Laminar heat transfer in the thermal entrance region of concentric annuli with moving heated cores

(Part I: The cases with the first and second kinds of thermal boundary condition)

by

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Consideration is given to the effects of viscous dissipation on the developing heat transfer between a fully developed laminar non-Newtonian fluid flow and a concentric annular geometry with a moving heated core. In this report, the results with the first and second kinds of thermal boundary condition are presented. Applying the shear stress described by the modified power-law model, the energy equation including the viscous dissipation term is solved numerically. The effects of radius ratio, flow index, relative core velocity, dimensionless shear rate parameter and Brinkman number on temperature distribution and Nusselt number are discussed.

1. Introduction

The problems of fully developed heat transfer to non-Newtonian fluids in a concentric annulus with an axially moving core have been studied numerically for the thermal boundary conditions of constant heat flux at either tube(1)(2).

In this paper, the entrance-region heat transfer between a fully developed laminar fluid flow and a concentric annular geometry with a moving heated core is studied numerically. Applying the fully developed velocity profile reported for the modified power-law model in the previous report(3), the energy equation including the viscous dissipation term is solved numerically using the finite difference method for the thermal boundary conditions of first kind and second kind. The effects of radius ratio, relative velocity of the core, flow index and dimensionless shear rate parameter and Brinkman number on developing temperature distribution and Nusselt number are discussed.

Nomenclature

c_p	specific heat at constant pressure
D_h	hydraulic diameter $\equiv 2(R_o - R_i)$
k	thermal conductivity
m	consistency index
n	flow index
Pe	Peclet number
r	radial coordinate
r^*	dimensionless radial coordinate $\equiv r/D_h$
R	radius
q	wall heat flux
T	temperature
u_m	average velocity of the fluid
u^*	dimensionless velocity $\equiv u/u_m$
U^*	dimensionless relative velocity of the moving core $\equiv U/u_m$
z	axial coordinate
z^*	dimensionless axial coordinate $= z/(PeD_h)$

Greek Symbols

α	radius ratio $\equiv R_i/R_o$
β	dimensionless shear rate parameter
η_0	viscosity at zero shear rate
η^*	reference viscosity
ρ	density
ξ	transformed dimensionless radial coordinate $\equiv [2(1-\alpha)r^* - \alpha]/(1-\alpha)$

Subscripts

b	bulk
e	inlet
i	inner tube
ii	at the inner wall with the inner heated
o	outer tube
oi	at the outer wall with the inner heated

2. Analysis

The physical model for the analysis is shown in Fig.1. The core tube moves axially at a constant velocity, U. The assumptions used in the analysis are:

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1. The flow is incompressible, steady-laminar, and fully developed hydrodynamically.
2. The fluid is non-Newtonian and the shear stress may be described by the modified power-law model[4], and the physical properties are constant except viscosity.
3. The body forces and axial heat conduction are neglected.

Heat transfer

The energy equation together with the assumptions above is written as

$$k\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\tau\left(\frac{du}{dr}\right)=\rho c_p u\frac{\partial T_b}{\partial z}. \qquad (1)$$

The velocity, u, and its gradient, $\frac{du}{dy}$, have been evaluated and reported in the previous report[3]. The thermal boundary conditions:

(1) The first kind (constant wall temperature at the moving core and the temperature of the outer tube is kept equal to the uniform entering fluid temperature):

$$\begin{cases} T^{(1)} = T_i^{(1)} & \text{at } r = R_i \\ T^{(1)} = T_e & \text{at } r = R_o \end{cases} \qquad (2)$$

(2) The second kind (constant heat flux at the moving core with the outer tube insulated):

$$\begin{cases} -k\frac{\partial T^{(2)}}{\partial r} = q_i & \text{at } r = R_i \\ \frac{\partial T^{(2)}}{\partial r} = 0 & \text{at } r = R_o \end{cases} \qquad (3)$$

The inlet condition is:

$$z=0: \quad T^{(k)} = T_e \quad \text{for } R_i \le r \le R_o \quad (k = 1 \text{ or } 2) \qquad (4)$$

τ in Eq.(1) is the shear stress defined as

$$\tau \equiv \eta_a \frac{du}{dr} \qquad (5)$$

where η_a is the apparent viscosity defined by

$$\eta_a = \frac{\eta_0}{1+\frac{\eta_0}{m}\left|\frac{du}{dr}\right|^{1-n}} \quad \text{for } n<1, \qquad (6)$$

$$\eta_a = \eta_0\left(1+\frac{m}{\eta_0}\left|\frac{du}{dr}\right|^{n-1}\right) \quad \text{for } n>1. \qquad (7)$$

Dimensionless apparent viscosity is

$$\eta_a^* \equiv \frac{\eta_a}{\eta^*} = \frac{1+\beta}{1+\beta\left|\frac{du^*}{dr^*}\right|^{1-n}} \quad \text{for } n<1, \qquad (8)$$

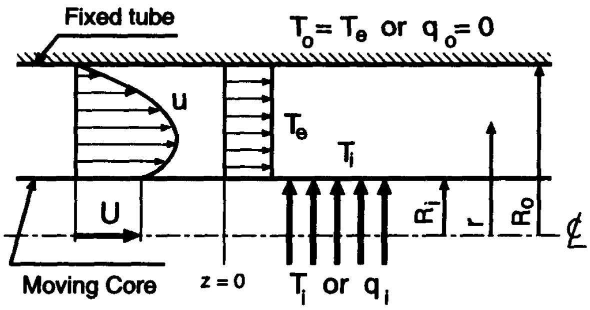

Fig.1 Schematic of a concentric annulus with an axially moving core

$$\eta_a^* \equiv \frac{\eta_a}{\eta^*} = \frac{\beta+\left|\frac{du^*}{dr^*}\right|^{n-1}}{\beta+1} \quad \text{for } n>1, \qquad (9)$$

where

$$\eta^* = \frac{\eta_0}{1+\beta} \quad \text{for } n<1, \qquad (10)$$

$$\eta^* = \eta_0\left(1+\frac{1}{\beta}\right) \quad \text{for } n>1, \qquad (11)$$

$$\beta = \frac{\eta_0}{m}\left(\frac{u_m}{D_h}\right)^{1-n}. \qquad (12)$$

Bulk temperature, T_b, is defined as

$$T_b^{(k)} \equiv \frac{\int_{R_i}^{R_o} uT^{(k)} r dr}{\int_{R_i}^{R_o} u r dr} = \frac{2}{u_m\left(R_o^2-R_i^2\right)}\int_{R_i}^{R_o} uT^{(k)} r dr. \qquad (13)$$

Average fluid velocity, u_m, is defined as

$$u_m \equiv \frac{1}{\pi(R_o^2-R_i^2)}\int_{R_i}^{R_o} u 2\pi r dr. \qquad (14)$$

Nusselt number at the tube walls:

$$Nu_{ii}^{(k)} = \frac{h_{ii}^{(k)}\cdot D_h}{k} \quad \text{for } k = 1 \text{ or } 2 \qquad (15)$$

$$Nu_{oi}^{(1)} = \frac{h_{oi}^{(1)}\cdot D_h}{k} \qquad (16)$$

Heat transfer coefficients are defined as:

$$h_{ii}^{(1)} \equiv \frac{-k\frac{\partial T^{(1)}}{\partial r}\Big|_{R_i}}{T_i^{(1)}-T_b^{(1)}} \qquad (17)$$

$$h_{io}^{(1)} \equiv \frac{k\frac{\partial T^{(1)}}{\partial r}\Big|_{R_o}}{T_o^{(1)}-T_b^{(1)}} \qquad (18)$$

$$h_{ii}^{(2)} \equiv \frac{q_i}{T_i^{(2)}-T_b^{(2)}} \qquad (19)$$

Thus, Nusselt numbers are calculated as:

$$Nu_{\rm ii}^{(1)} = \frac{D_{\rm h}}{T_{\rm i}^{(1)} - T_{\rm b}^{(1)}}\left[-\frac{\partial T^{(1)}}{\partial r}\bigg|_{R_{\rm i}}\right] \quad (20)$$

$$Nu_{\rm io}^{(1)} = \frac{D_{\rm h}}{T_{\rm o}^{(1)} - T_{\rm b}^{(1)}}\left[-\frac{\partial T^{(1)}}{\partial r}\bigg|_{R_{\rm o}}\right] \quad (21)$$

$$Nu_{\rm ii}^{(2)} = \frac{D_{\rm h}}{T_{\rm i}^{(2)} - T_{\rm b}^{(2)}}\left[\frac{q_{\rm i}}{k}\right] \quad (22)$$

Introducing a dimensionless temperature, θ, defined as

$$\theta^{(1)} \equiv \frac{T^{(1)} - T_e}{T_{\rm i}^{(1)} - T_e} \quad (23)$$

$$\theta^{(2)} \equiv \frac{k\left[T^{(2)} - T_e\right]}{q_{\rm i} D_{\rm h}} \quad (24)$$

the energy equation and the boundary conditions may be expressed in the dimensionless forms as

$$\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial \theta}{\partial r^*}\right) + Br \cdot \eta_a^*\left(\frac{du^*}{dr^*}\right)^2 = u^*\frac{\partial \theta}{\partial z^*} \quad (25)$$

(1) The boundary condition of the first kind:

$$\begin{cases} \theta^{(1)} = 1 \quad \text{at} \quad r^* = \frac{\alpha}{2(1-\alpha)} \\ \theta^{(1)} = 0 \quad \text{at} \quad r^* = \frac{1}{2(1-\alpha)} \end{cases} \quad (26)$$

(2) The boundary condition of the second kind:

$$\begin{cases} \frac{d\theta^{(2)}}{dr^*} = -1 \quad \text{at} \quad r^* = \frac{\alpha}{2(1-\alpha)} \\ \frac{d\theta^{(2)}}{dr^*} = 0 \quad \text{at} \quad r^* = \frac{1}{2(1-\alpha)} \end{cases} \quad (27)$$

The inlet condition is:

$z^* = 0 : \quad \theta^{(k)} = 0$

$$\text{for} \quad \frac{\alpha}{2(1-\alpha)} \le r^* \le \frac{1}{2(1-\alpha)} \quad (\text{k} = 1 \text{ or } 2) \quad (28)$$

Brinkman number is defined as follows

$$Br^{(1)} \equiv \frac{\eta^* u_{\rm m}^2}{k\left[T_{\rm i}^{(1)} - T_e\right]} \quad (29)$$

$$Br^{(2)} \equiv \frac{\eta^* u_{\rm m}^2}{D_{\rm h} q_{\rm i}} \quad (30)$$

Nusselt number at the tube walls:

$$Nu_{\rm ii}^{(1)} = -\frac{1}{1 - \theta_{\rm b}^{(1)}}\frac{\partial \theta^{(1)}}{\partial r^*}\bigg|_{\frac{\alpha}{2(1-\alpha)}} \quad (31)$$

$$Nu_{\rm oi}^{(1)} = \frac{1}{\theta_{\rm o}^{(1)} - \theta_{\rm b}^{(1)}}\frac{\partial \theta^{(1)}}{\partial r^*}\bigg|_{\frac{1}{2(1-\alpha)}} \quad (32)$$

$$Nu_{\rm ii}^{(2)} = \frac{1}{\theta_{\rm i}^{(2)} - \theta_{\rm b}^{(2)}} \quad (33)$$

where the dimensionless bulk temperature, $\theta_{\rm b}$, is defined as

$$\theta_{\rm b}^{(k)} = \frac{8(1-\alpha)}{1+\alpha}\int_{\frac{\alpha}{2(1-\alpha)}}^{\frac{1}{2(1-\alpha)}} u^*\, \theta^{(k)}\, r^* dr^* \quad (34)$$

3. Results and discussion

The calculation has been carried out by using the finite difference method. The range of parameters considered are:

The radius ratio: $0.2 \le \alpha \le 1.0$
The relative velocity: $0 \le U^* \le 1.0$
The flow index: $0.5 \le n \le 1.5$
The dimensionless shear rate parameter: $10^{-5} \le \beta \le 10^5$
Brinkman number: 0.0, 0.01, 0.05 and 0.1.

The mesh sizes used in the numerical calculation are shown below.

a. Axial direction (Δz^*):
$0 < z^* \le 10^{-3}$: $\Delta z^* = 10^{-9}$
$10^{-3} < z^* \le 1$: $\Delta z^* = 10^{-3}$

b. Radial direction ($\Delta \xi$)
$\Delta \xi = 1/100$.

The development of the non-dimensional temperature profiles in the thermal entrance region of a concentric annulus with a heated core for the two kinds of the boundary conditions (k = 1 and 2) are presented in Fig.2a and Fig.2b, respectively, for the same condition ($\alpha = 0.5$, $n = 0.5$, power law fluid ($\beta = 10^5$) and $U^* = 1.0$). The figures illustrate clearly how the temperature profiles develop for the two different boundary conditions.

The effects of the relative velocity, U^*, on the development of temperature profiles are demonstrated in Figs.Figs.2b and 2c for the second kind of boundary condition (k = 2). It is seen that the fluid temperature increase is less rapid for larger values of the core velocity.

The effects of viscous dissipation on the development of temperature profiles for $\alpha = 0.5$, $n = 0.5$, $\beta = 10^5$ and $U^* = 1.0$ for the second kind of boundary condition (k = 2) are shown in Fig.2b and 2d.

The effect of the moving core velocity on Nusselt number at the tube walls are shown in Figs.3a to 3c at given values of $Br = 0.0$ and $\alpha = 0.5$ for three different fluids ($n < 1.0$ pseudoplastic, $n = 1.0$ Newtonian and $n > 1.0$ dilatant).

The calculation results of the particular case of Newtonian fluids ($n = 1.0$) with neglected viscous dissipation ($Br = 0.0$) are compared with the predictions by Shah and London[5] for the stationary core ($U^* = 0$) and by Shigechi and Araki[6] for the moving core ($U^* = 1.0$), respectively. Even at small values of z^*, it can be seen in Figs.3a, 3b and 3c that the agreement is excellent. The effect of the relative velocity of the moving core tube is always to increase the values of Nusselt numbers, Nu_{ii} and to decrease the values of Nusselt numbers, Nu_{oi}, for the given conditions of α and Br.

The viscous dissipation effects on Nusselt number are shown in Figs.4 to 6 for three different fluids. With an increase in Brinkman number, Nu_{ii} decreases for $U^* = 0.0$ and Nu_{oi} increases for $U^* = 0.0$ and $U^* = 1.0$.

For the first kind of boundary condition, Br has a strong effect on Nu_{ii} at the thermally fully developed region while the core is fixed. But for the case of the moving core the effect of the Brinkman number is very small at both the thermal entrance region and the fully developed region. It is also seen that for dilatant fluids Br on Nu_{ii} is stronger than for Newtonian and pseudoplastic fluids.

For the second kind of boundary condition Br affects strongly on Nu_{ii} in both the thermally developing and developed regions when the core is fixed. Nu_{ii} decreases with an increase in Br. For the moving core in the thermal entrance region the effect of the Br on Nu_{ii} is almost negligible but in the fully developed region Nu_{ii} increases with an increase in Br.

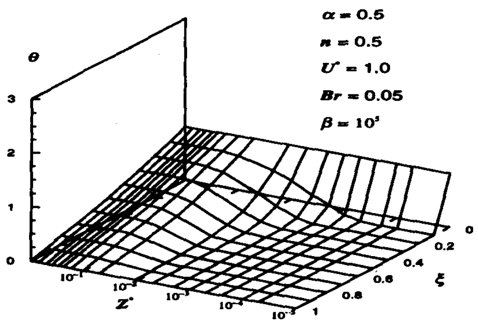

Fig.2.a Development of temperature profiles (1st kind)

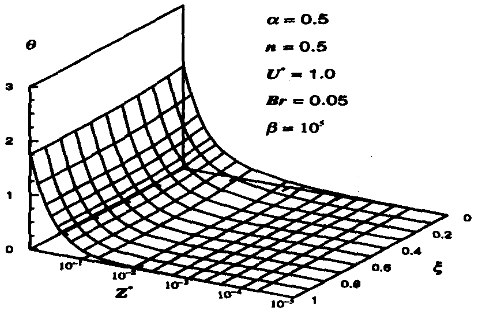

Fig.2.b Development of temperature profiles (2nd kind)

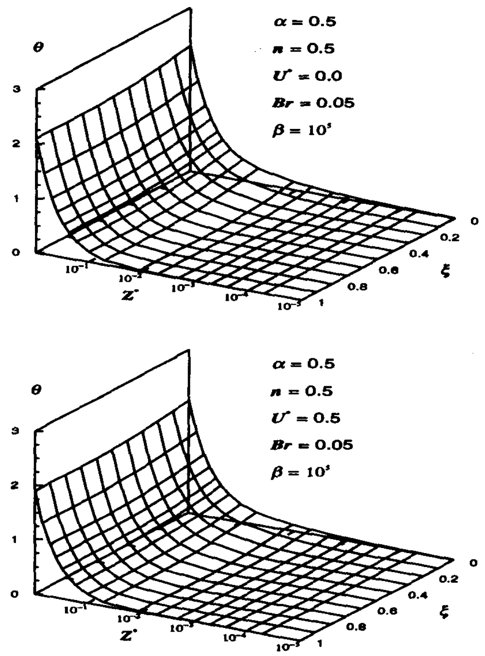

Fig.2.c Development of temperature profiles for different U^* (2nd kind)

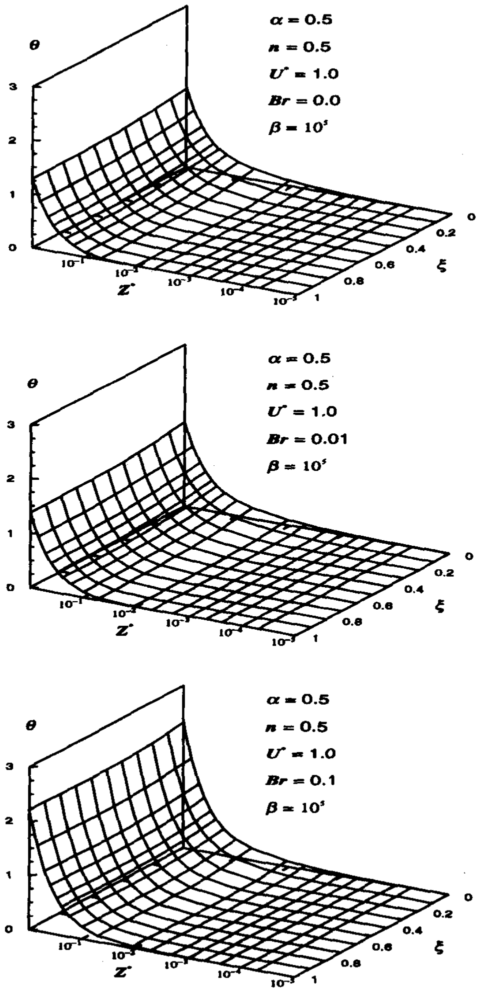

Fig.2.d Effect of Br on the development of temperature profiles (2nd kind)

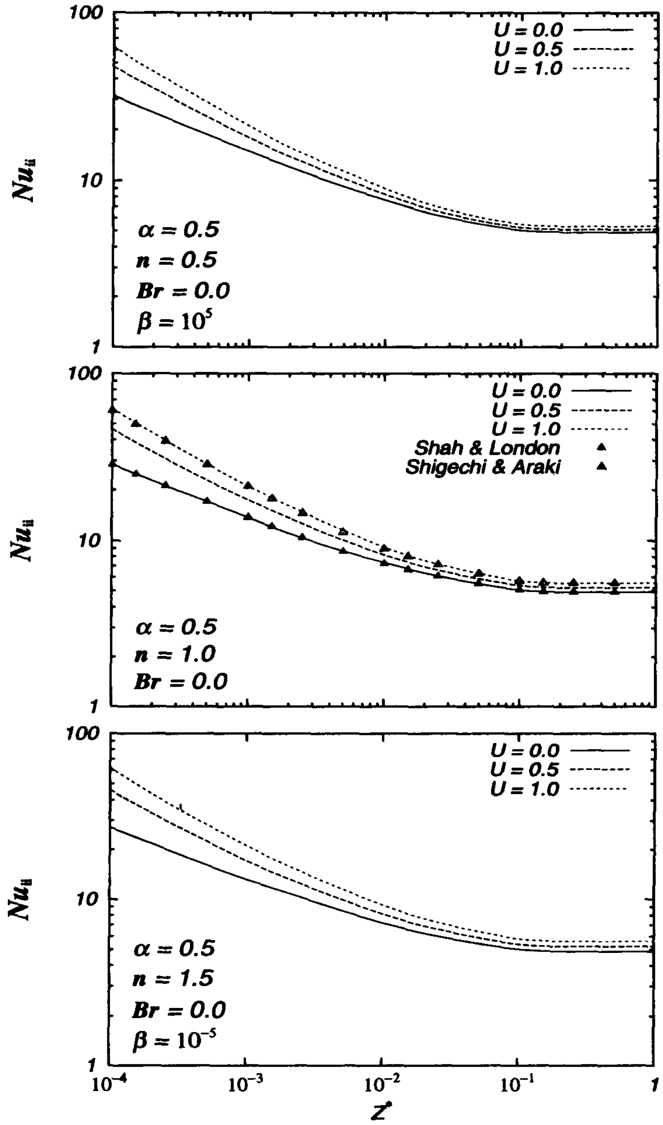

Fig.3.a Nusselt numbers, Nu_{ii}, at n = 0.5, 1.0 and 1.5 (1st kind)

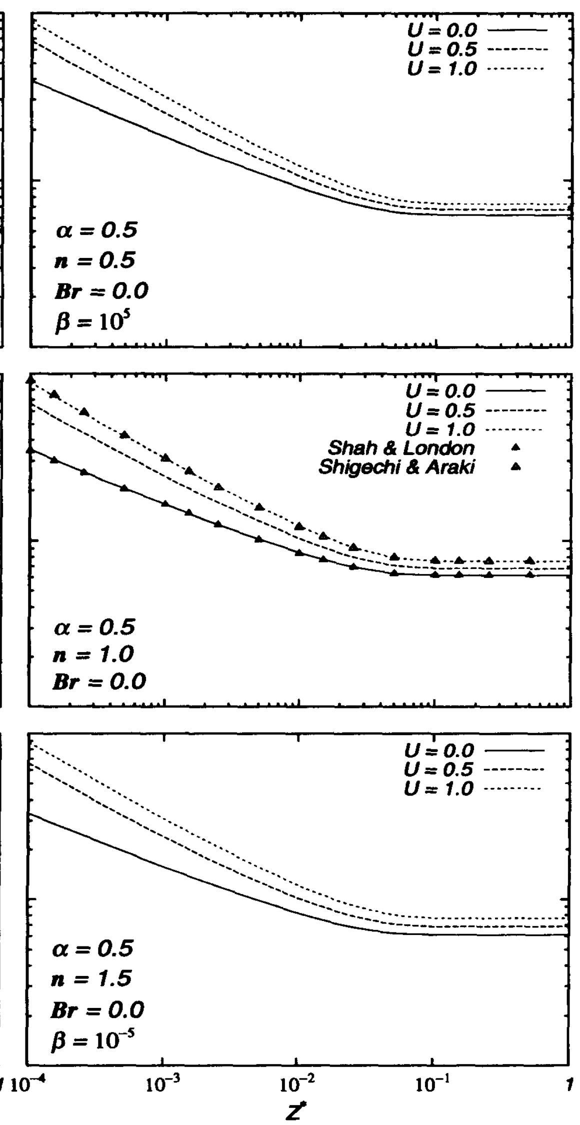

Fig.3.b Nusselt numbers, Nu_{ii}, at n = 0.5, 1.0 and 1.5 (2nd kind)

4. Conclusions

The heat transfer between a fully developed laminar fluid flow and concentric annular geometry with a moving heated core of fluid or solid body is studied with the two different boundary conditions.

It may be concluded that the viscous dissipation effect on heat transfer is stronger for dilatant fluids. Br affects strongly on Nusselt number at the unheated fixed tube.

In this report the heat transfer results for the first and second kinds of thermal boundary conditions are only discussed. The counterpart for the third and fourth kinds of boundary conditions will be reported in the next report.

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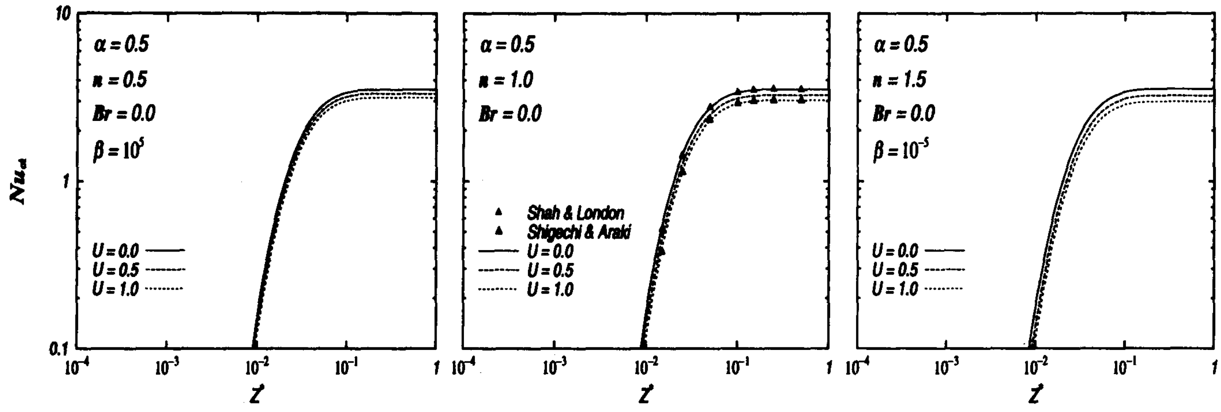

Fig.3.c Nusselt numbers, Nu_{oi} at n = 0.5, 1.0 and 1.5 (1st kind)

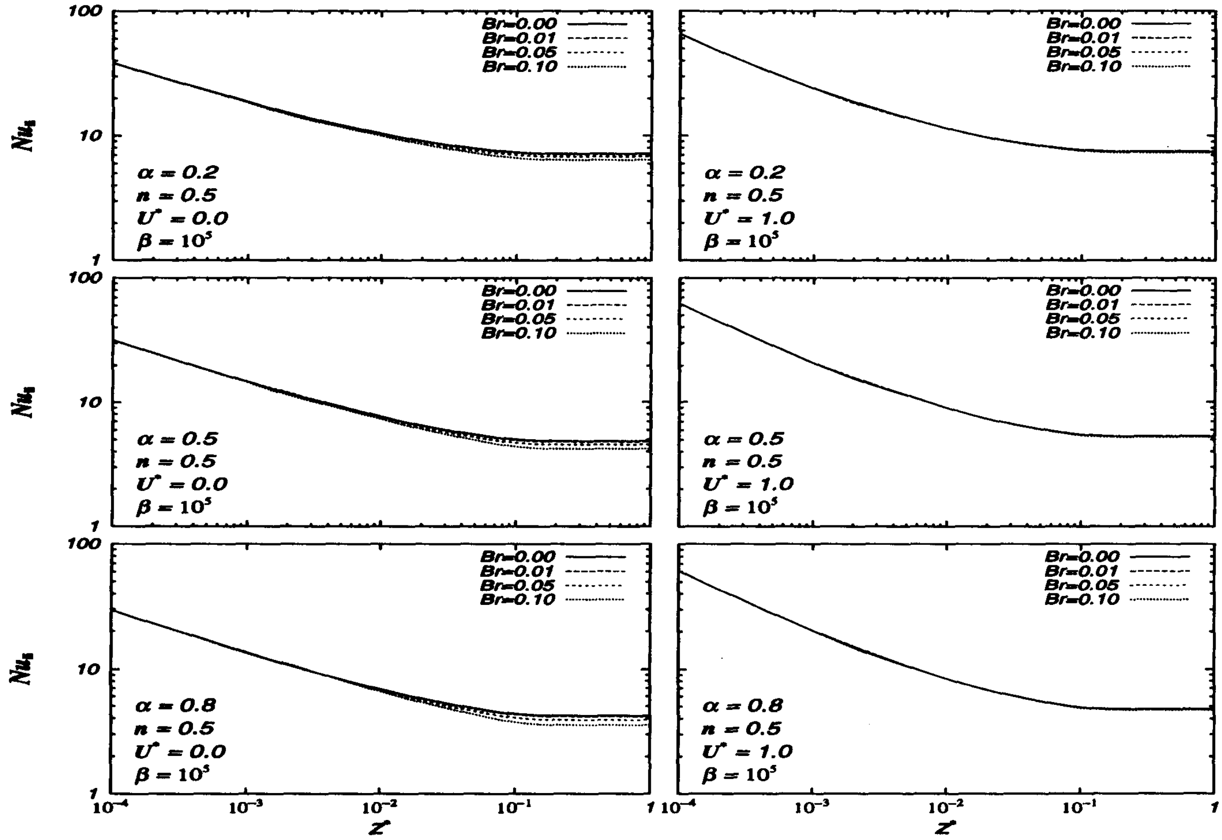

Fig.4.a Nusselt numbers, Nu_{ii}, for n = 0.5, U^* = 0.0 and 1.0 (1st kind)

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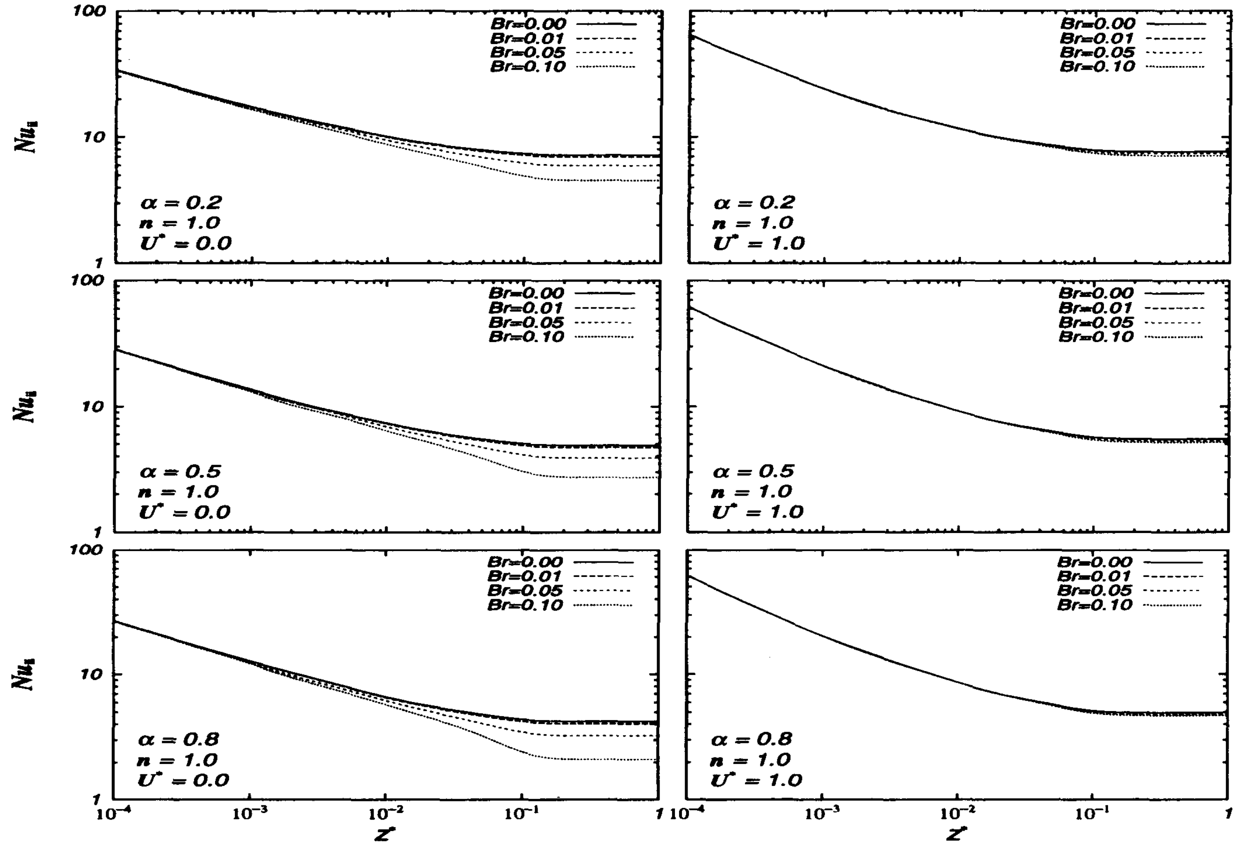

Fig.4.b Nusselt numbers, Nu_{ii}, for n = 1.0, U^* = 0.0 and 1.0 (1st kind)

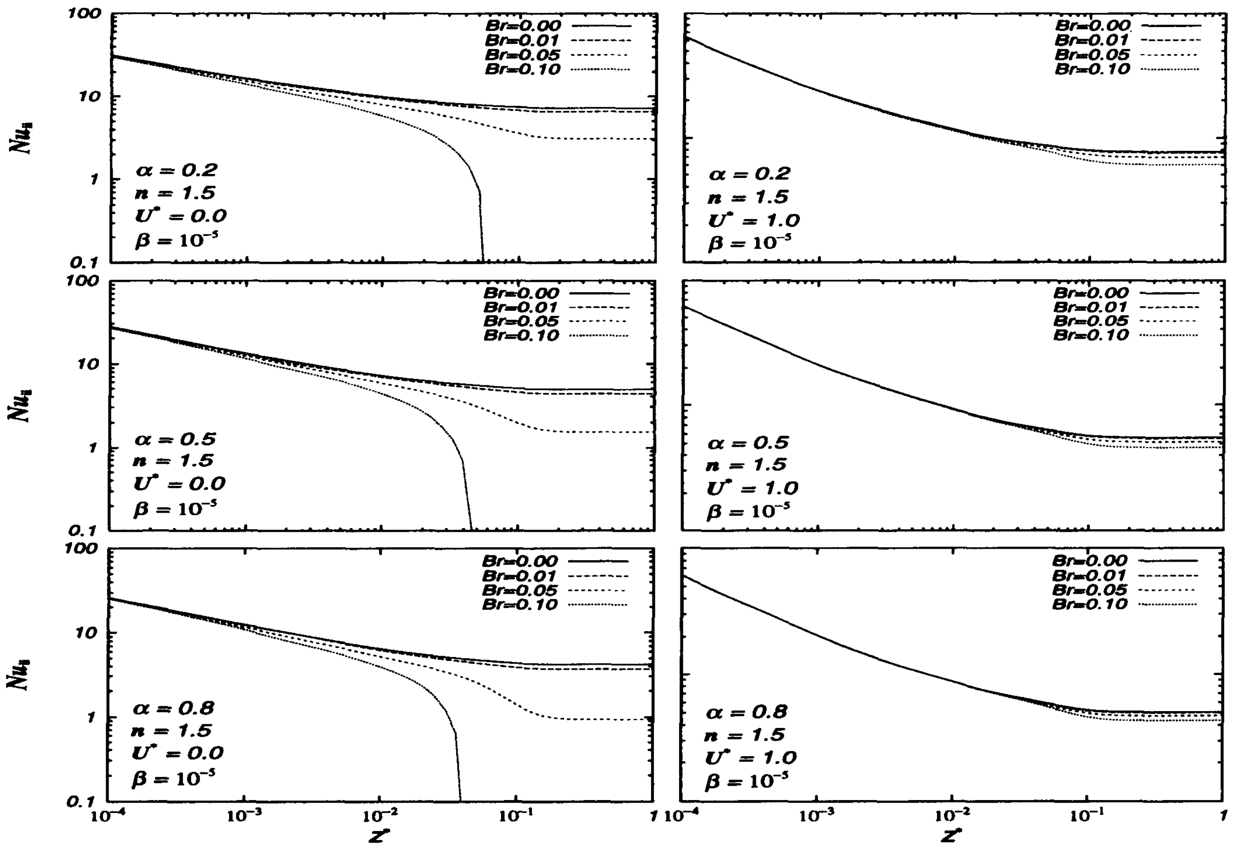

Fig.4.c Nusselt numbers, Nu_{ii}, for n = 1.5, U^* = 0.0 and 1.0 (1st kind)

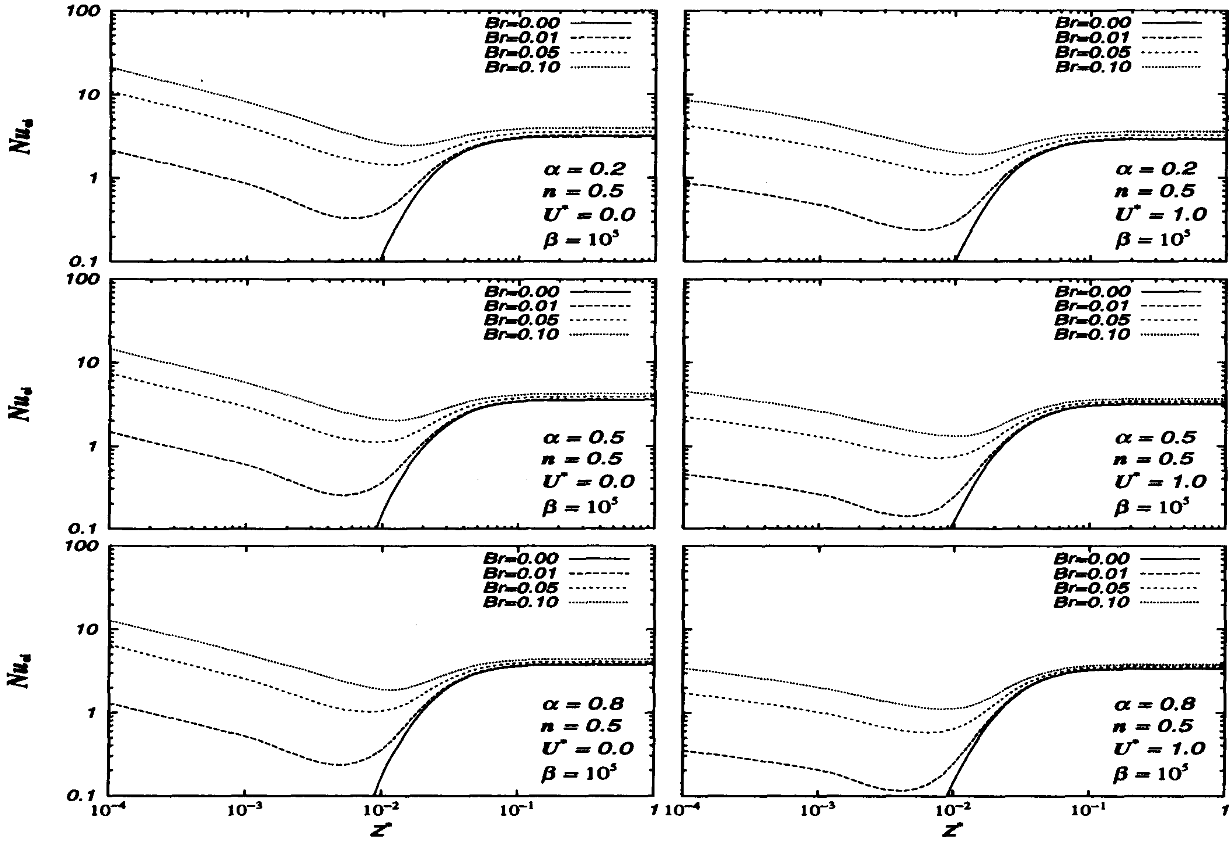

Fig.5.a Nusselt numbers, Nu_{oi}, for n = 0.5, U^* = 0.0 and 1.0 (1st kind)

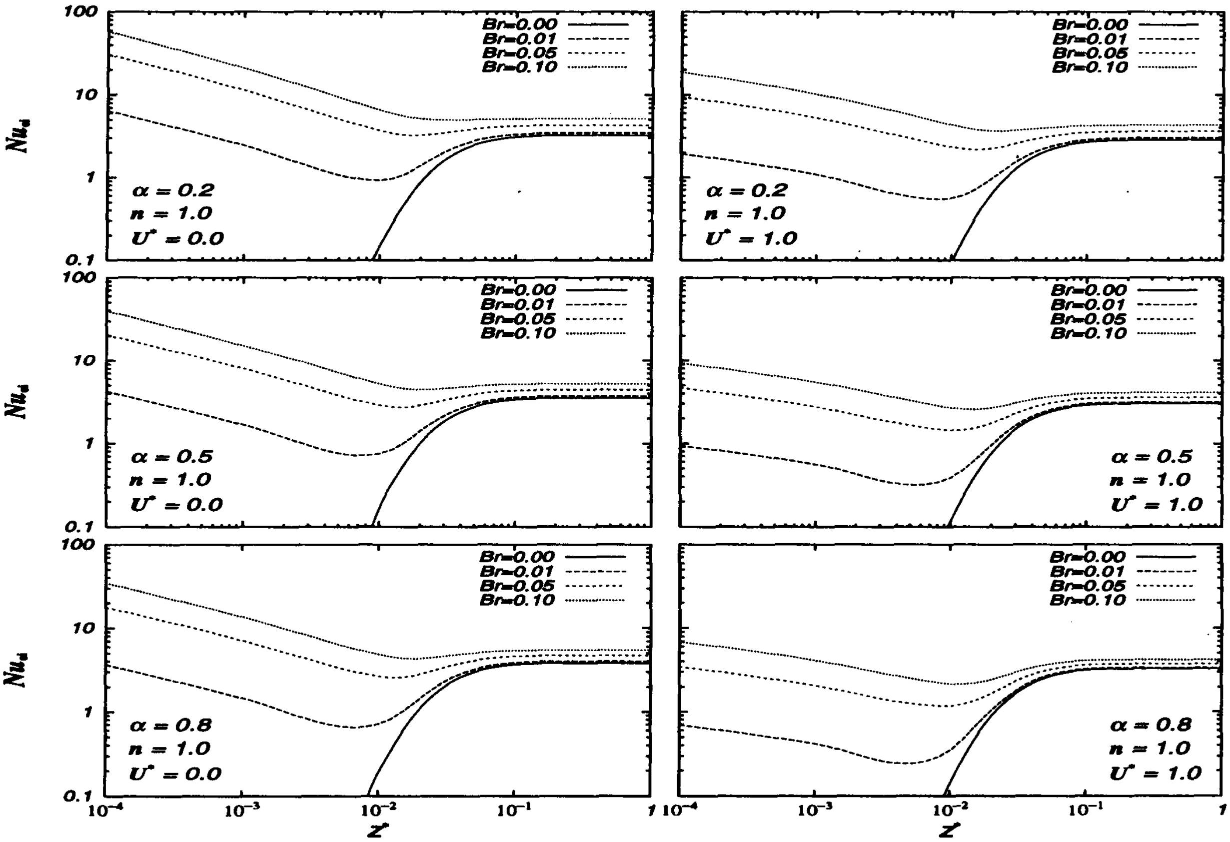

Fig.5.b Nusselt numbers, Nu_{oi}, for n = 1.0, U^* = 0.0 and 1.0 (1st kind)

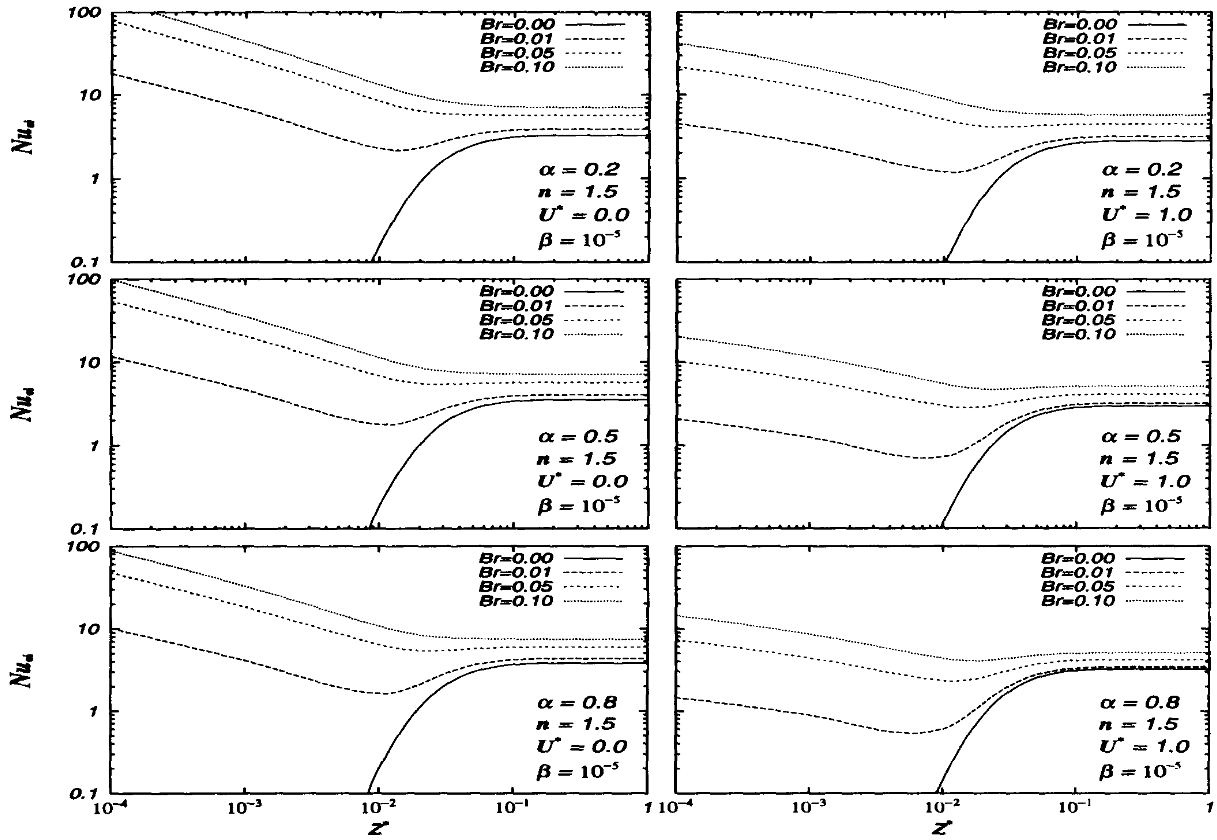

Fig.5.c Nusselt numbers, Nu_{oi}, for n = 1.5, U^* = 0.0 and 1.0 (1st kind)

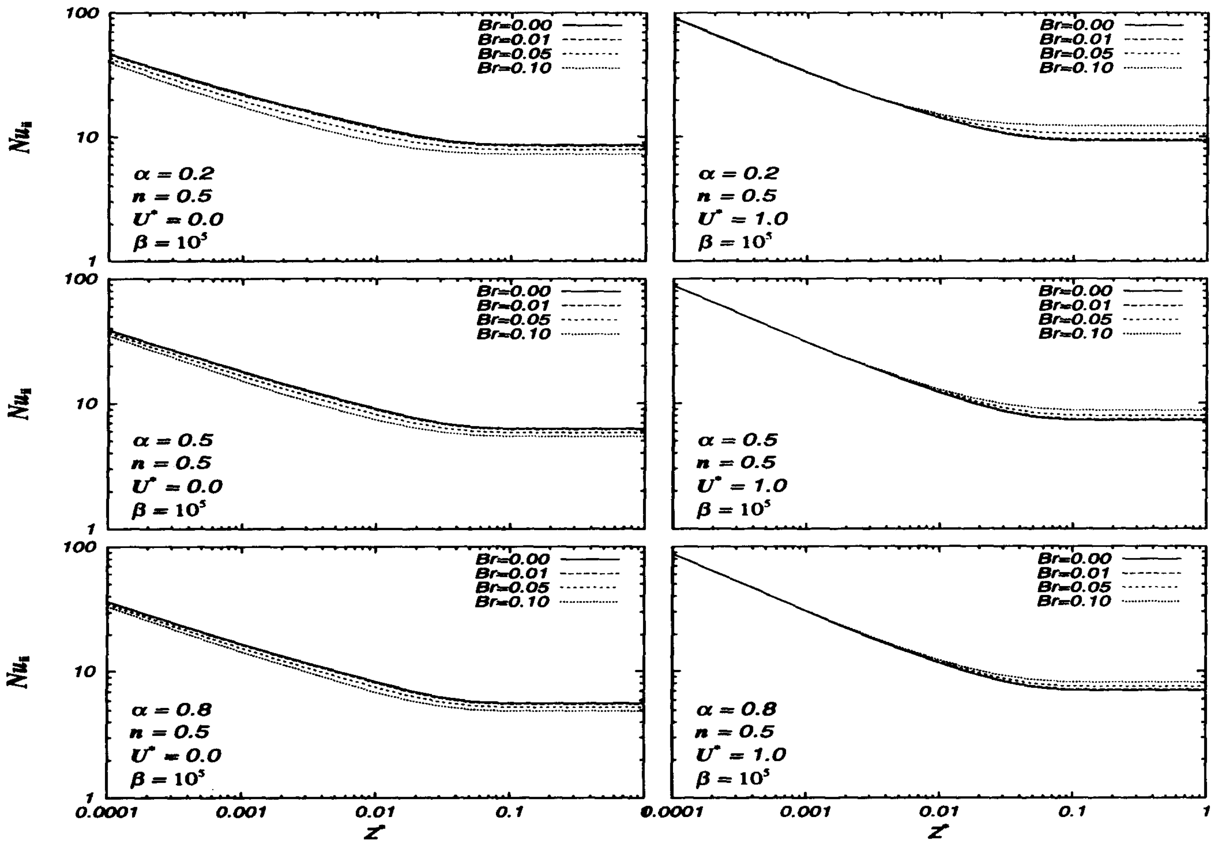

Fig.6.a Nusselt numbers, Nu_{ii}, for n = 0.5, U^* = 0.0 and 1.0 (2nd kind)

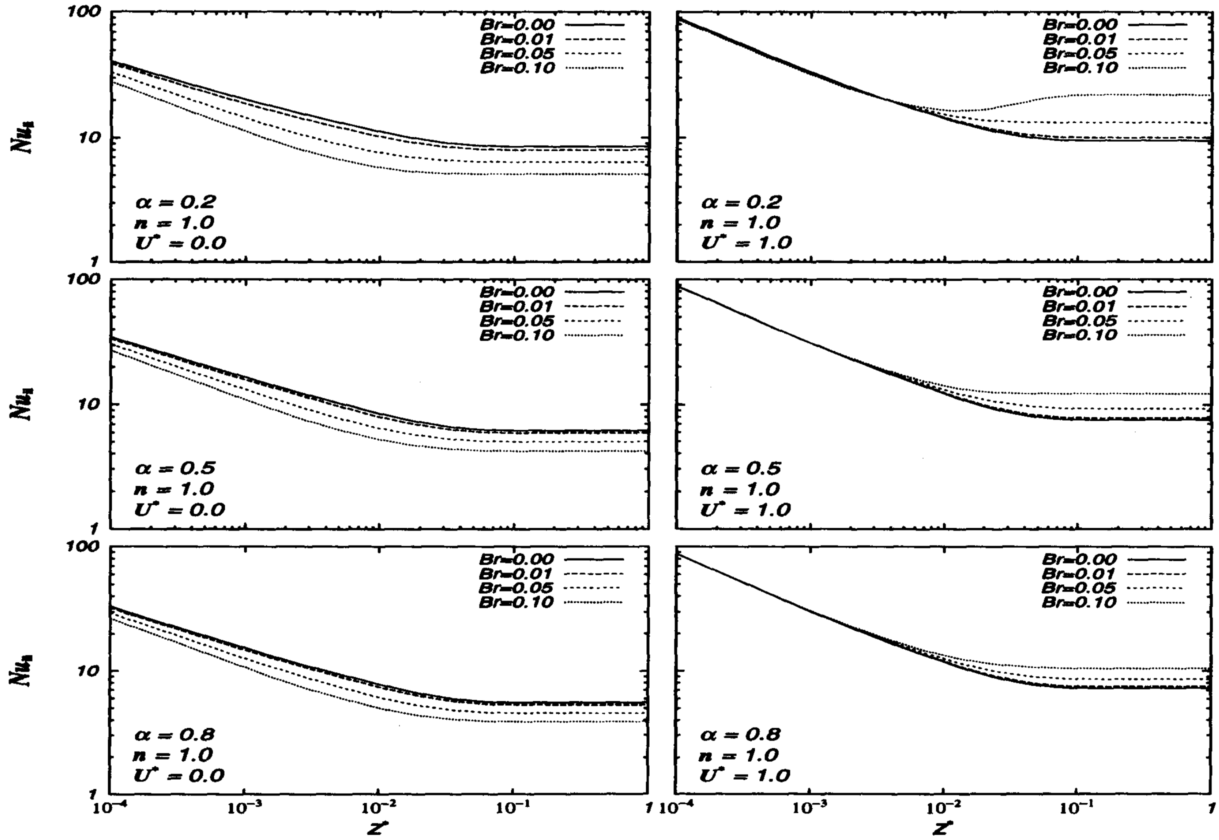

Fig.6.b Nusselt numbers, Nu_{ii}, for n = 1.0, U^* = 0.0 and 1.0 (2nd kind)

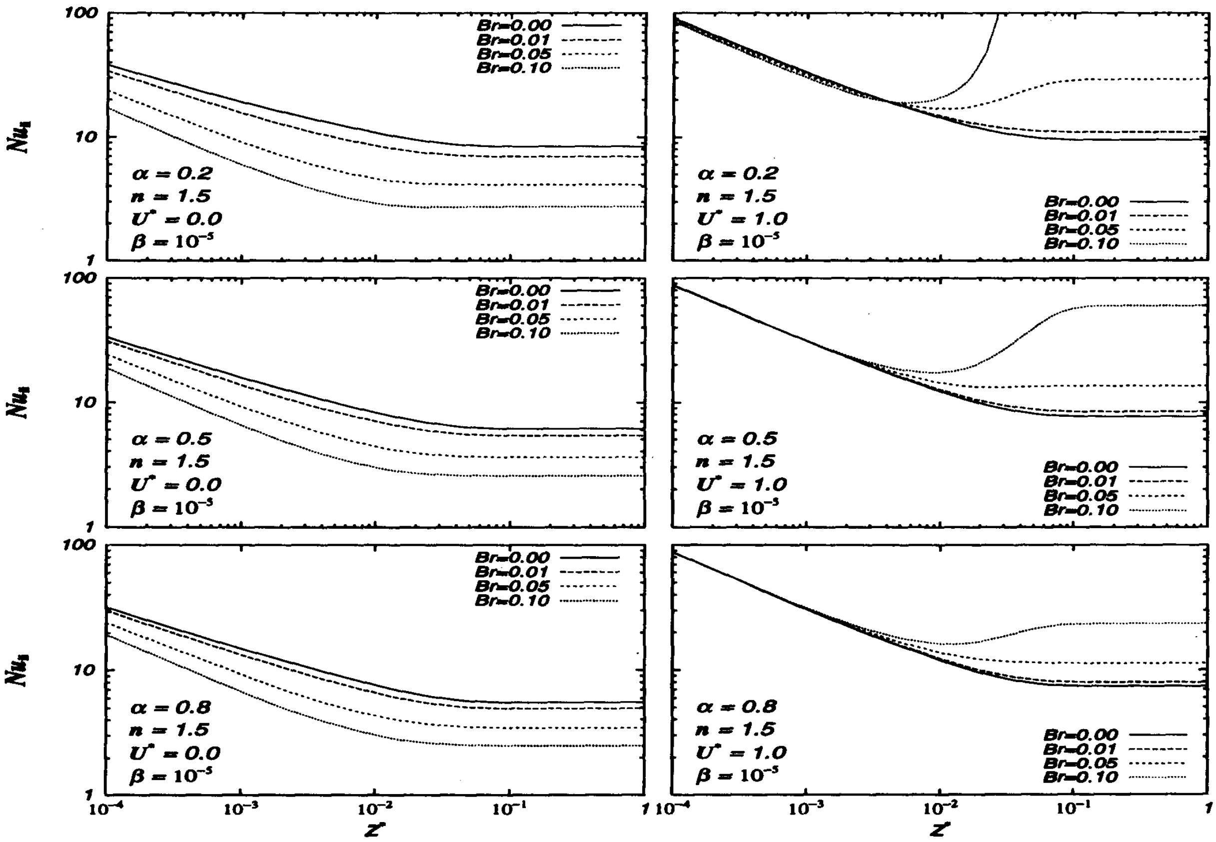

Fig.6.c Nusselt numbers, Nu_{ii}, for n = 1.5, U^* = 0.0 and 1.0 (2nd kind)