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An extended table of the Fourier coefficients of Eisenstein series of degree two.

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§ 1. Introduction

In [9] Resnikoff and Saldaña calculated the Fourier coefficients $a_k(T)$ of Eisenstein series of degree two and of weight k for the positive definite semi-integral symmetric matrices T. However their table is mainly confined to the case when k=4 and T has the from

$$T=\begin{pmatrix} m & 1/2 \\ 1/2 & 1 \end{pmatrix} \text{ with integral m such that } 1 \leqq m \leqq 130 \text{ or}$$

$$T=\begin{pmatrix} m & 0 \\ 0 & 1 \end{pmatrix} \text{ with integral m such that } 1 \leqq m \leqq 399$$

Besides the above case some other few cases are treated. Viewing the more matured present state of the arithmetical investigations of Siegel modular forms of degree two it would be desirable to make a further table of the same kind.

On the other hand in [3] H.Maass already gave explicit formulas for the Fourier coefficients $a_k(T)$ of Eisenstein series of degree two and of even weight k for the primitive binary T, and in [5] he also gave a recursion formula expressing $a_k(pT)$ by other quantities, where p is any prime number. As the "explicit" formula for $a_k(T)$ Maass formula is still involved. A result in [7] a little (but not "completely") diminishes this defect. In this paper we shall give general and explicit formulas for $a_k(T)$ mentioned above under a restriction on T. Using this formula together with a result in [6] we have obtained the values of $a_k(T)$ with k=4, 6, 10 and 12 and $\det(2T) \leqq 100$. Igusa's structure theorem [1] tells us that it is sufficient to know the Fourier coefficients $a_k(T)$ of Eisenstein series of degree two and of weight k=4, 6, 10 and 12 to obtain the whole information of the Fourier coefficients of Siegel modular forms of even weights.

§ 2. Explicit formulas for a_k (T)

We only give a brief sketch of the process. As general references one may consult [3], [4] or [11]. The normalized Eisenstein series $E_k(Z)$ of degree 2 and of weight k is expanded as

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$$E_k(Z) = \sum_T a_k(T)\, e^{2\pi i \sigma(TZ)}$$

where T runs over the set Q_2 of the positive semi-definite semi-integral symmetric binary matrices. Thanks to the work of Siegel [11] if T is of rank 2 $a_k(T)$ for even k is expressed by the formula

$$(1)\quad a_k(T) = (-1)^k\ 2^{2(k-1/2)}\ |T|^{k-3/2} \times \frac{\pi^{2k-1/2}}{\Gamma(k)\,\Gamma(k-1/2)} \prod_p S_p,$$

where $|T|$ is the determinant of T. In the above formula $\Gamma(k)$ is the value of gamma function at k and S_p is the p-adic density determined according to T.
The formula (1) is easily transformed into

$$(2)\quad a_k(T) = (-1)^k\,|2T|^{k-3/2}\ 2^{2k-1} \times \frac{\pi^{2k-1}}{(2k-1)!} \times \prod_p S_p$$

As Siegel [11] shows if the prime p does not divide $2\,|2T|$, then S_p is given by

$$S_p = (1-p^{-k})\,(1-p^{2-2k})\ /\ (1-\varepsilon p^{1-k})\ .$$

where $\varepsilon=\left(\frac{-|2T|}{p}\right)$ and $\left(\frac{*}{p}\right)$ is the Legendre's symbol. With this fact the formula (2) is transformed into

$$(3)\quad a_k(T) = -(|2T|\ /\ |d|)^{k-3/2} \times \frac{4k}{|d|\,B_k\ B_{2k-2}} \times \prod_p c_p(k) \times \sum_{q=1}^{|d|-1} \left(\frac{d}{q}\right)(q+|d|B)^{k-1}$$

We explain the notations used in (3). The quantity d is the discriminant of the imaginary quadratic field $Q(\sqrt{-|2T|})$ and $|d|$ is its absolute value. B_k is the k-th Bernoulli number and first few values of it are given by

$B_2 = 1/6,\ B_4 = -1/30,\ B_6 = 1/42,$

$B_8 = -1/30,\ B_{10} = 5/66,\ B_{12} = -691/2730$

and $B_m = 0$ for odd m.

The symbol $(q+|d|B)^{k-1}$ is the Bernoullian polynomial defined by

$$(q+|d|B)^{k-1} = \sum_{n=0}^{k=1} \binom{k-1}{n} q^{k-1-n}\ |d|^n\ B_n$$

$c_p(k)$ is the quotient $S_p(1-\varepsilon p^{1-k})/(1-p^{-k})(1-p^{2-2k})$, where p is any prime dividing $|2T|\,/\,|d|$ and $\varepsilon=\left(\frac{d}{p}\right)$.
The formula (3) is a slight modification of the formula given by Maass [3]. If we can give the concrete shape of $c_p(k)$ for each p dividing $|2T|\,/\,|d|$, we can say that the formula for $a_k(T)$ is made completely explicit. As Maass [3] shows the quantity S_p or $c_p(k)$ does not change if we replace T in Q_2 by another T' in Q_2 which is equivalent to T over the ring Z_p of p-adic integers. By this fact it is sufficient to give the shape of $c_p(k)$ for the p-adically canonical form of T in Q_2. By the local classification theory of the integral quadratic forms (see for example [2]) any 2T with T in Q_2 such that the rank of T is two is classified over Z_p as

$2T \cong \mathrm{diag}\,(u_1 p^r,\ u_2 p^s)$ over $Z_p (p \neq 2)$,

$$2T \cong \begin{cases} \mathrm{diag}\,(u_1 2^{r+1},\ u_2 2^{s+1}) & \text{or} \\ 2^r \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} & \text{or} \qquad \text{over } \mathbb{Z}_2 \\ 2^r \begin{pmatrix} 2 & 1 \\ 1 & 4 \end{pmatrix} & . \end{cases}$$

In the above relations the symbol " $\cong$ " means that the integral equivalence of quadratic matrices over $\mathbb{Z}_p$, and r and s are non-negative integers such that $r \leqq s$ and u_1 and u_2 are the units in $\mathbb{Z}_p$. As a direct consequence of one of the authors' result in [7] when $p \neq 2$ and $2T \cong \mathrm{diag}\,(u_1 p^r, u_2 p^s)$ over $\mathbb{Z}_p$ the shape of $c_p(k)$ is given by

(4) $c_p(k)$

$$= \sum_{\mu=0}^{(s-1)/2} p^{(3-2k)\mu} \qquad \text{if } r=0,\ s>0 \text{ and } s \equiv 1 \pmod 2,$$

$$= (1-\beta p^{1-k}) \sum_{\mu=0}^{s/2-1} p^{(3-2k)\mu} + p^{(3-2k)s/2}$$

if $r=0$, $s < 0$ and $s \equiv 0 \pmod 2$,

$$\begin{aligned} &= (1-\beta p^{1-k})\, p^{(2-k)(r-1)} + (1-\beta p^{1-k}) \sum_{\lambda=0}^{(r-3)/2} p^{(4-2k)\lambda} \left[\sum_{\mu=0}^{r-1-2\lambda} p^{(3-2k)\mu}\right] \\ &+ p^{2-k}(1-\beta p^{1-k}) \sum_{\lambda=0}^{(r-3)/2} p^{(4-2k)\lambda} \left[\sum_{\mu=0}^{r-2-2\lambda} p^{(3-2k)\mu}\right] \\ &+ p^{(1-2r)k+3r-1}(1+p^{1-k}) \left[\sum_{\mu=0}^{(r-1)/2} p^{(2k-2)\mu}\right]\left[\sum_{\mu=0}^{(s-r)/2} p^{(3-2k)\mu}\right] \\ &- \beta p^{r(3-2k)}(1+p^{1-k}) \left[\sum_{\mu=0}^{(r-1)/2} p^{(2k-2)\mu}\right]\left[\sum_{\mu=0}^{(s-r-2)/2} p^{(3-2k)\mu}\right] \end{aligned}$$

if $s \geqq r > 0$ and $s \equiv r \equiv 1 \pmod 2$,

$$\begin{aligned} &= p^{(r-1)(2-k)} + \sum_{\lambda=0}^{(r-3)/2} p^{(4-2k)\lambda} \left[\sum_{\mu=0}^{r-1-2\lambda} p^{(3-2k)\mu}\right] \\ &+ p^{2-k} \sum_{\lambda=0}^{(r-3)/2} p^{(4-2k)\lambda} \left[\sum_{\mu=0}^{r-2-2\lambda} p^{(3-2k)\mu}\right] \\ &+ \{p^{r(3-2k)} + p^{(1-2r)k+3r-1}\} \left[\sum_{\mu=0}^{(r-1)/2} p^{(2k-2)\mu}\right]\left[\sum_{\mu=0}^{(s-r-1)/2} p^{(3-2k)\mu}\right] \end{aligned}$$

if $s \geqq r+1 > 0$ and $s \equiv 0$, $r \equiv 1 \pmod 2$,

$$\begin{aligned} &= (1+p^{2-k}) \sum_{\mu=0}^{(r-2)/2} p^{(4-2k)\mu} + p^{3-2k} \sum_{\lambda=0}^{(r-2)/2} p^{(6-4k)\lambda} \left[\sum_{\mu=0}^{r/2-1-\lambda} p^{(4-2k)\mu}\right] \\ &+ (p^{6-4k}+p^{5-3k}+p^{8-5k}) \sum_{\lambda=0}^{r/2-2} p^{(6-4k)\lambda} \left[\sum_{\mu=0}^{r/2-2-\lambda} p^{(4-2k)\mu}\right] \\ &+ \left[\sum_{\mu=0}^{(s-r-1)/2} p^{(3-2k)\mu}\right]\left[p^{r(3-2k)} \sum_{\mu=0}^{r/2} p^{(2k-2)\mu} + p^{(1-2r)k+3r-1} \sum_{\mu=0}^{r/2-1} p^{(2k-2)\mu}\right] \end{aligned}$$

if $s \geqq r+1$ and $s \equiv 1$, $r \equiv 0 \pmod 2$,

$$\begin{aligned} &= (1-\beta p^{1-k})(1+p^{2-k}) \sum_{\mu=0}^{(r-2)/2} p^{(4-2k)\mu} \\ &+ p^{3-2k}(1-\beta p^{1-k}) \sum_{\lambda=0}^{(r-2)/2} p^{(6-4k)\lambda} \left[\sum_{\mu=0}^{r/2-1-\lambda} p^{(4-2k)\mu}\right] \\ &+ (1-\beta p^{1-k})(p^{6-4k}+p^{5-3k}+p^{8-5k}) \sum_{\lambda=0}^{r/2-2} p^{(6-4k)\lambda} \left[\sum_{\mu=0}^{r/2-2-\lambda} p^{(4-2k)\mu}\right] \\ &+ p^{(1-2r)k+3r-1} \left[\sum_{\mu=0}^{(s-r)/2} p^{(3-2k)\mu}\right]\left[p^{1-k}\sum_{\mu=0}^{r/2} p^{(2k-2)\mu} + \sum_{\mu=0}^{(r-2)/2} p^{(2k-2)\mu}\right] \\ &- \beta p^{r(3-2k)} \left[\sum_{\mu=0}^{(s-r-2)/2} p^{(3-2k)\mu}\right]\left[p^{1-k} \sum_{\mu=0}^{r/2} p^{(2k-2)\mu} + \sum_{\mu=0}^{(r-2)/2} p^{(2k-2)\mu}\right] \end{aligned}$$

if $s \geqq r > 0$ and $s \equiv r \equiv 0 \pmod 2$.

In the above formulas $\beta = \left(\frac{-u_1u_2}{p}\right)$ and we understand the sum vanishes if the upper bound of the sum is negative.

As to the shape of $c_2(k)$ we have not yet obtained the final form such as (4). The best knowledge about $c_2(k)$ in due to H.Maass [3] and [5]. From his results we can state a little weaker form of $c_2(k)$ than that of $c_p(k)$ $p \neq 2$. If T in Q_2 is non−degenerate and properly primitive and 2T is equivalent to diag $(2u_1, 2^{S+1} u_2)$ over $\mathbb{Z}_2$, where u_1 and u_2 are the units in $\mathbb{Z}_2$, then $c_2(k)$ is given by

$$(5)\quad c_2(k) = \{1 - (\tfrac{d}{2})\, 2^{1-k}\} \sum_{\mu=0}^{j} 2^{\mu(3-2k)} + (\tfrac{d}{2})^2\, 2^{(j+1)(3-2k)}$$

In the above d is the already explained quantity and j is the integer part of s/2. Here we should remark that because of the fact that $c_2(k) \neq 1$ if and only if 2 divides $|2T| \,/\, |d|$ $c_2(k) = 1$ for any even k if T is improperly primitive. Concerning the case when T is not primitive the recursion formula for $c_2(k)$ obtained by localizing the relation (3) in [5] does not fit the values in the table of [9]. It seems to us that Maass' formula should be remedied in some respects. Instead of using Maass' work [5] we employ the special values of $c_2(k)$ observed (a) from the table [9] or (b) from the values of $a_K(T)$ (k=4, 6, 8) which are gained by using Witt's relation (26) in [12] or equivalently — for k=4 and 8 — by using a theorem in [6].

(6) When $2T \cong 2\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ over $\mathbb{Z}_2$, then

$$c_2(k) = (1+2^{1-k})\,(1+2^{2-k}),$$

(7) when $2T \cong 2\begin{pmatrix} 2 & 1 \\ 1 & 4 \end{pmatrix}$ over $\mathbb{Z}_2$, then

$$c_2(k) = 1 + 2^{1-k} + 2^{3-2k},$$

(8) when $2T \cong 2\begin{pmatrix} u_1 & 0 \\ 0 & u_3 \end{pmatrix}$ over $\mathbb{Z}_2$ with $u_1u_2 \equiv 3 \pmod 8$, then

$$c_2(k) = 1 + 2^{1-k} + 2^{3-2k},$$

(9) when $2T \cong 2\begin{pmatrix} u_1 & 0 \\ 0 & u_2 \end{pmatrix}$ over $\mathbb{Z}_2$ with $u_1u_2 \equiv 7 \pmod 8$, then

$$c_2(k) = 1 - 2^{1-k} + 2^{3-2k}$$

(10) when $2T \cong 2^2\begin{pmatrix} u_1 & 0 \\ 0 & u_2 \end{pmatrix}$ over $\mathbb{Z}_2$ with $u_1u_2 \equiv 1 \pmod 8$, then

$$c_2(k) = 1 + 2^{2-k} + 2^{3-2k}$$

(11) when $2T \cong 2^2\begin{pmatrix} u_1 & 0 \\ 0 & u_2 \end{pmatrix}$ over $\mathbb{Z}_2$ with $u_1u_2 \equiv 3 \pmod 8$, then

$$c_2(k) = 1 + 2^{2-k} + 2^{1-k} + 2^{4-2k} + 2^{5-3k} + 2^{4-3k} + 2^{6-4k}$$

(12) when $2T \cong 2^2\begin{pmatrix} u_1 & 0 \\ 0 & u_1 \end{pmatrix}$ over $\mathbb{Z}_2$ with $u_1u_2 \equiv 5 \pmod 8$, then

$$c_2(k) = 1 + 2^{2-k} + 2^{3-2k} \;,$$

(13) when $2T \cong 2^2\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ over $\mathbb{Z}_2$, then

$$c_2(k) = 1 + 2^{2-k} + 2^{1-k} + 2^{5-2k} + 2^{5-3k} + 2^{4-3k} + 2^{6-4k},$$

(14) when $2T \cong 2^2\begin{pmatrix}1 & 0\\ 0 & 2\end{pmatrix}$ or $2^2\begin{pmatrix}1 & 0\\ 0 & 6\end{pmatrix}$ over $\mathbb{Z}_2$, then

$c_2(k) = 1 + 2^{2-k} + 2^{3-2k}$,

(15) when $2T \cong 2\begin{pmatrix}u_1 & 0\\ 0 & 2^2u_2\end{pmatrix}$ over $\mathbb{Z}_2$ with $u_1u_2 \equiv 1 \pmod 8$ or $u_1u_2 \equiv 5 \pmod 8$, then

$c_2(k) = 1 + 2^{3-2k}$

(16) when $2T \cong 2\begin{pmatrix}u_1 & 0\\ 0 & 2^2u_2\end{pmatrix}$ over $\mathbb{Z}_2$ with $u_1u_2 \equiv 3 \pmod 8$, then

$c_2(k) = 1 + 2^{1-k} + 2^{3-2k} + 2^{4-3k} + 2^{6-4k}$,

(17) when $2T \cong 2^2\begin{pmatrix}u_1 & 0\\ 0 & 2^2u_2\end{pmatrix}$ over $\mathbb{Z}_2$ with $u_1u_2 \equiv 1 \pmod 8$, then

$c_2(k) = 1 + 2^{2-k} + 2^{3-2k} + 2^{5-3k} + 2^{6-4k}$,

(18) when $2T \cong 2^3\begin{pmatrix}u_1 & 0\\ 0 & u_2\end{pmatrix}$ over $\mathbb{Z}_2$ with $u_1u_2 \equiv 1 \pmod 8$, then

$c_2(k) = 1 + 2^{2-k} + 2^{4-2k} + 2^{3-2k} + 2^{5-3k} + 2^{6-4k}$

These values of $c_2(k)$ together with the formula (4) suffice to calculate the Fourier coefficients $a_k(T)$ of Eisenstein series of degree two of any even weight k within the range that $3 \leqq \det(2T) \leqq 100$.

§ 3. The organization and the usage of the table.

To calculate $a_k(T)$ for given non−degenerate T in Q_2, we must first determine the discriminant d of the imaginary quadratic field $Q(\sqrt{-|2T|})$ then we should find the local canonical form of 2T over $\mathbb{Z}_p$ for such p dividing $|2T| \,/\, |d|$, and finally using the formulas (3) ~ (18) we have the value of $a_k(T)$. In making the table the first named author proposed the formulas to the second named author, then the second named author made up the whole programm to use the electronic computer and completed the project. The machine we used is "Computer, FACOM M−180 II AD, Information Processing Center of Nagasaki University". Here we explain the usage of the tables. Table I. consists of all the positive definite integral binary reduced quadratic forms $ax^2 + bxy + cy^2$ for which $D = 4ac - b^2 \leqq 100$ holds. We abbreviate such a form by (a, b, c). A binary quadratic form $ax^2 + bxy + cy^2$ is reduced (i) when the coefficients a, b and c satisfy $|b| \leqq a \leqq c$ and further (ii) we take only non−negative b if either $|b| = a$ or $a = c$ holds. It is clear that the matrix $\begin{pmatrix}a & b/2\\ b/2 & c\end{pmatrix}$ belongs to Q_2 if $ax^2 + bxy + cy^2$ is a positive definite integral quadratic form. We ordered the forms according to the quantity D which is integral and positive, then we grouped them according to the genera. For example, if D=23 there are three forms and they make a genus. We marked each genus as T_i, since the Fourier coefficients $a_k(T)$ is a genus invariant of T. Two different genera may sometimes have the same values of $a_k(T)$. This phenomenon can be explained by a property of $a_k(T)$ mentioned at page 7 in [3].

Table II consists of the values of Fourier coefficients $a_k(T_i)$ for k=4, 6, 10 and 12. Since $a_4(T_i)$ and $a_6(T_i)$ are integer valued, we exhibit them without modification. The values $a_{10}(T_i)$ and $a_{12}(T_i)$ are not integral, so we abbreviated the common denominators H of $a_{10}(T_1)$ and G of $a_{12}(T_1)$. H(resp. G) is the numerator of the product of Bernoulli numbers B_{10} B_{18} (resp. B_{12} B_{22}). In fact H is $5 \cdot 43867 = 21935$ and G is $691 \cdot 854513 = 691 \cdot 11 \cdot 131 \cdot 593 = 590468483$. For example the true value of $a_{10}(T_1)$ is $1136224320/219335 = 227244864/43867$.

Addendum: After typing the manuscript, we became aware that the formulas (8), (9), (15) and (16) were derivable from the formula (5).

Table I

D	reduced form	T_i, assigned to the genus	D	reduced form	T_i, assigned to the genus
3	(1, 1, 1)	T_{1}	36	(1, 0, 9)	T_{28}
4	(1, 0, 1)	T_{2}		(2, 2, 5)	T_{29}
7	(1, 1, 2)	T_{3}		(3, 0, 3)	T_{30}
8	(1, 0, 2)	T_{4}	39	(1, 1, 10)	T_{31}
11	(1, 1, 3)	T_{5}		(3, 3, 4)	
12	(1, 0, 3)	T_{6}		(2, ±1, 5)	T_{32}
	(2, 2, 2)	T_{7}	40	(1, 0, 10)	T_{33}
15	(1, 1, 4)	T_{8}		(2, 0, 5)	T_{34}
	(2, 1, 2)	T_{9}	43	(1, 1, 11)	T_{35}
16	(1, 0, 4)	T_{10}	44	(1, 0, 11)	T_{36}
	(2, 0, 2)	T_{11}		(3, ±2, 4)	
19	(1, 1, 5)	T_{12}		(2, 2, 6)	T_{37}
20	(1, 0, 5)	T_{13}	47	(1, 1, 24)	T_{38}
	(2, 2, 3)	T_{14}		(2, ±1, 6)	
23	(1, 1, 6)	T_{15}		(3, ±1, 4)	
	(2, ±1, 3)		48	(1, 0, 12)	T_{39}
24	(1, 0, 6)	T_{16}		(3, 0, 4)	T_{40}
	(2, 0, 3)	T_{17}		(2, 0, 6)	T_{41}
27	(1, 1, 7)	T_{18}		(4, 4, 4)	T_{42}
	(3, 3, 3)	T_{19}	51	(1, 1, 13)	T_{43}
28	(1, 0, 7)	T_{20}		(3, 3, 5)	T_{44}
	(2, 2, 4)	T_{21}	52	(1, 0, 13)	T_{45}
31	(1, 1, 8)	T_{22}		(2, 2, 7)	T_{46}
	(2, ±1, 4)		55	(1, 1, 14)	T_{47}
32	(1, 0, 8)	T_{23}		(4, 3, 4)	
	(3, 2, 3)	T_{24}		(2, ±1, 7)	T_{48}
	(2, 0, 4)	T_{25}	56	(1, 0, 14)	T_{49}
35	(1, 1, 9)	T_{26}		(2, 0, 7)	
	(3, 1, 3)	T_{27}		(3, ±2, 5)	T_{50}

Table I

D	reduced from	T_1, assigned to the genus
59	(1, 1, 15)	T_{51}
	(3, ±1, 5)	
60	(1, 0, 15)	T_{52}
	(3, 0, 5)	T_{53}
	(2, 2, 8)	T_{54}
	(4, 2, 4)	
63	(1, 1, 16)	T_{55}
	(4, 1, 4)	
	(2, ±1, 8)	T_{56}
	(3, 3, 6)	T_{57}
64	(1, 0, 16)	T_{58}
	(4, 4, 5)	T_{59}
	(2, 0, 8)	T_{60}
	(4, 0, 4)	T_{61}
67	(1, 1, 17)	T_{62}
68	(1, 0, 17)	T_{63}
	(2, 2, 9)	
	(3, ±2, 6)	T_{64}
71	(1, 1, 18)	T_{65}
	(2, ±1, 9)	
	(3, ±1, 6)	
	(4, ±3, 5)	
72	(1, 0, 18)	T_{66}
	(2, 0, 9)	T_{67}
	(3, 0, 6)	T_{68}
75	(1, 1, 19)	T_{69}
	(3, 3, 7)	T_{70}
	(5, 5, 5)	T_{71}

D	reduced form	T_1, assigned to the genus
76	(1, 0, 19)	T_{72}
	(4, ±2, 5)	
	(2, 2, 10)	T_{73}
79	(1, 1, 20)	T_{74}
	(2, ±1, 10)	
	(4, ±1, 5)	
80	(1, 0, 20)	T_{75}
	(4, 0, 5)	
	(3, ±2, 7)	T_{76}
	(2, 0, 10)	T_{77}
	(4, 4, 6)	T_{78}
83	(1, 1, 21)	T_{79}
	(3, ±1, 7)	
84	(1, 0, 21)	T_{80}
	(3, 0, 7)	T_{81}
	(2, 2, 11)	T_{82}
	(5, 4, 5)	T_{83}
87	(1, 1, 22)	T_{84}
	(4, ±3, 6)	
	(2, ±1, 11)	T_{85}
	(3, 3, 8)	
88	(1, 0, 22)	T_{86}
	(2, 0, 11)	T_{87}
91	(1, 1, 23)	T_{88}
	(5, 3, 5)	T_{89}
92	(1, 0, 23)	T_{90}
	(3, ±2, 8)	
	(2, 2, 12)	T_{91}
	(4, ±2, 6)	

Table I

D	reduced form	T_i, assigned to the genus
95	(1, 1, 24)	T_{92}
	(4, ±1, 6)	
	(5, 5, 6)	
	(2, ±1, 12)	T_{93}
	(3, ±1, 8)	
96	(1, 0, 24)	T_{94}
	(3, 0, 8)	T_{95}
	(5, 2, 5)	T_{96}
	(4, 4, 7)	T_{97}
	(2, 0, 12)	T_{98}
	(4, 0, 6)	T_{99}
99	(1, 1, 25)	T_{100}
	(5, 1, 5)	T_{101}
	(3, 3, 9)	T_{102}
100	(1, 0, 25)	T_{103}
	(2, 2, 13)	T_{104}
	(5, 0, 5)	T_{105}

Table II

T_i \ k	4	6	10	12
(1)	13440	44352	1136224320	24 4935250560
(2)	30240	166320	1 3130132400	502 4786326560
(3)	138240	2128896	153 0879989760	179155 1107906560
(4)	181440	3792096	475 4093873760	727674 9730810560
(5)	362880	15422400	7108 7579640000	20601734 2592181120
(6)	497280	23462208	14921 9203721280	51391750 9214229120
(7)	604800	24881472	14980 0950673120	51441913 6607376000
(8)	967680	65995776	99633 5909775360	535377927 3830507520
(9)	967680	65995776	99633 5909775360	535377927 3830507520
(10)	997920	85322160	172100 5844065200	1053774571 9104283680
(11)	1239840	90644400	172772 8471854000	1054803648 1501078560
(12)	1330560	178960320	740146 0144478400	6400125556 8548538240
(13)	1814400	234311616	1146985 8765140160	1 0972549250 9641008000
(14)	1814400	234311616	1146985 8765140160	1 0972549250 9641008000
(15)	2903040	453454848	3769975 5208980480	4 7626313118 0319042560
(16)	2782080	530228160	5402276 6344747200	7 4423172593 1058485120
(17)	2782080	530228160	5402276 6344747200	7 4423172593 1058485120
(18)	3279360	873024768	14673219 5025588480	25 6210923296 7838794240
(19)	3642240	883802304	14675455 9328879040	25 6215262251 2669746560
(20)	4008960	1058061312	20026512 7620433920	37 5532223226 5069706240

Table II

k / T_i	4	6	10	12
(21)	5114880	1126185984	20104893 8175191040	37 5899132893 4062341120
(22)	5806080	1724405760	47663141 0030899200	109 3950283060 0028805120
(23)	5987520	1945345248	62313334 6315344480	152 6045752821 8558335680
(24)	5987520	1945345248	62313334 6315344480	152 6045752821 8558335680
(25)	7439040	2066692320	62556744 2378709600	152 7536031166 7258362560
(26)	6531840	2818924416	133209153 8374988160	390 8364354877 2321058560
(27)	6531840	2818924416	133209153 8374988160	390 8364354877 2321058560
(28)	7650720	3287314800	169571359 8276390000	525 6133645905 5083339680
(29)	7650720	3287314800	169571359 8276390000	525 6133645905 5083339680
(30)	8467200	3327730560	169597203 8672419200	525 6222658487 8474464000
(31)	10644480	4864527360	335489910 5930803200	1218 6545790466 5488133120
(32)	10644480	4864527360	335489910 5930803200	1218 6545790466 5488133120
(33)	9555840	5258506176	415211051 9743623360	1588 9769238360 4753138560
(34)	9555840	5258506176	415211051 9743623360	1588 9769238360 4753138560
(35)	10039680	7057423296	766290079 3152749760	3393 8641110602 6830465920
(36)	13426560	8158449600	933586074 6541560000	4322 6084982803 4805818240
(37)	16329600	8651966400	937225758 7317240000	4326 8277334566 3592752000
(38)	17418240	1 1304437760	1638640701 9001958400	8644 3470306155 0807746560
(39)	15980160	1 2013404480	1955845976 2165262400	10777 6313253469 7659290240
(40)	15980160	1 2013404480	1955845976 2165262400	10777 6313253469 7659290240

Table II

T_i \ k	4	6	10	12
(41)	19958400	1 2764195136	1963485999 4470557760	10788 1563559356 8400528000
(42)	20818560	1 2809611584	1963515784 8858699840	10788 1666292646 9565338240
(43)	16208640	1 5281481600	3267987118 9734384000	20359 4004299169 7706223360
(44)	16208640	1 5281481600	3267987118 9734384000	20359 4004299169 7706223360
(45)	18264960	1 7118585792	3861776871 0531299520	24975 9939473151 7102266240
(46)	18264960	1 7118585792	3861776871 0531299520	24975 9939473151 7102266240
(47)	24192000	2 2751511552	6232910632 2157086720	45030 6307815341 7697920000
(48)	24192000	2 2751511552	6232910632 2157086720	45030 6307815341 7697920000
(49)	23950080	2 4105755520	7251060827 0046038400	54383 4652229833 0445925120
(50)	23950080	2 4105755520	7251060827 0046038400	54383 4652229833 0445925120
(51)	24312960	2 9564377920	1 1277115390 2998606400	94021 3990913377 5990426240
(52)	28062720	3 2799900672	1 3033767470 9083269120	112222 1199547506 7897390080
(53)	28062720	3 2799900672	1 3033767470 9083269120	112222 1199547506 7897390080
(54)	35804160	3 4911765504	1 3084779869 4888253440	112331 7653542787 2776791040
(55)	34974720	4 2077629440	1 9770813704 5537536000	187403 6316102150 5403079680
(56)	34974720	4 2077629440	1 9770813704 5537536000	187403 6316102150 5403079680
(57)	38707200	4 2594951168	1 9773826935 6375982080	187406 8052892561 7726464000
(58)	31963680	4 3685112240	2 2557567800 6444026800	220992 5451031601 1514405920
(59)	31963680	4 3685112240	2 2557567800 6444026800	220992 5451031601 1514405920
(60)	39947040	4 6415421360	2 2645683299 8605409200	221208 3581354873 7087382560

Table II

T_i \ k	4	6	10	12
(61)	41882400	4 6585733040	2 2646027498 4033274800	221208 5688903012 5723296800
(62)	30360960	5 1923906496	3 3231286382 3264237760	357323 1008367688 1369735040
(63)	38465280	5 7718894464	3 7768404885 1182944640	417672 7665274818 7570149120
(64)	38465280	5 7718894464	3 7768404885 1182944640	417672 7665274818 7570149120
(65)	49351680	7 2393108480	5 4619577054 4553113600	657532 7125614059 6645637120
(66)	42638400	7 4336457888	6 1391327091 8855957280	761169 4267659552 4180276800
(67)	42638400	7 4336457888	6 1391327091 8855957280	761169 4267659552 4180276800
(68)	47537280	7 5257937216	6 1400684574 8573175360	761182 3173098008 8078549120
(69)	42349440	8 6652764352	8 6687080516 6886224320	1167942 3121274683 2435250560
(70)	42349440	8 6652764352	8 6687080516 6886224320	1167942 3121274683 2435250560
(71)	44029440	8 6791364352	8 6687302435 5011224320	1167942 4317247586 3685250560
(72)	49230720	9 4670009280	9 7202635931 4203793600	1342859 6240505046 6318788480
(73)	59875200	10 0396739520	9 7581590690 8176734400	1344170 3697645485 3725104000
(74)	59996160	11 6088698880	13 5343846880 5912473600	2017363 9595882518 7310090240
(75)	59875200	12 0201859008	15 0338879792 3216191680	2301111 4579307116 8850224000
(76)	59875200	12 0201859008	15 0338879792 3216191680	2301111 4579307116 8850224000
(77)	74390400	12 7699830720	15 0926136561 0967953600	2303358 6360173091 3634608000
(78)	74390400	12 7699830720	15 0926136561 0967953600	2303358 6360173091 3634608000
(79)	56246400	13 7250324288	20 5172554729 8064841280	3385326 3325196611 6640476800
(80)	63624960	14 8848416640	22 7591053201 3219516800	3840808 4207063944 5678186240

Table II

T_i \ k	4	6	10	12
(81)	63624960	14 8848416640	22 7591053201 3219516800	3840808 4207063944 5678186240
(82)	63624960	14 8848416640	22 7591053201 3219516800	3840808 4207063944 5678186240
(83)	63624960	14 8848416640	22 7591053201 3219516800	3840808 4207063944 5678186240
(84)	78382080	17 9829974016	30 7286533557 2747151360	5554620 4155963691 8592005120
(85)	78382080	17 9829974016	30 7286533557 2747151360	5554620 4155963691 8592005120
(86)	67616640	18 2629605312	33 7957001218 7432521920	6259731 8476294234 1508543360
(87)	67616640	18 2629605312	33 7957001218 7432521920	6259731 8476294234 1508543360
(88)	66528000	20 6073141120	44 8501211410 0926038400	8896318 0966877718 1464057600
(89)	66528000	20 6073141120	44 8501211410 0926038400	8896318 0966877718 1464057600
(90)	84188160	22 5367059456	49 3176887717 3199452160	9983089 6089787097 4362250240
(91)	107412480	23 9877614592	49 5107115184 0197457920	9992843 4779052826 7761413120
(92)	101606400	26 8323922944	64 9086821679 7779978240	13989511 4847222203 8581888000
(93)	101606400	26 8323922944	64 9086821679 7779978240	13989511 4847222203 8581888000
(94)	91808640	27 2007046080	70 8092605310 5049745600	15607677 9673149709 5244863360
(95)	91808640	27 2007046080	70 8092605310 5049745600	15607677 9673149709 5244863360
(96)	91808640	27 2007046080	70 8092605310 5049745600	15607677 9673149709 5244863360
(97)	91808640	27 2007046080	70 8092605310 5049745600	15607677 9673149709 5244863360
(98)	114065280	28 8974347200	71 0858570947 3560312000	15622919 8330620390 3022389120
(99)	114065280	28 8974347200	71 0858570947 3560312000	15622919 8330620390 3022389120
(100)	85276800	30 2325307200	91 7979528746 2642920000	21550020 0454562471 8237353600

Table II

T_i \ k	4	6	10	12
(101)	85276800	30 2325307200	91 7979528746 2642920000	21550020 0454562471 8237353600
(102)	95074560	30 6072950400	91 8119450429 2697040000	21550384 9989981289 5346218240
(103)	93774240	32 4739966320	100 1749090377 0005132400	23960047 8346461609 3536326560
(104)	93774240	32 4739966320	100 1749090377 0005132400	23960047 8346461609 3536326560
(105)	97554240	32 5259716320	100 1751654855 9848882400	23960050 2881551094 5098826560

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