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A Note on the Relative Performance of Linear
Versus Nonlinear Compensation Plans

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ABSTRACT

This paper addresses the problem of compensating a salesperson in an uncertain selling environment. A numerical experiment was conducted to compare the performance of a linear compensation plan consisting of a salary and a straight commission with that of the optimal compensation plan derived from agency theory, which is usually nonlinear. The simpler, linear plan was found to perform almost as well as the more complex agency theoretic plan in high uncertainty environments while the agency theoretic plan performed significantly better than the linear plan when uncertainty was low.

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1. Introduction and Objectives

Selling activities constitute a major expense of running a business. In 1981, US companies spent about $150 billion on personal selling, far exceeding the $61 billion spent on advertising that year (Kotler, 1986, p.499). Not surprisingly, salesforce management and salesforce compensation have received considerable attention from researchers. An important aspect of the selling environment is its uncertainty : a given amount of effort does not always result in the same amount of sales. This fact has long been recognized and incorporated in models of salesforce compensation (e.g. Berger 1972, 1975, Weinberg 1975). More recently, Basu, Lal, Srinivasan and Staelin (1985), henceforth called BLSS, applied the paradigm of agency theory, developed in mathematical economics (e.g. Holmstrom 1979, Shavell 1979, Grossman and Hart 1983) to the problem of finding the optimal salesperson compensation plan. Refer to Lal and Staelin (1986), Lal (1986), Rao (1988), and Lal and Srinivasan (1988) for further work in this area.

The present research relates primarily to the BLSS model which considers one risk averse salesperson selling a single product in one time period. The BLSS model, in spite of the elegance of the approach, generally yields compensation plans which are complex in form, and can be approximated by a plan with a sliding commission rate. In contrast, a recent study by Wilson and Bennett (1986) shows that in 1984, 29.0 percent of US firms used salary and commission, 33.6 percent used salary plus bonuses and 17.1 percent used only salary to compensate their salesforces, and that over 40 percent of the commission based plans involved straight commissions. This indicates the prevalence of simple compensation plans in industry practice. Industry usage is not restricted entirely to simple compensation plans, however. The Wilson and Bennett study also found that more complex compensation plans such as those involving sliding commission rates were used frequently by US firms.

In this paper, we study how the linear compensation plan consisting of a salary plus a constant rate of commission performs compared to the optimal agency theoretic compensation plan which is often nonlinear. Common sense suggests that a salesperson will find a linear compensation plan easier to understand than one involving sliding commission rates. Also, a linear compensation plan avoids the administrative problem of creative accounting by a salesperson where the latter places sales in earlier or later periods in order to exploit the nonlinearity of the compensation plan. (For a discussion of this point, see Lal and Srinivasan, 1988.) Finally, a linear compensation plan with a constant commission rate can be easily incorporated into the fixed and variable costs of a firm even when multiple salespeople are involved. Therefore, it seems likely that a firm would choose a nonlinear compensation plan over a linear plan only if that would lead to a significant improvement in performance.

Theoretical literature suggests that there are situations when the linear compensation plan can be optimal as well. The BLSS paper identifies such a case where the salesperson has a logarithmic utility function for earnings. Lal and Srinivasan (1988), adopting the dynamic perspective developed by Holmstrom and Milgrom (1987), present a scenario where the salesperson has constant absolute risk aversion for earnings and the optimal compensation plan is linear. It should be noted that in these cases, the salesperson is assumed to be highly risk averse. Consequently, it would seem logical that (s)he would appreciate the relative stability of the linear compensation plan where his/her earnings would not be greatly affected by slight changes in sales realized.

In contrast, if uncertainty is low, the salesperson will be less concerned about the possibility of adverse outcomes which are now less likely. It is a commonly known result of agency theory that when uncertainty does not play a role, the firm can use a nonlinear compensation plan to obtain a **first best solution**, discussed later in this paper. Albers (1986) has shown that nonlinear salesforce compensation plans can perform significantly better than the linear plan in a deterministic environment. Basu (1984) has identified a class of nonlinear compensation plans which will be very efficient in low uncertainty environments. Factors such as uncertainty, therefore, help determine when a linear or a nonlinear plan would be preferred.

This paper adopts the BLSS framework and restricts its attention to the case where the salesperson's utility function for earnings s is the power function $\frac{1}{\delta}s^{\delta}$. In this case, BLSS shows that if certain assumptions are made regarding the probability distribution of the realized sales x, the agency theoretic compensation plan has the nonlinear form $(A+Bx)^{1/(1-\delta)}$, $A \geq 0$, $B > 0$. This paper uses a numerical procedure to find optimal compensation plans of the form $(A+Bx)^{\alpha}$, which includes both the linear and the agency theoretic compensation plans in the case considered. Thus, it is possible to determine how the realtive performance of the linear and the agency theoretic compensation plans would be affected by changes in parameters of the environment such as uncertainty. In specific, this paper aims to determine when a linear compensation plan performs almost as well as the agency theoretic plan and is likely to be used, and also when a nonlinear plan suggested by agency theory performs significantly better than the linear plan and is therefore likely to be adopted. We hope that future research will test the validity of these conclusions.

The rest of the paper is organized as follows. Section 2 very briefly states the problem of finding an optimal compensation plan of the form $(A+Bx)^{\alpha}$. Section 3 presents results of a numerical experiment comparing performances of linear and agency theoretic compensation plans. Section 4 summarizes the findings of the paper.

2. Assumptions And Model Development

In this section we present a brief overview of the model development. The notations closely follow the BLSS paper. t is the time (effort) devoted by the salesperson which the firm cannot observe directly, x the realized sales level, and $s(x)$ the compensation the salesperson receives from the firm. c is the (constant) marginal cost of production and distribution as a fraction of price. The problem structure and assumptions we use are generally identical to that of BLSS except for the following which are more specific :

(a) The salesperson's utility function for earnings $s \geq 0$ and effort t is,
$U(s) - V(t) = (1/\delta)s^{\delta} - dt^{\gamma_2}$, where $0 < \delta < 1$, $d > 0$, and $\gamma_2 > 1$.
If $s < 0$, $U(s) = -\infty$. This is equivalent to an institutional constraint that the salesperson cannot be assessed a penalty.
(b) The probability density function of x given t is the gamma function,

$$f(x|t) = \frac{1}{\Gamma(q)}\left(\frac{q}{g(t)}\right)\left(\frac{qx}{g(t)}\right)^{q-1} e^{-qx/g(t)}, \quad 0 < x < \infty. \tag{1}$$

$g(t) = E(x|t) = h + kt^{\gamma_1}$, where $h \geq 0$, $k > 0$, and $0 < \gamma_1 < 1$, $\mathrm{Var}(x|t) = g^2(t)/q$.
(c) $\gamma_2 \geq \gamma_1 + 1$, and $\delta < \gamma_2/(\gamma_1 + \gamma_2)$. This is a purely technical assumption.

As discussed in BLSS, it is assumed that the salesperson responds to a compensation plan $s(x)$ by selecting effort t to maximize his/her expected utility, and (s)he will accept the terms of employment if and only if (s)he can achieve the minimum expected utility m which we assume to be strictly positive. The firm's problem is to select $s(x)$ to maximize its expected profit π while incorporating the salesperson's response to $s(x)$, and satisfying the minimum expected utility requirement of the salesperson.
If $s(x)$ is not restricted to have a prespecified form, the resulting optimal compensation plan is the agency theoretic compensation plan. BLSS has shown that in the present case, if the agency theoretic problem of finding the optimal $s(x)$ (without a prespecified form) has a Lagrangean solution, then the optimal plan has the form

$$s^*(x) = (A + Bx)^{1/(1-\delta)}, \quad A \geq 0,\ B > 0. \tag{2}$$

Clearly, $s^*(x)$ belongs to the class of compensation plans of the form

$$s(x) = (A + Bx)^{\alpha}, \quad 1 \leq \alpha \leq 1/(1-\delta). \tag{3}$$

Also, any linear compensation plan consisting of a salary and a constant commission rate is a special case of (3) with $\alpha = 1$.

An efficient algorithm was developed to numerically obtain the optimal $s(x)$ of the form specified by equation (3) for a given set of values of the model parameters δ, m, q, h, k, γ_1, d, γ_2, and c, and a given α, with any degree of precision desired. (Details of the algorithm are available from the authors on request.) The optimal compensation plan (A^*, B^*) always has $A^* \geq 0$ and $B^* > 0$. An optimal solution will be called a boundary solution if $A^* = 0$, and an interior solution if $A^* > 0$. For a given set of parameter values, the best compensation plan determined using $\alpha = 1$ yields the optimal linear compensation plan. If the best $s(x)$ is determined using $\alpha = 1/(1-\delta)$ and the resulting optimal plan is interior, then it can be shown that this solution is the optimal agency theoretic compensation plan. The performances of the linear and the agency theoretic plans can then be compared.

3. Numerical Results

3.1 Study Design. In this section we study how the linear compensation plan performs compared to the agency theoretic compensation plan, and how the relative performance is affected by changes in the parameters of the selling environment, δ, m, q, h, k, γ_1, d, γ_2, and c. A larger δ represents a reduction in the risk aversion of the salesperson, and $\delta = 1$ implies risk neutral behavior. A larger m implies that for a given level of effort, the salesperson requires an increase in his/her earnings in order to accept employment, i.e. (s)he is more costly to employ. A larger q (called the 'certainty parameter' by BLSS) implies a reduction of uncertainty in sales for a given level of effort. An increase in h denotes an increase in base sales, i.e. a reduction in the elasticity of sales with respect to effort, while an increase in k or γ_1 represents a greater sales responsiveness to effort. An increase in γ_2 represents greater marginal disutility for effort, and an increase in d increases disutility for any given level of effort. A larger c means a reduction in the expected profit of the firm for any given level of sales.

A numerical 'experiment' was conducted using a full factorial design ($5 \times 2^8 = 1280$ units) with five levels of δ and two levels of each of the parameters m, q, h, k, γ_1, d, γ_2, and c. The levels of parameter values used are presented in Table 1. It may be noted that for the parameter δ, we used a range of $\pm 1/6$ around $\delta = .5$ used by BLSS. For numerical convenience, levels of m and k chosen were not the same across δs. The parameter values were so chosen that in each of the 1280 cases studied, the optimal compensation plan using $\alpha = 1/(1-\delta)$ was interior. (We found that we could always get an interior optimum by using an adequately large m.)

Table 1 about here

We would like to stress that apart from making sure that the optimal solution with $\alpha = 1/(1-\delta)$ was interior, i.e. it was also the optimal agency theoretic compensation plan, we did not choose parameter values to favor either the linear or the agency theoretic compensation plan. This, combined with the fact that we analysed a wide spectrum of possibilities, should establish that the results obtained would hold over a large range of cases if not everywhere. For each of the 1280 cases, the optimal $s(x)$ was determined using $\alpha = 1$ (linear plan) and $\alpha = 1/(1-\delta)$ (agency theoretic plan). Section 3.2 discusses how the two plans performed relative to each other.

3.2 Relative Performances of Linear and Agency Theoretic Plans. For each case studied, we define :

1. π_A = expected profit of the firm from the agency theoretic plan.
2. t_A = optimal selling effort for the agency theoretic plan.
3. π_L = expected profit of the firm from the linear plan.
4. t_L = optimal selling effort for the linear plan.
5. $\pi_0 = -(\delta m)^{1/\delta}$.

π_0 is the firm's expected profit in the worst case where it pays the salesperson a fixed amount (equal to π_0) to satisfy the minimum expected utility requirement, and receives no selling effort in return.

6. $R_1 = (t_L/t_A)$. This expresses the optimal selling effort under the linear plan as a fraction of the optimal selling effort for the agency theoretic plan.
7. $R_2 = \{\pi_L - \pi_0\}/\{\pi_A - \pi_0\}$. R_2 expresses the fraction of the agency theoretic plan's profit achieved by the linear plan, where both profits are measured from the base π_0.

R_1 and R_2 were computed in each case and used to measure how the linear plan performed compared to the agency theoretic compensation plan. We wanted to determine, for each value of δ selected, how R_1 and R_2 varied from case to case and depended on the model parameters m, q, h, k, γ_1, d, γ_2, and c. We used R_1 and R_2 as the dependent variables and performed dummy variable regressions with main effects and two-way-interaction effects. (For any parameter, e.g. m, the dummy variable is -1 if the parameter is at the low level, and 1 if the parameter is at the high level.)

Results about R_1. Table 2 presents the regression results relating R_1 to the independent variables for each of the five values of δ selected (after eliminating insignificant predictors using a subset F-test). For clarity of exposition, only the estimated coefficeients of the main-effect terms are presented in the table. In every case, the estimated intercept term is the average of R_1 for the sample of size 256. The following patterns emerge from Table 2 :

1. As δ increases, R_1 tends to go down, the decline being very slow for $\delta \leq .5$. Thus, as the salesperson becomes less risk averse, the agency theoretic compensation plan is more effective in inducing higher effort from the salesperson compared to the linear plan.

2. As the environment becomes more deterministic such that risk aversion of the salesperson plays less of a role in selection of effort level, the agency theoretic plan is once again more effective in inducing higher effort than the linear plan.
3. As m increases, i.e. the cost of inducing any level of effort goes up, R_1 increases.

The above discussions are of course limited by the fact that the analysis has been based on arbitrarily chosen parameter values.

Table 2 about here

Results about R2. The regression results relating R_2 to the predictors are presented in Table 3 (after eliminating insignificant predictors using a subset F-test). For clarity of exposition, only the estimated coefficeients of the main-effect terms are presented in the table. For each δ, the estimated intercept term is the average value of R_2 for the sample of size 256.

Table 3 about here

From an inspection of Table 3, it is clear that the linear plan performs almost as well as the agency theoretic plan when $\delta \leq .5$ with the average R_2 exceeding 99% in each case. If δ exceeds .5, the relative performance of the linear plan declines significantly.

Even though the regression results are limited by our arbitrary choice of parameter values, it is interesting to note the high explanatory power of the regression model for the higher values of δ (For the lower values of δ, the variation of R_2 is small.), and the fact that certain results tend to hold over the range of δ's considered. Table 3 shows that the relative profitability of the linear plan declines significantly when δ increases beyond .5, or when the certainty parameter q is high. Noting that $\delta = 1$ signifies risk neutrality, it is clear that when the salesperson is not greatly affected by uncertainty, the agency theoretic plan performs much better than the linear plan.

An inspection of Tables 2 and 3 reveals that when either k or γ_1 is high, the relative performance of the linear plan is adversely affected. Intutitively, in these situations, the output x depends strongly on the selling effort t. The agency theoretic plan, being more flexible than the linear plan, can motivate the salesperson more effectively in these situations. Conversely, when it is costly to induce additional effort (m is high, d is high, γ_2 is high, or q is low), or the revenue is not significantly affected by the salesperson's effort (h is relatively high), the relative performance of the linear plan improves.

Comparison with the First Best Solution. In order to explore the effect of uncertainty on performance further, we compared the results from the linear and the agency theoretic compensation plans with the 'first best' solution to the firm's problem of designing an optimal compensation plan. Here, the first best solution corresponds to the case where the firm can measure selling effort t perfectly and without cost, and thus can 'force' the salesperson to devote any specific amount of effort, subject only to the constraint that the salesperson's expected utility must equal or exceed the minimum, m. The first best solution corresponds to the hypothetical case where uncertainty has no effect on profitability, and the expected profit from the first best solution is an upper bound to the expected profit achievable from the agency theoretic compensation plan (also known as the 'second best solution'). See Holmstrom (1979) or Shavell (1979) for discussions of the first best solution.

Let t_F and π_F represent the optimal selling effort and expected profit, respectively, for the first best solution. It can be easily shown that

$$(4)\quad t_F = \text{argmax}\left\{(1-c)g(t) - U^{-1}(m+V(t))\right\}, \quad \pi_F = (1-c)g(t_F) - U^{-1}(m+V(t_F))$$

In each of the 1280 cases studied, the following additional quantities were computed:

1. t_F.
2. π_F.
3. $R_3 = (t_A - t_L)/(t_F - t_A)$.
4. $R_4 = (\pi_A - \pi_L)/(\pi_F - \pi_A)$.

Thus, R_3 and R_4 measure the loss in performance resulting from using the linear plan instead of the agency theoretic plan in terms of the loss in performance resulting from uncertainty in the selling environment.

For each of the five values of δ used, we performed dummay variable regressions using R_3 and R_4 as the dependent variables and the same set of independent variables as discussed earlier in this section. The results for R_3 and R_4 are presented in Tables 4a and 4b, respectively. (For clarity of exposition, only the estimated coefficeients of the main-effect terms are presented in the tables.)

Table 4a about here

Table 4b about here

The findings are consistent with the results presented in Tables 2 and 3 earlier: when the salesperson is more risk averse or the selling environment less certain, the linear plan performs almost as well as the agency theoretic plan. With a reduction in the effect of uncertainty (more certain environment or less risk averse salesperson), the more flexible agency theoretic plan can exploit the opportunities presented by a kinder environment more effectively than the linear compensation plan.

4. Conclusion

This paper investigates how the linear compensation plan performs compared to the agency theoretic plan when the salesperson's utility function $U(s)$ is $(1/\delta)s^{\delta}$. It was found that for $\delta \leq .5$, the linear compensation plan was almost as profitable as the agency theoretic plan. This provides theoretical justification for the extensive use of the linear compensation plan in industry practice.

When δ exceeds .5, the salesperson is less risk averse and hence less affected by uncertainty, and a nonlinear plan involving greater variations in possible earnings performs significantly better than the linear plan. Thus, it seems likely that in practice, we should observe the linear plan more often in high uncertainty environments, while nonlinear plans such as those involving increasing commission rates for higher sales would be used more frequently in low uncertainty settings. An empirical investigation is needed to firmly establish if there is indeed such a relationship between the use of the linear plan and the level of uncertainty in the environment.

This study also shows that under the linear compensation plan, the salesperson devotes significantly less effort compared to the agency theoretic plan. Thus, if we extend the model to incorporate economies of scale or experience effect, it is possible that the relative profitability of the linear compensation plan would be adversely affected, and hence the firm would use nonlinear plans more often. We leave that study to future research.

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Table 1

Factor no.	Parameter	Values used in full factorial design
1	δ	1/3; .4; .5; .6; 2/3
2	m	50; 55 for $\delta = 1/3$
		70; 80 for $\delta = .4$
		130; 150 for $\delta = .5$
		180; 200 for $\delta = .6$
		230; 250 for $\delta = 2/3$
3	q	2; 10
4	h	0; 4000
5	k	4000; 5000 for $\delta = 1/3, .4, .5$
		2500; 3000 for $\delta = .6, 2/3$
6	γ_1	.5; .6
7	d	1; 1.5
8	γ_2	2; 2.5
9	c	0; .2

Table 2

Estimated		$\delta = 1/3$	$\delta = .4$	$\delta = .5$	$\delta = .6$	$\delta = 2/3$
Constant		.9926	.9758	.9433	.8607	.7321
Coefficient of	m	.0052	.0144	.0207	.0196	.0156
Coefficient of	q	−.0052	−.0195	−.0430	−.0753	−.0768
Coefficient of	h	.0046	.0103	.0181	.0288	.0325
Coefficient of	k	−.0033	−.0078	−.0152	−.0199	−.0231
Coefficient of	γ_1	−.0020*	−.0057	−.0132	−.0268	−.0355
Coefficient of	d	.0016†	.0038	.0072	.0112	.0128
Coefficient of	γ_2	.0034	.0090	.0211	.0451	.0674
Coefficient of	c	−.0063	−.0169	−.0265	−.0535	−.0559
Adj R^2		.6153	.8539	.9630	.9781	.9960
$s(R_1)$		.0225	.0490	.0779	.1204	.1309

† → $p < .1$, * → $p < .05$, $p < .01$ otherwise.
$s(R_1)$ is the sample standard deviation of R_1.

Table 3

Estimated		$\delta = 1/3$	$\delta = .4$	$\delta = .5$	$\delta = .6$	$\delta = 2/3$
Constant		.9995	.9980	.9936	.9808	.9481
Coefficient of	m	.0003	.0013	.0030	.0038	.0049
Coefficient of	q	−.0001†	−.0012	−.0042	−.0115	−.0213
Coefficient of	h	.0003	.0010	.0026	.0057	.0106
Coefficient of	k	−.0002	−.0008	−.0024	−.0044	−.0086
Coefficient of	γ_1	−.0002	−.0008	−.0026	−.0071	−.0166
Coefficient of	d	.0001†	.0004	.0012	.0026	.0050
Coefficient of	γ_2	.0002	.0009	.0030	.0085	.0200
Coefficient of	c	−.0003	−.0011	−.0025	−.0059	−.0059
Adj R^2		.4251	.7113	.8831	.9834	.9977
$s(R_2)$		.0013	.0046	.0109	.0220	.0394

† → $p < .1$, $p < .01$ otherwise. $s(R_2)$ = sample standard deviation of R_2.

Table 4a

Estimated		$\delta = 1/3$	$\delta = .4$	$\delta = .5$	$\delta = .6$	$\delta = 2/3$
Constant		.0321	.1194	.3370	.7883	1.4940
Coefficient of	m	−.0231	−.0699	−.1147	−.0854	−.0488
Coefficient of	q	.0272	.1094	.3067	.6546	1.0503
Coefficient of	h	−.0202	−.0515	−.1012	−.1293	−.1072
Coefficient of	k	.0158	.0434	.0947	.1114	.1035
Coefficient of	γ_1	.0110	.0389	.1039	.1959	.2801
Coefficient of	d	−.0073†	−.0216	−.0430	−.0619	−.0566
Coefficient of	γ_2	−.0144	−.0432	−.1195	−.2218	−.2988
Coefficient of	c	.0261	.0700	.1017	.0662	−.1115
Adj(R^2)		.6150	.8405	.9448	.9756	.9972
$s(R_3)$		.1027	.2478	.4906	.8089	1.1833

† → $p < .1$, $p < .01$ otherwise. $s(R_3)$ is the sample standard deviation of R_3.

Table 4b

Estimated		$\delta = 1/3$	$\delta = .4$	$\delta = .5$	$\delta = .6$	$\delta = 2/3$
Constant		.0087	.0372	.1279	.4018	.9940
Coefficient of	m	−.0052	−.0244	−.0567	−.0653	−.0645
Coefficient of	q	.0050	.0300	.1101	.3406	.7589
Coefficient of	h	−.0047	−.0181	−.0500	−.0984	−.1399
Coefficient of	k	.0036	.0150	.0444	.0769	.1174
Coefficient of	γ_1	.0028*	.0130	.0456	.1195	.2311
Coefficient of	d	0	−.0076	−.0216	−.0441	−.0660
Coefficient of	γ_2	−.0033	−.0155	−.0547	−.1443	−.2810
Coefficient of	c	.0054	.0237	.0535	.1109	.0533
Adj(R^2)		.4542	.7544	.9097	.9817	.9947
$s(R_4)$		.0263	.0897	.2239	.4933	.9190

* → $p < .05$, $p < .01$ otherwise. 0 → insignificant parameter.
$s(R_4)$ is the sample standard deviation of R_4.

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