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Abstract

This paper studies various lot splitting policies in stochastic job shop and flow shop settings with the objective of minimizing long-run mean flow time (MFT). Using a simulation model, we estimate MFT for each policy in stochastic, dynamic situations. When lot splitting is combined with repetitive lots priority, MFT decreases, but there are few differences between the exact lot splitting policy used. Thus, in stochastic, dynamic situations the use of lot splitting is more important than the exact method used. Methods which perform well in static, deterministic environments do not necessarily perform well in other scenarios. We conclude our analysis with a discussion of our findings in relation to flow dominance and JIT/kanban issues.

Keywords: Lot Splitting, Scheduling, Simulation.

1 Introduction

In batch manufacturing, orders arrive to the shop floor in sizes that may not be desirable for the purpose of optimizing manufacturing system performance. Management may choose to split each order into smaller lots with the objective of reducing flow times. There are many ways to split an order: the splits may be equal or unequal, with the number of splits ranging from one to the number of units in the order. The objective of this study is to examine the influence of various lot splitting rules under different shop floor conditions. Specifically, we investigate the performance of various types of lot splitting heuristics in the stochastic environment for the two extreme flow dominance conditions — flow shops and job shops.

We accomplish our research objective in three steps. First, we examine lot splitting rules that have been shown to be optimal for deterministic flow shops (Kropp and Smunt [13]) to evaluate how their performance carries over to stochastic scenarios and environments with jumbled flow dominance. Second, we test several heuristic unequal-split policies, including geometric lots proposed by Baker [1]. These results indicate that differences between lot splitting policies diminish with increased variability in flowshops and in all jobshop scenarios. Finally, we test the effect of increasing the number of splits.

The role of this paper is to fill gaps in the literature on lot splitting. For realism, we consider environments that are stochastic, dynamic, and include multiple job types. We use simulation to test the flow time performance of various lot splitting approaches, most of which have been proposed by the prior research focused on deterministic single-job systems. We investigate conditions under which different approaches provide the best performance over time in these settings. We hypothesize that flow dominance, setup times, shop load, and operation time variance are among the important process design factors that influence the impact of lot splitting heuristics.

The remainder of this paper is organized as follows. In the following section we review the relevant literature. In Section 3, we describe the simulation model, the factors we varied, and the parameters considered. In Section 4 we describe the experiments to determine the effect of lot splitting heuristics. The frequency domain approach is used to determine the sensitivity of the system to the various factors. The results of a comprehensive Analysis of Variance (ANOVA) are presented and discussed. We then consider Baker's geometric lot splitting rule. Section 5 describes experiments that test the effect of different numbers of equal lot splitting rules. Section 6 discusses our conclusions and applications of the results. Finally, Section 7 summarizes the paper and suggests further research.

2 Literature Review

Some papers have dealt with the relationships between lot sizing and job flow times. Karmarkar, Kekre, Kekre, and Freeman ([11], [12]) use both a simulation model of a job shop and Q-LOTS, an analytical procedure based on queueing theory, to examine the impact of lot sizes on flow times. Their approach is to search for the combination of item lot sizes which yields the smallest mean flow time. Other authors, as well, consider the relationship between lot sizing and job flow times (Szendrovits [18], Santos and Magazine [15], and Dobson, Karmarkar, and Rummel [4]). However, none of these papers directly address lot splitting.

Numerous papers have described the effect of lot splitting under deterministic conditions. Graves and Kostreva [7] derived an expression for the optimal number of sublots under the conditions of constant demand, identical machine production rates, and equal sublot sizes. Baker and Pyke [2] and Trietsch ([19], [20]) develop algorithms for minimizing makespan of a single job in a flow shop. In both of these situations, unequal sublot sizes are permitted. Baker [1] proposed a geometric lot splitting rule, which performs well in deterministic flowshops. Finally, Kropp and Smunt [13] developed both optimal and heuristic procedures for minimizing either makespan or mean flow time for a single job in a flow shop. They suggested using equal size sublots when machine setup times were small and a "flag" heuristic to deal with situations in which setup times were large. With the "flag" heuristic, the first sublot has the smallest feasible nonzero size and all other sublots are equal in size. In their deterministic tests they found that these heuristic approaches had excellent performance when compared to the optimal procedures.

Other papers have focused explicitly on lot splitting in stochastic environments. Jacobs and Bragg [9] use a simulation model to examine lot splitting and flow times in a stochastic job shop. They were the first to use the concept of repetitive lots, in which jobs can be split into several transfer batches or sublots. When a work center finishes processing on a sublot, priority is given to another sublot of the same product. In this way the number of setups is decreased, thus increasing the effective capacity of the system and reducing flow times. Although they only considered equally sized splits, Jacobs and Bragg demonstrated that repetitive lots can indeed substantially reduce mean flow times. In another paper that studied lot splitting in a stochastic job shop, Hancock [6] examined a simple lot splitting heuristic and found it to improve job timeliness under the three different routing strategies he tested. The single lot splitting rule that he used allowed a job at any processing station to be split into two transfer batches. Since his focus was mainly on the impact of routing strategies, he did not test different lot splitting rules.

3 The Simulation Model

To test the performance of the various lot splitting heuristics a simulation model was developed and implemented in SIMSCRIPT II.5. In this model, entering jobs are split into smaller transfer batches, so that these transfer batches could be independently processed through their assigned task routing. Using the *repetitive lots* sequencing rule (RL), a transfer batch of the same job type as the current setup at a machine is always be processed next. If no batch with the current machine setup is in the machine queue, then the *first-come, first-served* rule (FCFS) is used for sequencing.

Our jobshop has the same structure as Jacobs and Bragg [9], with 10 departments, each with a single machine, and 10 job types. In general, we used parameter settings similar to those used in [9]. Each job type had an equally likely chance of arriving into the system and required 5 departments to complete its processing. Each department was utilized equally (no long-term bottlenecks at any machine) and was the first or last operation by any job an equal number of times. The flow shop scenario had 5 single-machine departments and 10 job types. Each job type had the same sequence over the 5 departments, and was distinguished from the other job types by virtue of the required setup to change a machine from one job type to another. A 5 department flow shop was required in order to compare results with the job shop, since the same number of tasks was required for each job. The interarrival rates were adjusted to give identical utilizations with the job shop scenarios.

Jobs arrived into the system with deterministic interarrival times, with job sizes varying uniformly by ±67% of the mean job size. Orders were released into the shop as they arrived. Deterministic interarrival times were used in order to mimic a steady release of work to the shop floor. Sensitivity tests indicated that random interarrival times had little impact on the differential effects of lot splitting rules, which is the primary focus of this study. Orders were not batched by job type on a periodic basis; rather, the time a machine spent processing a given job type was determined by the repetitive lots rule and the sequence of transfer batches of that type which happened to arrive during processing. Variable operation times were modeled using a gamma distribution with a coefficient of variation (CV) level specified by the experiment design. Empirical studies of task time distributions (see, for example, [5]) indicate that the distributions are unimodal and skewed to the right, making the gamma distribution and appropriate distribution. Mean operation task times were identical in each department in order to have a balanced shop (i.e. in order to avoid long-run bottlenecking problems) and were 0.0456, 0.0576, or 0.0696 hours per unit. These mean processing times were chosen to produce processing utilization levels of 57%, 72%, and 87%, respectively. In this way, we were able to test a range of ±15% of the processing utilization of 72% used by Jacobs and Bragg. Setup times were deterministic and were varied by multiplying by a setup ratio (SU). A setup ratio of 1.0 is the base case of 3 hours per setup, a setup ratio of .5 results in 1.5 hours per setup and so on. By increasing or decreasing the levels of setup ratio, total utilization also increased or decreased and ranged from 60% to 95% in our experiments. The ratio of setup time to total processing time (including setup) ranged from 22% to 57% per job, on the average.

Our primary performance measure was mean flow time (MFT), rather than the more traditional makespan. While makespan is a suitable criterion for static scenarios, MFT seems to be a more appropriate measure of performance in a dynamic setting. Another measure of interest to studies of this type is the average amount of work in process inventory (WIP), particularly in light of the recent focus in manufacturing toward reducing levels of WIP. However, in steady-state, MFT will be proportional to WIP by Little's formula (see [8]). Thus, results for MFT will usually translate into comparable results for WIP. In the context of this study, this fact means that lot splitting rules that reduce MFT will also reduce WIP, and consequently we need only consider MFT. This relationship was verified by our simulation experiments. In addition, we computed the standard deviation of flow time (SDFT) as a measure of the variability of flowtime.

We estimated MFT and SDFT for each experimental setting by first "warming up" the system for 10,000 hours of operation, followed by the data collection portion of the run. Plots of the output for several combinations of factor settings, including those with the highest processing utilization, CV, and SU, indicated that 10,000 hours of transient observation to be sufficient for each scenario to be in steady state. Flow times were then collected in blocks of 5000 hours separated by periods of 1000 hours with no data collection. Thus, when repeated observations were desired, the resulting block means were taken as the data points. This procedure is similar to that of "batch means" (see Law and Kelton [14]). We verified that there was no substantial serial correlation in these block means.

We conducted two sets of experiments on the model. The first was to determine the impact of different lot splitting heuristics on MFT performance, and may be seen as a continuation of the work of Kropp and Smunt [13]. Several of their rules that were optimal or performed well in deterministic environments were tested in our stochastic model. A comparison of our results with theirs will help determine the effect of randomness on the performance of their rules and measure the robustness of their results. The second set of experiments focused on the number of equal splits. The results of these experiments will be presented in the following two sections.

4 The Effect of Lot Splitting Heuristic

In this section we present our experiments to test different lot splitting heuristics. First we will describe the experimental design, followed by the use of Frequency Domain Experiments (FDE) to determine the sensitivity of MFT to the various experimental factors. We then present the results of an ANOVA for the primary set of lot splitting rules, followed by some tests of Baker's geometric lot splitting rule.

The primary lot splitting heuristics we considered for this experiment are shown in Table 1. The other experimental factors we varied were: setup ratio (SU), operation time coefficient of variation (CV), job size (JS), and processing utilization level (U). The factors (and levels) tested in this experiment are shown in Table 2. In addition to the lot splitting rules considered in Table 1, we considered two variants of Baker's geometric lot splitting rule (Baker [1]). However, these geometric lot splitting rules cannot be directly used in the flow shop as we have modeled in our experiments. We therefore consider these rules separately in Subsection 4.4.

4.1 Description of Experiments

We classified the lot splitting heuristics into three categories, 1) equal splits (RL3E), 2) equal splits preceded by a "flag" split (RL4F), and 3) unequal splits (RLU1, RLU2). In the Kropp and Smunt deterministic flow shop study (Kropp and Smunt, 1990), it was shown that a flag heuristic (one that initially sends a batch of one unit through the system) tends to work well if setup times are high. This result is due to the fact that the contribution of the setup to flowtime is mitigated by the overlap with processing of the following batches. Thus, the overlapping processing was extended to setups times, and the subsequent batches spent less time in queue waiting for a setup. In the deterministic study, it appeared that the following lot splits were nearly equal. Unequal lot splits worked well for the deterministic flow shop conditions of small setup times. As setup times approach zero, however, the optimal lot splits were close to equal. We chose RLU1 and RLU2 to test for robustness of lot splitting distributions. Both RLU1 and RLU2 have characteristics of being nearly equal splits, but with smaller initial transfer batches. Therefore, they are in between the equal split rule (RL3E) and the equal-with-flag rule (RL4F).

It is difficult to predict *a priori* what the effect of variability will have, although low variance conditions in a flow shop should result in similar behavior to the deterministic flow shop. Consequently, we tested levels of operation time coefficient of variation ranging from nearly deterministic (CV = 0.01) to extremely high (CV = 1.5). Based on empirical evidence (see, for example, Dudley[5]) CV = 0.5 is probably the closest to actual operation task times. Nevertheless, there are situations in which a high CV may be appropriate, such as having unreliable machines, parts that may jam a machine, or cases involving rework. In each of these situations, lumping the activities into operation time may result in a higher degree of variability than for the actual processing itself. Six levels of mean job size were chosen, ranging from 75 to 225.

We tested the processing utilization level of 72% since Jacobs and Bragg (1988) used that value as their base setting for shop load. We also tested processing utilization levels that were 15% lower and higher, *i.e.* 57% and 87%, respectively, since we hypothesize that shop load has an effect on the performance of the different lot splitting rules. Note that the total utilization level of the shop will be greater than the processing utilization level due to the effect of setup times.

The experimental design was full factorial and results in $(4 \times 4 \times 6 \times 2 \times 3 \times 6)$ 3456 combinations. Before embarking on such a study, however, it was important to have information about the sensitivity of the shops to changes in these factors. One method for doing this sensitivity analysis involves Frequency Domain Methodology, which we describe next.

4.2 Frequency Domain Approach

To test the sensitivity of the system to the factors listed in the previous section, we used the frequency domain approach (Schruben & Cogliano [16]). This method oscillates each experimental factor of interest throughout the simulation run and measures the effects by the corresponding impact on the power spectrum of the output. Each factor is oscillated at a different frequency, called the driving frequency. If a factor affects the output linearly, a peak in the output power spectrum will be observed at the corresponding driving frequency. A quadratic effect will be detected by a peak at twice the driving frequency. Interactions between two factors may be detected at the sum and difference of their respective driving frequencies. Frequencies at which peaks may occur due to factors are called *term indicator frequencies*, since they indicate the influence of the corresponding "term" in a hypothetical polynomial response function.

Specifically, let θ be the factor under consideration, with $[a - b, a + b]$ the region of interest for θ. The *nominal* value for θ is a and the *amplitude* is b. In an FDE in which θ has driving frequency ω_θ, the value of θ for the n^{th} job is given by

$$\theta(n) = a + b \cos 2\pi\omega_\theta n\,. \tag{1}$$

If the response of the system to θ is linear, then a peak will be observed in the estimated frequency spectrum at ω_θ, and if the response is quadratic, a peak should occur at frequency $2\omega_\theta$.

The design of a frequency domain experiment begins with the set of factors. For the simulation described in Section 3, the factors are setup ratio (SU), coefficient of variation of operation times (CV), jobsize (JS), and processing utilization (U). Flow dominance and lot splitting heuristics are qualitative variables, so a separate frequency domain experiment must be performed for each combination of these factors. Next, the region of interest is determined for each factor. For example, the region of interest for SU is interval $[0.1, 1.5]$. The nominal value for setup ratio is therefore 0.8 and the amplitude is 0.7. Finally, a driving frequency is assigned to each factor. The driving frequencies must be chosen so that there is no confounding between term indicator frequencies of interest. For a given degree of response polynomial, driving frequencies may always be chosen to avoid confounding. See Jacobson, Buss, & Schruben [10] for a discussion of the problem of frequency selection and tables of driving frequencies. For the present study, we chose frequencies which allowed observation of all second-order effects. Table 3 shows the design parameters for the frequency domain experiments. Table 4 shows the term indicator frequencies used. For example, Mean Job Size corresponds to Factor 4 in Table 4, and was oscillated throughout the signal run at frequency 0.130. Thus, the value of JS for the n^{th} job of the simulation was $150 + \cos 2\pi(0.130)n$.

Figure 1 shows the results of frequency domain experiments for the flow shop and job shop under three different lot splitting heuristics: RL0, RL3E, and RL4F. Each plot is the result of two runs. First, control run is performed with the factor levels held fixed at their nominal values. Second, a signal run is performed, in which the parameters are oscillated at their respective driving frequencies. The power spectrum is calculated for each run, and the ratio is taken at each frequency. The plots in Figure 1 are of this "signal to noise" ratio.

The plots show that the lot splitting rules RL3E and RL4F show more sensitivity to the experimental factors than RL0. As results in the next section indicate, the mean flowtime for RL0 is considerably worse than both RL3E and RL4F. The large peaks at frequencies 0.130, 0.174, 0.196, and 0.238 indicate substantial linear effects in the four factors mean jobsize, operation coefficient of variation, processing utilization, and setup ratio, respectively in the RL3E and RL4F cases. Furthermore, a moderate sized peak at frequency 0.022 indicates an interaction between CV and Processing Utilization. The indicated interaction occurs over all values of the other factors, rather than at specific values. Thus, there *may* be other interactions for particular factor settings of remaining factors. The important point is that all four factors are shown to have impact on the flowtime across the ranges indicated in Table 3. Observe that for sufficiently small factor intervals there will be no observed peak and that, typically, for a sufficiently wide range there will always be a peak. On the basis of the frequency domain experiments, it is clear that all experimental factors should be included in the experimental design (Smunt, Kropp, & Buss [17]).

4.3 Results

An Analysis of Variance (ANOVA) was conducted to test the significance of the main and second-order interaction effects for the different factors on mean flow time (MFT). The resulting F-Values and associated significance levels are shown in Table 5.

The R^2 of .96 indicates a reasonably good fit of the model and that the main effects and interactions explain most of the variance. Higher-order interactions, are difficult to interpret, do not contribute to understanding the effects of lot splitting, and do not increase the fit of the model appreciably. Consequently, we considered only main effects and second-order interactions in the model. All main effects are significant at the 0.01 level, and all interactions are significant except for lot splitting method $\times$ setup factor. The results of Duncan's multiple comparisons test are shown in Table 6.

The results in Table 6 show that RL0 performs significantly poorer than the lot splitting rules (about 55% worse) and that there is overall no significant difference between the different rules. These results are therefore evidence that no single lot splitting rule will be universally superior in all situations we have considered. Furthermore, the presence of significant interactions indicates that a more detailed analysis is in order.

In order to compare the effects of different flow dominance conditions (job shop vs. flow shop), we conducted a separate ANOVA for each type of shop, shown in Tables 7 and

9. Because RL0 is really a base case and, based on the results of the first ANOVA, has a significantly higher MFT than any of the lot splitting rules, we omitted it from our subsequent analysis. As with the overall ANOVA, the R^2 of .98 for the flowshop and .99 for the jobshop indicate good fit. In the flow shop ANOVA, the results are similar to the overall ANOVA, except that lot splitting method is now significant at the 0.05 level. The results of Duncan's multiple range test for this scenario are shown in Tables 8 and 10. Although RL4F is statistically indistinguishable from all other heuristics except RLU1, the order of means is nearly the same as the overall means — the difference between RL3E and RLU3 is very small in both cases. Indeed, the difference between the largest and the smallest is under 4% of the mean flow time.

The ANOVA for the job shop (Table 9), in contrast with the flow shop, shows *no* significance for the lot splitting method (Table 7). The other difference between the job shop and the flow shop is the setup × job size interaction, which is significant for the flow shop but not the job shop. The results of Duncan's multiple range test in Table 10 indicate no significant differences between the mean flow times of the lot splitting heuristics. These results are as expected, since the lot splitting method was insignificant to begin with. Note that the mean flow times for the job shop are even more closely bunched than in the flow shop and that RL3E, rather than RL4F, has the lowest MFT. This suggests that in a job shop setting the use of the flag transfer batch may be counter-productive.

From these results we conclude that, while lot splitting is beneficial to MFT in a job shop, the exact method used is unimportant. On the other hand, in a flow shop, the method may matter. Since RL4F performs well in the deterministic flow shop setting, it seems likely that it also performs well for stochastic flow shops with conditions close to the deterministic case, *i.e.*, low levels of CV.

In Figure 2, we plot MFT vs. jobsize under different processing utilization and CV levels, averaged over all other factors. This figure supports the somewhat better performance of RL4F for the flow shop under nearly deterministic (CV = 0.01) settings. On the other hand, both for high processing utilization levels and moderate to high variability the advantage of RL4F vanishes. The lowest processing utilization level of 57% shows much similarity between the deterministic flow shop and stochastic flow shops which have CV's under 1.5. In these cases, RL4F clearly gives the smallest MFT over all job sizes. For the high CV case, there are no clear differences among the lot splitting methods, although RL4F does slightly better. For the medium processing utilization level of 72%, RL4F dominates for CV of 0.01, but is only slightly better for CV's of 0.5 and 1.0. For the highest CV of 1.5, there is no clear difference between the methods. At the highest processing utilization level of 87%, RL4F gives the smallest MFT only for CV of 0.01, with no differences at higher CV's. Thus, as the flow shop moves away from the deterministic setting, differences between lot splitting heuristics disappear. There are no perceptible differences between lot splitting rules in the job shop.

In the deterministic flow shop setting, higher setup levels resulted in better performance of the flag heuristic. We would expect that a similar phenomenon would exist in the stochastic flow shop at low variance levels, and this is indeed the case. Figure 3 shows the MFT for different levels of setup. In the flow shop, for a processing utilization level of 57% and, to a lesser extent 72%, it can be seen that RL4F is increasingly differentiated from the other rules as the setup factor increases. On the other hand, for 87% processing utilization there is no difference between any of the lot splitting rules. Again, there are no discernible differences between the different lot splitting heuristics in a job shop.

Figure 5 shows SDFT for various levels of setup ratios and Figure 4 shows SDFT for various levels of CV. The contrast between low-utilization flowshops and other scenarios is even more apparent here. Note that SDFT in jobshops is higher than that of flowshops both for the same levels of CV and SU. Also, SDFT increases with both CV and SU . In low utilization flowshops, however, differences in SDFT tend to decrease with increasing job sizes. Another interesting contrast is how SDFT increases with increased mean job size to a far greater extent in the job shop than in the flow shop. In no scenario did the lot splitting rule have an appreciable impact on SDFT.

Observe that the results for both MFT and SDFT in the job shop are unanimous and conclusive: the choice of lot splitting rule have virtually no impact on the outcome. We will discuss some implications of this observation in Section 6.

4.4 Geometric Lot Splitting

Baker [1] proposed the use of geometric lot splitting in flow shops and showed that for the 2-machine, single-job, deterministic shop makespan can be reduced by determining the optimal geometric lot splits. He also indicated how geometric lot splitting can be determined for the multiple machine problem. In a balanced shop as we previously examined, the geometric heuristic resulted in equal lot splits. Therefore, we designed additional simulation experiments in which the task times for a job are unequal and ran this experiment to test the use of geometric lot splits in both the stochastic flow shop and job shop settings. We tested four different unbalanced task time scenarios as shown in Table 11 with the corresponding ratio of lot splits used. For example, the resulting GEO3 splits in the first design were 0.4098, 0.3279, and 0.2623 for successive transfer batches and for GEO3F they were 0.01, 0.4057, 0.3246, and 0.2597. For a detailed description of the geometric lot splitting calculation see Baker [1].

In these experiments, we kept the operation time CV at the low level of 0.01. We tested three lot splitting heuristics, RL4F, GEO3, and GEO3F. The RL4F is our flag-plus-three-equal-splits heuristic, the GEO3 is derived from the geometric lot splitting scheme proposed by Baker, and the GEO3F is a flag plus three geometric splits.

In order to operationalize the geometric lot splitting rules in our model, we adjusted the mean task times based upon the sequence in the job routing. For example, in Scenario 1, the fourth task in each job sequence has the highest mean time, whereas for Scenario 2, the first task has the highest mean time, and so on (see Table 11). Due to the jumbled routings in the job shop, the resulting workstation utilizations are equal, on the average. However, since the flowshop routings are sequential, this assignment method results in the bottlenecking of a workcenter, causing it to react as a single-station capacity constrained process. Therefore, the geometric lot splits will not reduce MFT in the multiple job, flow shop setting as we tested in this study.

The MFT results of this experiment are shown in Figure 6 only for the job shop. There was no clear evidence that either the GEO3 or GEO3F heuristics performed any better that RL4F. Our findings from this stochastic experiment seem to confirm the observation in Baker and Pyke [2] that using equal sublots results in nearly the same performance as using geometric splits.

4.5 Discussion

We can draw several general conclusions from the preceding results. The paramount one is that lot splitting is an effective way of reducing MFT, particularly when combined with repetitive lots. As had been pointed out elsewhere ([13], [2], [6]), lot splitting can reduce MFT in deterministic flow shop environments by use of overlapping processing of items from the same job. We have shown here that this reduction extends to stochastic job shops and flow shops.

However, the differences between lot splitting heuristics diminish as the environment moves further from the deterministic flow shop. The only job shop scenario in which there was *any* noticeable difference had very low processing utilization and high setup levels; in other scenarios the exact method used does not seem to matter. This result is consistent with the observations of Baker and Pyke [2] for the deterministic flow shop case, where they found that equal splits were usually nearly optimal. Thus, we find that their results appear to hold in stochastic jobshops and flowshops, in addition to the deterministic flowshops they considered.

The results are similar in the flow shop scenarios. For low CV levels and/or high setup levels and low to medium processing utilization levels the RL4F heuristic performed best, as in the deterministic flow shop ([13]). However, as the flow shop parameters approached more realistic levels (*i.e.*, higher utilization and CV), this advantage disappears. Extending conclusions regarding lot splitting policies which perform well in deterministic settings to more realistic stochastic situations may not be justified. Whereas splitting the lots still has a beneficial effect on MFT in the stochastic environments, the exact method used does not seem to matter. It may be, then, the *number* of splits that matters most in a stochastic environment, in which case using equal splits is a simple, nearly optimal policy. In the following section we describe experiments that determine the effect of the number of equal splits on flowtime performance.

5 The Effect of the Number of Equal Splits

We will now examine the problem of determining the desired number of splits. We expect that the impact of increasing the number of transfer batches will be more dramatic for higher levels of the setup ratio, for higher processing utilization levels, and higher levels of variability (CV). We also expect that as the number of transfer batches gets very large, so the expected transfer batch size approaches one, deleterious effects of lot splitting will appear in the job shop. Conceivably with many small splits in the job shop, with no dominant flow to coordinate the sequencing, there is increased likelihood that at least one straggler will arrive at a machine with the "wrong" setup and get delayed due to repetitive lots working *against* that small batch. As we shall see, however, this delay does not seem occur.

5.1 The Experiment

Using the same simulation model, we ran a series of experiments varying the number of equal splits. Except for the lot splitting rule now being "RLnE," where "n" is the number of equal splits, the other parameters are identical to the experiments above in which we studied the lot splitting heuristics. The number of splits was run at 1 (RL0), 2, 4, 8, 16, 32, 48, and 64. Due to the lengthy computer time required, especially in the 32–64 split cases, and with the results from the previous section in mind, we only considered a subset of the other parameters. We used high, medium, and low processing utilization levels as before (57%, 72% and 87%) and CV levels of 0.01, 0.5, and 1.0. The incoming jobs had a mean of 75 units to allow the larger number of splits to be closer to 1. The setup ratio factor was kept at 1.00 for all experiments.

5.2 Results

The results for the CV/Number of Splits interaction are shown in Figure 7 for processing utilization levels of 57%, 72%, and 87%.

There is a dramatic difference in the flow shop between low CV (0.01) and the higher CV's. For near-deterministic CV (0.01), the MFT is significantly lower for all number of splits. However, the MFT's in the job shop converge for all CV levels as the number of splits increases. Also, the biggest reduction in MFT occurs as the number of splits increases for the high CV situations for both the flow shop and the job shop.

Especially for the flow shop, as the processing utilization increases to 87% the difference between the low-CV flow shop and the higher-CV flow shops is even more dramatic. The divergence between the CV levels in the job shop appears, but is much smaller.

In all scenarios, there was considerable improvement to MFT due to lot splitting. However, the incremental benefits of lot splitting become negligible after about 8 splits. For example, for the job shop with utilization of 72% and CV = 1.0, splitting from one lot to two decreased MFT by 22%, splitting from two to four decreased MFT by 14% more, and splitting from four to eight decreased MFT by 8% more. The final 8% improvement in MFT was achieved with 64 equal splits. The amount of improvement increased with higher utilization and with greater variability (higher CV). The overall benefits of lot splitting are greater in flow shops than job shops. Finally, it is interesting that the MFT for jobshops tended to converge with increased lot splitting, whereas in flowshops they stayed distinct for each CV level.

Figure 8 shows the effect of the number of splits on the standard deviation of flow time (SDFT). As for MFT, there is convergence as the number of splits grows large and the lot size approaches 1. Also, SDFT is larger in the job shop than the flow shop. Finally, it is interesting that increasing the number of equal splits tends to reduce SDFT. The majority of this improvement comes with the first few splits, with little improvement beyond 8 splits.

6 Conclusions

From the various results presented, several conclusions may be drawn.

Flow Dominance. In a flow shop environment, using equal size lots with a flag is superior provided that the variability is not too high, the system is not highly congested, and the setup times are relatively high. In contrast, for a purely random job shop, the differences between the lot splitting heuristics are minimal. The managerial implications are as follows. Consider a job shop which begins moving to more line-oriented flow. Until there is clear flow dominance, the use of unequal lot splitting rules offers little benefit over equal splits. Once there is clear flow dominance, the variability, congestion, and setup times should be considered to determine if there are possible improvements due to unequal lot splitting rules. In these circumstances, our results indicate that the use of the flag tends to improve MFT.

Deterministic/Static vs. Stochastic/Dynamic Settings. Our results indicate there is a large discrepancy between the orderly world of the deterministic, static flow shop models and the chaotic world of the stochastic, dynamic flow shop and job shop models. Since many production facilities processing in batch mode resemble the latter more than the former, methods which work well *only* in deterministic settings have limited applicability. As we have demonstrated, rules such as the "flag" heuristic, which performed well in a deterministic flow shop, seem to have no particular advantage when there is even a moderate amount of variability or congestion.

Interaction Between Lot Splitting and Repetitive Lots. The results we have obtained give insight into the effect of lot splitting in stochastic environments and further serve to point out the relationship between lot splitting and repetitive lots. Lot splitting gains its advantage by increasing the amount of overlapping processing, thus reducing the mean flow time. However, if each split (*i.e.* transfer batch) required its own (minor) setup, the result would be a degradation of performance, high congestion, and bottlenecking of machines. Repetitive lots alleviates this degradation by giving splits of the type currently being processed the highest priority. On the other hand, repetitive lots by itself decreases mean flow time by saving setups for some jobs. With the addition of lot splitting, the overlapping processing gives an additional boost to performance.

JIT/Kanban Issues. Current views of manufacturing, influenced by some Japanese companies, advocate smaller lot sizes, reduced WIP inventory, use of "pull" systems and the related production triggering mechanisms, such as kanbans. Our results on the number of splits indicate that perhaps many of the benefits of such systems may be simply due to reduced lot sizes. If splitting orders into small transfer batches reduces mean flow time, as indicated above, then WIP will be correspondingly reduced as well. Note that in our models we used a classic "push" system with batch processing, yet were able to reduce the flow time significantly by splitting. Furthermore, this worked well in both flow shops and in job shops, where the application of Just-in-Time methods can be problematic. Clearly the above comments are speculative, since we do not test important JIT issues such as variability and setup time reduction.

When Lot Splitting is Not Beneficial. However, lot splitting is not necessarily all benefit. One consequence of many smaller batches in the shop is that material handling costs could skyrocket. Furthermore, the likelihood of a batch getting misplaced in a job shop increases dramatically with the number of such batches in the shop. Thus, a facility which implemented lot splitting with repetitive lots would be wise to rationalize their layout, routing and tracking mechanisms, and make sure their material handling capabilities were sufficiently flexible to handle the resulting load. On the other hand, a flow shop already has a layout that is matched with the routing of its parts. Consequently, lot splitting may still be more desirable in flow shops than job shops, despite the fact that the improvements in MFT tend to be greater in job shops.

In a similar vein, the existence of minor setups may also counteract the potential benefits of lot splitting. In this case, we would consider a minor setup to be one associated with the processing of *any* new batch on a machine, even if of the same type. With more splits, the effect of such setups on MFT would increase. Since we have not considered such setups in this study, we leave this issue to further research.

7 Summary and Further Research

We extensively tested various lot splitting rules in job shop and flow shop environments in scenarios with different levels of setup times, processing time variability, processing utilization, jobsize, and type of shop. We found that as the environment moves away from a deterministic flow shop the differential impact of lot splitting rules diminishes, and there is virtually no difference in most job shop settings. As the number of splits increases, MFT tends to keep improving, but with decreasing returns. Repetitive lots and lot splitting appear to work together in a complementary way. The benefits of lot splitting in these more realistic environments may be even greater than the simpler deterministic cases. It provides a relatively easy way to obtain some of the benefits of smaller batches under the classic push system still employed by most batch production facilities without the need to radically change procedures. As discussed, improved mean flow time goes hand in hand with decreased WIP, another practice that is currently advocated.

Our initial tests constrained the number of machines per department to one, but this simulation model could be easily modified to allow multiple machines per department. We plan to test this environment in future research since we hypothesize that the repetitive lots rule will mimic a cellular manufacturing environment, given a sufficient number of like machines per department. Since the repetitive lots rule scans the queue of jobs waiting to use a machine in a department for one that could be processed without requiring a setup, the availability of multiple, like machines should cause dedication of machines to similar job types.

It is also possible to test the geometric lot splitting rule in flowshops in a special environment (Baker [3]). This environment requires that alternating jobs have exactly opposite task time distributions. For example, job 1 would have task times of 3-2-1, and job 2 would have task times of 1-2-3. Thus, the utilization of each workstation would remain equal, avoiding the bottleneck problem we encountered in Section 4.4.

Even though we found that increasing the number of equal splits did not degrade MFT performance, a number of situations in which MFT would increase with more splits can be envisioned. We have discussed some issues regarding layout, minor setups, and material handling issues above. In certain settings there may indeed be an optimal number of splits that would minimize MFT, and in others a transfer batch size of 1 may be optimal.

Finally, we have only considered flow shops and pure job shops, which are extreme cases of flow dominance. It would be interesting to determine how our results would change for intermediate cases of flow dominance (*i.e.*, between flow shops and pure job shops). We leave the exploration of these important issues to further study.

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Lot Splitting Heuristic	*Definition*
RL0	Repetitive Lots, No Splitting
RL3E	Repetitive Lots, 3 Equal Splits
RL4F	Repetitive Lots, 3 Equal Splits plus Flag
RLU1	Repetitive Lots, 3 splits of 20%, 40%, 40%
RLU2	Repetitive Lots, 3 splits of 25%, 35%, 40%

Table 1: **Definition of Lot Splitting Heuristics**

Factor	*Levels*
Setup Ratio	0.1, 0.5, 1.0, 1.5
Operation CV	0.01, 0.50, 1.00, 1.50
Mean Job Size	75, 105, 135, 165, 195, 225
Flow Dominance	Job Shop, Flow Shop
Processing Utilization	57%, 72%, 87%
Lot Splitting Rule	RL0, RL3E, RL4F, RLU1, RLU2

Table 2: **Factors and Levels for First Experiment**

Factor	*Nominal Value*	*Amplitude*	*Driving Frequency*
Setup Ratio	0.8	0.7	0.283
Processing Utilization	0.72	0.15	0.174
Mean Job Size	150	75	0.130
Operation Coefficient of Variation	0.755	0.0745	0.196

Table 3: **Design of the Frequency Domain Experiment**

		Driving Frequency			
		0.130	0.174	0.196	0.283
	0.130	0.260	0.304	0.326	0.413
Driving	0.174	0.044	0.348	0.370	0.457
Frequency	0.196	0.066	0.022	0.392	0.479
	0.283	0.153	0.109	0.087	0.434

Table 4: **Term Indicator Frequencies For Second-Order Terms**

Source	df	ANOVA SS	Mean Square	F Value	P
Type of Shop (S)	1	6330909.7848313	6330909.7848313	3695.09	0.0
Processing Utilization (U)	2	68292028.0611269	34146014.0305635	19929.60	0.0
Lot Splitting Method (M)	5	5377515.8362393	1075503.1672479	627.73	0.0
Setup Ratio (SU)	3	11635777.1952844	5861251.7687516	2263.77	0.0
Coefficient of Variation (CV)	3	17583755.3062547	3878592.3984281	3420.97	0.0
Mean Job Size (JS)	5	3436362.4751761	687272.4950352	401.13	0.0
S×U	2	4170138.1484389	2085069.0742195	1216.97	0.0
S×M	5	31593.8315973	6318.7663195	3.69	0.0025
S×SU	3	67703.1297404	22567.7099135	13.17	0.0001
S×CV	3	187590.2817836	62530.0939279	36.50	0.0001
S×JS	5	1523555.2710963	304711.0542193	177.85	0.0
U×M	10	1970969.8047876	197096.9804788	115.04	0.0
U×SU	6	5156849.1478169	859474.8579695	501.64	0.0
U×CV	6	10680739.9517084	1780123.3252847	1038.98	0.0
U×JS	10	2111077.4792715	211107.7479272	123.21	0.0
M×SU	15	6557.7844289	437.1856286	0.26	0.9982
M×CV	15	2433751.4003900	162250.0933593	94.70	0.0
M×JS	25	497233.8684283	19889.3547371	11.61	0.0
SU×CV	9	366879.5191605	40764.3910178	23.79	0.0
SU×JS	15	164023.0951700	10934.8730113	6.38	0.0001
CV×JS	15	989189.9238248	65945.9949216	38.49	0.0
Model	163	143014201.2965560	877387.7380157	512.09	0.0
Error	3292	5640286.9746173	1713.3314018		
Corrected Total	3455	148654488.2711732			

$R^2 = 0.962058$

Table 5: **ANOVA Results for Both Shops**

Method	Mean
RL0	290.132
RLU1	186.693
RLU2	185.913
RLU3	183.691
RL3E	183.026
RL4F	182.498

Table 6: **Duncan's Multiple Range Test for Both Shops**
(Means connected by a line are not significantly different)

Source	df	ANOVA SS	Mean Square	F Value	P
U	2	13069705.19863772	6534852.5993189	13644.65	0.0
M	4	5256.46082501	1314.11520625	2.74	0.0273
SU	3	5345676.10072099	1781892.0335737	3720.56	0.0
CV	3	5576568.79713815	1858856.2657127	3881.26	0.0
JS	5	41512.23472691	8302.4469454	17.34	0.0001
U×M	8	4509.53522012	563.6919025	1.18	0.3094
U×SU	6	2892888.23107627	482148.0385127	1006.72	0.0
U×CV	6	3136162.41529595	522693.7358827	1091.37	0.0
U×JS	10	34923.99426781	3492.3994268	7.29	0.0001
M×SU	12	1158.57382651	96.5478189	0.20	0.9984
M×CV	12	5820.61051574	485.0508763	1.01	0.4342
M×JS	20	5289.62464777	264.4812324	0.55	0.9443
SU×CV	9	502347.19995606	55816.3555507	116.54	0.0
SU×JS	15	224473.54338377	14964.9028923	31.25	0.0
CV×JS	15	180597.41108605	12039.8274057	25.14	0.0
Model	130	31026889.93132483	238668.38408711	498.33	0.0
Error	1309	626921.50164489	478.93162845		
Corrected Total	1439	31653811.43296971			

$R^2 = 0.980194$

Table 7: **ANOVA Results: Flow Shop Only**

Method	Mean
RLU1	143.350
RLU2	141.404
RL3E	139.247
RLU3	139.131
RL4F	138.001

Table 8: **Duncan's Multiple Range Test for Flow Shop**
(Means connected by a line are not significantly different)

Source	df	ANOVA SS	Mean Square	F Value	P
U	2	39010192.83130684	39666.20	19505096.4156534	0.0
M	4	3242.02932473	1.65	810.5073312	0.1597
SU	3	4338808.20023008	2941.18	1446269.4000767	0.0
CV	3	4804088.13149537	3256.58	1601362.7104985	0.0
JS	5	3267052.46175667	1328.80	653410.4923513	0.0
U×M	8	2836.77493458	0.72	354.5968668	0.6730
U×SU	6	1593823.14794543	540.21	265637.1913242	0.0
U×CV	6	3273725.07336451	1109.59	545620.8455608	0.0
U×JS	10	2211731.31719906	449.78	221173.1317199	0.0
M×SU	12	4821.84818433	0.82	401.820682	0.6329
M×CV	12	4573.33562947	0.78	381.1113025	0.6769
M×JS	20	9186.10010952	0.93	459.3050055	0.5429
SU×CV	9	13071.28936317	2.95	1452.3654848	0.0018
SU×JS	15	7944.25954558	1.08	529.617303	0.3730
CV×JS	15	383315.41355250	51.97	25554.3609035	0.0
Model	130	58928412.21394184	453295.47856878	921.84	0.0
Error	1309	643675.82031101	491.73095517		
Corrected Total	1439	59572088.03425284			

$R^2 = 0.989195$

Table 9: **ANOVA Results: Job Shop Only**

Method	Mean
RLU2	199.714
RLU3	199.386
RLU1	199.218
RL4F	198.057
RL3E	196.994

Table 10: **Multiple Comparisons for Job Shop**
(Means connected by a line are not significantly different)

Scenario	Task 1	Task 2	Task 3	Task 4	Task 5	Lot Splitting Ratio
1	0.0320	0.0480	0.0640	0.0800	0.0640	1.25
2	0.0960	0.0768	0.0576	0.0384	0.0192	2.00
3	0.0320	0.0640	0.0960	0.0480	0.0480	2.00
4	0.0960	0.0480	0.0240	0.0960	0.0240	4.00

Table 11: **Mean Task Times for Geometric Lot Splitting Scenarios**

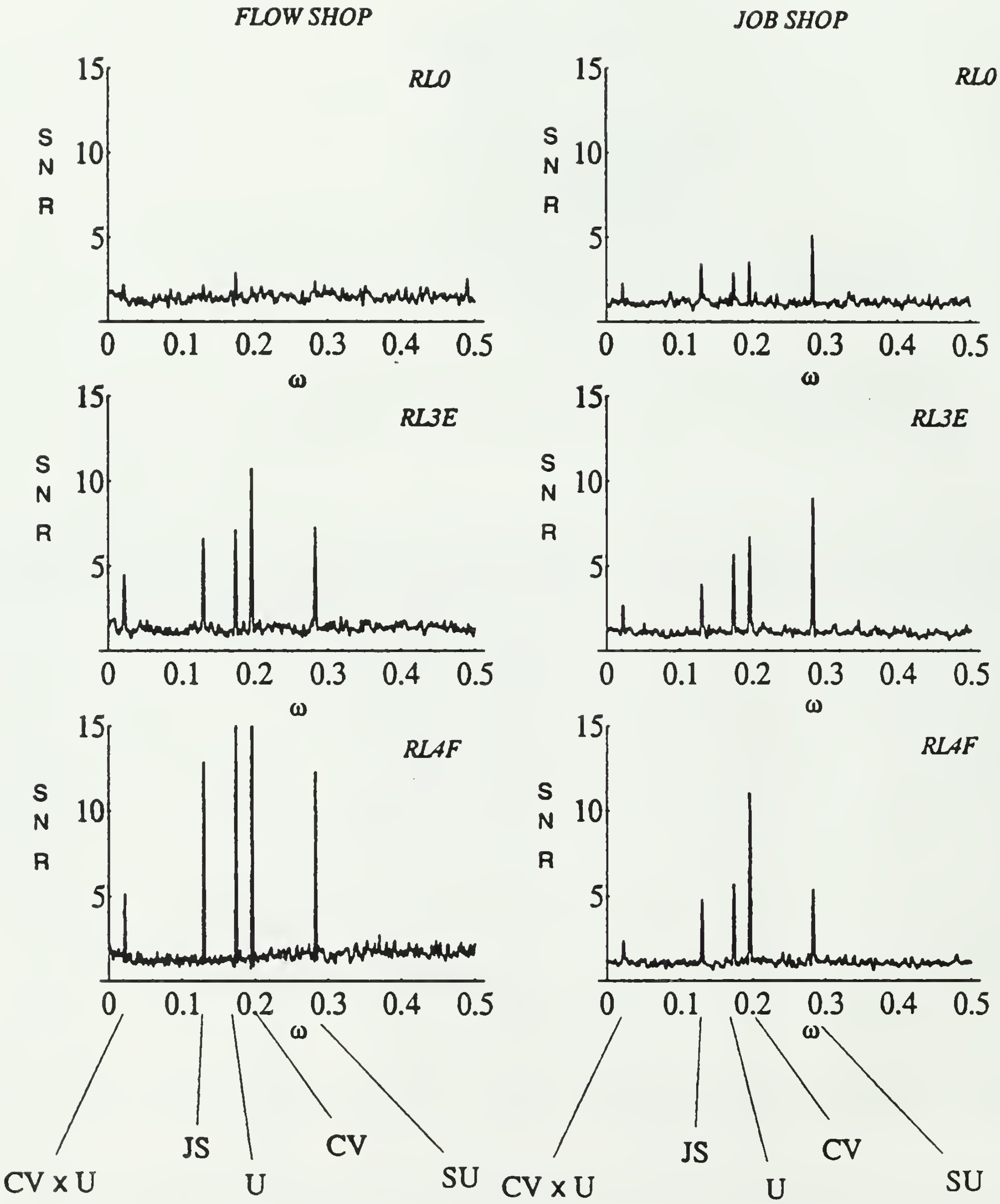

Figure 1: Frequency Domain Experiments: Signal/Noise Ratios (SNR)

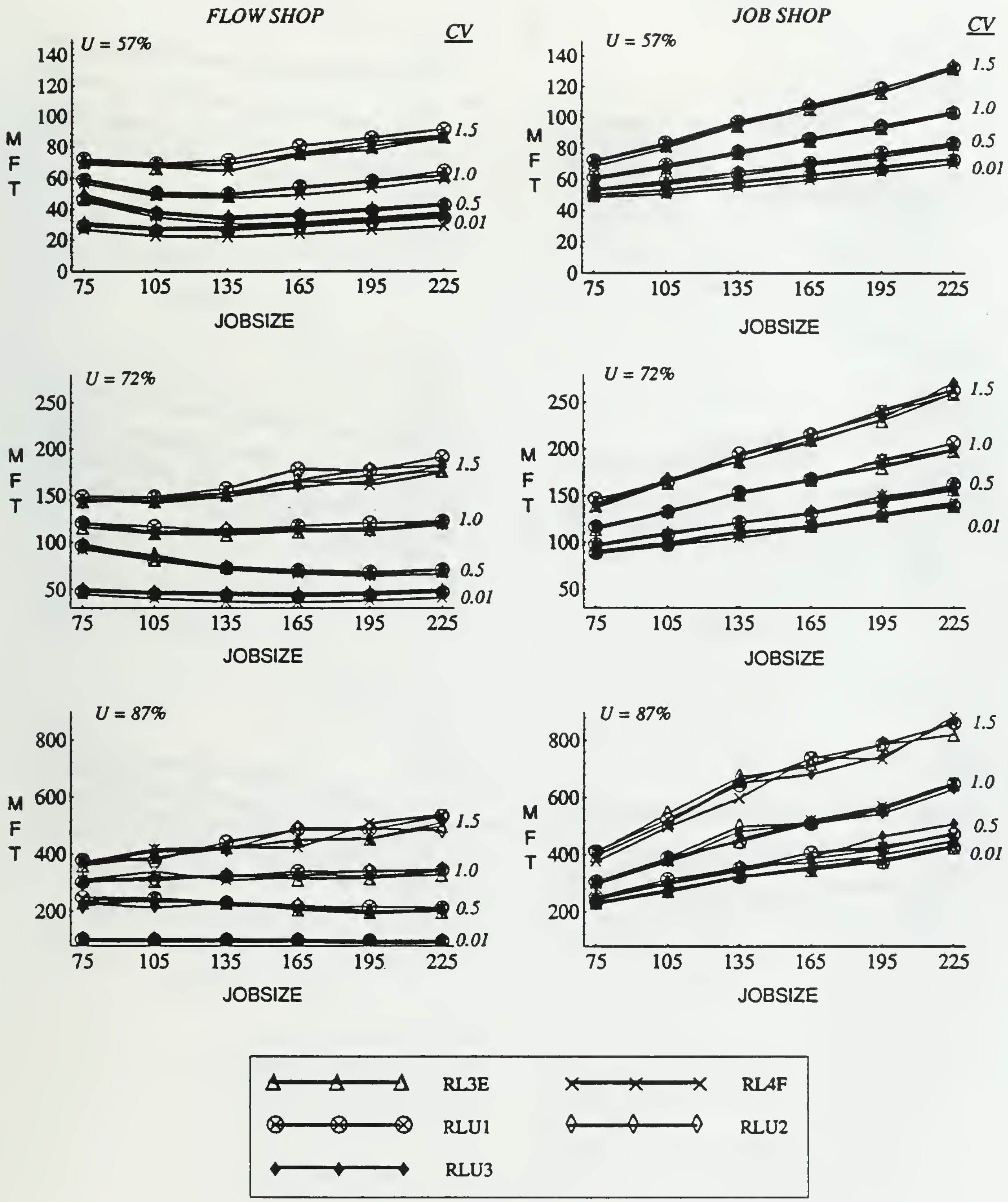

Figure 2: Mean Flow Time (MFT) for Different Jobsizes, Operation CV (CV), and Processing Utilization (U) Levels

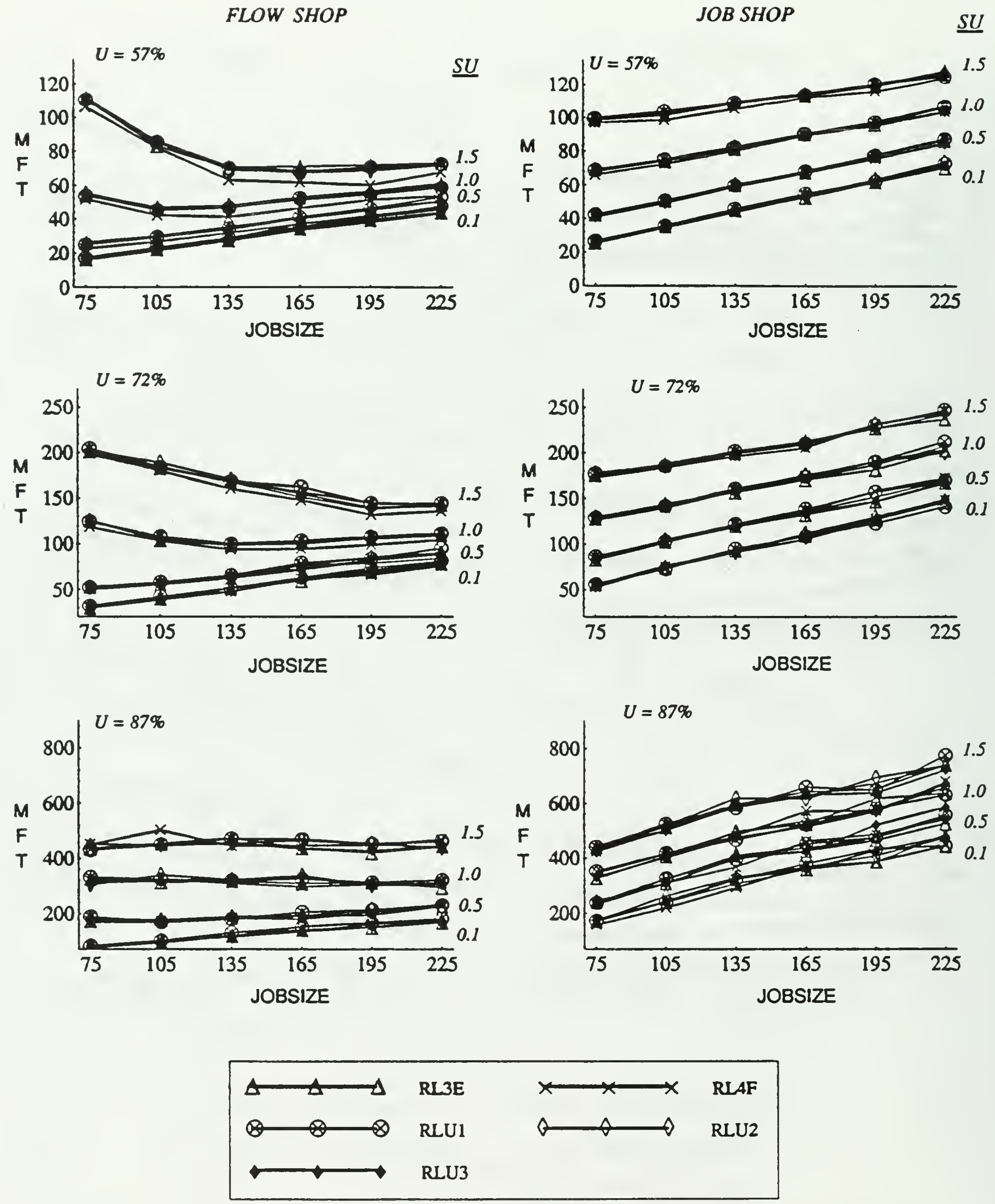

Figure 3: Mean Flow Time (MFT) for Different Jobsizes, Setup Ratios (SU), and Processing Utilization (U) Levels

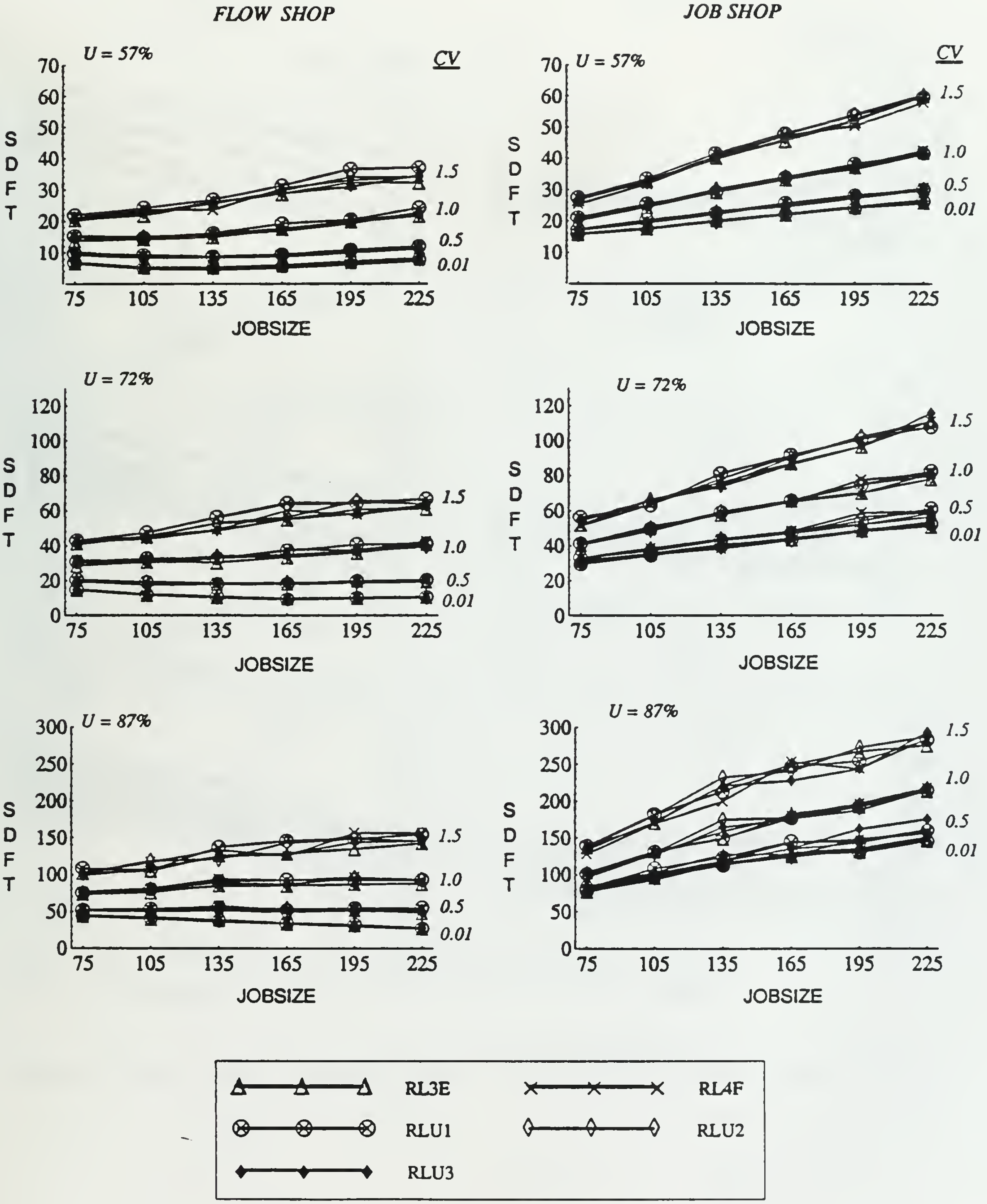

Figure 4: Standard Deviation of Flow Time (SDFT) for Different Jobsizes, Operation CV (CV), and Processing Utilization (U) Levels

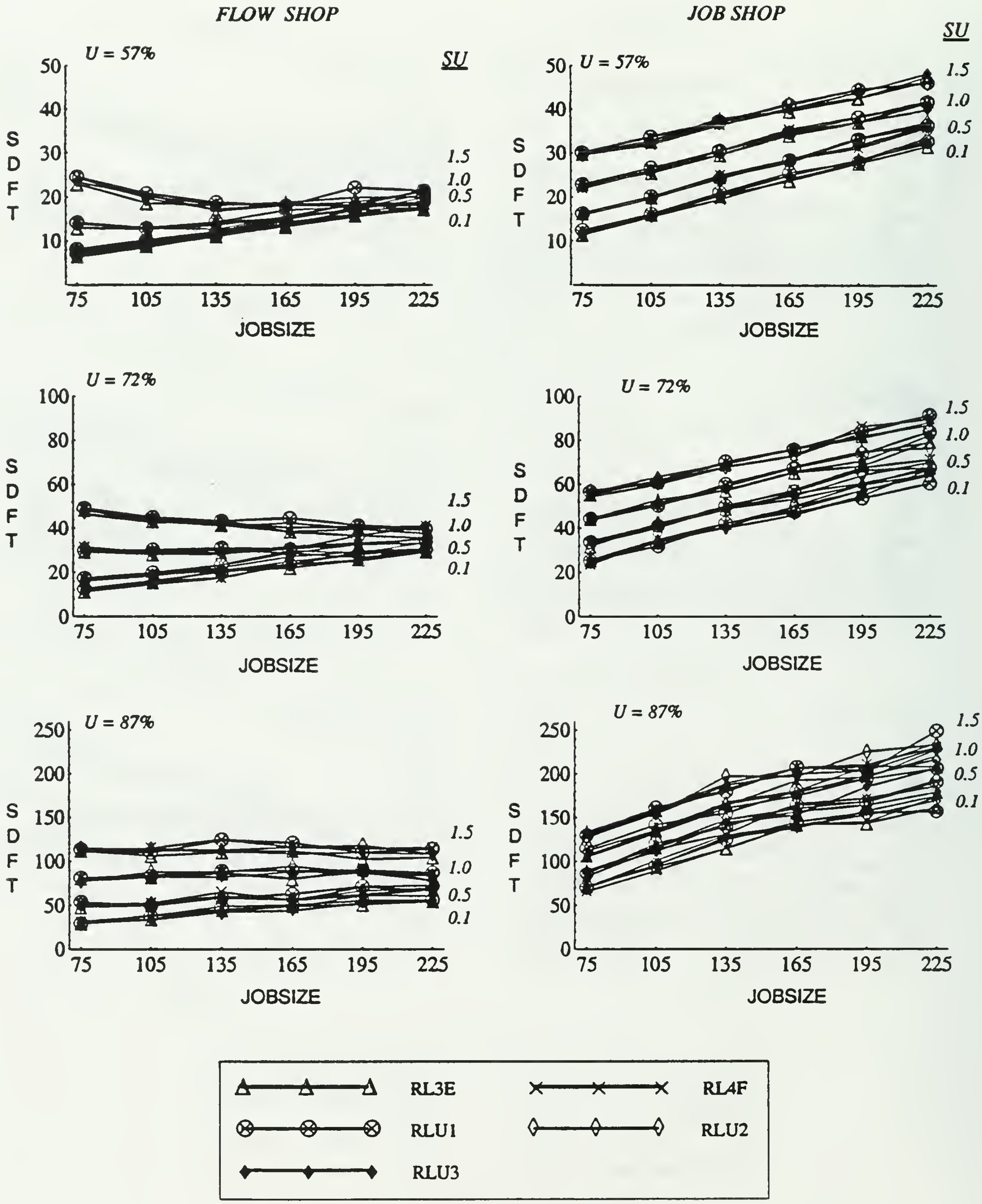

Figure 5: Standard Deviation of Flow Time (SDFT) for Different Jobsizes, Setup Ratios (SU), and Processing Utilization (U) Levels

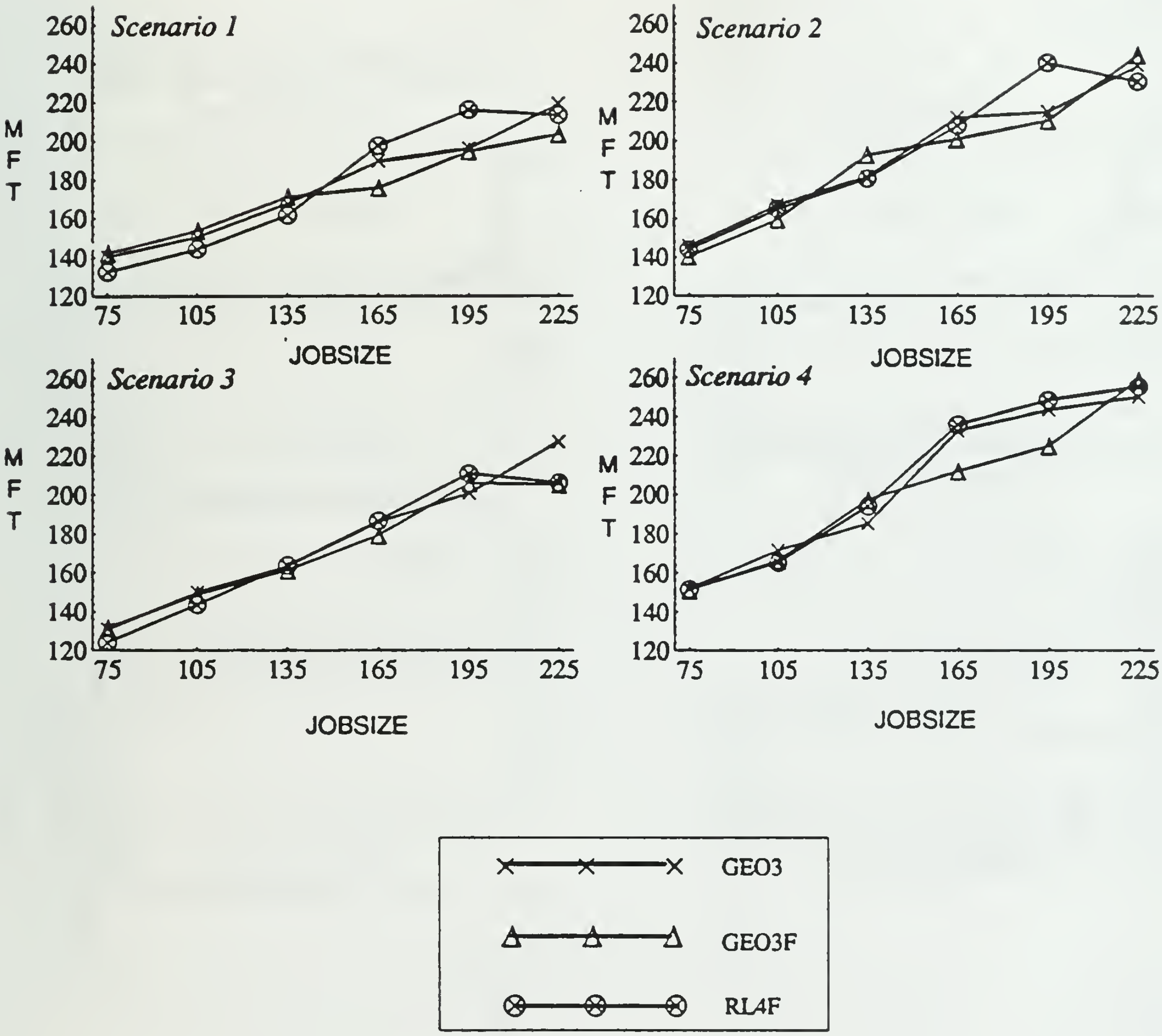

Figure 6: Mean Flow Time (MFT) for Geometric Lot-Splits: Jobshop Only

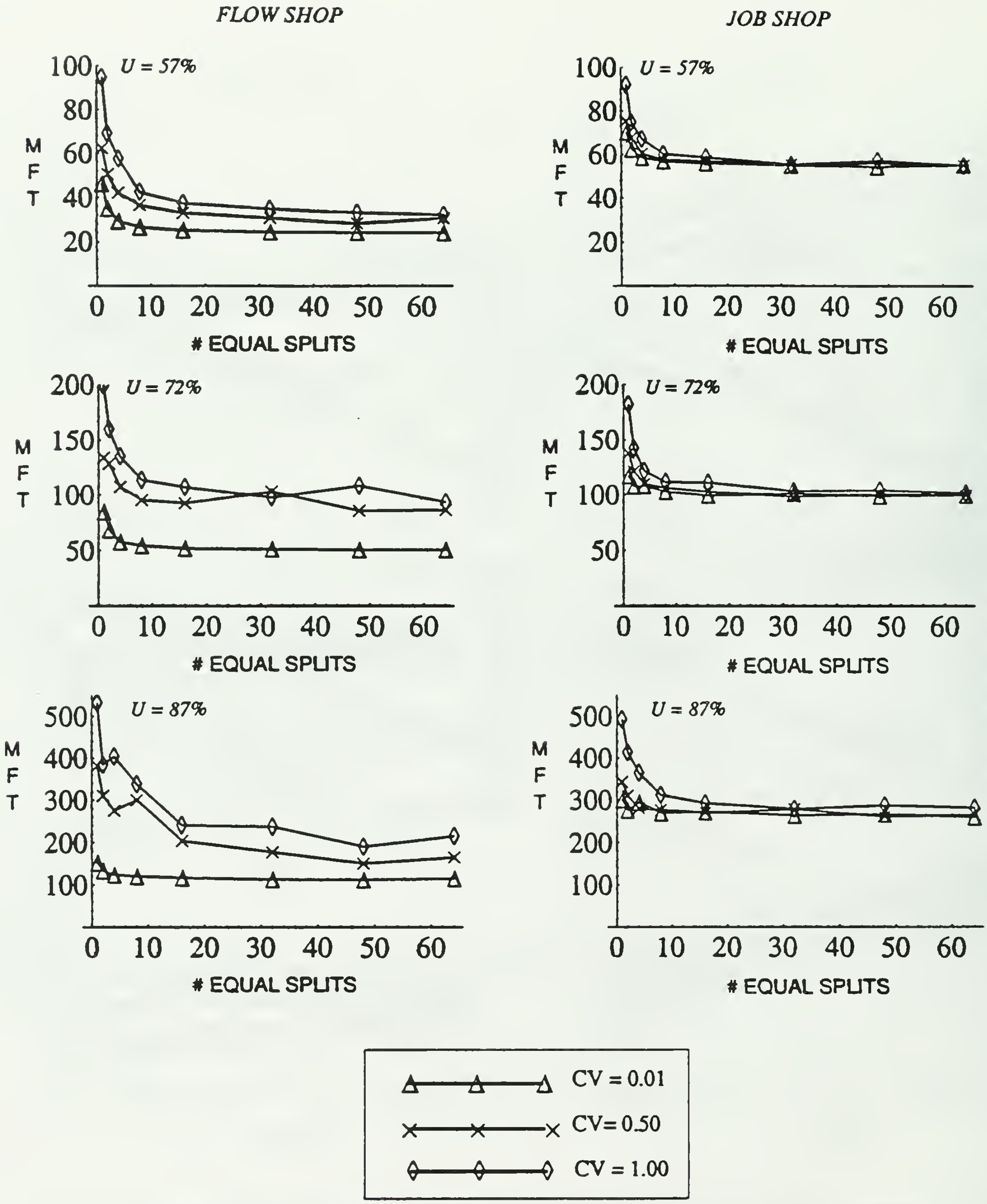

Figure 7: Mean Flow Time vs # of Equal Splits for Different Levels of Processing Utilization (U) and Operation CV (CV)

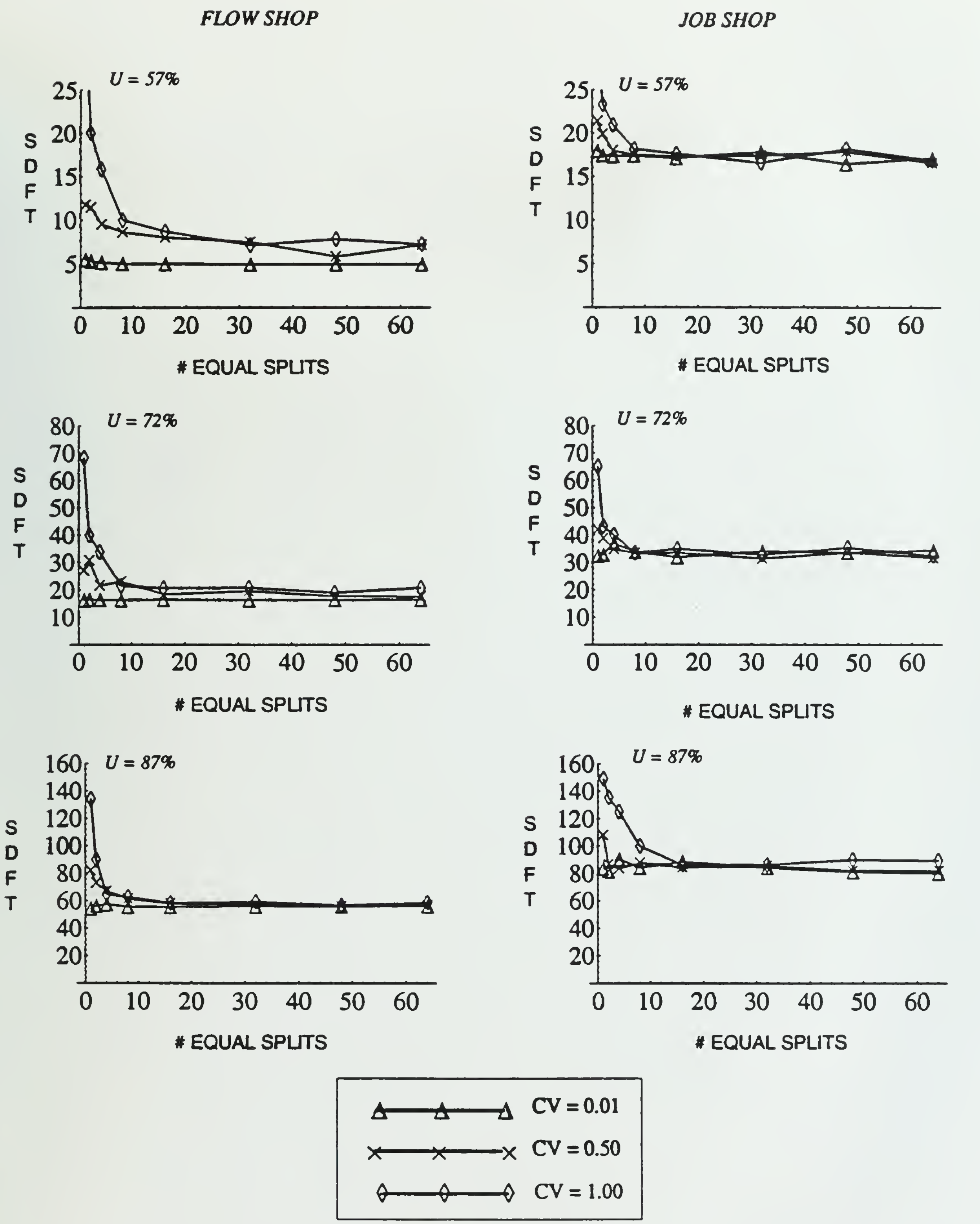

Figure 8: **Standard Deviation of Flow Time vs # of Equal Splits for Different Levels of Processing Utilization (U) and Operation CV (CV)**