Heat transfer for modified power law fluids in a concentric annulus with a heated fixed outer tube

by

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The present paper is an extension of the previous study on fully developed laminar heat transfer of modified power-law fluids in a concentric annulus with an axially moving core and deals with the case for the boundary conditions of constant heat flux at the fixed outer tube with the moving core insulated. Applying the shear stress described by the modified power-law model, the energy equation including the viscous dissipation term is solved numerically. The numerical results are presented graphically for temperature profiles and Nusselt number at the outer tube with a number of parameters such as viscous dissipation effect, rheological properties and the boundary conditions. The effects of radius ratio, the flow index, the relative core velocity, the dimensionless shear rate parameter and Brinkman number on the temperature distribution and Nusselt number are discussed.

1. Introduction

In the previous report[1], the numerical solutions of the momentum equation were presented for fully developed laminar flow of non-Newtonian fluids flowing in a concentric annulus with an axially moving core. The shear stress for non-Newtonian fluids was described by the modified power law model.

The problem of fully developed heat transfer to non-Newtonian fluids in a concentric annulus with an axially moving core has been studied numerically for the thermal boundary conditions of constant heat flux at the moving core with the fixed outer tube insulated[2]. This case was referred to as Case A.

In the present paper the results for the thermal boundary conditions of constant heat flux at the fixed outer tube with the moving core insulated are reported. Applying the shear stress described by the modified power-law model and the fully developed velocity profile reported in the previous report[1], the energy equation including the viscous dissipation term is solved numerically. The effects of radius ratio, relative velocity of the core, flow index and dimensionless shear rate parameter and Brinkman number on the temperature distribution and Nusselt number are discussed.

Nomenclature

A area normal to the flow direction

Br Brinkman number
c_p specific heat at constant pressure
D_h hydraulic diameter $\equiv 2(R_o - R_i)$
k thermal conductivity
m consistency index
n flow index
Nu Nusselt number
r radial coordinate
r^* dimensionless radial coordinate $\equiv r/D_h$
R radius
q wall heat flux
T temperature
u axial velocity of the fluid
u_m average velocity of the fluid
u^* dimensionless velocity $\equiv u/u_m$
U axial velocity of the moving core
U^* dimensionless relative velocity of the moving core $\equiv U/u_m$
V dimensionless parameter
z axial coordinate

Greek Symbols

α radius ratio $\equiv R_i/R_o$
β dimensionless shear rate parameter
η_a apparent viscosity
η_a^* dimensionless apparent viscosity $\equiv \eta_a/\eta^*$
η_0 viscosity at zero shear rate
η^* reference viscosity
ρ density
τ shear stress
θ dimensionless temperature
ξ transformed dimensionless radial coordinate $\equiv [2(1 - \alpha)r^* - \alpha]/(1 - \alpha)$

Subscripts

b   bulk
i   inner tube
o   outer tube
oo   constant heat flux at the outer wall with
      the inner insulated

2. Analysis

The physical model for the analysis is shown in Fig.1. The inner core tube moves axially at a constant velocity, U. The assumptions used in the analysis are:

1. The flow is incompressible, steady-laminar, and fully developed, hydrodynamically and thermally.

2. The fluid is non-Newtonian and the shear stress may be described by the modified power-law model[3], and the physical properties are constant except viscosity.

3. The body forces and axial heat conduction are neglected.

Heat transfer

The energy equation together with the assumptions above is written as

$$k\frac{1}{r}\frac{d}{dr}\left[r\frac{dT}{dr}\right] + \tau\left(\frac{du}{dr}\right) = \rho c_{\mathrm{p}} u\frac{dT_{\mathrm{b}}}{dz}. \tag{1}$$

The thermal boundary conditions:
Case B (constant heat flux at the fixed outer tube with the moving core insulated):

$$\begin{cases} -k\frac{\partial T}{\partial r} = 0 & \text{at } r = R_{\mathrm{i}} \\ k\frac{\partial T}{\partial r} = q_{\mathrm{o}} & \text{at } r = R_{\mathrm{o}} \end{cases} \tag{2}$$

The velocity, u, and its gradient, $\frac{du}{dy}$, have been evaluated and reported in the previous paper[1]. τ in Eq.(1) is the shear stress defined by

$$\tau \equiv \eta_a\frac{du}{dr} \tag{3}$$

where η_a is the apparent viscosity defined as

$$\eta_a \equiv \frac{\eta_0}{1 + \frac{\eta_0}{m}\left|\frac{du}{dr}\right|^{1-n}} \quad \text{for } n < 1, \tag{4}$$

$$\eta_a \equiv \eta_0\left(1 + \frac{m}{\eta_0}\left|\frac{du}{dr}\right|^{n-1}\right) \quad \text{for } n > 1. \tag{5}$$

Dimensionless apparent viscosity, η_a^*, is defined as

$$\eta_a^* \equiv \frac{\eta_a}{\eta^*} = \frac{1 + \beta}{1 + \beta\left|\frac{du^*}{dr^*}\right|^{1-n}} \quad \text{for } n < 1, \tag{6}$$
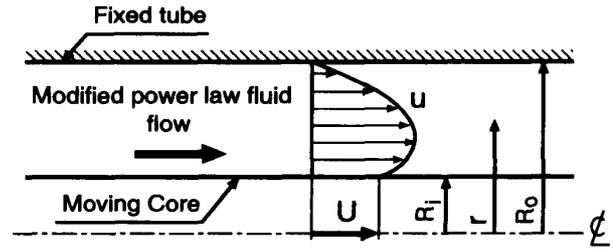
Fig.1 Schematic of a concentric annulus with an axially moving core

$$\eta_a^* \equiv \frac{\eta_a}{\eta^*} = \frac{\beta + \left|\frac{du^*}{dr^*}\right|^{n-1}}{\beta + 1} \quad \text{for } n > 1, \tag{7}$$

where

$$\eta^* = \frac{\eta_0}{1 + \beta} \quad \text{for } n < 1, \tag{8}$$

$$\eta^* = \eta_0\left(1 + \frac{1}{\beta}\right) \quad \text{for } n > 1, \tag{9}$$

$$\beta = \frac{\eta_0}{m}\left(\frac{u_{\mathrm{m}}}{D_{\mathrm{h}}}\right)^{1-n}. \tag{10}$$

Bulk temperature, T_{b}, is defined as

$$T_{\mathrm{b}} \equiv \frac{\iint_A uT\,dA}{\iint_A u\,dA} = \frac{2}{u_{\mathrm{m}}(R_{\mathrm{o}}^2 - R_{\mathrm{i}}^2)}\int_{R_{\mathrm{i}}}^{R_{\mathrm{o}}} uTr\,dr. \tag{11}$$

By integrating Eq.(1) together with Eq.(2), dT_{b}/dz is obtained as:

$$\frac{dT_{\mathrm{b}}}{dz} = \frac{2R_{\mathrm{o}}q_{\mathrm{o}}}{\rho c_{\mathrm{p}} u_{\mathrm{m}}(R_{\mathrm{o}}^2 - R_{\mathrm{i}}^2)}\left[1 + \frac{\int_{R_{\mathrm{i}}}^{R_{\mathrm{o}}}\tau r\left(\frac{du}{dr}\right)dr}{R_{\mathrm{o}}q_{\mathrm{o}}}\right] \tag{12}$$

The average fluid velocity, u_{m}, is defined as

$$u_{\mathrm{m}} \equiv \frac{1}{\pi(R_{\mathrm{o}}^2 - R_{\mathrm{i}}^2)}\int_{R_{\mathrm{i}}}^{R_{\mathrm{o}}} u2\pi r\,dr. \tag{13}$$

Introducing a dimensionless temperature, θ, defined as

$$\theta \equiv T\big/[q_{\mathrm{o}}D_{\mathrm{h}}/k], \tag{14}$$

the energy equation and the boundary conditions may be expressed in the dimensionless forms as

$$\frac{1}{r^*}\frac{d}{dr^*}\left(r^*\frac{d\theta}{dr^*}\right) = \frac{4}{(1 + \alpha)}u^* + Br_{\mathrm{o}} \cdot V. \tag{15}$$

$$\begin{cases} \frac{d\theta}{dr^*} = 0 & \text{at} \quad r^* = \frac{\alpha}{2(1-\alpha)} \\ \frac{d\theta}{dr^*} = 1 & \text{at} \quad r^* = \frac{1}{2(1-\alpha)} \end{cases} \quad (16)$$

where Br_0 is Brinkman number defined as

$$Br_0 \equiv \frac{\eta^* u_m^2}{D_h q_0} \quad (17)$$

and

$$V = \left\{ \frac{8(1-\alpha)}{(1+\alpha)}^{\frac{1}{2(1-\alpha)}} \int_{\frac{\alpha}{2(1-\alpha)}} r^* \eta_a^* \left(\frac{du^*}{dr^*}\right)^2 dr^* \right\} u^*$$

$$- \eta_a^* \left(\frac{du^*}{dr^*}\right)^2. \quad (18)$$

Nusselt number, Nu_{oo}, on the outer wall is calculated as

$$Nu_{oo} \equiv \frac{[q_0/(T_o - T_b)]D_h}{k} = \frac{1}{\theta_o - \theta_b} \quad (19)$$

where the dimensionless bulk temperature, θ_b, is defined as

$$\theta_b \equiv \frac{T_b k}{q_0 D_h} = \frac{8(1-\alpha)}{1+\alpha}^{\frac{1}{2(1-\alpha)}} \int_{\frac{\alpha}{2(1-\alpha)}} u^* \theta r^* dr^* \quad (20)$$

3. Results and discussion

The effect of viscous dissipation on temperature difference across the channel is demonstrated in Figs.2(a), 2(b) and 2(c) for three different values of U^*. $\xi = 0$ corresponds to the insulated inner tube and $\xi = 1$ the heated outer tube. It is seen that the results on Newtonian ($n = 1.0$) and non-Newtonian fluids are similar qualitatively. The temperature difference $(\theta - \theta_b)$ increases with an increase in Br_0 near the walls at $U^* = -1.0$ and $U^* = 0.0$. But it decreases in the middle section. This is attributed to that since the parameter V is large near the walls, the effect of viscous dissipation is the strongest there. The heat transferred axially by convection is large in the middle section. For $U^* = -1.0$, $(\theta - \theta_b)$ greatly increases with an increase in Br_0 near the moving core, but increases slightly near the fixed tube. For $U^* = 1.0$, $(\theta - \theta_b)$ increases with an increase in Br_0 near the fixed tube and decreases near the moving core. It can be seen that $(\theta - \theta_b)$ decreases with an increase in U^* near the moving core.

Nusselt numbers Nu_{oo}, are shown in Figs.3(a), 3(b) and 3(c). Nu_{oo} decreases with an increase in

Br_0 for $U^* = -1.0$, $U^* = 0.0$ and $U^* = 1.0$. These behaviors of Nusselt number can be explained by the viscous dissipation effects on temperature difference $(\theta - \theta_b)$, as mentioned above. For the case of $U^* = -1.0$, it is seen that the relationship between Nu_{oo} and β is not similar for the small values of Brinkman number ($Br_0 = 0 \sim 0.01$) for both fluids with $n < 1$ and $n > 1$. It may be explained by how Br_0 influences on Nu_{oo} and the explanation has been reported in the previous report[2].

4. Conclusions

The fully developed laminar heat transfer of modified power-law fluids in a concentric annulus with an axially moving core was analyzed taking into account the viscous dissipation of the flowing fluid. In this paper, the numerical solutions for the thermal boundary condition of constant heat flux at the fixed outer tube with the moving core insulated have been reported. The effects of radius ratio, flow index, relative core velocity, dimensionless shear rate parameter and Brinkman number on the temperature distribution and Nusselt numbers have been studied.

References

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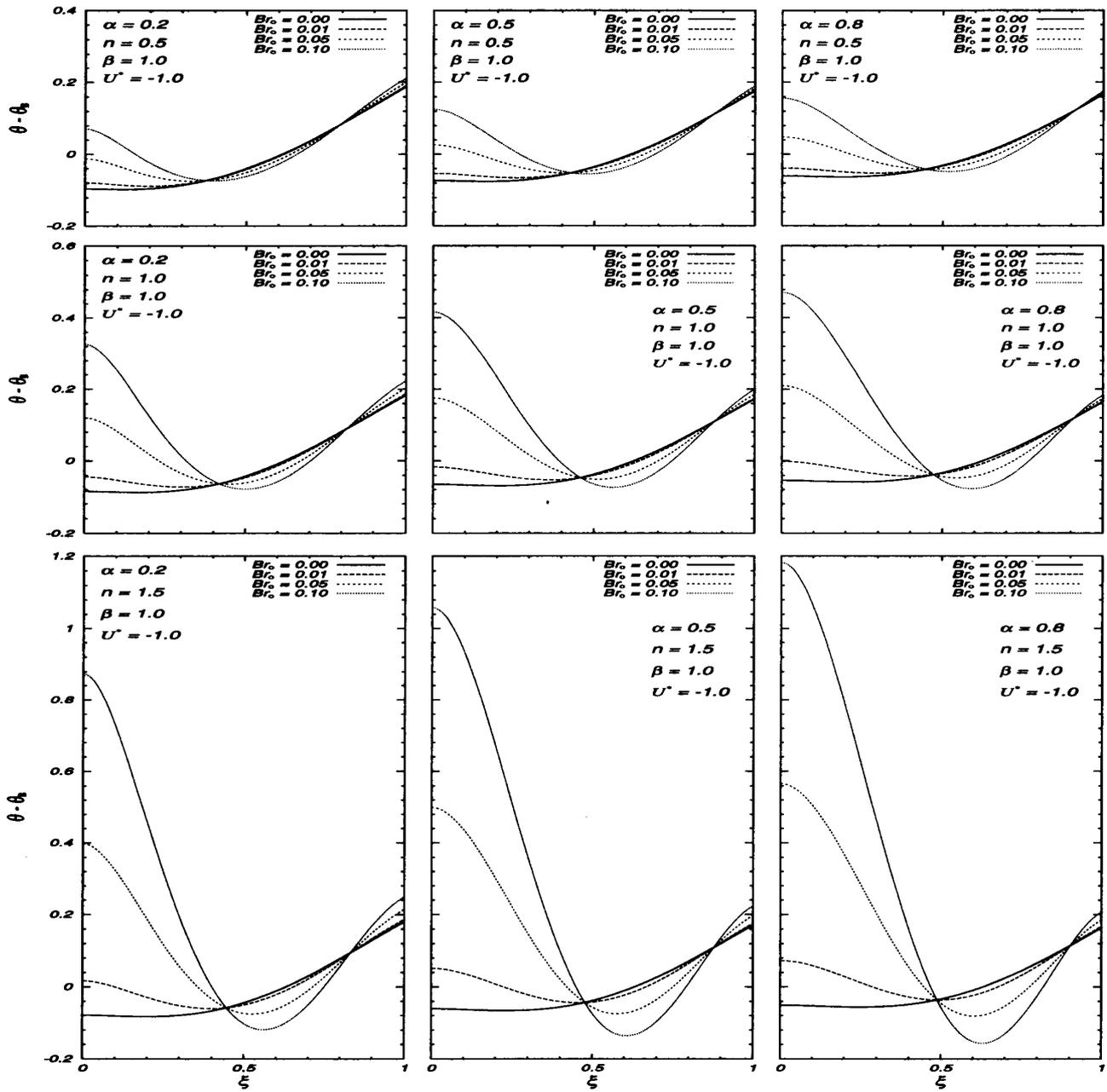
Fig.2(a) Dimensionless temperature difference for Case B ($U^* = -1.0$, $\beta = 1.0$)
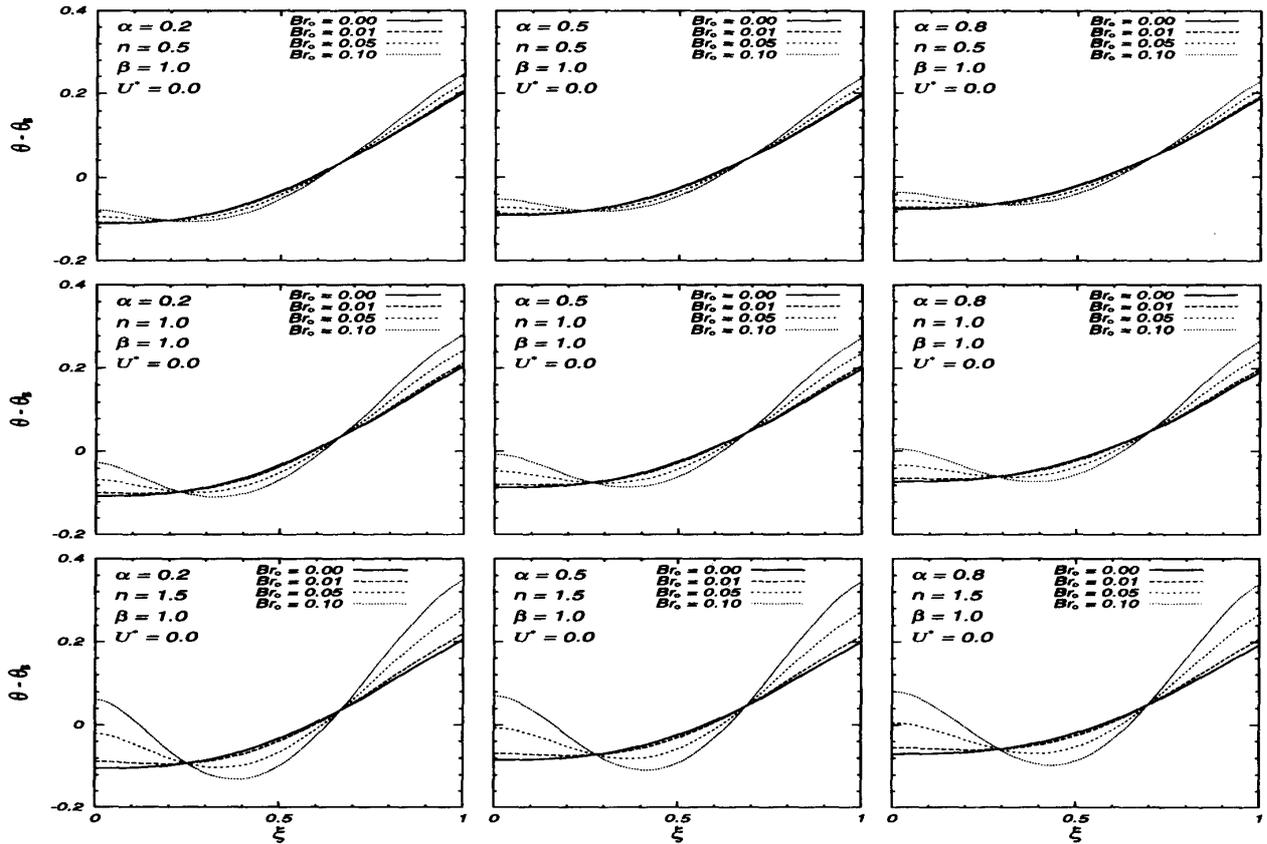
Fig.2(b) Dimensionless temperature difference for Case B ($U^* = 0.0$, $\beta = 1.0$)
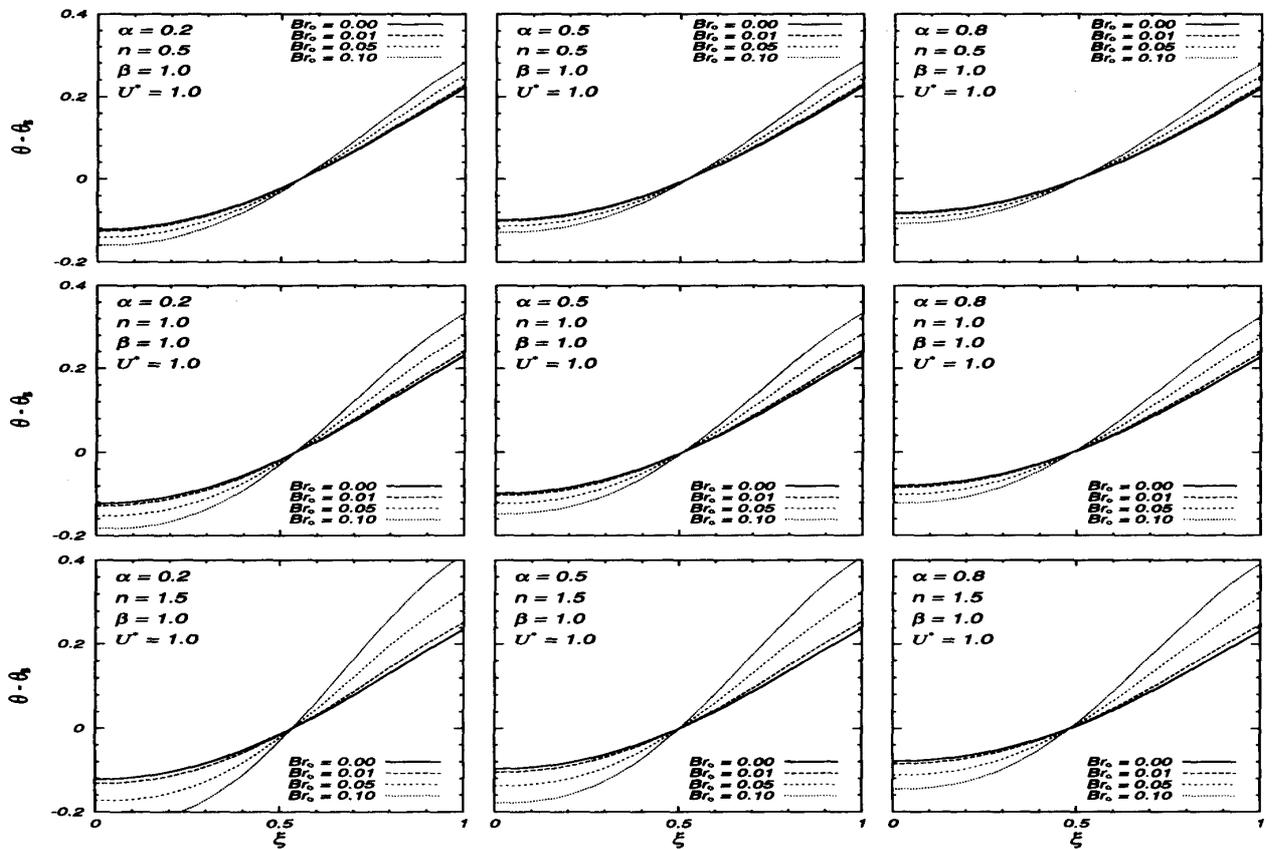
Fig.2(c) Dimensionless temperature difference for Case B ($U^* = 1.0$, $\beta = 1.0$)
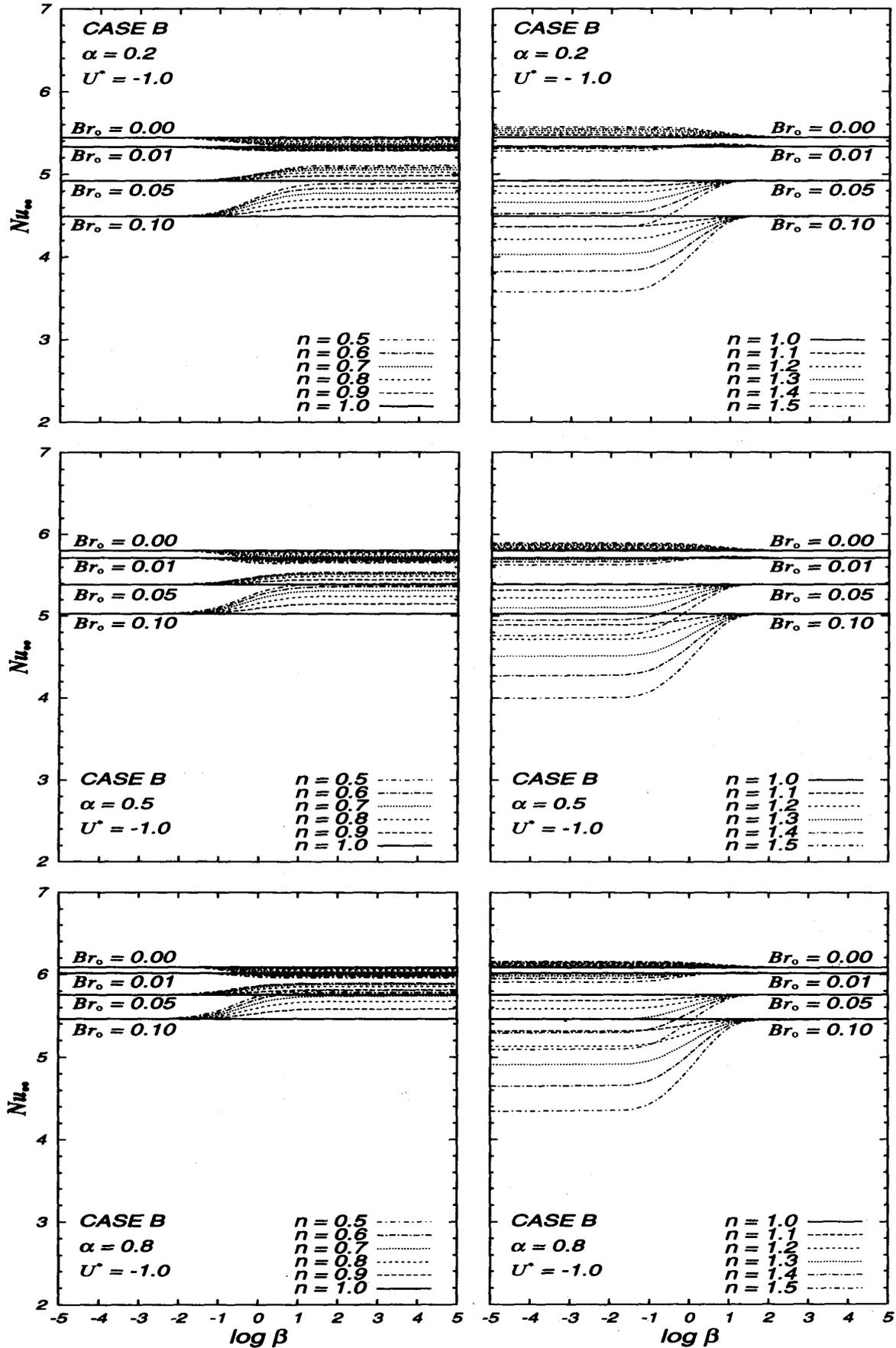
Fig.3(a) Nusselt numbers for $U^* = -1.0$ (Case B)
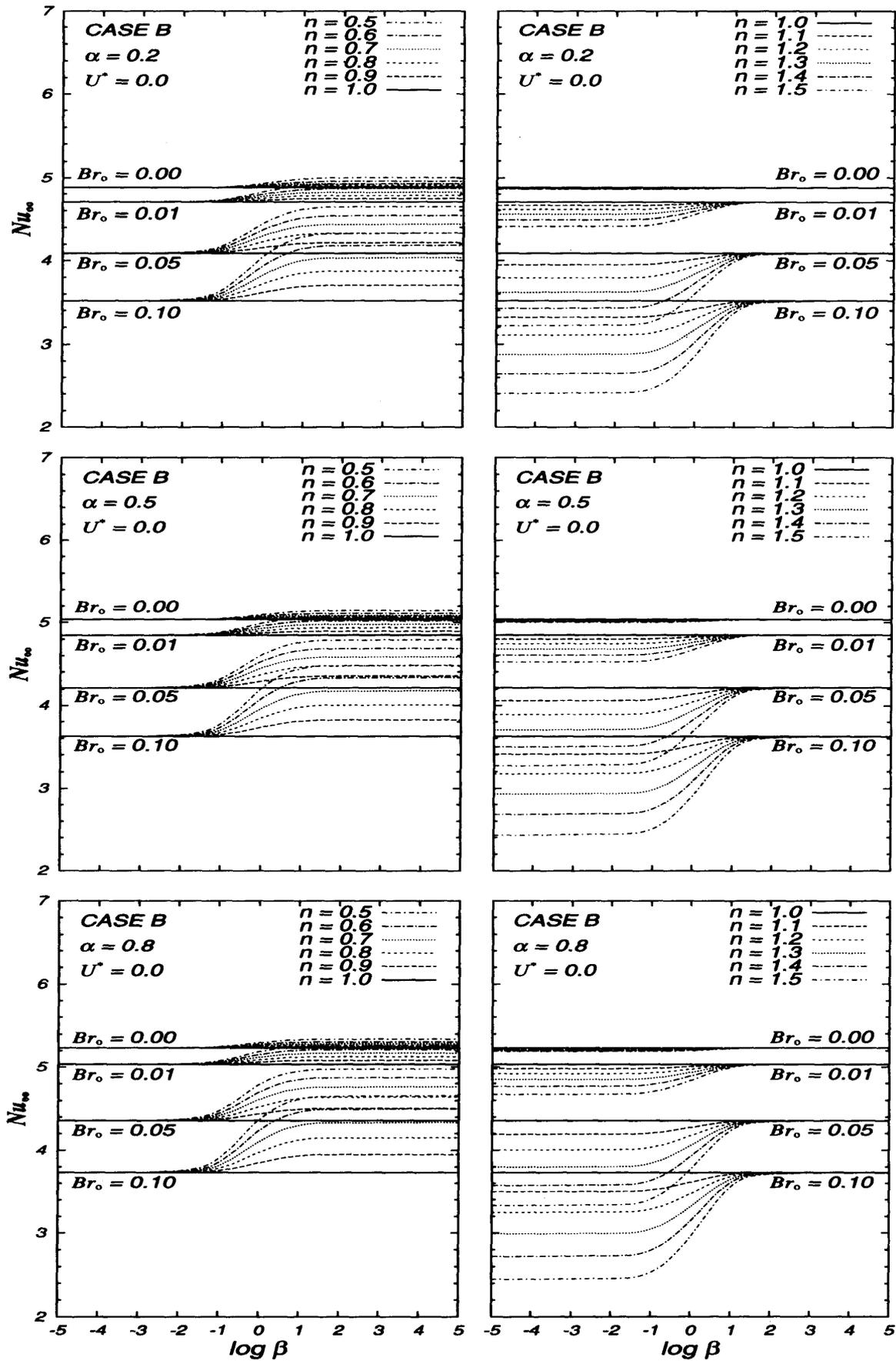
Fig.3(b) Nusselt numbers for $U^* = 0.0$ (Case B)

Fig.3(c) Nusselt numbers for $U^* = 1.0$ (Case B)