Effects of viscous dissipation and fluid axial heat conduction on entrance-region heat transfer in parallel plates (Part I: The thermal boundary condition of the first kind)

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Consideration is given to the effects of viscous dissipation and fluid axial heat conduction on the heat transfer in the thermal entrance region of a laminar plane Couette-Poiseuille flow. The temperature distribution of the fluid for $-\infty < z < \infty$ is determined by solving the energy equation including the viscous dissipation term and the fluid axial heat conduction term subject to the constant wall temperature boundary condition. The results indicate that viscous heating has a strong effect where velocity gradient is large and, because of the heat conducted upstream, transverse variation in fluid temperature exists in the region $z \leq 0$ and the temperature profile at $z = 0$ is greatly affected by fluid axial heat conduction. The effects of relative velocity, Brinkman number and Peclet number on developing temperature distribution and Nusselt number at the walls are discussed.

1. Introduction

The problem of entrance region heat transfer for non-Newtonian fluids in parallel plates with a moving boundary has been studied numerically. In this paper the results of the study on the developing heat transfer subject to the thermal boundary condition of constant wall temperature are presented and the attention has been focused on the effects of viscous dissipation of the flowing fluid and fluid axial heat conduction.

Without the fluid axial heat conduction term, the familiar Graetz problem of laminar heat transfer in a conduit is simplified from an elliptic type problem to a parabolic one. The relative importance of fluid axial heat conduction in heat transfer depends on the magnitute of Peclet number. The contribution of fluid axial heat conduction plays a significant role in the analysis and design of heat transfer equipment using low Peclet number condition or low Prandtl number fluids. Low Prandtl number fluid implies a fluid with a high thermal conductivity (liquid metal fluids). For moderate values of Prandtl number (gases) the development of the velocity profile and the temperature profile are similar. For very low Prandtl numbers the temperature development is much faster than that of the velocity profile. In this study the flow is assumed to be fully developed hydrodynamically and the conduit is considered in two semi-infinite regions $-\infty < z < 0$ and $0 \leq z < \infty$. The two semi-infinite regions are necessary to be considered for the investigation on the fluid axial heat conduction because in reality the temperature profile of the fluid at $z = 0$ (where heating at the wall commences) is affected by fluid axial heat conduction from downstream.

The problem is controlled by the magnitudes of: Peclet number, Pe, which characterizes the ratio of axial heat convection to axial heat conduction, Brinkman number, Br, which represents the ratio of overall dissipation to heat conduction, dimensionless relative velocity of the moving plate, U^*, which is the ratio of the moving plate velocity to the average velocity of the fluid and rheological parameters such as flow index n and dimensionless shear rate parameter β.

Nomenclature

Br	modified Brinkman number
c_p	specific heat at constant pressure, [J/ (kg· K)]
D_h	hydraulic diameter, $D_h = 2L$ [m]
E	constant of the axial transformation
f	friction factor
k	thermal conductivity, [W / (m· K)]
L	distance between the parallel plates, [m]
n	flow index
Nu	Nusselt number
Pe	Peclet number
Re_M	modified Reynolds number

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Pr_{M} modified Prandtl number
T temperature, [K]
u fully developed velocity profile, [m/s]
u_{m} average velocity of the fluid $u_{\mathrm{m}} \equiv \frac{1}{L}\int_0^L u dy$ [m/s]
u^* dimensionless velocity $\equiv u/u_{\mathrm{m}}$
U axial velocity of the moving plate, [m/s]
U^* dimensionless relative velocity of the moving plate $\equiv U/u_{\mathrm{m}}$
y coordinate normal to the fixed plate, [m]
y^* dimensionless coordinate
z axial coordinate, [m]
z^* dimensionless axial coordinate
z_t transformed axial coordinate

Greek Symbols

β dimensionless shear rate parameter
η_a apparent viscosity, [kg/(m· s)]
η_a^* dimensionless apparent viscosity $\equiv \eta_a/\eta^*$
η_0 viscosity at zero shear rate, [kg/(m· s)]
η^* reference viscosity, [kg/(m· s)]
ρ density, [kg/m^3]
τ shear stress, [N/m^2]
θ dimensionless temperature

Subscripts

I the first kind of the boundary condition
b bulk
e entrance or inlet
fd fully developed
lw lower plate
uw upper plate

2. Analysis

The physical model for the analysis is shown in Fig.1. The lower plate moves axially at a constant velocity, U. The assumptions and conditions used in the analysis are:

- The flow is incompressible, steady-laminar, and fully developed hydrodynamically.
- The fluid is non-Newtonian and the shear stress may be described by the modified power-law model, and physical properties are constant except viscosity.
- The body forces are neglected.
- The entering fluid temperature, T_e, is uniform at upstream infinity ($z \to -\infty$) and the upper wall temperature is equal to T_e everywhere. The lower plate is kept at a constant temperature $T_e < T_{lw}$ for $0 \le z$, whereas for $z < 0$ the temperature is equal to T_e.

The energy equation together with the assump-

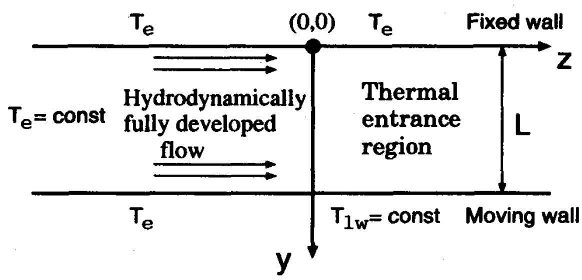

Fig.1 Geometrical configuration

tions above is written as

$$\rho c_p u \frac{\partial T}{\partial z} = k\left[\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] + \eta_a\left[\frac{du}{dy}\right]^2 \quad (1)$$

$$\text{in} \quad 0 \le y \le L, \text{ and } -\infty \le z \le \infty$$

$$\begin{cases} T{=}T_e & \text{at} & y{=}0, & 0 \le z \\ T{=}T_{lw} & \text{at} & y{=}L, & 0 \le z \\ T{=}T_e & \text{at} & y{=}0, & z < 0 \\ T{=}T_e & \text{at} & y{=}L, & z < 0 \\ \lim_{z\to-\infty} & T{=}T_e, & 0 < y < L & \\ \lim_{z\to+\infty} & T{=}T_{fd}, & 0 < y < L. & \end{cases} \quad (2)$$

From the solutions of Eq.(1) with Eq.(2), bulk temperature and Nusselt number are obtained as

$$T_b \equiv \frac{\int_0^L uTdy}{\int_0^L udy} \quad (3)$$

$$Nu \equiv \frac{hD_h}{k} \quad (4)$$

Heat transfer coefficients are:

$$h_{uw} = \frac{-k\frac{\partial T}{\partial y}\Big|_{y=0}}{T_{uw} - T_b} \qquad h_{lw} = \frac{k\frac{\partial T}{\partial y}\Big|_{y=L}}{T_{lw} - T_b} \quad (5)$$

The following dimensionless variables are introduced

$$y^* = \frac{y}{D_h} \qquad z^* = \frac{z}{PeD_h} = \frac{z\alpha}{u_m D_h^2} \quad (6)$$

$$u^* = \frac{u}{u_m} \qquad \beta = \frac{\eta_0}{m}\left(\frac{u_{\mathrm{m}}}{D_{\mathrm{h}}}\right)^{1-n} \quad (7)$$

$$\theta = \frac{T - T_e}{T_{lw} - T_e} \quad (8)$$

Where

$$Pe = Re_{\mathrm{M}} \cdot Pr_{\mathrm{M}} = \frac{u_m D_h}{\alpha} \quad (9)$$

$$Re_{\mathrm{M}} \equiv \frac{\rho u_{\mathrm{m}} D_{\mathrm{h}}}{\eta^*} \qquad Pr_{\mathrm{M}} \equiv \frac{c_p \eta^*}{k} \tag{10}$$

$$Br_I = \eta^* \frac{u_m^2}{k(T_{lw} - T_e)} \tag{11}$$

With the substitution of the above quantities into the dimensional formulation, the dimensionless energy equation and boundary conditions are obtained as

$$u^* \frac{\partial \theta}{\partial z^*} = \frac{\partial^2 \theta}{\partial y^{*2}} + \frac{1}{Pe^2} \frac{\partial^2 \theta}{\partial z^{*2}} + Br_I \eta_a^* \left(\frac{du^*}{dy^*}\right)^2 \tag{12}$$

$$\text{in} \quad 0 \le y^* \le \tfrac{1}{2} \text{ and } -\infty \le z^* \le \infty$$

$$\left\{\begin{array}{llll} \theta=0 & \text{at} & y^*=0, & 0 \le z^* \\ \theta=1 & \text{at} & y^*=1/2, & 0 \le z^* \\ \theta=0 & \text{at} & y^*=0, & z^* < 0 \\ \theta=0 & \text{at} & y^*=1/2, & z^* < 0 \\ \lim\limits_{z^* \to -\infty} & \theta=0, & 0 < y^* < 1/2 & \\ \lim\limits_{z^* \to +\infty} & \theta=\theta_{fd}, & 0 < y^* < 1/2. & \end{array}\right. \tag{13}$$

For infinitely large values of the axial distance ($z^* \to \infty$), the terms $\frac{\partial^2 \theta}{\partial z^{*2}}$ and $\frac{\partial \theta}{\partial z^*}$ vanish. Then the dimensionless temperature θ_{fd} corresponding to the boundary condition of constant wall temperature is the particular solution of the following equation:

$$\frac{\partial^2 \theta}{\partial y^{*2}} = -Br_I \eta_a^* \left(\frac{du^*}{dy^*}\right)^2 \tag{14}$$

$$\left\{\begin{array}{l} \theta = 1 \quad \text{at} \quad y^* = 1/2, \\ \theta = 0 \quad \text{at} \quad y^* = 0 \end{array}\right. \tag{15}$$

The bulk temperature in the dimensionless form

$$\theta_{\mathrm{b}} \equiv 2 \int_0^{1/2} u^* \theta dy^* \tag{16}$$

Then Nu at the walls are

$$Nu_{uw} = -\frac{1}{(\theta_{uw} - \theta_b)} \frac{\partial \theta}{\partial y^*}\bigg|_{y^*=0} \tag{17}$$

$$Nu_{lw} = \frac{1}{(\theta_{lw} - \theta_b)} \frac{\partial \theta}{\partial y^*}\bigg|_{y^*=1/2} \tag{18}$$

3. Numerical calculation

In order to convert the upstream and downstream infinities, the dimensionless axial coordinate z^* is transformed according to the relation employed by Verhoff and Fisher[1] as follows:

$$z^* = E \tan \pi z_t \quad \textit{or} \quad z_t = \frac{1}{\pi} \arctan \frac{z^*}{E} \tag{19}$$

By introducing the transformed coordinate z_t, the energy equation and the boundary conditions become

$$A \frac{\partial \theta}{\partial z_t} = \frac{\partial^2 \theta}{\partial y^{*2}} + B \frac{\partial^2 \theta}{\partial z_t^2} + \eta_a^* Br_I \left(\frac{du^*}{dy^*}\right)^2 \tag{20}$$

$$\text{in} \quad 0 \le y^* \le \frac{1}{2} \text{ and } -0.5 \le z_t \le 0.5$$

where

$$A = \frac{\cos^2 \pi z_t}{\pi E}\left(u^* + \frac{1}{Pe^2} \frac{\sin 2\pi z_t}{E}\right) \tag{21}$$

$$B = \frac{1}{Pe^2}\left(\frac{\cos^2 \pi z_t}{\pi E}\right)^2 \tag{22}$$

$$\left\{\begin{array}{llll} \theta = 0 & \text{at} & y^* = 0, & 0 \le z_t \le 0.5 \\ \theta = 1 & \text{at} & y^* = 1/2, & 0 \le z_t \le 0.5 \\ \theta = 0 & \text{at} & y^* = 0, & -0.5 \le z_t < 0 \\ \theta = 0 & \text{at} & y^* = 1/2, & -0.5 \le z_t < 0 \\ \lim\limits_{z_t \to -0.5} & & \theta = 0, & 0 < y^* < 1/2 \\ \lim\limits_{z_t \to +0.5} & & \theta = \theta_{fd}, & 0 < y^* < 1/2. \end{array}\right. \tag{23}$$

Eqs.(14) and (20) along with the associated boundary conditions have been solved numerically. The solution zone was laid in the range of $0 \le y^* \le 0.5$ and $-0.5 \le z_t \le +0.5$. The numerical approach employed for the system equations was based on Gauss-Seidel method. An irregular mesh system consisting of denser grids near $z_t = 0$ was applied to allow more accurate representation of the fluid axial heat conduction effect. The finest mesh was used next to $z_t = 0$ and as the location of the node goes farther from the origin mesh size is increased with a ratio $\Delta z_n / \Delta z_{n-1}$. Along the vertical axis the solution zone was divided evenly. The calculation has been carried out with two steps in order to get the results with high accuracy. First, the temperature at every node within the whole calculation zone has been calculated. Then by using the calculation results of the temperature at $z^* = 0.0$ and $z^* = 1.001$, more accurately calculated temperature profiles at the thermally developing region have been obtained. In the first step the finest mesh spacing was $\Delta z = 1.059 \cdot 10^{-7}$ and, mesh size was changed with the ratio $\Delta z_{n-1} / \Delta z_n = 1.33$ for $z_t \le 0$ and

$\Delta z_n/\Delta z_{n-1} = 1.33$ for $0 \leq z_t$. In the second step, the finest mesh spacing was $\Delta z = 4.93 \cdot 10^{-7}$ and the ratio was $\Delta z_n/\Delta z_{n-1} = 1.02$. Constant of the axial transformation, E, was chosen as 4.62 in the both calculation steps.

4. Results and discussion

The temperature distribution of the fluid for $-\infty < z < \infty$ in parallel plates was calculated for the boundary condition of constant wall temperature. The calculation has been carried out by using the finite difference method. The range of parameters considered are:

The relative velocity: $U^* = 0$ and $U^* = 1$

Brinkman number: 0.0, 0.05 and 0.1

Peclet number: ∞, 100, 50, 20, 10, 5, 1.

The case with $Br = 0.0$ and $Pe \to \infty$ is the limiting case of neglected viscous dissipation and axial heat conduction. It is worthwhile, to compare the results for this particular case with those reported by Shah and London(3) whose predictions were for stationary wall boundaries ($U^* = 0$) and by Shigechi and Araki(4) for the moving boundary ($U^* = 1.0$) case, respectively. Even at small values of z^*, it can be seen in Figs.2 and 3 that the agreement is excellent.

It is also seen that Nu remains almost constant throughout the thermal entrance region if Pe is small. The same trend was observed for Newtonian and non-Newtonian fluid flows. This behaviour is attributed to that the fluid temperature increases due to axial heat conduction (at $z \leq 0$) before the fluid enters into the heated wall region. This effect of fluid axial heat conduction is shown obviously in Figs.4 and 5 which illustrate respectively developing temperature profiles of a pseudoplastic fluid ($n = 0.5$ and $\beta = 1$) for the cases of stationary walls and moving lower wall.

For $Pe = 10$ the fluid temperature increase is occured at negative values of z^*. For $Pe \to \infty$ even at $z^* = 0$, where the step change in wall temperature, the dimensionless temperature of the fluid is zero except the cases with considerable viscous dissipation. This indicates the vanishing influence of axial heat conduction in the fluid for $z^* \leq 0$ with an increasing Peclet number.

For $Br = 0.0$ fluid is considered as it experiences no gain of heat due to viscous dissipation. For larger values of Br, it can be seen that the dimensionless temperature of the fluid at $z^* \leq 0$ deviates significantly from zero. This increase is due to the contribution of viscous dissipation to the flowing fluid.

Since the highest shear rate occurs near the stationary wall, the effect of viscous dissipation is most significant near the fixed wall and it is seen that the temperature increase due to viscous dissipation is greater for $U^* = 0$.

For the case of the stationary walls, the increase of temperature of the fluid caused by the effect of fluid axial heat conduction and viscous dissipation is higher than in the case of moving boundary. As soon as z^* becomes positive the fluid temperature profile undergoes rather rapid change causing a decrease in Nu.

Figs. 6a through 8b present the combined effects of viscous dissipation and fluid axial heat conduction on bulk temperature θ_b and Nu at the walls for a pseudoplastic fluid whose $n = 0.5$ and $\beta = 1$ for $U^* = 0$ and $U^* = 1$ respectively. With increasing values of Peclet number, the values of the bulk temperature and the Nusselt number approach to those of the parabolic problem. The figures indicate that Pe does not effect on Nu at a location farther downstream and, Nu at the walls remains fairly uniform throughout the thermal entrance region for smaller values of Pe. It also can be explained that, the fluid temperature increases due to fluid axial heat conduction.

With the neglected viscous dissipation Nu at the walls increases with a decrease in Pe number. In comparing Nusselt number at the lower plate ($\theta_{lw} = 1$) in Figs. 6a, 6b, 7a, 7b, 8a and 8b, it is observed that the differences among the three different degrees of viscous dissipation ($Br_I = 0.0;0.05$ and 0.1) are negligible in the thermal entrance region for both $U^* = 0$ and 1 while Nu_{lw} decreases with an increase in Br_I in the fully developed region for $U^*{=}0$. For Nusselt number at the upper wall ($\theta_{lw} = 0$) this behavior is complicated. The curves for Nu_{uw} do not change monotonically along the axial distance in the thermal entrance region.

5. Conclusions

The problem of Couette-Poiseuille laminar flow in the thermal entrance region including viscous dissipation of the flowing fluid and fluid axial heat conduction with constant wall temperature condition is analyzed by considering an infinite axial domain.

The results are presented graphically in dimensionless form and the effects of moving boundary, fluid axial heat conduction and viscous dissipation are mainly demonstrated.

An inspection of the temperature profile deve-

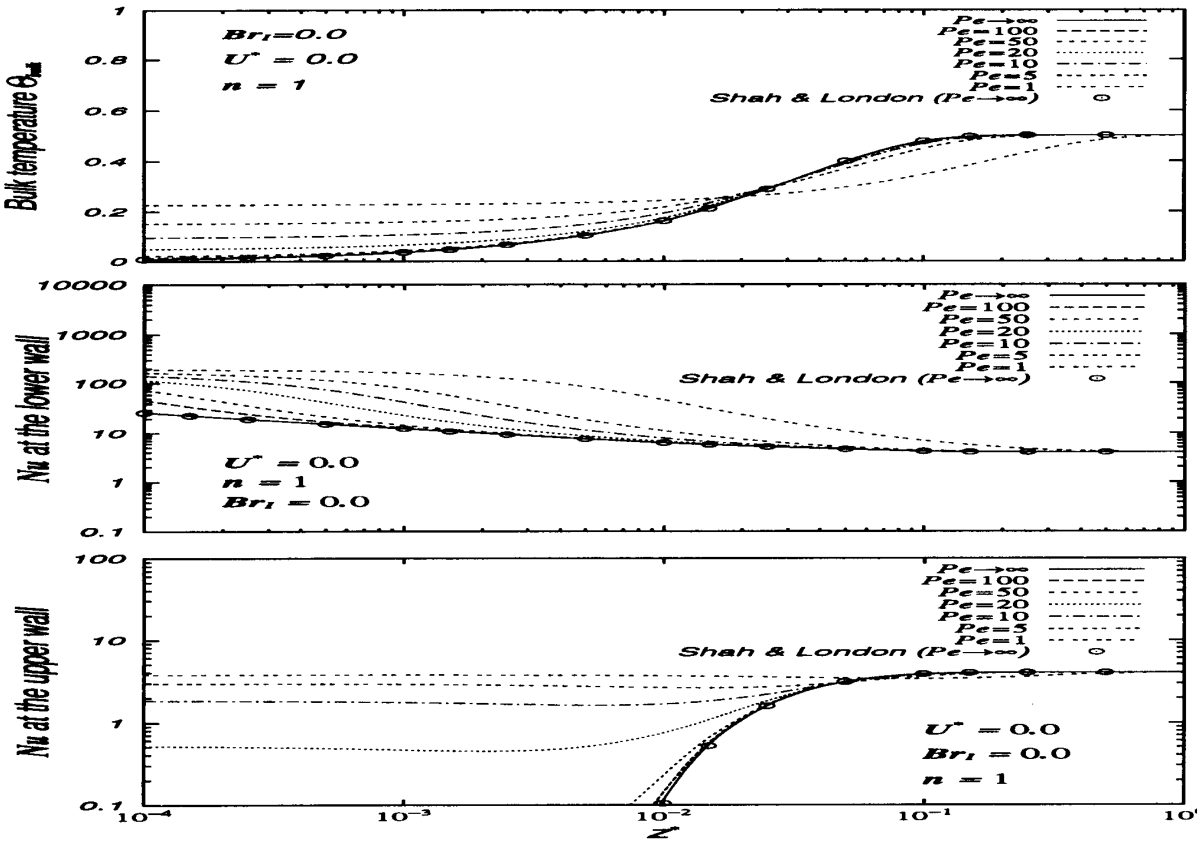

Fig.2 Bulk temperature and Nu for various Pe ($U^* = 0$)

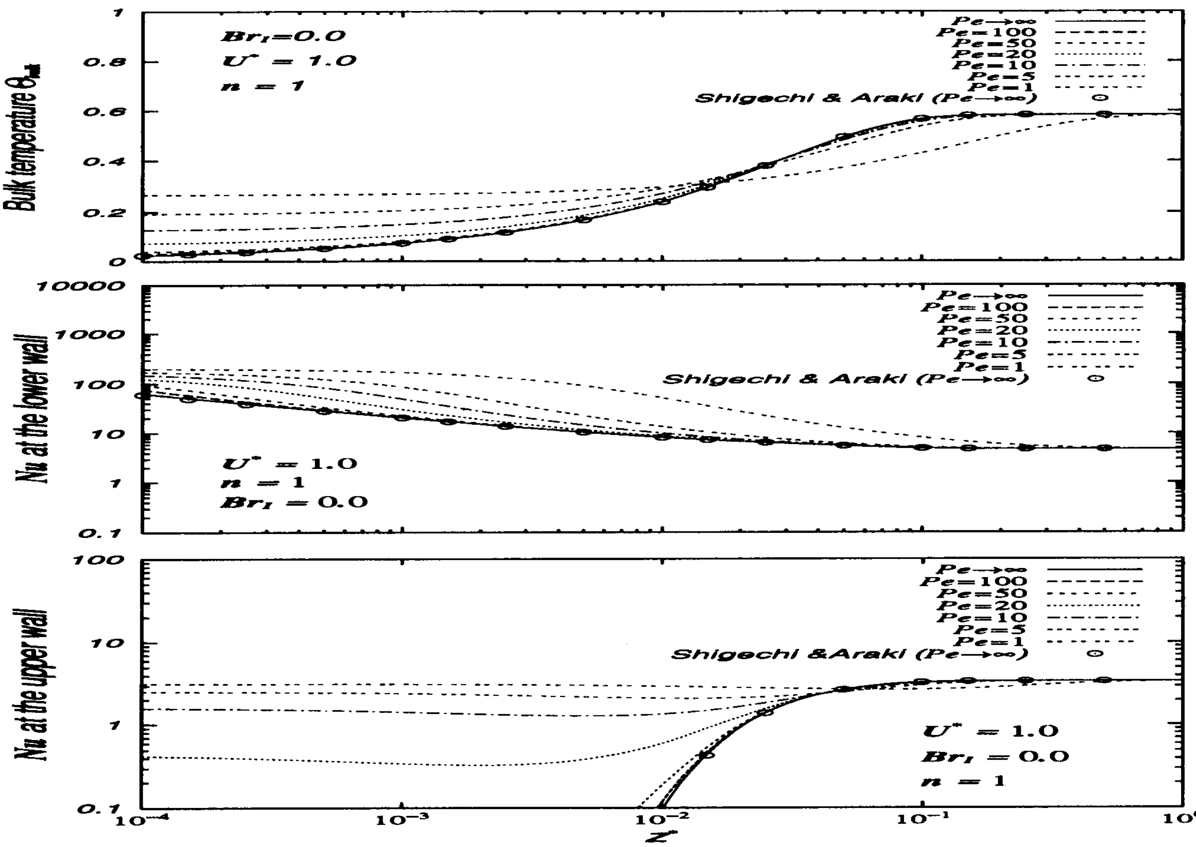

Fig.3 Bulk temperature and Nu for various Pe ($U^* = 1$)

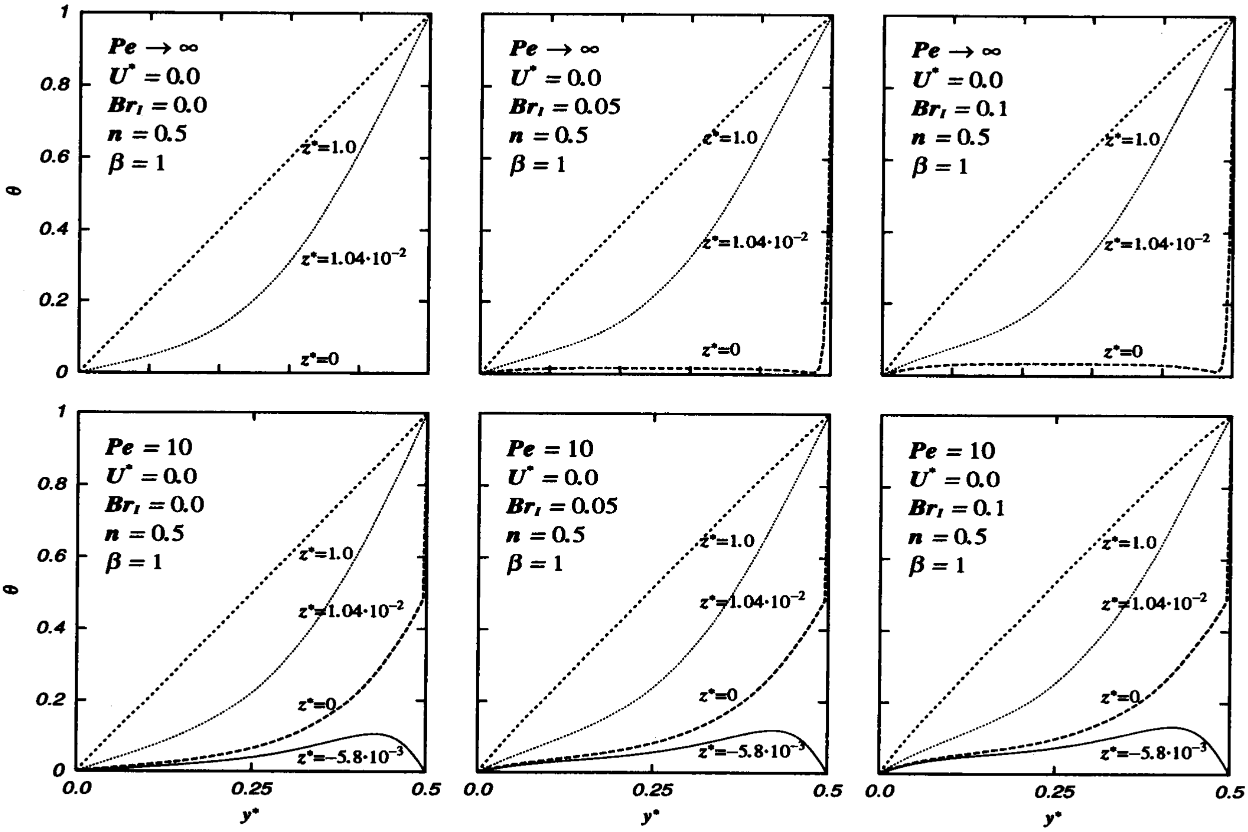

Fig.4 Developing temperature profiles ($U^* = 0$)

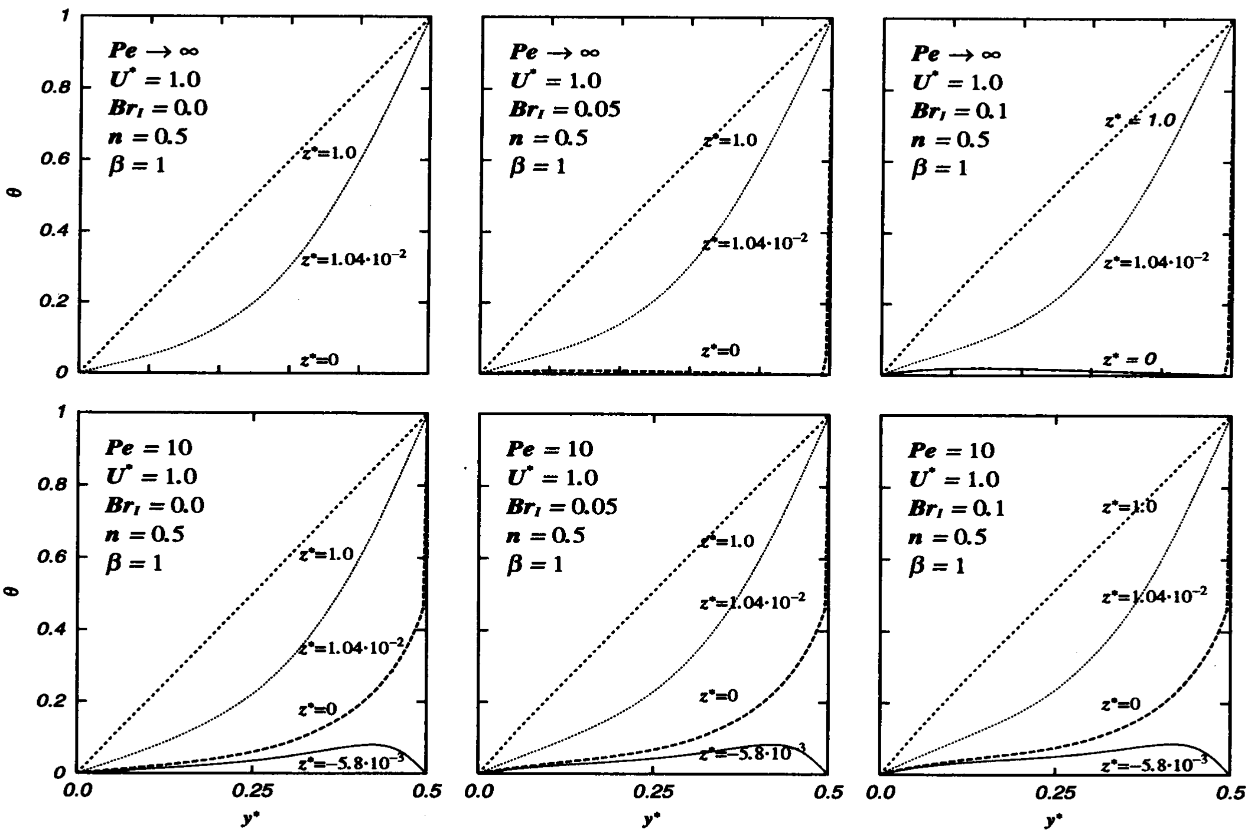

Fig.5 Developing temperature profiles ($U^* = 1$)

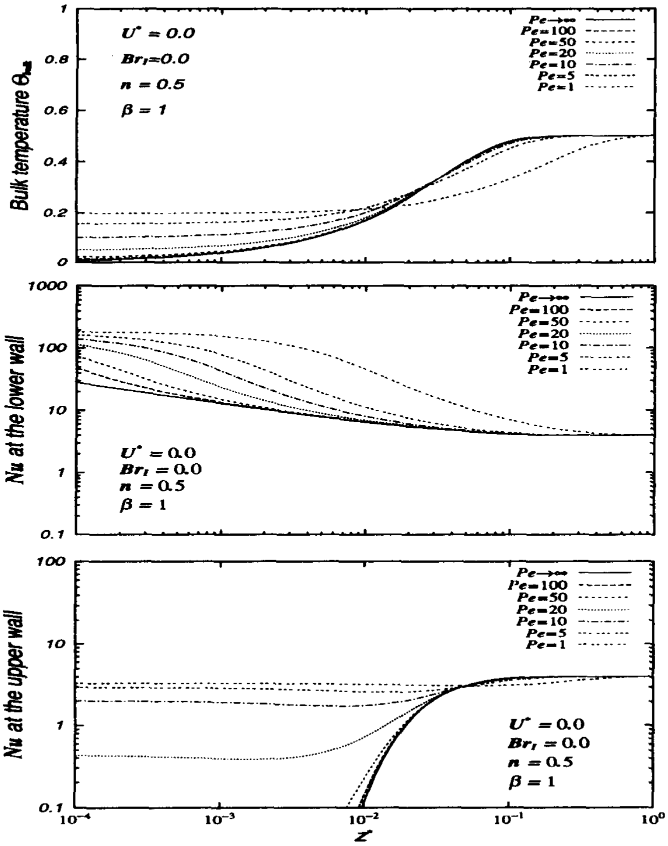

Fig.6a θ_b and Nu ($U^* = 0$)

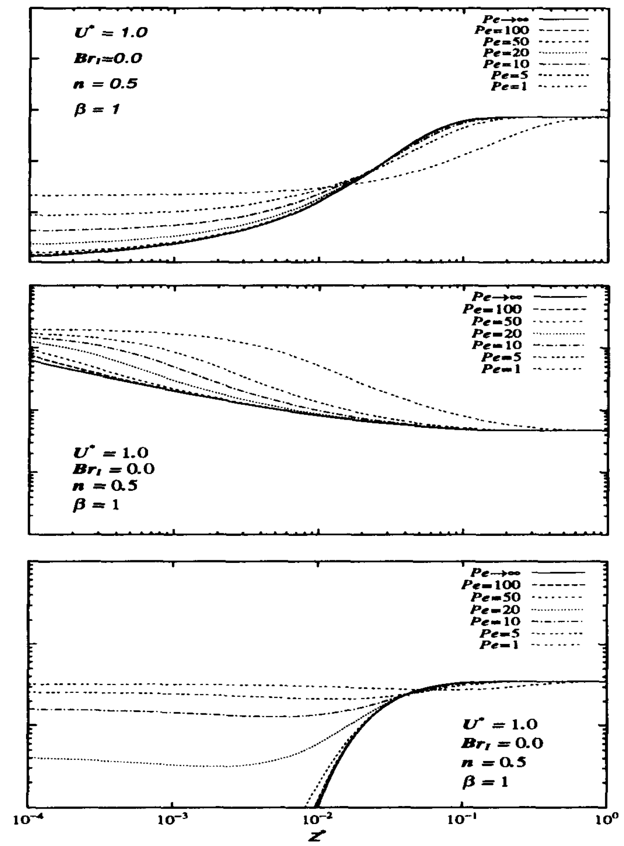

Fig.6b θ_b and Nu ($U^* = 1$)

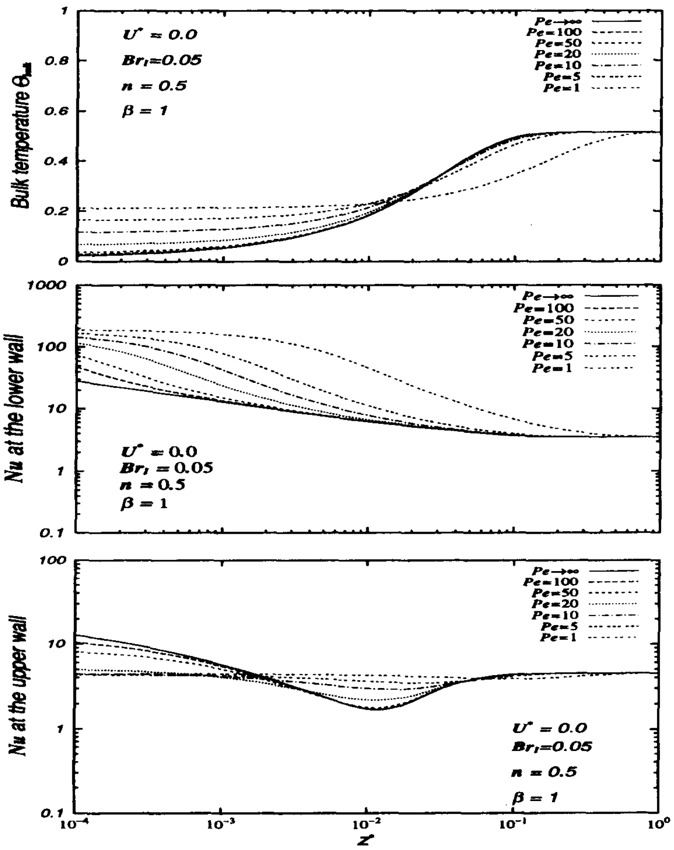

Fig.7a θ_b and Nu ($U^* = 0$)

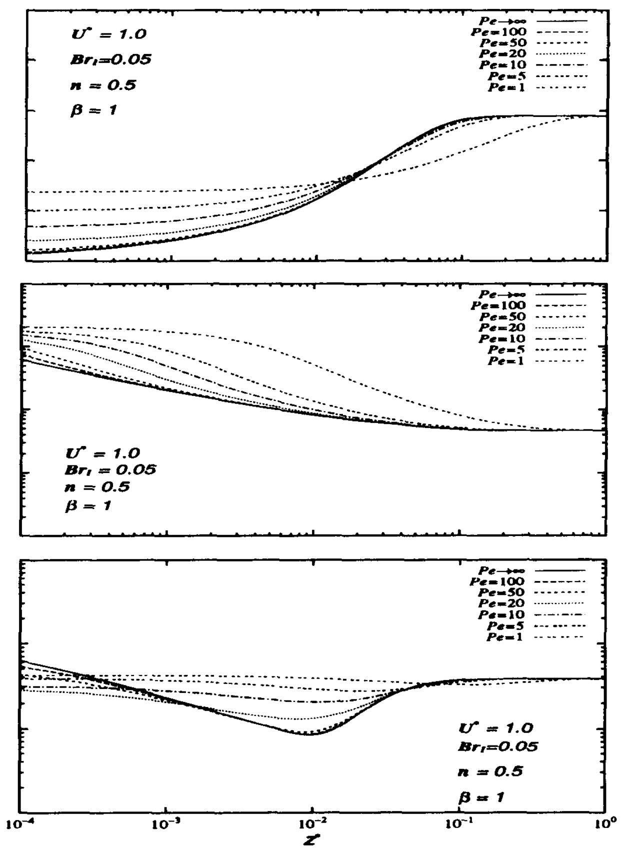

Fig.7b θ_b and Nu ($U^* = 1$)

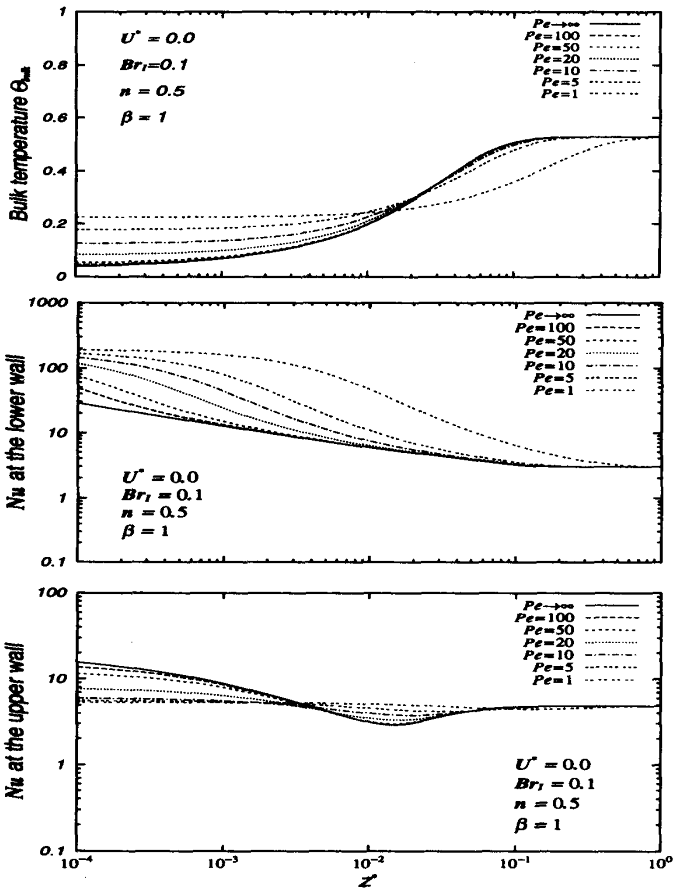

Fig.8a θ_b and Nu ($U^* = 0$)

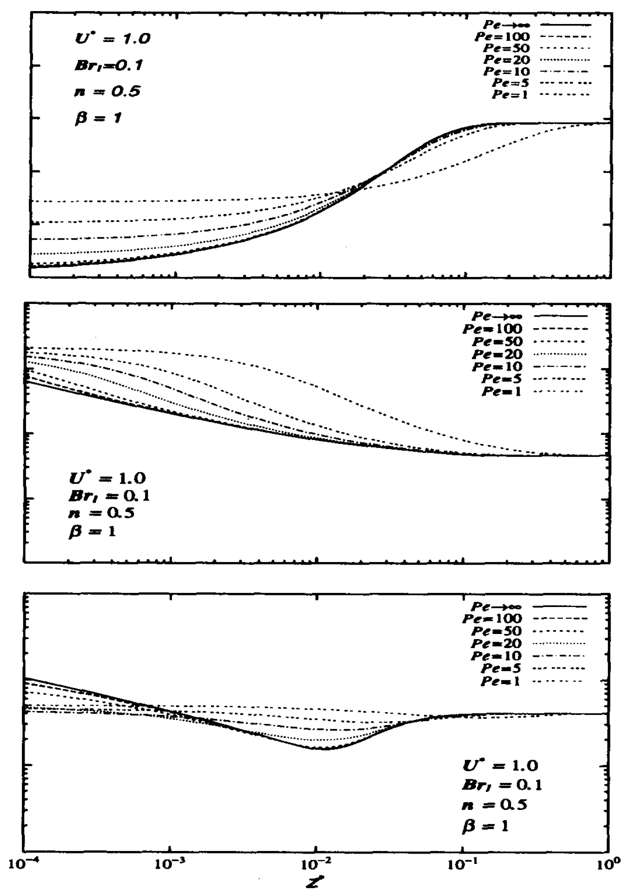

Fig.8b θ_b and Nu ($U^* = 1$)

lopment reveals that the fluid temperature increases at $z \leq 0$ due to fluid axial heat conduction and viscous heating. Including the effect of fluid axial heat conduction in the analyses results in higher values for the Nusselt number at the heated wall in the thermal region than in the case without axial heat conduction for a given z.

It may be concluded that the effect of fluid axial heat conduction is negligible only at high Pe. For moderate values of Pe number, fluid axial heat conduction is important in the thermal entrance region. The effect of Br is stronger on Nusselt number at the unheated, fixed wall.

For thermally developing flow when $Br \neq 0$, it is shown that the viscous dissipation effect is different depending on U^*. Nu_{lw} at the the moving heated wall ($\theta_{lw} = 1$) is little sensitive to Br. It was found that the curves for Nu_{uw} at the upper wall ($\theta_{uw} = 0$) do not change monotonically along the axial distance particularly in the thermally developing region.

The counterpart for the second kind of the boundary condition will be given in the next report.

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