Geometrically and Materially Nonlinear Analysis of Nonprismatic Arches of Any Shape

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Published in the Proc. of JSCE, No. 310, June 1981, pp. 11–20 (In Japanese).

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Introduction

In this paper a semi-analytical method is developed to study the in-plane geometrical and material nonlinearity of nonprismatic arches of any shape. The incremental differential equations of an arch are translated into the integral equations, and by applying the approximate solution of integral equation semi-analytical solutions of the original differential equations are obtained. The solutions have discrete type expressions concerned with the equally spaced points of arch axis. The method is applied to the analysis of stability limit loads, inelastic and elastic bifurcation buckling loads of two hinged, parabolic and circular arches with pipe section, by considering the effects of initial displacement and residual stress.

Basic Equation

The incremental differential equations which analyze the geometrical and material nonlinearity of arches with any shape can be derived from the equilibrium equations of the deformed arch element. In case that the load retains its original direction like the dead load of an arch bridge, the basic equations for the incremental loads Δp, Δq and Δm are derived as follows.

$$d\Delta Q/ds+\Delta N/R+N\cdot d\Delta\theta/ds-q\cdot\Delta\theta+(\Delta p+\Delta p_0)=0 \quad \cdots\cdots(1\cdot\mathrm{a})$$

$$d\Delta N/ds-\Delta Q/R-Q\cdot d\Delta\theta/ds+p\cdot\Delta\theta+(\Delta q+\Delta q_0)=0 \quad \cdots\cdots(1\cdot\mathrm{b})$$

$$d\Delta M/ds-\Delta Q-Q\cdot\Delta\epsilon_0-(\Delta m+\Delta m_0)=0 \quad \cdots(1\cdot\mathrm{c})$$

$$\Delta N=\beta EA\cdot\Delta\epsilon_0+\gamma EI/L\cdot\Delta K \quad \cdots\cdots(1\cdot\mathrm{d})$$

$$\Delta M=-\alpha EI\cdot\Delta K-\gamma EI/L\cdot\Delta\epsilon_0 \quad \cdots\cdots(1\cdot\mathrm{e})$$

$$\Delta\theta=d\Delta u/ds+\Delta w/R \quad \cdots\cdots(1\cdot\mathrm{f})$$

where

$$\Delta K=d\Delta\theta/ds-\Delta\epsilon_0/R, \quad \Delta\epsilon_0=d\Delta w/ds-\Delta u/R$$

$$\alpha=\int E_t y^2 dA/EI, \quad \beta=\int E_t dA/EA,$$

$$\gamma=L\int E_t y dA/EI$$

$$\Delta p_0=\Delta N\cdot d\Delta\theta/ds-\Delta q\Delta\theta+(p+\Delta p)\cdot(\cos\Delta\theta-1)-(q+\Delta q)(\sin\Delta\theta-\Delta\theta)$$

$$\Delta q_0=-\Delta Q\cdot d\Delta\theta/ds+\Delta p\Delta\theta+(p+\Delta p)\cdot(\sin\Delta\theta-\Delta\theta)+(q+\Delta q)(\cos\Delta\theta-1)$$

$$\Delta m_0=\Delta Q\Delta\epsilon_0$$

Semi-analytical Solution of Incremental Equations

For the convenience of numerical works, following nondimensional variables are introduced.

$$X_1=-L^2\Delta Q/EI_0, \quad X_2=-L^2\Delta N/EI_0, \quad X_3=-L\Delta M/EI_0, \quad X_4=\Delta\theta, \quad X_5=\Delta w/L, \quad X_6=\Delta u/L, \quad \eta=s/l$$

By using the nondimensional variables X_t, the differential equations $(1\cdot a)-(1\cdot f)$ are rearranged together as follows.

$$dX_t/d\eta=\sum_{k=1}^{7} G_{tk}\cdot X_k \quad (t=1\sim 6,\ X_7=1)\cdots\cdots(2)$$

The semianalytical solutions of differential equations (2) concerning with the discrete point i which is one of the equally spaced points on arch axis shown in Fig. 1 are given by

$$X_{ti}=\sum_{n=1}^{7} d_{tni}\cdot X_{n0} \quad (i=1\sim m,\ X_{70}=1)\cdots\cdots(3)$$

where

$$d_{tni}=\delta_{tn}+\sum_{j=0}^{i}\sum_{k=1}^{7}\beta_{ij}G_{tkj}d_{knj}, \quad d_{7nj}=\delta_{n7}$$

$\beta_{ij}=\alpha_{ij}/24m$, δ_{tn}: Kronecker delta

α_{ij}: weight coefficient of numerical integration

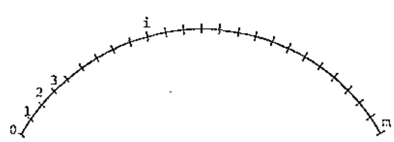

Fig. 1 Discrete points on arch axis.

Numerical Results

(1) Comparison with Existing Experimental Results

Fig. 2 shows a comparison of the calculated stability limit loads with the experimental results which were obtained by Shinke, Zui and Namita. The calculated results from the proposed semi-analytical method are in good agreement with the existing experimental results. In addition,

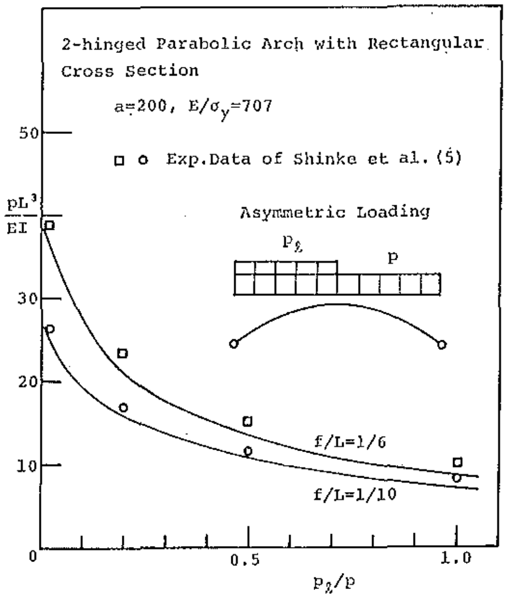

Fig. 2 A compression between experimental results and theoretical ones.

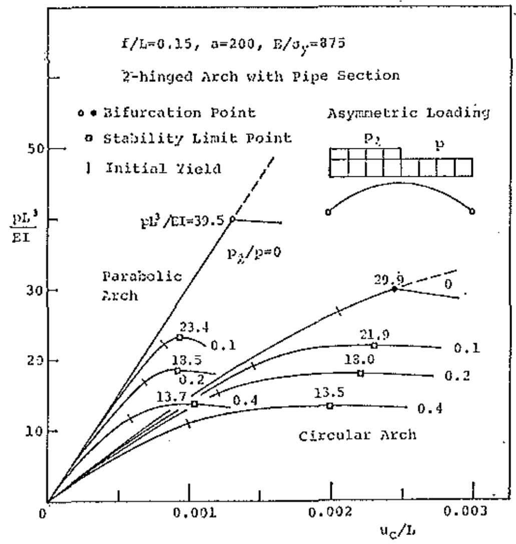

Fig. 3 Load-deflection curves.

it has been made sure that the semi-analytical solutions (3) converge satisfactorily by setting 21 equally spaced points on the arch axis.

(2) In-plane Load Carrying Capacity of 2-hinged Arch with Pipe Section

The load-deflection curves have been calculated for parabolic and circular arches from the semi-analytical solution (3), and the results are shown in Fig. 3. When the ratio p_l/p is equal to zero, the parabolic arch is stressed in pure compression and the nonlinearity between load and deflection is not recognized before its elastic bifurcation buckling. On the other hand, the circular arch is stressed in combined compression and bending, and inelastic bifurcation buckling occurs, when p_l/p is equal to zero. When p_l/p is not equal to zero, both arches are stressed in combined compression and bending, and they reach the inelastic unstable state according to the increasing of the load.