Fluid flow for modified power law fluids in concentric annuli with axially moving cores

by

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Fully developed laminar flow of modified power law fluids in a concentric annulus with an axially moving core is studied numerically. Applying the shear stress described by the modified power law model, the effects of the radius ratio, relative velocity of the core, the flow index and dimensionless shear rate parameter on the velocity distribution and friction factor are discussed.

1. Introduction

Problems involving fluid flow and heat transfer with an axially moving core of solid body or fluid in an annular geometry can be found in many manufacturing processes, such as extrusion, drawing and hot rolling, etc. In such processes, a hot plate or cylindrical rod continuously exchanges heat with the surrounding environment. For such cases, the fluid involved may be Newtonian or non-Newtonian and the flow situations encountered can be either laminar or turbulent.

In the previous study[1], the exact solutions of the momentum and energy equations were obtained for fully developed laminar flow of Newtonian fluids flowing in an annular geometry. There the effect of viscous dissipation on heat transfer have not been examined.

In the previous report[2], fully developed laminar heat transfer of a Newtonian fluid in a concentric annulus with an axially moving core was analyzed taking into account the viscous dissipation of the flowing fluid.

In this study, numerical solutions of the momentum equation are obtained for fully developed laminar flow of non-Newtonian fluids flowing in a concentric annulus with an axially moving core. The shear stress for non-Newtonian fluids is described by the modified power law model proposed by Capobianchi and Irvine[3]. The effects of the radius ratio, relative velocity of the core, the flow index and dimensionless shear rate parameter on the velocity distribution and friction factor are discussed.

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Nomenclature

A	area normal to the flow direction
D_h	hydraulic diameter $\equiv 2(R_o - R_i)$
f	friction factor
m	consistency index
n	flow index
P	pressure
r	radial coordinate
r^*	dimensionless radial coordinate $\equiv r/D_h$
R	radius
Re_M	modified Reynolds number
u	axial velocity of the fluid
u_m	average velocity of the fluid
u^*	dimensionless velocity $\equiv u/u_m$
U	core velocity
U^*	relative core velocity $\equiv U/u_m$
z	axial coordinate

Greek Symbols

α	radius ratio $\equiv R_i/R_o$
β	dimensionless shear rate parameter
η_a	apparent viscosity
η_a^*	dimensionless apparent viscosity $\equiv \eta_a/\eta^*$
η_0	viscosity at zero shear rate

η^*	reference viscosity
ρ	density
τ	shear stress
ξ	transformed dimensionless radial coordinate $\equiv [2(1-\alpha)r^* - \alpha]/(1-\alpha)$

Subscripts

i	inner tube
o	outer tube

2. Analysis

The physical model for the analysis is shown in Fig. 1. The inner core tube is axially moving at a constant velocity, U. The assumptions used in the analysis are:

1. The flow is incompressible, steady-laminar, and hydrodynamically fully developed.
2. The fluid is non-Newtonian and the shear stress may be described by the modified power-law model[3], and physical properties are constant except viscosity.
3. The body forces are neglected.

Fluid Flow

With the assumptions described above, the governing momentum equation is

$$\frac{1}{r}\frac{d}{dr}(r\tau) = \frac{dP}{dz}. \tag{1}$$

The boundary conditions are:

$$\begin{cases} u = U & \text{at} \quad r = R_i \\ u = 0 & \text{at} \quad r = R_o. \end{cases} \tag{2}$$

The shear stress, τ, is given by the modified power law model.

$$\tau = \eta_a \frac{du}{dr} \tag{3}$$

where η_a is the apparent viscosity defined by

$$\eta_a = \frac{\eta_0}{1 + \dfrac{\eta_0}{m}\left|\dfrac{du}{dr}\right|^{1-n}} \qquad \text{for } n<1, \tag{4}$$

$$\eta_a = \eta_0\left(1 + \frac{m}{\eta_0}\left|\frac{du}{dr}\right|^{n-1}\right) \qquad \text{for } n>1. \tag{5}$$

The average velocity, u_m, is defined as:

$$u_m \equiv \frac{1}{A}\int_A u \cdot dA = \frac{1}{\pi(R_0^2 - R_i^2)}\int_{R_i}^{R_0} u \cdot 2\pi r dr. \tag{6}$$

The momentum equation and its boundary conditions are reduced to

$$\frac{1}{r^*}\frac{d}{dr^*}\left(r^* \eta_a^* \frac{du^*}{dr^*}\right) = -2f \cdot Re_M \tag{7}$$

$$\begin{cases} u^* = U^* & \text{at} \quad r^* = \dfrac{\alpha}{2(1-\alpha)} \\ u^* = 0 & \text{at} \quad r^* = \dfrac{1}{2(1-\alpha)}. \end{cases} \tag{8}$$

Friction factor, f, and modified Reynolds number, Re_M are defined as

$$f \equiv \frac{D_h}{2\rho u_m^2}\left(-\frac{dP}{dz}\right) \tag{9}$$

$$Re_M \equiv \frac{\rho u_m D_h}{\eta^*}. \tag{10}$$

Dimensionless apparent viscosity η_a^* is defined as

$$\eta_a^* \equiv \frac{\eta_a}{\eta^*} = \frac{1+\beta}{1 + \beta\left|\dfrac{du^*}{dr^*}\right|^{1-n}} \qquad \text{for } n<1, \tag{11}$$

$$\eta_a^* \equiv \frac{\eta_a}{\eta^*} = \frac{\beta + \left|\dfrac{du^*}{dr^*}\right|^{n-1}}{\beta + 1} \qquad \text{for } n>1, \tag{12}$$

where

$$\eta^* = \frac{\eta_0}{1+\beta} \qquad \text{for } n<1, \tag{13}$$

$$\eta^* = \eta_0\left(1 + \frac{1}{\beta}\right) \qquad \text{for } n>1, \tag{14}$$

$$\beta = \frac{\eta_0}{m}\left(\frac{u_m}{D_h}\right)^{1-n}. \tag{15}$$

The dimensionless form of the Eq.(6) is written as:

$$1 = \frac{8(1-\alpha)}{(1+\alpha)}\int_{\frac{\alpha}{2(1-\alpha)}}^{\frac{1}{2(1-\alpha)}} u^* r^* dr^*. \tag{16}$$

3. Results and discussion

The governing momentum equation together with the integrated continuity equation, Eq.(16) is solved numerically.

Parameter β defined by Eq.(15) represents the dimensionless average shear rate under the specified

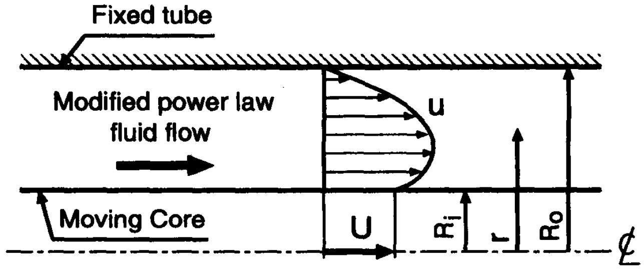

Fig.1 Schematic of a concentric annulus with an axially moving core

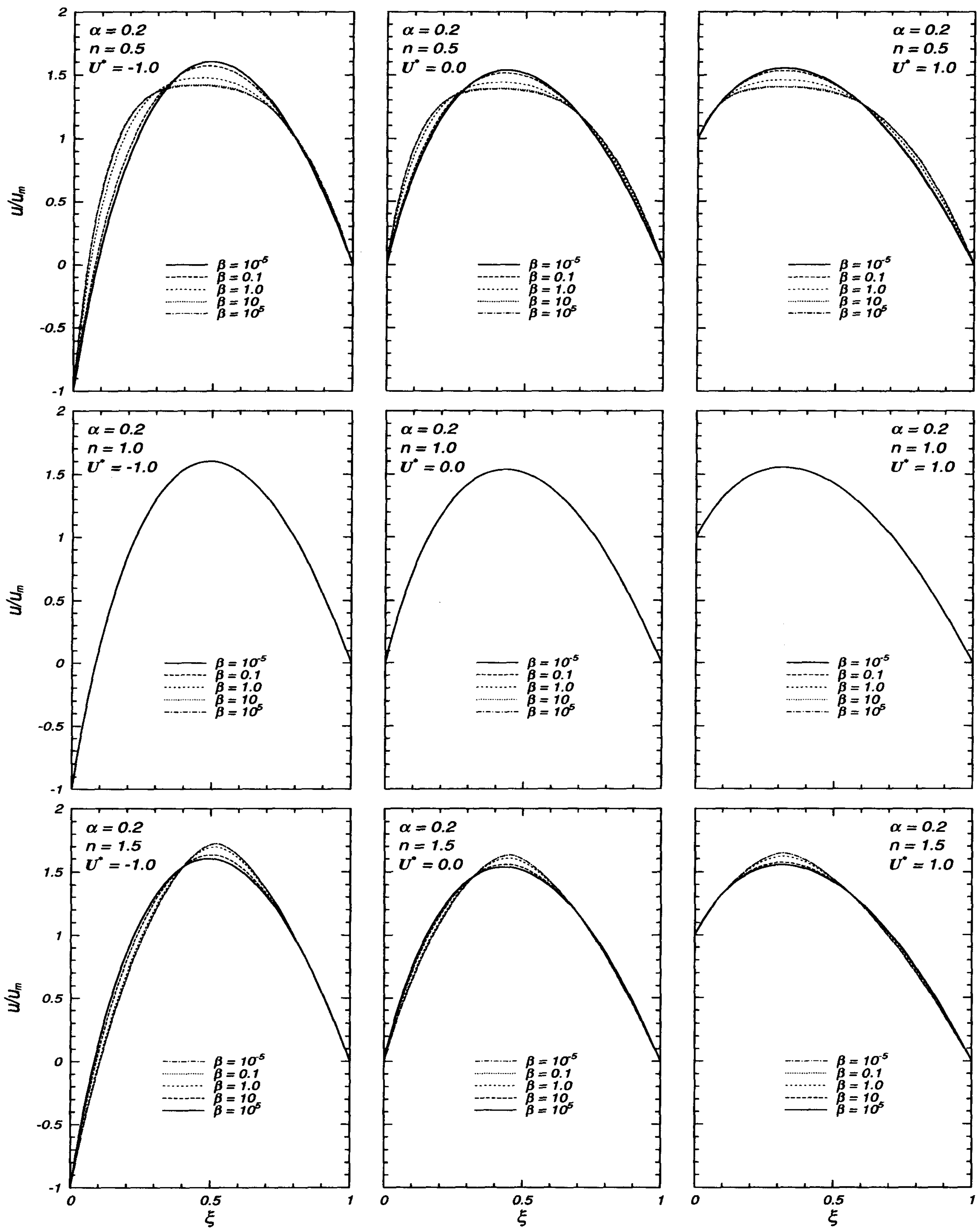

Fig.2 Velocity profile ($\alpha = 0.2$)

fluid flow condition. For pseudoplastic fluids ($n<1$) the extreme at $\beta \to 0$ corresponds to a Newtonian fluid and at $\beta \to \infty$ to a power law fluid. For dilatant fluids ($n>1$) the extreme at $\beta \to \infty$ corresponds to a Newtonian fluid and at $\beta \to 0$ to a power law fluid. Figures 2, 3 and 4 show the effect of parameter β on the velocity profile across the annuli for the nine combinations of $n = 0.5$, 1.0, 1.5 and $U^* = -1.0$, 0.0, 1.0 for three different radius ratio $\alpha = 0.2$, 0.5, 0.8, respectively.

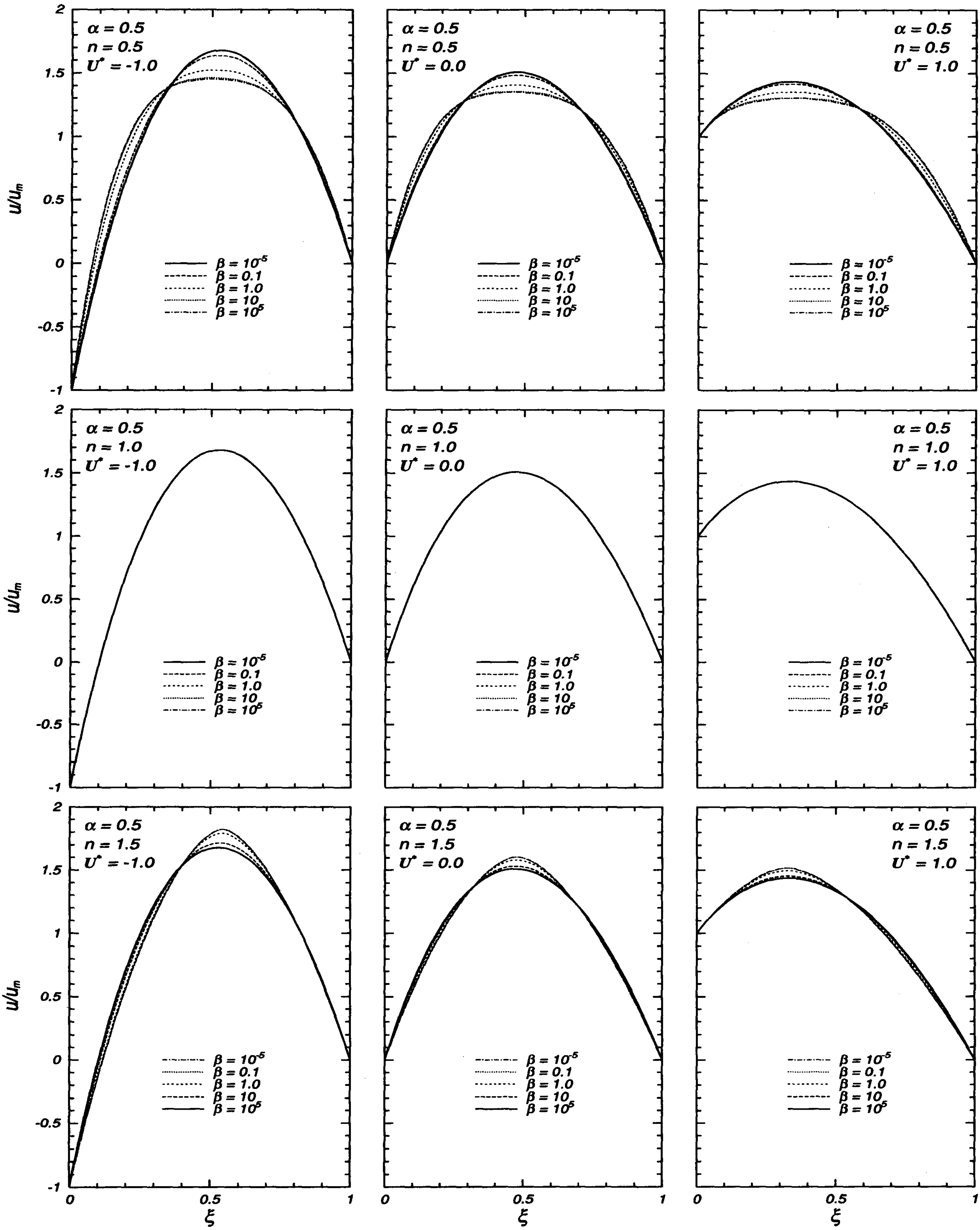

Fig.3 Velocity profile ($\alpha = 0.5$)

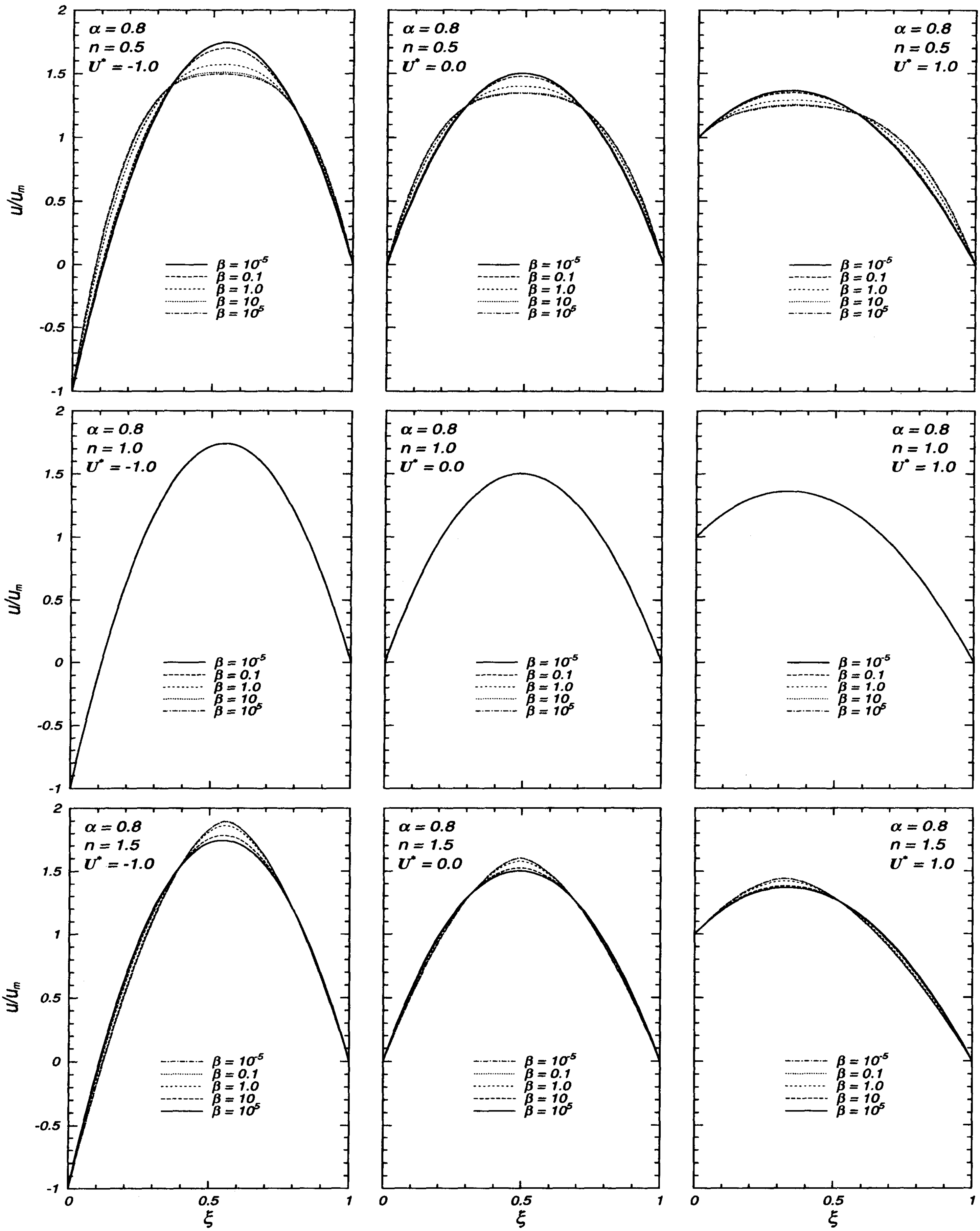

Fig.4 Velocity profile ($\alpha = 0.8$)

The results of the analysis for $\alpha = 0.2$, 0.5 and 0.8 are presented in Fig.s 5, 6 and 7 that show the effect of parameter β on friction factor in terms of $f\,Re_M$. The values of $f\,Re_M$ at the extremes of $\beta \to 0$ and $\beta \to \infty$ approach, respectively, to the values of Newtonian fluid and power law fluid. It is seen that the values of $f\,Re_M$ become greater with a decrease in U^*. $f\,Re_M$ increases with an increase in radius ratio, α, for $U^* = -2.0, -1.0$, and 0.0, and $f\,Re_M$ decreases with an increase in α for $U^* = 1.0$ and 2.0 (see table 1). The power law asymptotic values of $f\,Re_M$ are tabulated in Table 1. The values corresponding to $\alpha = 1.0$, are obtained by the previous study[4] for the case of the parallel plate geometry. It is seen from the table $f\,Re_M = 0$ at $U^* = 2.0$. Because velocity profiles are linear. For $U^* > 2.0$ velocity profiles become concave while for $U^* < 2.0$ they are convex.

4. Conclusions

Fully developed laminar flow in concentric annuli is analyzed by using the modified power-law model proposed by Capobianchi and Irvine[3]. Velocity distributions and friction factors are obtained by the numerical method.

In the analysis of fluid flow of non-Newtonian fluids, the results calculated by adopting the simple power-law fluid model do not predict correctly the values of viscosity and friction factor in the region of lower shear rate. In order to calculate the whole region of shear rate from zero to infinity, the modified power-law model as adopted in this study should be used.

In this report the fluid flow study results are discussed. The heat transfer part will be shown in the following report.

Reference

1. T.Shigechi and Y. Lee "An analysis on fully developed laminar fluid flow and heat transfer in concentric annuli with moving cores" *Int. J. Heat Mass Transfer,* vol. 34, No. 10, p.2593 - 2601 (1991).
2. G.Davaa, T.Shigechi and S.Momoki "Effects of moving core velocity and viscous dissipation on fully developed laminar heat transfer in concentric annuli" *Reports of the Faculty of Engineering, Nagasaki University,* vol. 31, No. 56, p.13 - 22 (2001).
3. M.Capobianchi and T.F.Irvine, "Predictions of pressure drop and heat transfer in concentric annular ducts with modified power law fluids" *Wärme-und Stoffübertragung,* 27, p.209 - 215 (1992).
4. G.Davaa, T.Shigechi, S.Momoki and O.Jambal "Fluid flow and heat transfer to modified power law fluids in plane Couette-Poiseuille laminar flow between parallel plates" *Reports of the Faculty of Engineering, Nagasaki University* vol. 31, No. 57, p.31 - 39 (2001).

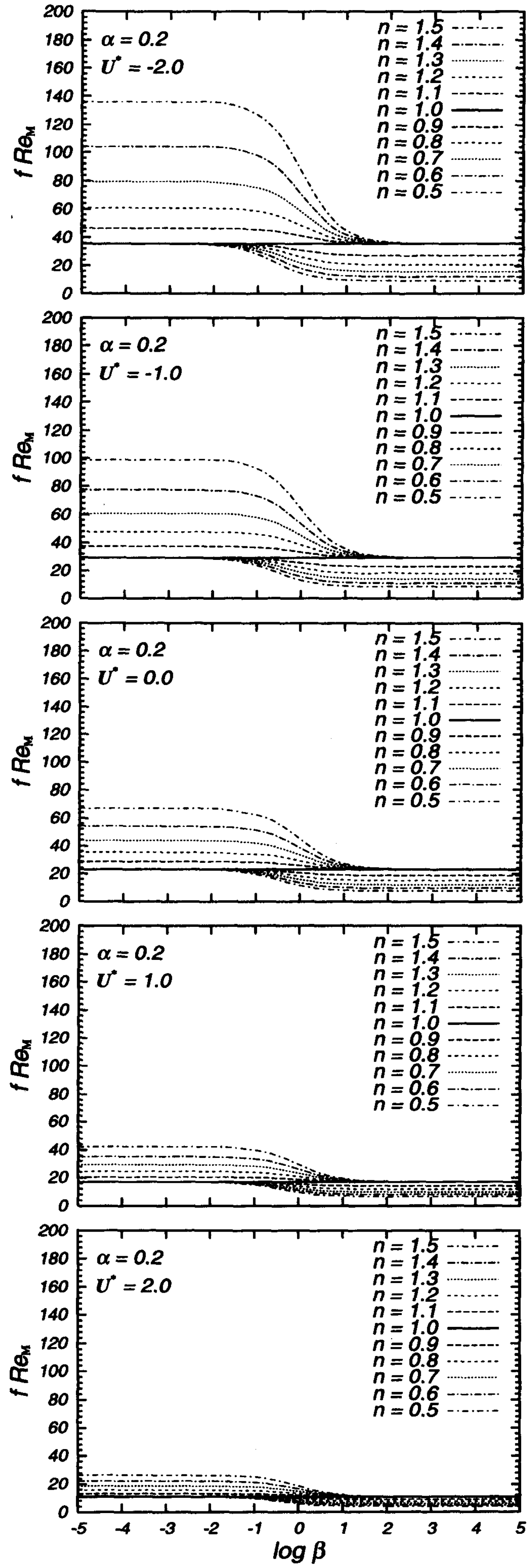

Fig.5 Friction factor ($\alpha = 0.2$)

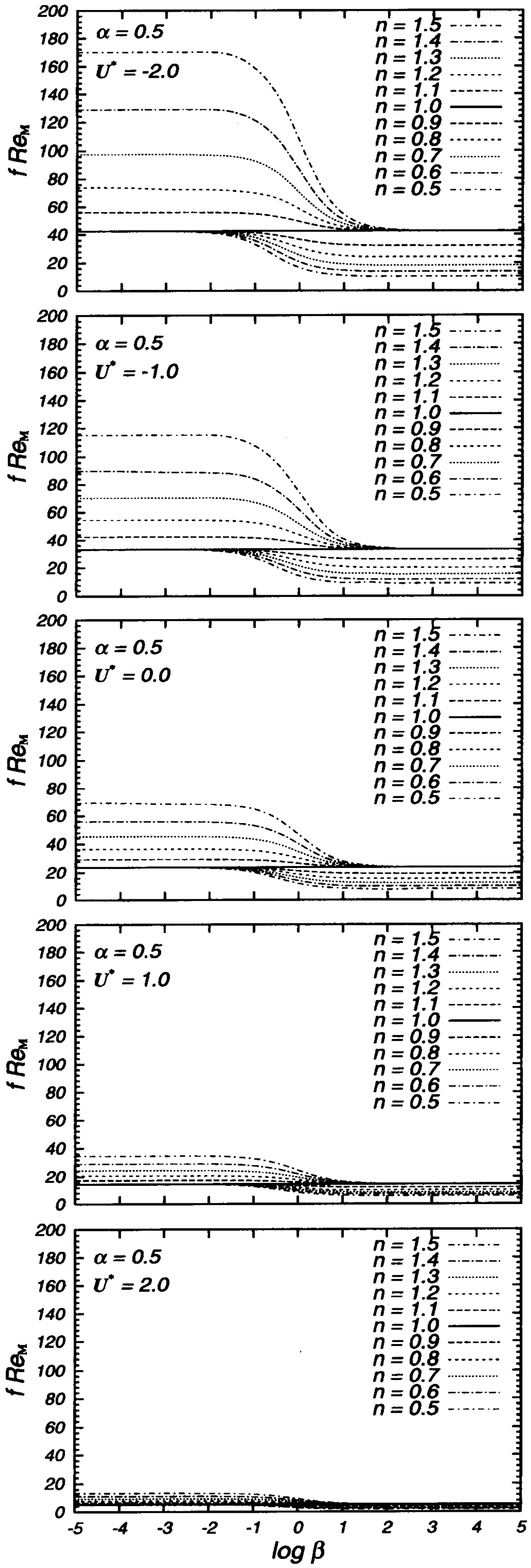

Fig.6 Friction factor ($\alpha = 0.5$)

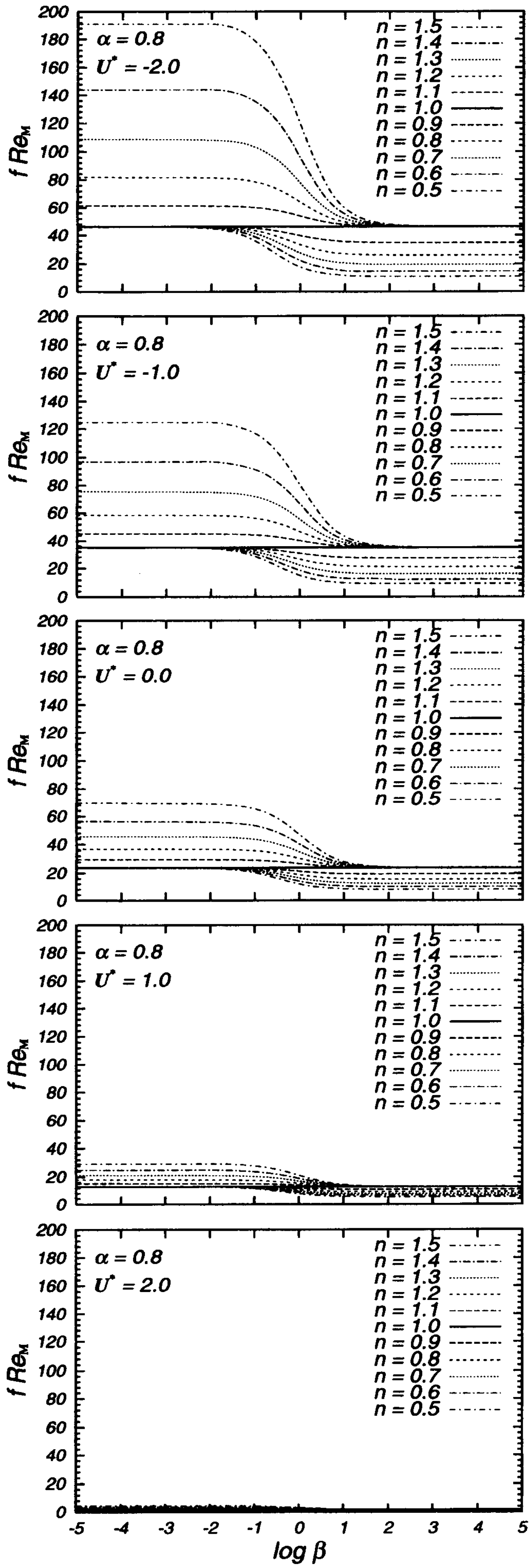

Fig.7 Friction factor ($\alpha = 0.8$)

Table 1 Numerical values of fRe_{M} at the power law asymtote

		fRe_{M}										
U^{*}	α	n										
		0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
	0.1	8.497	11.047	14.362	18.675	24.290	31.595	41.120	53.510	69.641	90.634	117.934
	0.2	9.193	12.066	15.817	20.721	27.132	35.510	46.473	60.797	79.521	103.988	135.931
	0.3	9.664	12.761	16.822	22.149	29.139	38.307	50.344	66.129	86.830	113.974	149.527
	0.4	10.017	13.288	17.590	23.250	30.700	40.500	53.404	70.376	92.697	122.040	160.583
-2.0	0.5	10.296	13.707	18.205	24.138	31.967	42.292	55.921	73.890	97.581	128.800	169.942
	0.6	10.523	14.051	18.712	24.875	33.024	43.794	58.042	76.865	101.735	134.580	177.923
	0.7	10.712	14.338	19.138	25.497	33.921	45.075	59.858	79.425	105.318	139.574	184.830
	0.8	10.871	14.581	19.501	26.030	34.693	46.183	61.434	81.653	108.457	143.960	191.009
	0.9	11.008	14.790	19.815	26.492	35.364	47.149	62.814	83.612	111.218	147.852	196.443
	1.0	11.132	14.975	20.089	26.893	35.949	48.000	64.034	85.364	113.734	151.462	201.627
	0.1	8.040	10.265	13.085	16.664	21.206	26.969	34.296	43.591	55.388	70.358	89.345
	0.2	8.523	10.948	14.030	17.952	22.945	29.299	37.395	47.698	60.809	77.492	98.714
	0.3	8.833	11.391	14.651	18.809	24.115	30.884	39.529	50.556	64.624	82.566	105.429
	0.4	9.059	11.716	15.111	19.449	24.997	32.089	41.164	52.764	67.591	86.541	110.743
-1.0	0.5	9.232	11.969	15.471	19.954	25.698	33.052	42.480	54.550	70.005	89.784	115.104
	0.6	9.371	12.172	15.762	20.366	26.272	33.846	43.569	56.036	72.020	92.516	118.767
	0.7	9.484	12.339	16.003	20.708	26.752	34.512	44.488	57.295	73.739	94.838	121.919
	0.8	9.578	12.479	16.206	20.997	27.160	35.081	45.276	58.378	75.218	96.862	124.651
	0.9	9.657	12.597	16.379	21.245	27.510	35.572	45.958	59.319	76.509	98.617	127.044
	1.0	9.723	12.701	16.527	21.457	27.811	36.000	46.554	60.154	77.676	100.249	129.325
	0.1	7.489	9.360	11.666	14.509	18.018	22.343	27.689	34.281	42.413	52.444	64.813
	0.2	7.714	9.653	12.041	14.984	18.614	23.088	28.614	35.427	43.830	54.193	66.972
	0.3	7.828	9.801	12.230	15.223	18.912	23.461	29.078	36.002	44.542	55.071	68.055
	0.4	7.895	9.889	12.340	15.362	19.087	23.678	29.347	36.336	44.955	55.583	68.687
0.0	0.5	7.937	9.942	12.408	15.448	19.194	23.813	29.514	36.543	45.211	55.900	69.078
	0.6	7.963	9.975	12.451	15.502	19.262	23.897	29.619	36.673	45.372	56.099	69.319
	0.7	7.979	9.997	12.478	15.535	19.304	23.950	29.684	36.754	45.472	56.222	69.476
	0.8	7.989	10.009	12.494	15.555	19.329	23.980	29.722	36.801	45.531	56.294	69.563
	0.9	7.994	10.015	12.502	15.565	19.341	23.996	29.741	36.825	45.560	56.331	69.607
	1.0	8.000	10.019	12.504	15.566	19.342	24.000	29.746	36.835	45.580	56.368	69.674
	0.1	6.707	8.184	9.957	12.085	14.644	17.717	21.419	25.868	31.218	37.652	45.389
	0.2	6.549	7.952	9.627	11.627	14.018	16.877	20.304	24.402	29.307	35.177	42.204
	0.3	6.359	7.688	9.266	11.143	13.378	16.038	19.213	22.995	27.504	32.879	39.285
	0.4	6.169	7.429	8.920	10.687	12.782	15.268	18.223	21.732	25.899	30.850	36.731
1.0	0.5	5.988	7.186	8.599	10.267	12.240	14.573	17.338	20.610	24.485	29.073	34.507
	0.6	5.818	6.960	8.303	9.885	11.750	13.949	16.548	19.615	23.237	27.514	32.566
	0.7	5.658	6.751	8.032	9.536	11.305	13.387	15.840	18.728	22.131	26.140	30.862
	0.8	5.510	6.557	7.782	9.218	10.902	12.879	15.205	17.936	21.146	24.921	29.358
	0.9	5.371	6.377	7.552	8.926	10.535	12.419	14.631	17.223	20.265	23.834	28.021
	1.0	5.260	6.223	7.347	8.662	10.200	12.000	14.109	16.578	19.471	22.860	26.831
	0.1	4.876	5.976	7.303	8.897	10.809	13.091	15.809	19.028	22.839	27.354	32.715
	0.2	3.970	4.871	5.955	7.255	8.810	10.667	12.882	15.513	18.628	22.306	26.639
	0.3	3.212	3.941	4.816	5.865	7.119	8.615	10.401	12.524	15.045	18.035	21.574
	0.4	2.561	3.141	3.837	4.671	5.668	6.857	8.276	9.963	11.968	14.347	17.166
2.0	0.5	1.995	2.446	2.987	3.635	4.410	5.333	6.436	7.747	9.305	11.154	13.346
	0.6	1.498	1.836	2.242	2.727	3.308	4.000	4.826	5.809	6.976	8.362	10.006
	0.7	1.058	1.297	1.583	1.926	2.335	2.824	3.406	4.100	4.923	5.901	7.061
	0.8	0.667	0.817	0.997	1.213	1.470	1.778	2.145	2.581	3.100	3.715	4.445
	0.9	0.316	0.387	0.472	0.574	0.697	0.842	1.016	1.223	1.468	1.760	2.106
	1.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000