Analytical model for grouted rock bolts

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This paper presents an analytical model of the grouted rock bolt in soft rock. The behavior of both the rock bolt system and the single rock bolt are discussed respectively. The coupling mechanism of rock bolt and rock mass is discussed from the viewpoint of displacement. A simple method is suggested for the rock bolt design in tunneling. According to the analysis, displacement of rock mass controls the initial force in rock mass. Case simulations confirm the previous findings that a bolt in-situ has a pick-up length, an anchor length and at least one neutral point. The theoretical prediction of single rock bolt agrees with the measured data. The position of the neutral point is not only related to the length of the rock bolt and the internal radius of the tunnel, but is also strongly influenced by the properties of rock mass. Neutral point and maximum axial load in the rock bolt tend to be constant when the anchor length of the bolt is sufficiently long, which means that increasing the length of the rock bolt may result in only a slight improvement in displacement control under certain conditions.

1 INTRODUCTION

In Japan, soft rock mass is often encountered during underground excavation. Rock bolting is considered to be an effective and economical means of support under various conditions. Unfortunately, the coupling mechanism has not yet been clarified. The design of rock bolts for use in tunnels or other excavations is still empirical, and in most cases, few methods exist by which to evaluate the bolting effect. Research on rock bolting includes study of the behavior of rock bolt system and modeling of the mechanism of the single rock bolt. Monitoring in field is also common in practice. B.Indraratna (1990) established the analytical model for the design of grouted rock bolts according to the Elasto-plastic constitutive law. The rock bolt and rock mass are considered as a system and isotropic behavior is assumed. Based on the strain softening constitutive law, Y.Jiang and T.Esaki (1995) have suggested another model for the rock bolt system. However, the position of the neutral point, at which the shear stress on the rock bolt is zero and the axial force of the rock bolt becomes maximum, is very important in theoretical analysis. The neutral point can only be determined by an empirical formula, which is only correct when the rock bolt is short. At the same time, no decoupling behavior is discussed in the models and it is not easy to predict the bolting effect in design. It is widely accepted that the displacement of the rock bolt is equal to that of the rock mass at the neutral point. However, the position of neutral point is not easy to determine (C.Li and B.Stillborg, 1999). Considering the coupling behavior of the rock bolt system, the bolt works compatibly with the surrounding rock mass, and the neutral point is strongly influenced by the type of rock deformation. Obviously, the mechanical properties of rock mass influence the initial mechanics of the rock bolt, especially for soft rock mass.

Studies on the single rock bolt often focus on the modeling of the interface between rock bolt and rock mass. The pullout test is usually used as a verification method (Madhav et al., 1998, N.Gurung, 2001). However, the pullout test itself may not be accurate because of the concentration of the normal stress on and the nonuniform shear stress along the rock bolt. At the same time, it is difficult to determine the initial tangent shear stiffness (k) because the tested shear deformation along the interface comprising both the inelastic deformation of the rock mass before slipping and the relative displacement between the rock mass and reinforcement during slipping. The present paper suggests an analytical method of the rock bolt system that considers the behavior of the single rock bolt in order to evaluate the performance of both a single rock bolt and a bolt system for a variety of rock conditions.

2 ROCK BOLT SYSTEM AROUND A CIRCULAR TUNNEL

2.1 *Soft rock mass behavior without rock bolt*

In the previous researches, the behavior of rock mass and bolt is discussed separately. The strain

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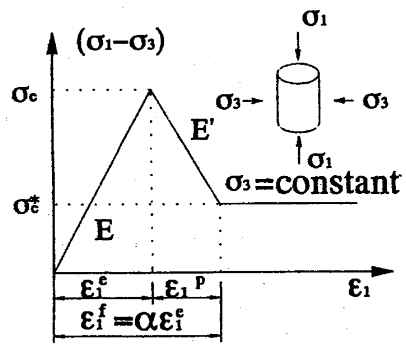
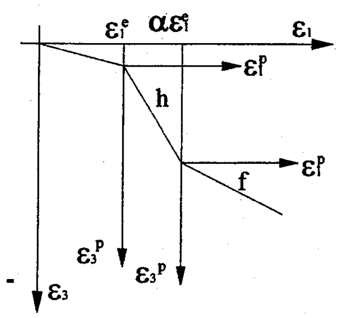
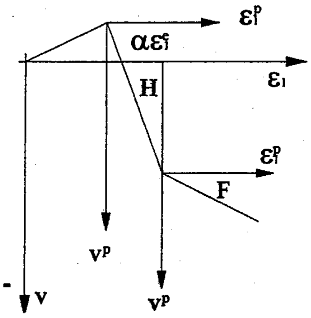
softening model has been suggested by the first author of this paper as a means for describing the behavior of soft rock mass. The constitutive law of strain softening is described in Figure 1,



(a) Simplified linear stress-strain relationship



(b) Major and minor principal strains



(c) Volumetric and major principal strains

Fig.1 Stress-strain relation and dilatancy behavior of material.

where α is the brittleness rate of the rock mass; σ_c is the axial strength of the rock mass, and σ_c* represents the strength of the rock mass under the strain softening condition. According to the Mohr-Coulomb failure criterion, we have Equation 1,

$$\sigma_t = \sigma_c + K_p \sigma_r \quad \text{(1a)}$$

$$K_p = (1+\sin\phi)/(1-\sin\phi) \quad \text{(1b)}$$

In the plastic flow zone, Equation 1 becomes

$$\sigma_t = \sigma_c^* + K_p \sigma_r \quad \text{(2)}$$

where ϕ is the internal friction angle of the rock mass. Accordingly, the soft rock mass around a tunnel may be divided into three zones: the plastic flow zone, the strain softening zone and the elastic zone, as shown in Figure 2. The outer radii of the plastic flow and strain softening zones are R_f and R_e, respectively.

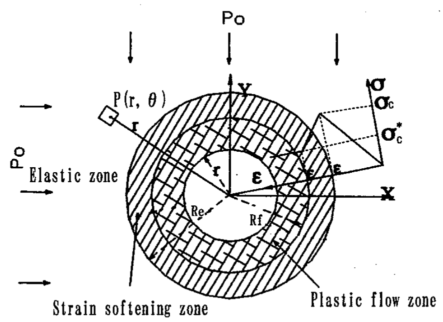


Fig.2 Tunnel excavation in soft rock mass.

The displacement formulas for each zone have been derived by Y.Jiang and T.Esaki (1997). Although the strain softening constitutive law describes the behavior of homogenous materials, it also can be used to describe the rock bolting system considering the anisotropic characteristics of the rock bolt.

2.2 *Modeling the rock bolt system*

The rock bolt system model shows the "mean performance" for both rock mass behavior and bolt performance. No decoupling is considered in the present models. Based on the assumptions of round tunnel profile, homogeneous rock mass and hydrostatic stress conditions, the theoretical formulas of the rock bolting system have been established considering the anisotropic characteristics of the rock bolt (Y.Jiang and T.Esaki, 1997). Since the direction of shear stress at the interface between the rock bolt and the rock mass is changed, the position of neutral point is critical to that obtained by the interaction model of the rock bolting system. Their relative positions are shown in Figure 3, where L is the length of the rock bolt, and Lt and Lz are the distances between rock bolts in radial direction and z direction, respectively.

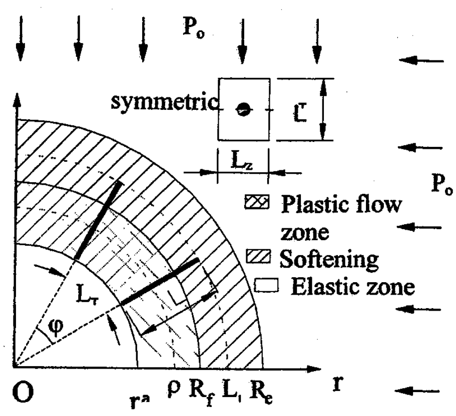


Fig. 3 Relationship among plastic radius and bolt length and neutral point.

Next, the composite element of rock bolt and rock mass is analyzed, as shown in Figure 4. The coupling equation is given as Equation 3, and the constitutive equation of the rock bolting section is given as Equation 4

$$dF = 2\pi r_b \lambda \sigma_t dr \tag{3}$$

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - (1-\beta)\sigma_t}{r} = 0 \tag{4}$$
$$\beta = 2\pi r_b \lambda r_a / L_z L_t$$

where λ is the friction coefficient between the rock bolt and the rock mass, r_a is the radius of the tunnel, σ_r and σ_t are the stresses of the rock bolting section in the radial and tangential directions respectively, and r_b is the radius of the rock bolt.

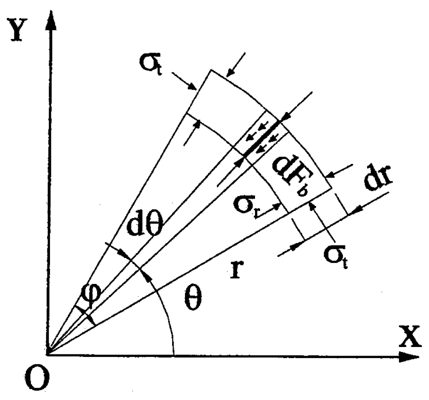


Fig.4 Equilibrium consideration for bolt-ground interaction.

Considering the positions of the neutral point and the length of rock bolt, nine cases are analyzed. The rock bolting section is shown in Figure 5. The detailed equations of stress and strain distributions in the rock bolting system have been presented by Y.Jiang and T.Esaki (1997).

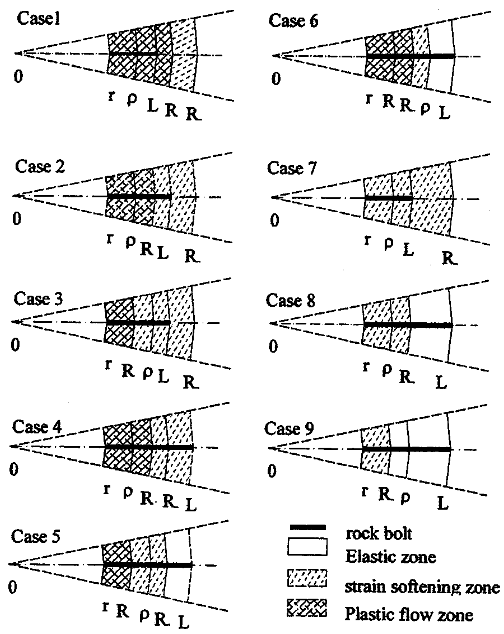


Fig.5 Analytical cases of the bolt-ground interaction.

3 INTERACTION MODEL OF SINGLE ROCK BOLT AND MATRIX

3.1 *Coupling behavior of rock bolt and rock mass*

The interaction between the rock mass and the grouted bolts involves complex mechanics, and there have been few reports on the subject which can explain in theory the satisfactory results obtained using grouted rock bolts. A coupling model has been suggested to describe the behavior of a single rock bolt based on the shear-lag model (SLM) or fiber-loading theory. The SLM was originally developed by H.Cox (1952) and has been widely used by material scientists and structural geologists as a powerful analytical method. The concept of the suggested model is described in Figure 6. According to the balance conditions between the infinitesimal reinforcement element, the surrounding matrix and the matrix with reinforcement in cylinder coordinate system, the basic constitutive law can be expressed as Equation 5, and the assumption of SLM is expressed as Equation 6. The model is discussed in detail elsewhere (Y.Cai et al, 2003).

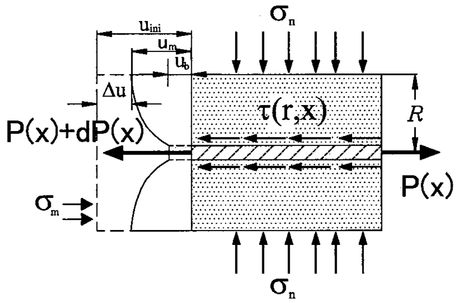


Fig. 6 Coupling behavior of reinforcement and rock mass.

$$\frac{dP(x)}{dx} = -2\pi r_b \tau_b \qquad (5a)$$

$$\frac{\partial \sigma_m(r,x)}{\partial x} + \frac{\partial \tau(r,x)}{\partial r} + \frac{\tau(r,x)}{r} = 0 \qquad (5b)$$

$$\frac{dP(x)}{dx} + 2\pi \int_{r_b}^{R} r \frac{d}{dx} \sigma_m(r,x) dr = 0 \qquad (5c)$$

$$\frac{dP(x)}{dx} = H(u_b - u_m) \qquad (6)$$

where *P(x)* is pullout force at *x*, R is the influence radius, σ_n is the normal stress perpendicular to reinforcement, τ(r,x) is the shear stress at (r,x) (as shown in Figure 6), u_b and u_m are the displacement of reinforcement and matrix at the edge of influence radius *R*, and *H* is the material parameter. Different stress distributions result in different models and parameters. In order to simplify the analysis and satisfy the boundary conditions, a uniform stress distribution is assumed in the influence area of the reinforcement. Correspondingly, parameter *H* is expressed as Equation 7.

$$H = \frac{2\pi G_g G_m}{[\ln(R/r_b) - 1/2]G_g + \ln(r_g/r_b)G_m} \qquad (7)$$

where G_g and r_g are the shear modulus and radius of grout, respectively, and G_m is the shear modulus of the rock mass. The in-situ mechanical properties of the rock mass are very complicated and Equation 5 provides a simple method of evaluating the coupling properties of the rock bolt and rock mass.

3.2 *Decoupling behavior of single rock bolt and rock mass*

Decoupling influences the bolting effect dramatically while it has not been considered in the general models of the rock bolt. Different types of rock bolt have different decoupling behaviors. Only the grouted rock bolt is discussed herein. The shear strength at the interface is made up of three components: adhesion, interlock and friction in the axial direction. These three components are lost in sequence if no deformation arose along the interface. After decoupling, the shear stress at the interface becomes to the residual strength at slipping section. As a result, rock bolt behavior varies according to the residual shear stress. L.Malvar (1992) and M.Moosavi (2001) revealed that the confining pressure influences the strength of the interface dramatically. Although the full mechanism of bond failure during axial slip in a deformed bar can be explained by the shearing mechanism in the cement annulus (Moosavi, 2001), failure may occur at the bolt-grout interface, in the grout medium or grout-rock interface, or in the rock mass. The shear strength can be expressed as

$$\tau_m = c + \sigma_{nb} \tan \phi^* \qquad (8)$$

where ϕ^* and c are the friction angle and cohesive force of interface, respectively, which can be evaluated by the direct shear or pullout tests, and σ_{nb} is the normal stress perpendicular to the rock bolt.

4 APPLICATION OF THE PROPOSED MODEL

The coupling behavior of the rock bolt and rock mass can be described by Equations 5 and 6 for different boundary conditions. Considering the effect of the rock bolt system, the strain of rock mass at the edge of influence radius *R* can be expressed as

$$\varepsilon_m = \varepsilon_{ini} - \Delta\varepsilon_m \qquad (9a)$$

$$\Delta\varepsilon_m = P/(SE_m) \qquad (9b)$$

$$\frac{d^2P(x)}{dx^2} = H\left[\frac{P(x)}{E_b A_b} - \varepsilon_m\right] \qquad (9c)$$

where ε_{ini} is the rock mass strain without the bolt, E_m is the deformation modulus of the rock mass, and *S* is the influence area of a single rock bolt. The initial strain of the rock mass determines the axial force of the rock bolt. Since the displacement function of the rock mass around a circular tunnel is relatively complicated, it is not easy to obtain a theoretical expression for the stress distribution or the position of the neutral point. Numerical methods are favorable in this case. Therefore, the neutral point for rock bolts installed around a tunnel can be obtained using Equation 7 together with the boundary conditions. Ignoring the effect of external fixtures, the boundary conditions of the rock bolt can be written as *P(0)*=0 and *P(L)*=0.

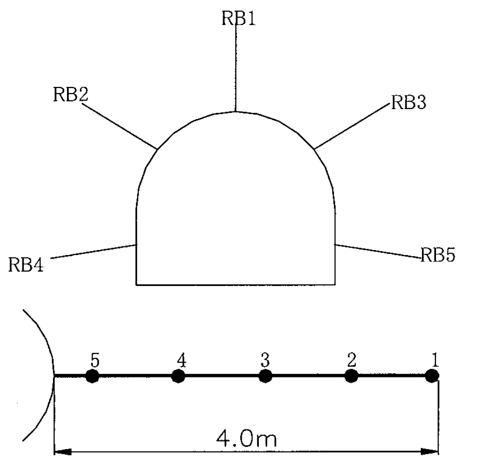
Axial load measurement of rock bolts in the field was performed at the Holland slope Tunnel in

Nagasaki. The tunnel is situated at a depth of 18.35 m. The rock mass around the tunnel is classified as DI, which is a type of soft rock. The positions of the measured rock bolts are shown in Figure 7. The rock mass and rock bolt parameters are listed in Table 1.

Table 1 Parameters of rock bolt and rock mass.

Radius of Tunnel	4.75 m
Hydraulic water pressure, po	0.38 MPa
Axial strength of rock mass	0.5 MPa
Deformation modulus of rock mass	1.0 GPa
Poisson's ratio of rock mass	0.25
Length of rock bolt	4.0 m
Young's modulus of rock bolt	210 GPa
Radius of rock bolt	12.7 mm
Distance between rock bolt (Lz × Lt)	0.2 m × 1.4 m
Shear strength on interface	0.35 Mpa

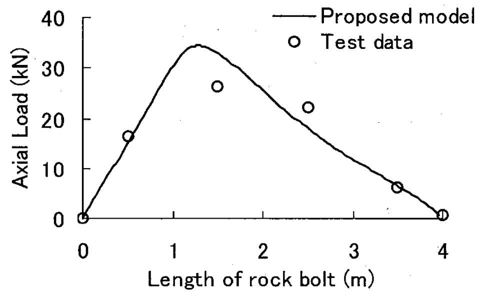
Comparison of the calculated results and the measured data for a single rock bolt is presented in Figure 8. Theoretical predictions agree well with the measurements. Grouted rock bolts installed around a tunnel in soft rock mass can be divided into three lengths: pick-up length, neutral point and anchor length. This also agrees with the in-situ conditions. The neutral point in this case is 1.26 m from the end of the rock bolt.



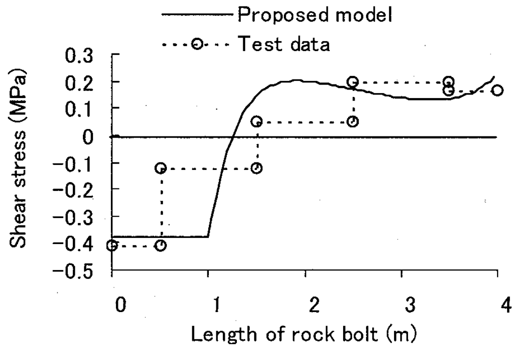
Distance from tunnel wall, 0.5m,1.5m,2.5m,3.5m,4.0m

Fig. 7 Test arrangement of rock bolts in Holland Slope Tunnel (No.68+28, Section C).

Since the mechanical properties of rock mass are not easy to obtain in practice, discussion is needed in order to determine the inclination of the neutral point. Z.Tao and J.Chen (1984) have suggested Equation 10 for calculating the neutral point of the grouted rock bolt.



(a) Axial force along rock bolt



(b) Shear stress along rock bolt

Fig.8 Theoretical prediction for a single rock bolt.

$$\rho = L/(\ln[1+(L/r_a)] \qquad L = (40r_b \sim 60r_b) \tag{10}$$

where L is the length of the rock bolt, r_a is the radius of the tunnel, r_b is the radius of the rock bolt, and ρ is radius of the neutral point.

In the range of deformation modulus of the rock bolt from 0.50 GPa to 5.0 GPa, the other parameters remain the same as those in Table 1, and the neutral point changes from 0.74 m to 0.38 m toward tunnel wall, as listed in Table 2. As the length of the rock bolt changes from 0.80 m to 5.0 m, the neutral point changes from 0.37 m to 2.20 m, as listed in Table 3.

Table 2 Neutral points for rock bolts of various deformation modulus.

E_m (GPa)	0.50	1.00	2.00	3.00	5.00
Proposed model	0.74	0.62	0.5	0.46	0.38
Tao and Chen	1.80	1.80	1.80	1.80	1.80

Table 3 Neutral points for rock bolts of different length.

L(m)	0.80	1.00	2.00	4.00	5.00
Proposed model	0.37	0.45	0.61	0.64	0.66
Tao and Chen	0.39	0.48	0.94	1.80	2.20

According to the above example, when the rock bolt is relatively short, the neutral point obtained by the proposed model is approximately equal to that of Z.Tao and J.Chen (1984). However, according to the model proposed herein, the neutral point tends to be constant if the length of the rock bolt exceeds 2.0 m.

5 CONCLUSIONS

An analytical model for the rock bolt has been proposed. The rock bolting system behavior with decoupling can be described by the strain softening constitutive law according to the placement of the neutral position. The coupling and decoupling behavior of the grouted reinforcement system has also been discussed. Theoretical predictions of axial force and shear stress distribution agree well with the field data for the Holland Slope Tunnel in Nagasaki.

According to the proposed model, it is clarified that the neutral point of the rock bolt is related to the displacement in the rock mass around the tunnel. Correspondingly, the axial force of a single rock bolt and the shear stress at the interface between bolt and rock mass are significantly influenced by the mechanical properties of rock mass and the boundary conditions. Decoupling takes place when the shear stress exceeds the strength of the interface medium or the rock mass. The position of the neutral point becomes constant when the length of the rock bolt exceeds a certain value without decoupling.

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