Design of Vibration Control and Electricity Generating Device

by

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This paper deals with a vibration control device which generates electricity by using absorbed vibration energy. This device consists of a Hula-Hoop and generator. In this paper, we researched the design of the device. As the result of the numerical calculation, the following were made clear: (1) The influences of parameters of this device, damping force related to the capacity of the generator, the length of Hula-Hoop, and moment of inertia of the Hula-Hoop on the vibration control and on generating electricity were made clear. (2) The operating curve was shown by composing the resonance curve when the Hula-Hoop rotates and that when it does not. (3) The design diagram that describes the vibration control effect of this device was shown.

Keywords: *Forced Vibration, Nonlinear Vibration, Vibration Control Device, Generation of Electricity, Hula-Hoop, Design*

1. Introduction

Although many kinds of dynamic absorbers have been used widely as equipments which control the vibrations of machines and structures[1)~3)], all the absorbed energy is thrown away as heat. Moreover, many active dampers[4)~6)] have been used in machines and structures, but they need the external energy. The aim of this research is to develop a device that generates power using the absorbed vibration energy.

The authors have researched the vibration control and electricity generating device consisting of a Hula-Hoop and a generator[7)~9)] experimentally and numerically. This device uses the principle of a synchronous rotation of a Hula-Hoop. The Hula-Hoop rotates the generator and electricity is generated. Concerning the quenching problem of self-excited vibration[7)], it is realized that vibration quenching and generation are possible using the chaotic motion of the Hula-Hoop. In the quenching problem of forced vibration[8)9)], the amplitudes near the resonance point are well controlled by the steady rotation of the Hula-Hoop. In these researches, the experimental and the numerical results were in good qualitative agreement with each other. However, since commercial direct-current motors were used as generators, the effects of vibration control and power generation were not enough. In this paper, the effect of each parameter of the device on the vibration control and generation of electricity is researched numerically, and the design of the device to control the vibration in the much wider frequency region near the resonance point is performed.

2. Analytical Model and Equation of Motion

2.1 Equation of Motion

The analytical model is shown in Fig. 1. Figs. 1 (a) and (b) are the side view and the overhead view of the model, respectively. The vibration control and electricity gener-

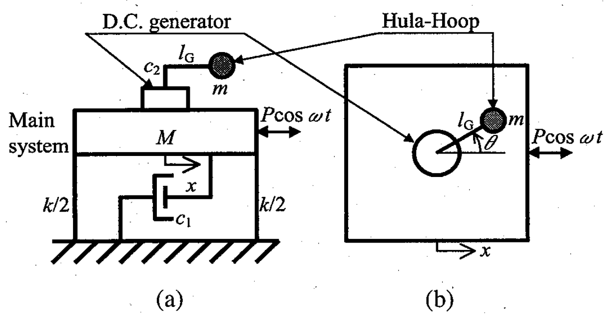

Fig. 1 Analytical model

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ating device with a Hula-Hoop and D.C. motor is attached to a single-degree-of-freedom forced vibration system. Each parameter in the system is defined as follows:

M: Mass of the main system
m : Mass of the Hula-Hoop
c_1 : viscous damping coefficient of the main system
c_2 : viscous damping coefficient of the generator
k : spring constant of the main system
θ : angular displacement of the Hula-Hoop
x : displacement of the main system
P : amplitude of the external force
l_G: length from the central axis of the generator to the center of gravity of the Hula-Hoop
I_G: moment of inertia about the center of gravity of the Hula-Hoop

Where, the damping of the generator is expressed by viscous damping. In the experiments of the previous report[8], it has become clear from the measurement result of free deceleration rotation of the generator that the damping of a generator can be approximately expressed by the viscous damping within a moderate frequency region near the resonance of the main system.

Kinetic energy T, potential energy V and dissipative energy F are shown as follows :

$$T=\frac{1}{2}(M+m)\dot{x}^2-ml_G\dot{x}\dot{\theta}\sin\theta+\frac{1}{2}(I_G+ml_G^2)\dot{\theta}^2 \quad (1)$$

$$V=\frac{1}{2}kx^2 \quad (2)$$

$$F=\frac{1}{2}c_1\dot{x}^2+\frac{1}{2}c_2\dot{\theta}^2 \quad (3)$$

Using the equation of Lagrange, the equations of motion are shown as follows.

$$(M+m)\ddot{x}+c_1\dot{x}+kx-ml_G\ddot{\theta}\sin\theta-ml_G\dot{\theta}^2\cos\theta=P\cos\omega t \quad (4)$$

$$(I_G+ml_G^2)\ddot{\theta}-ml_G\ddot{x}\sin\theta+c_2\dot{\theta}=0 \quad (5)$$

In this paper, to analyze the effects of parameters of the device on the vibration control and the electricity generation, the parameters are non-dimensionalized using the parameters of the main system as follows:

$$\mu=m/M,\ \delta=l_G/\delta_{st},\ \lambda^2=I_G/m\delta_{st}^2,$$

$$\gamma_2=c_2/M\delta_{st}^2\omega_n,\ \omega_n^2=k/M,\ \delta_{st}=P/k$$

When Eqs. (4) and (5) are formed non-dimensionally, they are shown as follows:

$$(1+\mu)y''+\gamma_1 y'+y=\mu\delta(\theta''\sin\theta+\theta'^2\cos\theta)+\cos\nu\tau \quad (6)$$

$$(\lambda^2+\delta^2)\theta''-\delta y''\sin\theta+\frac{\gamma_2}{\mu}\theta'=0 \quad (7)$$

where,

$$'=d/d\tau,\ \tau=\omega_n t,\ \nu=\omega/\omega_n,\ y=x/\delta_{st},\ \gamma_1=c_1/M\omega_n$$

2.2 Numerical Computation Method[8]

In numerical computation, the shooting method is used. An outline to calculate a periodic solution of Eqs. (6) and (7) is shown below. This method is the same as the usual shooting method[10)~12)] except for dealing with the rotational angle of the Hula-Hoop.

First, Eqs.(6) and (7) are rewritten to the nonlinear simultaneous ordinary differential equations of the first degree, relating with variables $\boldsymbol{y}={}^t(y,\theta,y',\theta')$. Here, the upper-subscript t expresses a transposition sign. The numerical integration for the equations related $\boldsymbol{y}$ is carried out for one period $\tau=T=2\pi/\nu$ from the assumed initial value $\boldsymbol{y}^0={}^t(y(0),\theta(0),y'(0),\theta'(0))$ at $\tau=0$, and the solution is set to $\boldsymbol{y}^1={}^t(y(T),\theta(T),y'(T),\theta'(T))$. Then, the conditions of a periodic solution will serve as the following equation:

$$\boldsymbol{y}^1-\boldsymbol{y}^0=\boldsymbol{0} \quad (8)$$

Considering that the Hula-Hoop rotates synchronizing with the vibration of the main system, the variable $\theta(T)$ corresponding to the rotational angle θ is replaced by $\theta(T)-2\pi j$. Here, when the Hula-Hoop rotates in the positive direction, j is 1. When it rotates in the negative direction, j is -1. Also, when it doesn't rotate and vibrate, j is 0. Moreover, considering that $\boldsymbol{y}^1$ is obtained by carrying out the numerical integration from the assumed $\boldsymbol{y}^0$, $\boldsymbol{y}^1$ is the function of $\boldsymbol{y}^0$. Therefore, obtaining a periodic solution is equivalent to finding $\boldsymbol{y}^0$ that satisfies Eq.(8). If the Newton-Raphson method is applied to Eq.(8), the equation for repetition of calculation becomes the following:

$$(\boldsymbol{B}-\boldsymbol{I}_4)\tilde{\boldsymbol{y}}=\boldsymbol{y}^0-\boldsymbol{y}^1 \quad (9)$$

where, $\boldsymbol{I}_4$ is the unit matrix of 4×4, $\tilde{\boldsymbol{y}}$ is the amount of correction for the next repetition, and $\boldsymbol{y}^0+\tilde{\boldsymbol{y}}$ is the corrected value. The matrix $\boldsymbol{B}$ is constructed by the solutions of variational equations that are calculated from the initial values, i.e., unit vectors which form a unit matrix. When the periodic solution is obtained, the stability of a periodic solution is judged as follows: If all the absolute values of the eigenvalues of matrix $\boldsymbol{B}$ are less than unity when $\boldsymbol{y}^0$ converges, the solution is judged stable. If at least one of the absolute values is larger than unity, the solution is judged unstable.

2.3 Amount of Power Generation

If the Hula-Hoop rotates, the D.C. generator also rotates and electricity is generated. However, there are energy losses of copper loss, torque loss by friction, windage loss, and so on. Electric power is lost in the resistance inside the D.C. generator. The viscous damping coefficient of the generator c_2 is defined including these losses.

In fact, the above mentioned energy losses should be considered and the efficiency of generation should be defined, if all the viscous damping energy is transformed to electric energy for simplicity, the generated energy is expressed as follow.

$$E=\frac{c_2}{T}\int_0^{2\pi/\omega}\dot{\theta}^2dt \tag{10}$$

The non-dimensional generated power e per unit time is defined as follows.

$$e=\frac{E}{M\delta_{st}^2\omega_n^3}=\frac{\nu\gamma_2}{2\pi}\int_0^{2\pi/\nu}\theta'^2d\tau \tag{11}$$

If the rotational speed of the Hula-Hoop is approximated constant, namely, $\theta'=\nu$, the following equation is obtained.

$$e=\gamma_2\nu^2 \tag{12}$$

3. Design of the Device

3.1 Characteristic of Non-dimensional Parameter

The non-dimensional parameters in the non- dimensional Eqs. (6) and (7) are μ, γ_1, γ_2, δ and λ. First, we researched the effects of each parameter on the vibration control and the power generation. Then, considering the characteristics of each parameter, we designed a device that can control the vibration of the main system in a much wider region near the resonance point and can generate a great amount of electricity.

In this paper, we treated a case when the mass ratio μ of the vibration control device to the main system is 0.03 and the damping ratio γ_1 of the main system is 0.05. Specifically, we designed the device which has a much wider region where the amplitude of the main system is less than one third of the resonance amplitude without a Hula-Hoop and which can generate a great amount of electricity. Although an appropriate objective function with weighting factors is usually used in an optimum design, we did not use this function and gave priority rather to the vibration control over the power generation.

We designed the device in the following way: A parameter was set as a variable, and other parameters were set as constants. Then, considering the effect of vibration control in the wider frequency region near the resonance point and the generated electric power, we decided the values of each parameter. We designed all parameters in this way. However, when one parameter is considered, the other parameters need to be made the optimum values.

Therefore, after many numerical calculations varying the values of the parameters γ_2, δ, and λ, the next values are obtained as optimum ones :

$$\mu=0.03,\ \gamma_1=0.05,\ \delta=25,\ \lambda=5,\ \gamma_2=0.5$$

The resonance curves which have γ_2, δ and λ as parameters are shown in Figs. 2 (a), 3 (a), and 4 (a), respectively. The enlargements near the resonance point $\nu=1$ are shown in Figs. (b) of Figs. 2~4. Here, the ordinate a is the displacement amplitude of the main system, and the abscissa ν is the ratio of the angular velocity of the external force to the natural angular frequency of the main system. In Figs.(a) and (b) of Figs. 2~4, the dashed lines are the solutions in the case without a Hula-Hoop. The solid thick lines and the dotted line are the stable solution and the unstable solution in the case of a Hula-Hoop rotating, respectively. It is well known in the nonlinear vibration system that there exist stable and unstable solutions and the former is confirmed through the experiment and the latter is not realized by the experiment. Moreover, the solutions depend on the initial conditions. In the system treated in this paper, the above mentioned stable solution is realized by giving the Hula-Hoop an initial velocity. In the region where the unstable solution exists, the Hula-Hoop does not rotate and the vibration is not controlled. As shown in the next section, there exists a narrow region of unsteady rotating solution of the Hula-Hoop that connects the upper frequency limit of the stable solution. However, it is omitted in the Figs. 2~4 for simplicity. In Figs. (c) of Fig. 2~4, the displacement amplitude of the main system $a_{|\nu=1}$ and the amount of power generation e per unit time at $\nu=1$ in the case of a Hula-Hoop rotating are shown. The ordinates are $a_{|\nu=1}$ and e, and the abscissa is the value of each parameter of the device. Moreover, the symbols ●

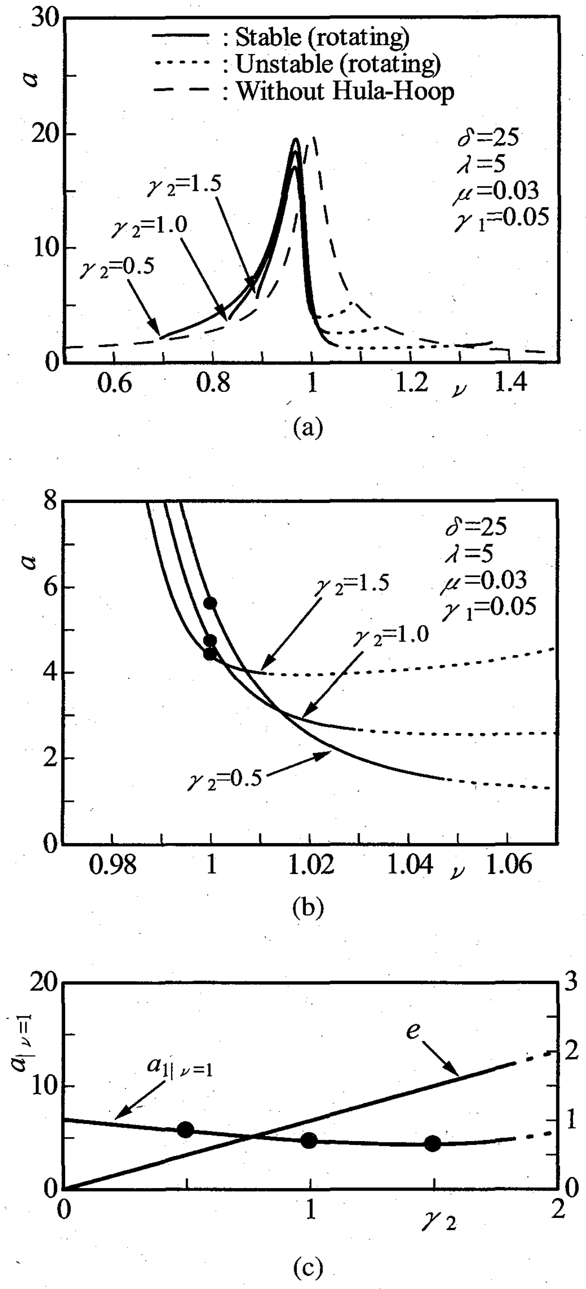


Fig. 2 Effect of damping ratio γ_2

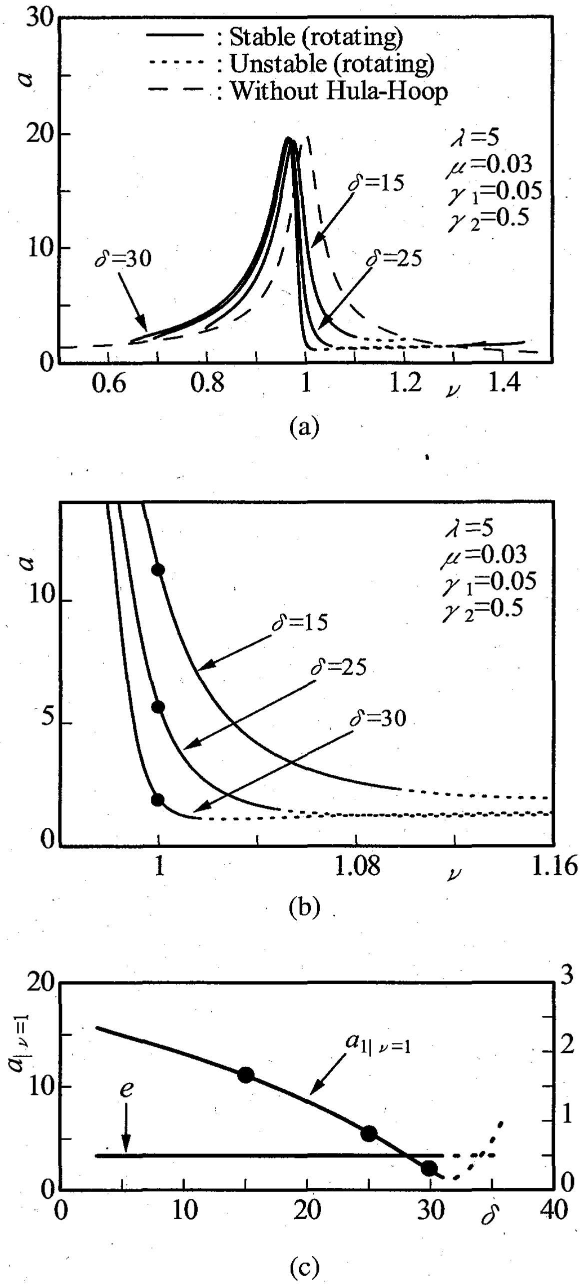


Fig. 3 Effect of length δ

in Figs. (b) and (c) of Fig. 2~4 correspond to each other.

First, we explain the effect of the damping ratio γ_2 of the generator which is related to the electric generating capacity of the generator. In Figs. 2 (b) and (c), it turns out that increasing the value of the damping ratio reduces $a_{|\nu=1}$ slightly, and lowers the upper frequency limit of a Hula-Hoop rotating. In Fig. 2 (c), the amount of generated electronic power increases in proportional to the parameter γ_2. Therefore, the value of γ_2 should be set large in order to generate greater electricity. The adequate value of γ_2 is considered to be 0.5 to make the controlled region wider near the resonance point of the main system.

Secondly, we discuss the effect of the length of the Hula-Hoop δ. From Figs. 3 (a) and (b), it turns out that increasing the value of δ greatly reduces the amplitude of the main system near the resonance point. Increasing the value δ much more makes $a_{|\nu=1}$ almost zero as shown in Fig. 3 (c). Thus, the parameter δ greatly influences the vibration of the main system directly. This is because the parameter δ appears in the right side of Eq. (6) which shows the motion of the main system. However, it is not related to the amount of power generation. Therefore, $\delta = 25$ is desirable in order to obtain

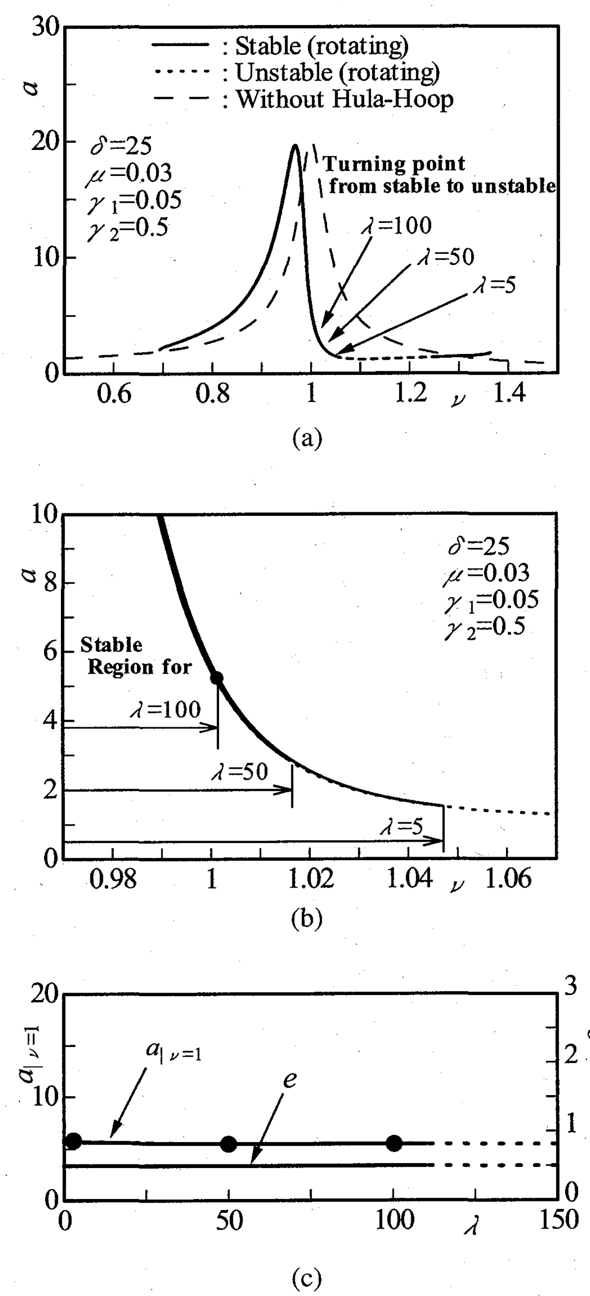


Fig. 4 Effect of moment of inertia λ

the wider controlled region with the amplitude less than one third of the resonance amplitude without a Hula-Hoop.

Lastly, we explain the effect of the non- dimensional amount of the moment of inertia about the center of gravity of Hula-Hoop λ. From the Figs. 4 (a) and (b), it turns out that increasing the value of λ doesn't change the form of the resonance curve and the value of $a_{|\nu=1}$. However, the value of λ greatly influences the region of the stable solution in Fig. 4 (b). Moreover, λ hardly influences not only the value of $a_{|\nu=1}$ but also the amount of power generation e in Fig. 4 (c). Therefore, the value of λ is set to be 5 in order to make the controlled region wide.

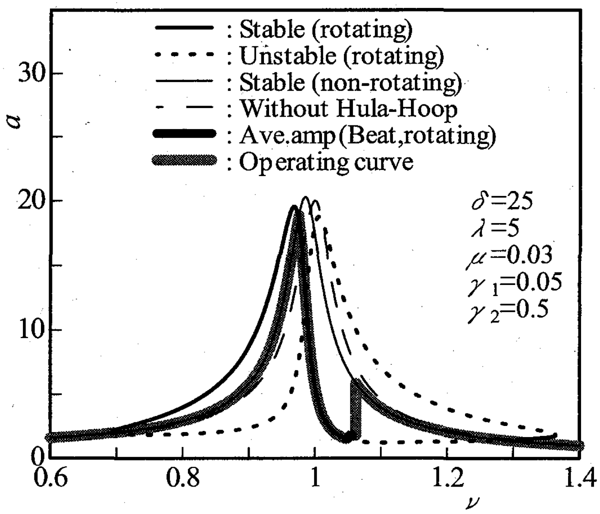


Fig. 5 Resonance curve

3.2 Resonance Curve and Bifurcation of Solution

The resonance curve in the case of attaching the vibration control and electricity generating device designed in consideration of the characteristics of each parameter is shown in Fig. 5. The ordinate a is the displacement amplitude of the main system, and the abscissa ν is the ratio of the angular velocity of the external force to the natural angular frequency of the main system. The dashed line in the Fig. 5 is the solution in the case without a Hula-Hoop, and the solid thin line is the stable solution in the case of a non-rotating Hula-Hoop. The solid thick line and the dotted line are the stable solution and the unstable solution in the case of a Hula-Hoop rotating, respectively. A gray solid line is an operation curve.

In this figure, it turns out that when the parameter γ_1 is 0.05, this device can control the amplitude less than one third of the resonance amplitude in the case without a Hula-Hoop. The controlled region is 0.997～1.047 of ν. The above mentioned controlled region is wider than the one treated before[8]. It is the strong point of this device that this device can generate power as described in the next section, and the amount of power generation also can be increased by tuning.

Moreover, the solution that the Hula-Hoop rotates without synchronizing with the external force was found. The bifurcation diagram that a stable periodic solution shifts to a unstable one is shown in Fig. 6. The Poincare sections

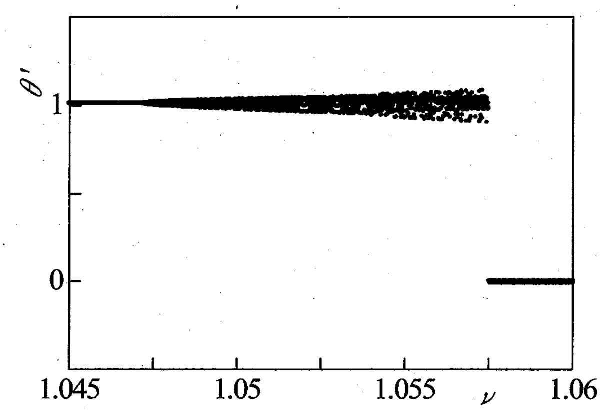
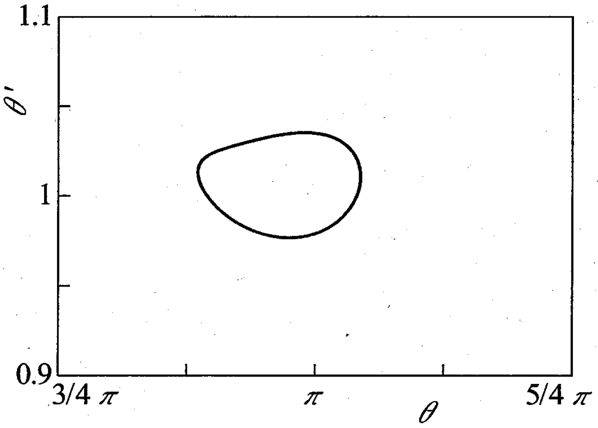
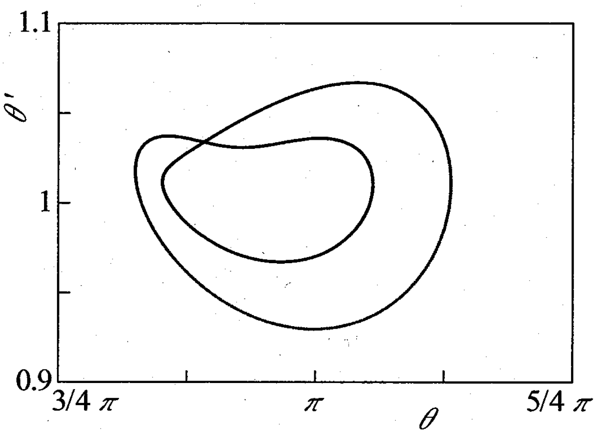


Fig. 6 Bifurcation diagram



(a) $\nu = 1.05$



(b) $\nu = 1.055$

Fig. 7 Poincare section

of the Hula-Hoop at $\nu = 1.05$ and 1.055 are shown in Figs. 7 (a) and (b), respectively. In Fig. 6, the ordinate is the non-dimensional angular velocity of a Hula-Hoop at the moment that the phase of the external force is zero, and the abscissa ν is the frequency ratio. The ordinate and the abscissa in Fig. 7 are the non-dimensional angular velocity and angle of the Hula-Hoop, respectively,

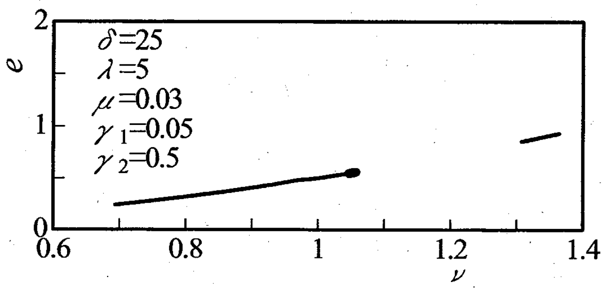


Fig. 8 Electric power

at the moment that the phase of external force is zero. The characteristic multipliers at $\nu = 1.047$ are complex and their absolute value are unity. The maximum Lyapunov exponent keeps zero until $\nu = 1.0575$. Thus, if the frequency ratio is increased, the periodic solution becomes an aperiodic solution at $\nu = 1.047$ by Hopf bifurcation. In the region of this almost periodic motion, the averaged amplitudes for 100 periods of external force are shown as the short solid thickest line in Fig. 5.

3.3 Generation of Electricity

The amount of power generation calculated numerically in the case of a Hula-Hoop rotating is shown in Fig. 8. This amount of power generation is obtained by carrying out the numerical integration of the viscous damping energy shown as Eq. (11). Also, in the situation of the almost periodic motion, the averaged amounts of power generation for 100 periods of external force are adopted. In the Fig.8, it turns out that the generated power is almost proportional to the second power of the frequency ratio ν shown as Eq. (12).

3.4 Design Diagram

In the section 3.2, the design of the device with due consideration for the characteristics of its parameters was performed to obtain a wider vibration controlled region and a greater generation of electricity. In this section, using the result, drafting the design diagram of this device according to the parameter γ_1 of the main system is performed. It will be possible to forecast the vibration control effect of the device using the design diagram. In the section 3.2, it turned out that the parameter λ hardly affected the vibration control at the resonance point of the main system. Thus, we consider the other parameters, δ and γ_2.

The vibration control effect of the device is shown in Fig. 9. The ordinate $(a_1/a_0)_{\nu=1}$ is the ratio of the resonance amplitude a1 in the case of the Hula-Hoop rotating

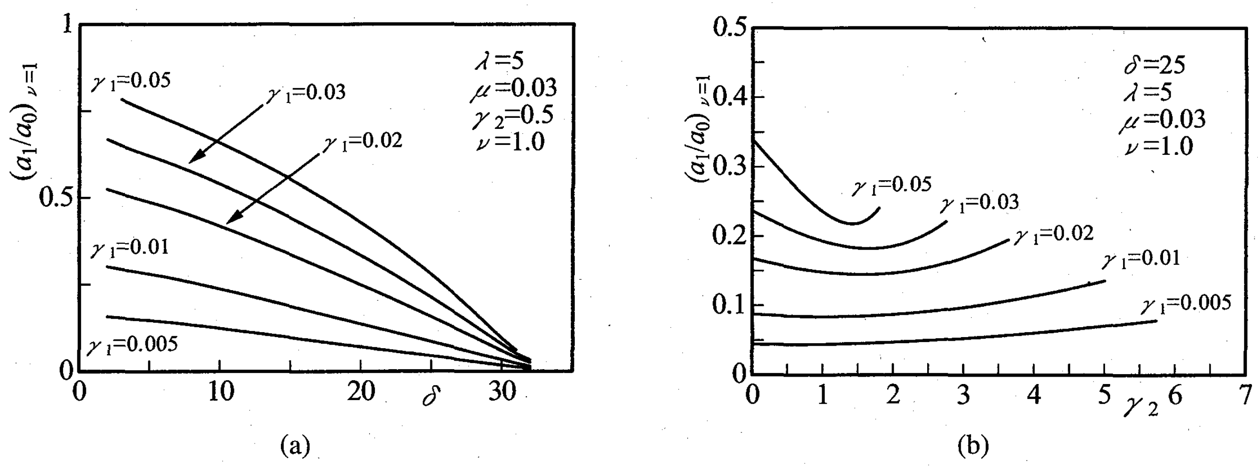


Fig. 9 Design diagram

to that a_0 without the Hula-Hoop. The abscissas of Figs. 9 (a) and (b) are δ and γ_2, respectively. In these figures., it turned out that when γ_1 is small, the vibration control effect is very large.

Using these figures, we can forecast the vibration control effect of the device with some values of δ and γ_2 at the resonance point of the main system with any damping ratio γ_1. Therefore, these figures are considered the design diagram of the device.

3.5 Application

In the application, the following is noted[9]. If only a single device is used, the component force of the centrifugal force which is perpendicular to the excitation direction acts to the main system. Therefore, if the main system is flexible in such a direction, the next measure is required. The same two devices composed of Hula-Hoops and generators must be used as one set, and the Hula-Hoops must be rotated simultaneously in opposite directions. As a result, the component forces of the centrifugal force perpendicular to the excitation direction will cancel each other.

It is thought that if this device is put into practical use, the electrical energy will be able to be used for various purposes and energy conservation will be realized.

4. Conclusion

From the numerical calculation using the shooting method, the following were made clear about the vibration control and electricity generating device.

(1) The effects of the parameters of the device on the vibration control and generation of electricity were made clear.

(2) The operating curve was shown by composing the resonance curve when the Hula-Hoop rotated and that when it did not.

(3) The design diagram of this device that forecasted the vibration control effect for any main system was shown.

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