Discussion of "A new approach of selecting real input ground motions for seismic design: The most unfavourable real seismic design ground motions" by C-H Zhai and L-L Xie

Abbas Moustafa

Department of Civil Engineering, Nagasaki University, Nagasaki 852-8521, Japan

SUMMARY

This discussion consists of two parts. The first part raises a few comments and questions on the method presented in the above paper. The second part proposes a measure for identifying resonant accelerograms in a set of earthquake records without the need for pre-processing of the records or inclusion of the structure dynamic analysis.

KEY WORDS: design accelerograms; entropy; resonant acceleration; frequency Content; Kanai-Tajimi; critical record

1. COMMENTS AND QUESTIONS

The above paper offers a useful application for identifying resonant accelerograms among a set of records. The following comments and questions are raised:

1. The use of nonlinear dynamic analysis in selecting most unfavourable accelerograms as design inputs for structures is highly computational. This is true when a large set of records is involved. I quote the following from the above paper "*It is a very complicated process to select the most unfavourable real seismic design ground motions from a large number of ground motion records*". Additionally, the application of the method to nonlinear multi-degree-of-freedom systems posses more questions.

2. The proposed method, even with the high computations involved, is capable of identifying unfavourable records for the selected structures only. The discretizations considered in the paper for the middle- and the short-period ranges are coarse. Consider a narrow-band record with its energy concentrated at 2.25 Hz which is common for a stiff soil site. The proposed method will not identify this record since a structure with a natural frequency of 2.25 Hz is not included in the short-period range. The same comment applies to records with dominant frequencies of 2.70, 3.60, and 4.50 Hz. Furthermore, the method identifies a single resonant accelerogram for each period range which is questionable. How can we average resonant signals ?

3. It is mentioned that the method is based on the critical excitation approach proposed by Drenick in 1973. Actually, the concept was introduced in 1970 [1-3]. Additionally, it is not clear how the method is based on Drenick's approach.

4. The use of Park and Ang damage index (D_{PA}) in selecting the most unfavourable records is mentioned several times in the paper. However, it is not explained how D_{PA} is used. Even the expression for D_{PA} is not provided in the paper.

5. The measure adopted for finding the most unfavourable accelerograms is taken as the plastic hysteretic energy per unit mass $V_p = \sqrt{2E_p / m}$. Normalizing E_p with the yield energy $E_y = f_y u_y$ is more appropriate since two structures may have the same mass but different yield characteristics. A robust measure would be the damage index ($D_{PA} = \dfrac{\mu_{max}}{\mu_u} + \beta \dfrac{E_H}{f_y u_y \mu_u}$) since it accounts for damage due to maximum ductility and hysteretic cumulative energy produced by the earthquake.

6. The basis of grouping the available records into two sets is not clear. In fact, this adds more computations to the process of finding most unfavourable records.

7. Scaling the ground motions using peak ground acceleration is not appropriate. Arias intensity measure is more appropriate for this purpose [4].

8. The verification of the method by comparing the structure maximum responses is not robust. The fact that structures are damaged by stress reversals and not only due to maximum response is ignored. A robust measure would be the damage index.

The question that can be asked here is "*can we identify unfavourable accelerograms without using the structure and thus eliminating nonlinear dynamic analysis?*". In other words "*Is there a practical way for finding the most unfavourable accelerograms at a site?*". We propose a simple and practical method to extract resonant accelerations here. The method does not require pre-classification of the records. More importantly, it does not require dynamic analysis and is capable of identifying all peculiar records.

Before we outline the method, we consider the four time histories shown in figure (1). The first three time histories represent samples from a narrow-band model, a band limited model, and Kanai-Tajimi model [5]. The fourth time history is the 1940 Elcentro NS component. Table 1 lists the mathematical expressions and numerical parameters adopted for these models. The envelope function for the first three models is taken as $e(t) = A_0 [\exp(-\alpha_1 t) - ex(\alpha_2 t)]$. The parameters A_0, α_1, α_2 are calculated such that they match the transient trend of the Elcentro record. All accelerograms are normalized such that the Arias intensity of $\ddot{x}_g(t)$ (i.e. $\int_0^\infty [\ddot{x}_g(t)]^2 dt$) is set to unity [4]). Simple remarks can be made from figure 1. For instance, all accelerations are rich in frequency content

except of that from the narrow-band model. However, the frequency contents cannot be extracted from the time histories.

The power spectral density (PSD) functions for the four acceleration models are shown in figure 2. This figure reveals that the first model is highly resonant at a single frequency of 2.5 Hz (see table 1). In other words, the energy of the ground acceleration is located at a single frequency. Clearly this ground acceleration is unfavourable (more precisely '*critical*' or '*resonant*') for structures with fundamental frequency close to 2.5 Hz. It is also seen that the other three models are rich in frequency content. However, the band-limited model is not a realistic model for actual ground motions. Furthermore, the Kanai-Tajimi model and the actual record possess similar features and have a dominant frequency of about 2.5 Hz.

The most important observation that can be made from figure 2 is that the center of mass of the PSD function for the narrow-band model is located exactly at the central frequency ω_c. Such acceleration among a set of records can be expected to produce the maximum damage to structures with fundamental frequency close to 2.5 Hz. Also, the center of mass for the Kanai-Tajimi model and for the actual record are located away from ω_c. This property was recognized earlier as the measure of '*disorder*' in recorded earthquakes within the context of critical excitations method [6-9]. This measure is called the '*entropy*' and was introduced by this author as an explicit constraint in developing critical earthquakes [7-9]. We define this measure below.

2. IDENTIFYING RESONANT GROUND MOTIONS USING ENTROPY PRINCIPLE

The entropy principle offers a useful tool for quantifying uncertainty in random processes [10, 11]. As shown above, the PSD function for the narrow-band model is resonant or *ordered*. Such an ordered signal cannot serve as a realistic earthquake model. However, with the high uncertainty involved in the earthquake phenomenon, actual records show this resonance nature. We thus use the entropy to measure the resonance in accelerograms. The entropy for a stationary Gaussian process is defined as [11]:

$$\overline{H}_W = \log_e \sqrt{2\pi e} + \frac{1}{2(\omega_u - \omega_0)} \int_{\omega_0}^{\omega_u} \log_e S(\omega)\, d\omega \qquad (1)$$

where (ω_0, ω_u) defines the frequency range of the PSD function $S(\omega)$. Equation (1) can be used to calculate the entropy from samples of Gaussian random processes. For mathematical convenience, we measure the entropy with reference to an ideal white

noise process $\ddot{\zeta}_g(t)$ of intensity s_0. Thus, under the assumption that $\ddot{u}_g(t)$ is independent of $\ddot{\zeta}_g(t)$, the increase in entropy when $\ddot{u}_g(t)$ is added to $\ddot{\zeta}_g(t)$ is [7,8]:

$$\Delta \overline{H}_W = \frac{1}{2(\omega_u - \omega_0)} \int_{\omega_0}^{\omega_u} \log_e \left[1 + \frac{S(\omega)}{s_0} \right] d\omega \tag{2}$$

The use of equation (2) to estimate the entropy for the first three models is straightforward since the PSD functions for the stationary components are known. To compute the entropy for the Elcentro acceleration, we first estimate the PSD function for the stationary part. Thus, the Elcentro acceleration is represented as:

$$\ddot{x}_g(t) = e(t)\ddot{u}_g(t) = A_0 \left[\exp(-\alpha_1 t) - \exp(-\alpha_2 t) \right] \ddot{u}_g(t) \tag{3}$$

The parameters A_0, α_1, α_2 are calculated such that they match the transient trend of $\ddot{x}_g(t)$. The sample of the stationary acceleration $\ddot{u}_g(t)$ is then obtained by dividing $\ddot{x}_g(t)$ by $e(t)$. This is followed by the estimation of the PSD function of $\ddot{u}_g(t)$.

The numerical results on the entropy for the four earthquake models of table 1 ($s_0 = 0.02$) are found to be 0.0299, 0.8670, 0.6271 and 0.5745 for the narrow-band, the band-limited, the Kanai-Tajimi and the Elcentro earthquake, respectively. Thus, the narrow-band model possesses the lowest entropy and the band-limited signal possesses the highest entropy. The entropy of the actual earthquake and for the Kanai-Tajimi model are significantly similar and are bounded by the ideal narrow-band and the band-limited signals. Thus the entropy successfully predicts resonant accelerations.

We now examine the applicability of this measure to a broader spectrum of earthquake records. Figure 3 shows the first horizontal acceleration for four recorded ground motions, namely, 1986 Dharmsala (India), 1995 Kobe (Japan), 1999 Chichi (Taiwan), and 1999 Izmit (Turkey) earthquakes. The Kobe and Chichi earthquakes represent long duration earthquakes. The Dharmsala earthquake represents an acceleration that is rich in frequency content. These records are normalized to the same Arias intensity. The entropy of these records from a narrow-band white noise process are computed and listed in figure 3. The highest value is 0.62 for the Dharmsala earthquake while the lowest is 0.33 for the Kobe earthquake. These results, again, confirms that the entropy successfully identifies resonant ground accelerations (see figure 3).

To summarize, the method proposed by the authors requires nonlinear dynamic analysis

of all available records for several single-degrees-of-freedom systems [12]. The construction of two groups of records requires significant effort. A practical application for identifying resonant accelerograms, at a site, without the recourse to the structure or to nonlinear dynamic analysis is proposed. The method is based on the entropy principle that quantifies the effective frequency range of recorded ground motions. The proposed methodology is simple, practical, and promising since it avoids including the structure and does not require pre-processing of the set of available records.

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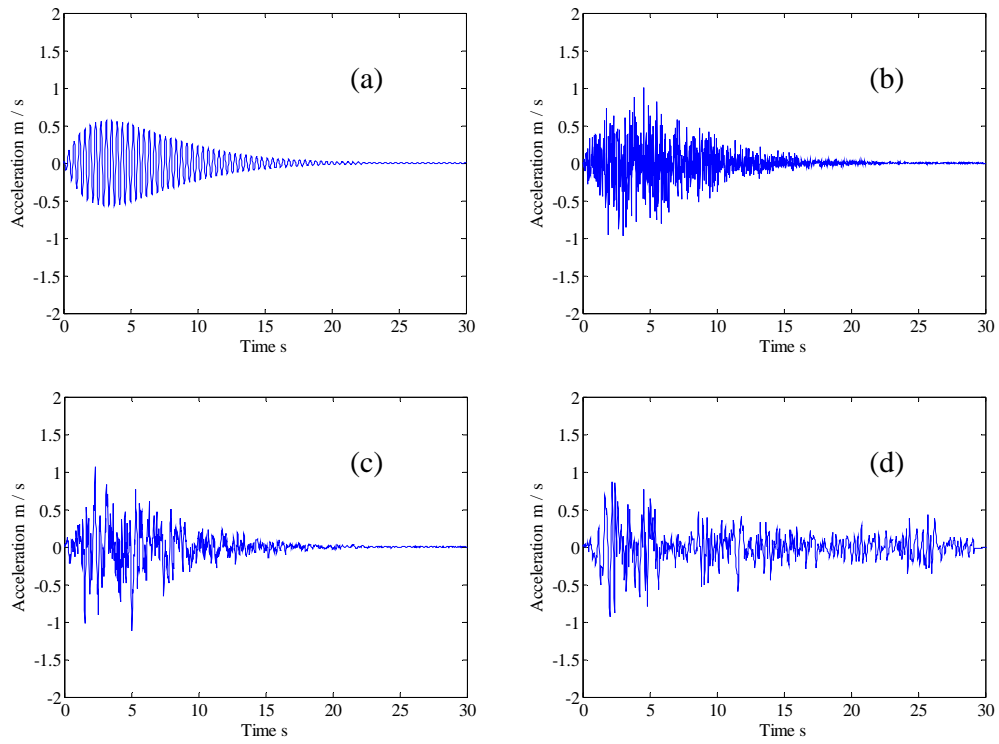
Figure 1: Sample accelerations of (a) Narrow-band model (b) Band-limited model (c) Kanai-Tajimi model (d) Actual earthquake acceleration.
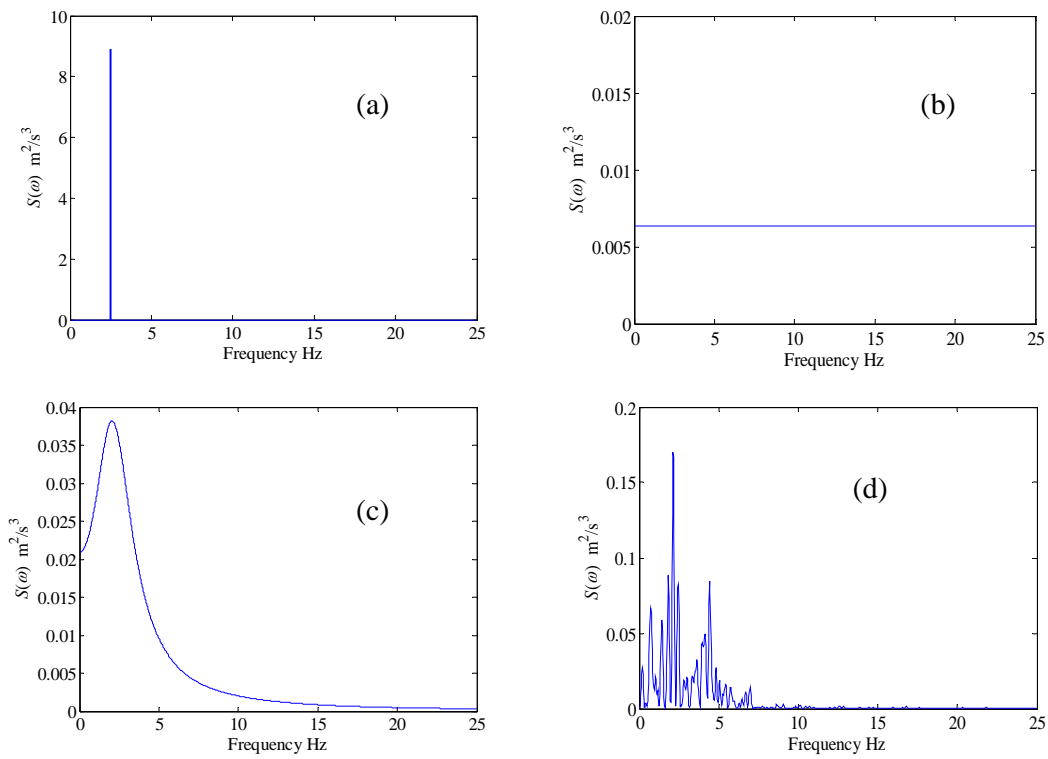
Figure 2: PSD function for (a) Narrow-band acceleration (b) Band-limited acceleration (c) Kanai-Tajimi acceleration (d) Actual earthquake acceleration.

1986 Dharmsala
(India)

$\Delta \overline{H}_W = 0.6154$

1995 Kobe
(Japan)

$\Delta \overline{H}_W = 0.3268$

1999 Chichi
(Taiwan)

$\Delta \overline{H}_W = 0.4274$

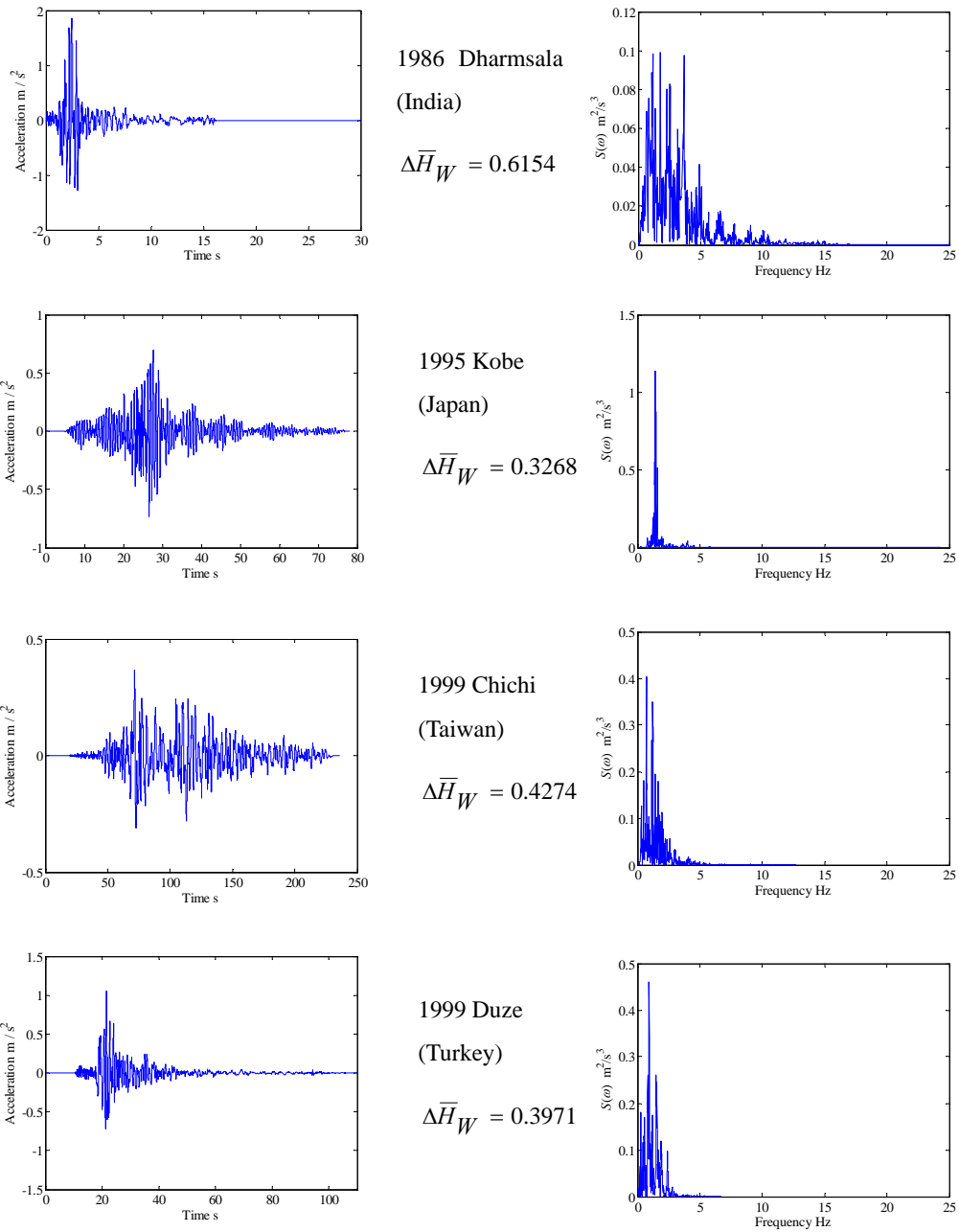1999 Duze
(Turkey)

$\Delta \overline{H}_W = 0.3971$

Figure 3: Time history and PSD function for recorded ground motions.

Table 1: Mathematical expressions and numerical values for the acceleration models

Acceleration model	Mathematical model	Numerical parameters	Entropy
Narrow-band (Figure 1-a)	$S(\omega) = s_0 \delta(\omega - \omega_c)$	$\omega_c = 5\pi$ rad/s	0.0299
Kanai-Tajimi (Figure 1-c)	$S(\omega) = s_0 \dfrac{1 + 4\zeta_g^2 \omega_g^2}{(1 - \dfrac{\omega^2}{\omega_g^2}) + 4\zeta_g^2 \omega_g^2}$	$\omega_g = 5\pi$ rad/s, $\zeta_g = 0.60$	0.6271
Elcentro record (Figure 1-d)	actual recorded ground motion	-	0.5745
Band-limited (Figure 1-b)	$S(\omega) = s_0$	$(\omega_0, \omega_u) = 2\pi(0,25)$ rad/s	0.8670