Bending Analysis of Rectangular Plates with Variable Thickness

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Introduction

This paper presents an approximate solution of partial differential equations concerning the bending of plates with variable thickness, various edge conditions and various load conditions. The approximate solution is obtained by converting the differential equations into integral equations and applying numerical integration. From the results of numerical works, it is confirmed that the approximate solution is accurate enough for practical usage.

Basic Equation

The basic equations of bending of plates are given as follows.

$$\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}+q=0 \quad \cdots\cdots (1.a)$$

$$\frac{\partial M_x}{\partial x}+\frac{\partial M_{xy}}{\partial y}-Q_x=0 \quad \cdots\cdots (1.b)$$

$$\frac{\partial M_y}{\partial y}+\frac{\partial M_{xy}}{\partial x}-Q_y=0 \quad \cdots\cdots (1.c)$$

$$\frac{\partial \theta_x}{\partial x}+\nu\frac{\partial \theta_y}{\partial y}=\frac{M_x}{D} \quad \cdots\cdots (1.d)$$

$$\frac{\partial \theta_y}{\partial y}+\nu\frac{\partial \theta_x}{\partial x}=\frac{M_y}{D} \quad \cdots\cdots (1.e)$$

$$\frac{\partial \theta_x}{\partial y}+\frac{\partial \theta_y}{\partial x}=\frac{2}{D(1-\nu)}M_{xy} \quad \cdots\cdots (1.f)$$

$$\frac{\partial w}{\partial x}+\theta_x=\frac{Q_x}{Gt_s} \quad \cdots\cdots (1.g)$$

$$\frac{\partial w}{\partial y}+\theta_y=\frac{Q_y}{Gt_s} \quad \cdots\cdots (1.h)$$

where Q_x and Q_y are shearing forces, M_{xy} twisting moment, M_x, M_y bending moments, θ_x, θ_y deflection angles, w deflection, q the intensity of distributed load, E the modulus of elasticity in tension and compression, G the modulus of elasticity in shear, ν the Poisson's ratio, D the flexural rigidity of plate, $t_s=h/1.2$ and h the thickness of plate.

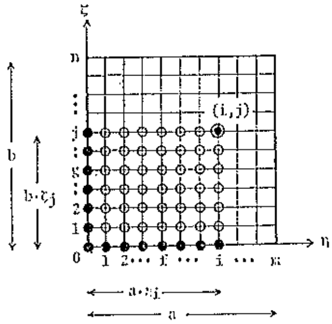


Fig. 1 Discrete points on rectangular plate.

These are used to analyse the bending behaviour of rectangular plates with variable thickness, various edge conditions and various load conditions.

Approximate Solution

The approximate solutions of basic differential equations are obtained, by converting the differential equations into integral equations and applying numerical integration, concerning the discrete points on a plate shown in Fig. 1. The results are as follows.

$$X_{pij}=\sum_{d=1}^{6}\left(\sum_{f=0}^{i} a_{1pijfd}\cdot X_{rfo} + \sum_{g=0}^{j} a_{2pijgd}\cdot X_{sog}\right)+q_{pij} \quad \cdots\cdots (2)$$

where

$$X_1=\frac{a^2}{D_0(1-\nu^2)}Q_y, \quad X_2=\frac{a^2}{D_0(1-\nu^2)}Q_x,$$

$$X_3=\frac{a}{D_0(1-\nu^2)}M_{xy}, \quad X_4=\frac{a}{D_0(1-\nu^2)}M_y,$$

$$X_5=\frac{a}{D_0(1-\nu^2)}M_x, \quad X_6=\theta_y, \quad X_7=\theta_x,$$

$$X_8=\frac{w}{a} \cdots\cdots$$

The integral constants X_{rfo} and X_{sog} in the approximate solutions (2) mean the shearing forces, twisting moment, bending moments, deflection angles and deflection along the two edges of a rectangular plate. The numerical value of integral constants are determined by using the boundary conditions of two opposite edges.

Integral Constant and Boundary Condition

For examples, the integral constants and boundary conditions of plates with four simply supported edges and with four fixed edges are

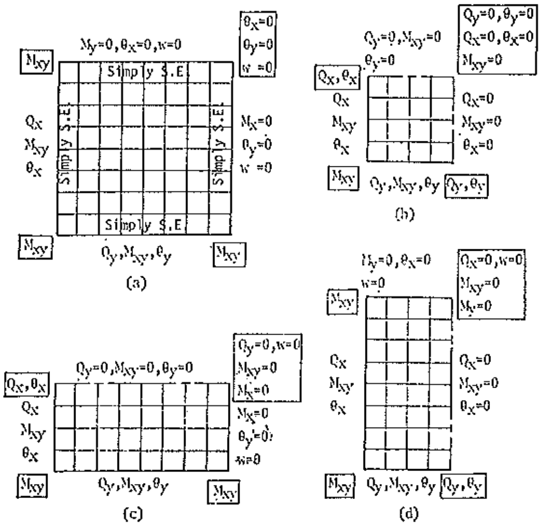


Fig. 2 Integral constants and boundary conditions of plate with four simply supported edges.

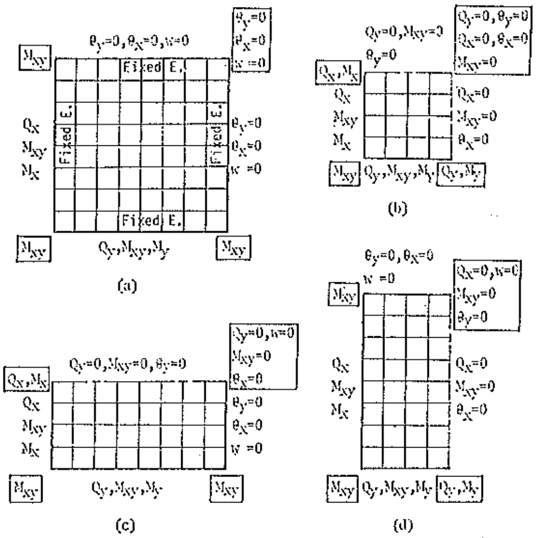
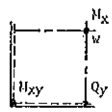


Fig. 3 Integral constants and boundary conditions of plate with four fixed edges.

Table 1 Convergency of numerical solutions of plate with four simply supported edges under uniform load ($\nu=0.3$).



m	Q_y/qa	M_{xy}/qa^2		M_x/qa^2		$w/qa^4/D$		
		Author	Kurata	Author	Kurata	Author	Kurata	Kubo
4	0.334	−0.0340	−0.0308	0.0542	0.0472	0.00412	0.00400	0.00394
8	0.336	−0.0328	−0.0319	0.0492	0.0479	0.00408	0.00406	0.00407
12	0.337	−0.0325	−0.0321	0.0484	0.0479	0.00407	0.00406	0.00408
16	0.337	−0.0325	−0.0323	0.0482	0.0499	0.00407	0.00406	0.00408
20	0.338	−0.0325	−0.0324	0.0481	0.0479	0.00407	0.00406	
N.A.S.	0.338	−0.0325		0.0479		0.00406		

N.A.S. : Navier's Analytical Solution

shown in Fig. 2 and Fig. 3.

Numerical Result

For an example of numerical results, Table 1 shows the convergency of approximate solutions of a square plate with four simply supported edges under uniform load.