	.0826	.0155
5A	15.24	164.025	.3114	.0039
5B	15.24 12.35	202,500 164,025	.2957	.0311
5C	15.24 12.35 10.00	250,000 202,500 164,025	.2764	.1092
5D	15.24 12.35 10.00 8.10	308,642 250,000 202,500 164,025	.2524	.2189
5E	15.24 12.35 10.00 8.10 6.56	381,039 308,642 250,000 202,500 164,025	.2229	.2739
5F	12.35 10.00 8.10 6.56	381,039 308,642 250,000 202,500	.2056	.2189
5G	10.00 8.10 6.56	381,039 308,642 250,000	.1414	.1092
5H	8.10 6.56	381,039 308,642	.0859	.0311
5I	6.56	381,039	.0172	.0039