

《研究ノ一ト》

On the Factor Price Frontier in the Presence  
of Product Market Imperfections

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I

In a recent paper Ikema [3] successively attempts to apply the two-dimensional factor price frontier to prove some basic theorems in the pure theory of international trade. The factor price frontier (FPF), which gives the relationship between the wage rate and the profit level or rate of interest has received a good deal of attention from the pure theory of capital and growth (See Burmeister [1] Hicks [2], Samuelson [7].), while this idea has by and large been ignored in the pure theory of international trade. To my knowledge, recent tasks by Shimomura in [8], Steedman-Mainwaring-Metcalf in [9], and Kouri in [6], following Kemp [5], Jones [4], and Ikema [3], constitute a few exceptions to this neglect.

It has been demonstrated by Ikema that the FPF is a promising tool with several advantages also in the analysis of international trade. Indeed, more attention should be paid to the application of this concept from trade theorists. In this context, Some analyses in Steedman [9] must be remarked.

The FPF is not an unambiguous concept. For example, if both factor prices are deflated by the price of the same commodity, the FPF depends only upon the production function for that commodity. Given the assumption of linear homogeneous production functions, each price of commodity turns out to be a linear homogeneous function of each factor prices. Nonetheless, replacing the market structure under consideration, then it follows that the FPF

has no longer the property of linear homogeneity. This paper is intended not as further analysis using the FPF, but as examining the implications of monopoly on the FPF.

On the other hand, the assumption of perfect competition has long since been a major part of the corpus of trade theory. A more interesting approach would be start with the assumption that product markets are now characterised by imperfect competition. This paper will thus serve as a preliminary analysis of the impact of product market imperfections on the contents of international trade theory, by use of a well-defined FPF.

## II

We first present the Samuelson-Kemp type model under monopolistic conditions. The  $j$ th production relationship is written

$$X_j = F_j(K_j, L_j) = L_j f_j(k_j) \quad (1)$$

where  $f_j(k_j) = F_j(k_j, 1)$  and  $k_j = K_j/L_j$ . It is assumed that  $F_j$  is homogeneous of the first degree in  $L_j$  and  $K_j$ . Under the assumption of monopolistic product markets, the reward of each factor is not equal its marginal value product, but its marginal revenue product, which, in turn, equals the marginal revenue times the marginal product. If we denote by  $p_j$  the price of the  $j$ th commodity, by  $r$  the rental per unit of capital, and  $w$  the wage rate, each in terms of some arbitrary unit of account, the required equalities may be written as

$$\begin{aligned} w &= p_j(1 - 1/\varepsilon_j)(f_j - k_j f_j') \\ r &= p_j(1 - 1/\varepsilon_j)f_j' \end{aligned} \quad (2)$$

It follows that the wage-rental ratio is equal to

$$\omega \equiv w/r = (f_j/f_j') - k_j \quad (3)$$

It is also assumed that all marginal products are positive but diminishing, so that  $f_j'(k_j) > 0$ , if  $k_j > 0$  and  $f_j''(k_j) < 0$ .

In view of the restrictions placed on  $f_j(k_j)$ , Eqs. (3) determine  $k_j$  uniquely in terms of  $\omega$ , with

$$dk_j/d\omega = -(f_j')^2/f_j f_j'' > 0 \quad (4)$$

We are now ready to derive the FPF in the presence of product market imperfections. However, at this point it would be convenient to summarize the results of the previous studies. Those analyses tell that the FPF has some fundamental properties. That is to say, first, the FPF,  $r = r_j(w)$  is convex to the origin, and its negative slope indicates the capital-labour ratio. Furthermore, it is also clear that there is a close connection between the elasticity of the FPF and the competitive distribution of income. Secondly,  $p_j$  is homogeneous of the first degree with respect to  $w$  and  $r$ .

We can imagine the following function.

$$p_j = p_j(w, r) \quad (5)$$

We define the FPF as the relationship between  $w$  and  $r$  at constant prices  $p_j$ . Thus, Differentiating Eqs.(5) totally, and setting  $dp_j = 0$ , we obtain

$$dr/dw = -(\partial p_j/\partial w)/(\partial p_j/\partial r) \quad (6)$$

Here, we must explore the effect of changes in individual factor rewards on commodity price. For this purpose we need refer only to Eqs.(2). Differentiating, first with respect  $w$  and then with respect to  $r$ , and solving, we can get

$$\frac{\partial p_j}{\partial w} = \frac{\frac{\varepsilon_j}{\varepsilon_j - 1} \cdot \frac{1}{f_j}}{1 + \frac{1}{\varepsilon_j - 1} \cdot \frac{p_j}{\varepsilon_j} \cdot \frac{\partial \varepsilon_j}{\partial p_j}} \quad (7a)$$

$$\frac{\partial p_j}{\partial r} = \frac{\frac{\varepsilon_j}{\varepsilon_j - 1} \cdot \frac{k_j}{f_j}}{1 + \frac{1}{\varepsilon_j - 1} \cdot \frac{p_j}{\varepsilon_j} \cdot \frac{\partial \varepsilon_j}{\partial p_j}} \quad (7b)$$

Hence Eqs. (6) may be rewritten

$$dw/dr = -k_j < 0 \quad (6')$$

Another important feature is the fact that the elasticity of the function  $r_j(w)$  is equal to the ratio of the two factor's income shares. By using Eqs. (6') we have

$$(w/r)(dr/dw) = (w/r)/(-k_j) = -(wL_j)/(rK_j) \quad (8)$$

We are now in a position to prove the convexity of the FPF. Since it is apparent that

$$d^2w/dr^2 = (w/r^2)(\partial k_j/\partial w) > 0 \quad (9)$$

Thus, it can be easily concluded.

The final step is to investigate the linear homogeneity we are heavily concerned with. The answer for this question is derived from the results of Eqs. (7), with making use of Eqs. (2), that is,

$$\frac{\partial p_j}{\partial r} \cdot \frac{r}{p_j} + \frac{\partial p_j}{\partial w} \cdot \frac{w}{p_j} = \frac{1}{1 + \left[ \frac{1}{\varepsilon_j - 1} \cdot \frac{p_j}{\varepsilon_j} \cdot \frac{\partial \varepsilon_j}{\partial p_j} \right]} \quad (10)$$

Eqs. (10) indicate that a commodity price is no longer homogeneous of degree one in factor prices. As long as  $\partial \varepsilon_j / \partial p_j \neq 0$ , a 1% change in factor prices will not induce the same 1% change in a commodity price.

### III

For expositional purposes, we turn now to consider the same problem in an alternative model, which we call the Amano-Jones type model in turn. We begin to tell the story in the case of perfect competition.

In a competitive equilibrium with both output being positive, these unit costs equal market prices, as in Eqs. (11).

$$a_{Lj}w + a_{Kj}r = p_j \quad (11)$$

where  $a_{ij}$  denotes the quantity of factor  $i$  required to produce a unit of a commodity  $j$ . Differentiating Eqs. (11) totally, and using the minimum unit cost conditions,  $wa_{Lj} + rda_{Kj} = 0$ , yield

$$a_{Lj}dw + a_{Kj}dr = dp_j \quad (12)$$

Apparently, this expression provides the negatively sloped FPF. Now it can be easily derived that

$$(w/r)(dr/dw) = -(wa_{Lj})/(ra_{Kj}) \quad (13)$$

Let us us now identify the validity of the convexity. It is shown in Eqs. (14).

$$d^2w/dr^2 = (w/r^2)[\partial(a_{Kj}/a_{Lj})/\partial\omega] \quad (14)$$

Substitute the definition of the elasticity of substitution between labor and capital in each sector [*e.g.*  $\sigma_j = (a_{Kj}^* - a_{Lj}^*) / (w^* - r^*)$ ] to obtain

$$d^2w/dr^2 = (\sigma_j/r)(a_{Kj}/a_{Lj}) > 0 \quad (14')$$

Finally, we consider the linear homogeneity. For this purpose we only need an inspection of Eqs. (11). Suppose that  $w$  and  $r$  increase by factor  $\lambda$ . Then the wage-rental ratio  $\omega$  remains unchanged, it is reduced that  $p_j$  increases by the same factor  $\lambda$ . The fact can be sufficiently proved if we recall that in a competitive equilibrium the input mix used in production depends solely upon the ratio of factor prices. The expression for the relative change in  $a_{Lj}$  and  $a_{Kj}$  can be obtained by the interaction of the minimum cost conditions and the definitions of the two factor elasticities of factor substitution  $\sigma_j$ . It proves convenient to write cost minimization in relative terms (denoted by an asterisk). Thus  $a_{Lj}^*$  is  $da_{Lj}/a_{Lj}$ . It entails that

$$\mu_{Lj} a_{Lj}^* + \mu_{Kj} a_{Kj}^* = 0$$

where  $\mu_{ij}$  is the  $i$ th factor distributive share in the  $i$ th commodity, for example,  $\mu_{Lj}$  is  $wa_{Lj}/p_j$ . The solutions for the  $a_{ij}^*$ 's can then be as follows.

$$\begin{aligned} a_{Lj}^* &= -\mu_{Kj} \sigma_j (w^* - r^*) \\ a_{Kj}^* &= \mu_{Lj} \sigma_j (w^* - r^*) \end{aligned}$$

Let us turn now to the case of monopoly. We concentrate on the linear homogeneity, which is a major concern throughout the entire analysis. The price equations now include monopoly profits. Let  $a_{\pi j}$  stand for monopoly profits per unit of output in the  $j$ th sector. Then

$$a_{Lj}w + a_{Kj}r + a_{\pi j} = p_j \quad (15)$$

where  $a_{\pi j} = p_j / \varepsilon_j$ .

With monopoly, we have the problem of determining  $\varepsilon_j$  in the context of general equilibrium. We assume that the social utility function is of C. E. S.

$$U = (ax_1^{\beta-1} + bx_2^{\beta-1})^{-1/\beta}, \quad a, b > 0, \quad -1 < \beta, \quad \beta \neq 0$$

where assumed  $a/b = 1$  for the sake of simplicity. Solving the above function for  $\varepsilon_j$  and restating the results in equations of change, we obtain

$$\begin{aligned} \varepsilon_1^* &= \frac{-\beta^2 \sigma_D^2 (p_2/p_1)^{\beta \sigma_D}}{[1 + (p_2/p_1)^{\beta \sigma_D}][1 + \sigma_D (p_2/p_1)^{\beta \sigma_D}]} (p_2^* - p_1^*) \\ &= -\theta_1 (p_2^* - p_1^*) \end{aligned} \quad (16a)$$

$$\begin{aligned} \varepsilon_2^* &= \frac{\beta^2 \sigma_D^2 (p_2/p_1)^{-\beta \sigma_D}}{[1 + (p_2/p_1)^{-\beta \sigma_D}][1 + \sigma_D (p_2/p_1)^{-\beta \sigma_D}]} (p_2^* - p_1^*) \\ &= \theta_2 (p_2^* - p_1^*) \end{aligned} \quad (16b)$$

where  $\sigma_D$  is the elasticity of substitution between two commodities in consumption. Differentiating Eqs. (15) totally and using the minimum unit cost condition, we obtain

$$\mu_{Lj}w^* + \mu_{Kj}r^* = p_j^* (1 - \mu_{\pi j}) + \mu_{\pi j} \varepsilon_j^* \quad (15')$$

where  $\mu_{\pi j} = a_{\pi j} / p_j = 1 / \varepsilon_j$ . Substituting Eqs. (16a) and (16b) in Eqs. (15'), and solving those two equations together, we have

$$p_1^* / w^* = |\mu| / [|\mu| + 1] \quad (17a)$$

$$p_1^* / r^* = |\mu| / [|\mu| - \Pi] \quad (17b)$$

where  $|\mu|$  is the notation for the matrix of production coefficients shown in (15'),

$$|\mu| = \begin{vmatrix} \mu_{L1} & \mu_{K1} \\ \mu_{L2} & \mu_{K2} \end{vmatrix}$$

$$\text{and } A = [(1 - \mu_{\pi 2})\mu_{K1} + \mu_{K2}\mu_{\pi 1}\theta_1 + \mu_{K1}\mu_{\pi 2}\theta_2] / |\mu|$$

$$II = [(1 - \mu_{\pi 2})\mu_{L1} + \mu_{L2}\mu_{\pi 1}\theta_1 + \mu_{L1}\mu_{\pi 2}\theta_2] / |\mu|$$

From Eqs. (17a) and (17b) we have

$$\frac{p_1^*}{w^*} + \frac{p_1^*}{r^*} = 1 + \frac{|\mu|^2 + AII}{|\mu|^2 + (A - II)|\mu| - AII} \quad (18)$$

By similar methods we can also obtain the expressions concerning  $(p_2^*/w^*) + (p_2^*/r^*)$ . An inspection of Eq. (18) reveals that each commodity price is no longer homogeneous of degree one in factor prices once again.

### References

- [1] Burmeister, D., *Mathematical Theories of Economic Growth*, Macmillan, 1970.
- [2] Hicks, J. R., *Capital and Growth*, Oxford University Press, 1965.
- [3] Ikema, M., "On the Factor-Price Frontier in the Pure Theory of International Trade," *Hitotsubashi Journal of Economics*, 18 (Feb. 1978), pp. 62-75.
- [4] Jones, R. W., "A Three-Factor Model in Theory, Trade, and History," in his *International Trade: Essays in Theory*, North-Holland, 1979.
- [5] Kemp, M. C., *The Pure Theory of International Trade and Investment*, Prentice-Hall, 1969, esp., chap. 1.
- [6] Kouri, P. J., "Profitability and Growth in a Small Open Economies," in A. Lindbeck ed., *Inflation and Employment in Open Economies*,



North-Holland, 1979.

- [7] Samuelson, P. M., "Parable and Realism in Capital Theory: The Surrogate Production Function," *Review of Economic Studies*, 29 (June 1962), pp. 193—206.
- [8] Shimomura, K., "Capital Accumulation, Industrial Structure, and the Industrial Composition of Trade," (in Japanese), *Kokumin-Keizai-Zasshi*, 140 (July 1979), pp. 54—72.
- [9] Steedman, I., *Fundamental Issues in Trade Theory*, Macmillan, 1979.