

Optimal Consumption Taxes and the Gain from Trade

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I

Perfect competition in the market is, indeed, one of the basic assumptions in the traditional theory of international trade. While monopoly and other forms of imperfect competition are certainly a fact of life, with a few exceptions, most of the international-trade models can work only under perfect competition.

"Broadly speaking, there have been two separate lines of attack on the problem of relaxing the assumptions of perfect competition in the theory of international trade. The first has treated the nation as the monopolizing unit, attempting to show the impact on trade of one nation's goods being generally imperfect substitutes for those another produces. The second analytical strand—and more interesting one, from the present viewpoint—explores the behavior of imperfectly competitive firms in international setting, tracing both partial and general equilibrium aspects."¹

The latter line of thought has not been developed so much. The theoretical reasons stem from the contrasting nature of imperfect competition theory and international-trade theory. That is, international-trade theory is mainly concerned with general equilibrium and with macroeconomic propositions. However, recently, a few mathematical models have contributed to examine the impli-

1 Caves (1963), p. 174.

cations of monopolistic elements existing either in domestic or international market, in the context of the two-country, two-good model of international trade.² The first framework for the analysis has been developed in the argument of optimum tariffs.³ Moreover, it is also well-known that a given country can reproduce an optimum tariff by appropriate combinations of consumption and production taxes.⁴ In the real world, although GATT rules restrict a country from imposing an optimal tariff, a country could still circumvent the free-trade orientation of this agreement by imposing appropriate consumption taxes. Since various forms of consumption taxes are widely used throughout the world, highlighting the impact of internal consumption taxes upon international trade seems to be relevant to the theory of international trade.

The implication of the imposition of domestic taxes has been analyzed by Friedlaender, Vandendorpe, and Dornbusch.⁵ They derived the welfare-maximizing tax rates on consumption and/or production for a country that possesses some monopoly power in international trade, and then gave a comparison of the effectiveness of tariffs and domestic taxes as tools for the maximization of a country's welfare, using Baldwin's envelope as known. It is the purpose of this paper to suggest the framework to examine explicitly to what extent a country can improve its welfare at the expense of its trading partners by imposing an appropriate consumption

2 See Melvin and Warne (1973), Sweeney, R. J., "Monopoly, the Law of Comparative Advantage, and Commodity Price Agreements: A Simple General Equilibrium Analysis," *Weltwirtschaftliches Archiv*, 110, No. 2, 1974. etc.

3 For a review of optimum tariffs arguments, See Amano (1964), Takayama (1972).

4 Lerner (1936).

5 See Friedlaender and Vandendorpe (1968), Dornbusch (1971).

tax. For this purpose, we specify a social utility function and a transformation curve representing a country's tastes and production possibilities. The application to the case of a production tax can be done in a similar way.

The following procedure of measuring welfare changes is based upon H. G. Johnson's.⁶ Section II concentrates on the aspect of consumption with a consumption tax imposed. And Section III deals with the aspect of production and the total (welfare) effectiveness of a consumption tax. Finally, Section IV makes remarks on parameters which effects the level of welfare improvement by a relevant consumption tax.

II

The analytical model is characterized by two-country and two-commodity, according to the standard model of international-trade theory.⁷ The home country consumes and produces, respectively, the quantities (C_1, C_2) and (X_1, X_2) of the two commodities and imports the second commodity. We define three price relations as follows:

$$\begin{array}{ll} \text{Consumption side:} & MRS = P \\ \text{Production side:} & MRT = P^* \\ \text{World prices:} & P^* = \frac{P}{1 + t} \end{array}$$

6 Johnson (1971), chapter 9. For the formula of welfare gain from optimum tariffs, see chapter 6.

7 The framework for the examination of the welfare implications of a consumption tax must be two-country, two-commodity model. It is the reason why a consumption tax becomes a tariff when the home country imports all of one good. In other words, the optimal-tariff formula emerges as a special case of the optimal consumption tax in the boundary case where the home country specializes completely. See Appendix.

where $M R S$ and $M R T$ are, respectively, the marginal rates of substitution and transformation, P and P^* , are, respectively, the home and world prices of the second commodity in terms of the first, t is the consumption tax rate on importables.

Now, the social utility function is assumed to be of the Cobb-Douglas type. Of course, we can alternatively use the C. E. S. utility function which permits the elasticity of substitution to vary from zero to infinity. Unfortunately, it is not possible using this function to derive a clear solution for the relative welfare gain from an optimal tax policy by ordinary algebraic methods. Instead, we can get a simpler expression using the Cobb-Douglas utility function.

Let the utility function be

$$U = C_1^\alpha C_2^{1-\alpha} \quad (1)$$

The marginal utilities of the two goods are

$$U_1 = \alpha \left(\frac{U}{C_1} \right)$$

$$U_2 = (1 - \alpha) \left(\frac{U}{C_2} \right) \quad (2)$$

And the $M R S$ between C_1 and C_2 is

$$\frac{U_2}{U_1} = \frac{1 - \alpha}{\alpha} \cdot \frac{C_1}{C_2} \quad (3)$$

Given the tax t on the consumption of C_2 , the home country maximizes its welfare by setting

$$\frac{1 - \alpha}{\alpha} \cdot \frac{C_1}{C_2} = P = P^* (1 + t) \quad (4)$$

The budget constraint of the country is given by

$$Y = C_1 + P^* C_2 \quad (5)$$

implying that all tax revenue collected on the consumption of C_2 is redistributed in a manner implicit in this social utility function.

In this case, consumption of the two goods are determined by equations (4) and (5) as follows,

$$C_1 = \frac{\alpha(1+t)}{1+\alpha t} Y$$

$$C_2 = \frac{1}{P^*} \cdot \frac{(1-\alpha)}{(1+\alpha t)} Y \quad (6)$$

Secondly, we assume the consumption of importables given by

$$C_2 = (1-\alpha) Y P \xi_2 \quad (7)$$

where ξ_2 is the price elasticity of the consumption of importables, and the constant term $(1-\alpha) Y$ is chosen so that the free-trade price of importables will be unity. We can find out that by the following equation derived from equations (6) and (7):

$$P^* = (1+\alpha t)^{-\frac{1}{\xi_2+1}} (1+t)^{-\frac{\xi_2}{\xi_2+1}} \quad (8)$$

Hence, substituting (6), (8) into (1), the level of welfare achieved with domestic tax can be obtained as,

$$U = (1+t)^{\frac{\alpha+\xi_2}{\xi_2+1}} (1+\alpha t)^{-\frac{\alpha+\xi_2}{\xi_2+1}} \alpha^\alpha (1-\alpha)^{1-\alpha} Y \quad (9)$$

III

In order to permit the direct manipulation of U in terms of the tax rate t , it is instructive to specify the transformation curve $X_1 = f(X_2)$. It is assumed to be of the form:

$$X_1^2 + mX_1X_2 + X_2^2 = T^2 \quad (10)$$

where T is a constant, and m is a parameter which may vary from 0 to +2 with satisfying the usual restrictive conditions on the shape of the transformation curve. In other words, the equation reflects an increasing-cost production-possibility frontier, denoted as, $f' < 0$, $f'' < 0$. In the case $m = +2$, the transformation curve

is a straight line, in another case $m=0$, the transformation curve can be shown as a quarter circle.

The real income in terms of production values is shown as follows.

$$Y = X_1 + P^*X_2 \quad (11)$$

Since we have $MRT = P^*$ at the equilibrium prices, as already stated, it follows that:

$$\frac{mX_1 + 2X_2}{2X_1 + mX_2} = P^* \quad (12)$$

Solving X_1 and X_2 for P^* and substituting the results into (11) yields,

$$Y = \frac{p^{*2} - mp^* + 2}{2\sqrt{(4-m^2)(p^{*2} - p^*m)}} \cdot T \quad (13)$$

Here, from equations (9) and (13), the social welfare index can be shown to depend upon some parameters.

$$U_t = \frac{1}{2} \alpha^\alpha (1-\alpha)^{1-\alpha} T \cdot \frac{B}{\sqrt{(4-m^2)A}} \quad (14)$$

Where, $A = (1+t) \frac{-2\xi_2}{\xi_2+1} (1+\alpha t) \frac{-2}{\xi_2+1} - m(1+t) \frac{-\xi_2}{\xi_2+1}$

$$\times (1+\alpha t) \frac{-1}{\xi_2+1}$$

$$B = (1+t) \frac{\alpha - \xi_2}{\xi_2+1} (1+\alpha t) \frac{-\alpha + \xi_2 + 2}{\xi_2+1} - m(1+t) \frac{\alpha}{\xi_2+1}$$

$$\times (1+\alpha t) \frac{-\alpha + \xi_2 + 1}{\xi_2+1} + 2$$

Thus, substituting the possible value of the optimal consumption tax into this equation and $t = 0$ for free trade, dividing and subtracting unity yields relative welfare gain from the optimum consumption-tax policy as compared with free trade, denoted by

$$\frac{U_{ot}}{U_{t0}} - 1 = f(m, \xi_2, \alpha) \quad (15)$$

IV

The model used in this paper is simplified so that the possible optimal-consumption tax can be determined by three parameters, m , ξ_2 , α . Our notation of parameters is different from Friedlaender & Vandendorpe's. In their analysis, the formula for the optimal consumption tax is shown to contain three different parameters, i. e. the price elasticity of the production of exportables, the foreign price elasticity of imports, and the ratio of exports to the production of exportables. In this paper, ξ_2 , the price elasticity of the consumption of importables refers to the nature of the preference system, and the shape of the transformation curve is denoted by a parameter, m . We conclude the analysis by making some remarks on the parameters which effect the level of welfare improvement by a relevant consumption tax, with a comparison to Friedlaender & Vandendorpe's formula.⁸

We assumed implicitly trade balance at the equilibrium. The home country maximizes a social welfare index, $U = U(C_1, C_2)$, subject to the income-expenditure equality

$$C_1 + P^*C_2 = X_1 + P^*X_2 \quad (16)$$

Since C_1 and X_1 represent consumption and production of commodity i in the home country, letting E_1 , the excess demand, defined as $C_1 - X_1$, the requirement of trade balancing is given by

$$E^*_1 + P^*E^*_2 = 0 \quad (17)$$

where starred variables refer to the corresponding quantities in the foreign country or the rest of the world. Differentiating, $C_2 = X_2 - E^*_2$ totally by P and rearranging terms yields

8 Friedlaender and Vandendorpe (1968), p. 1061.

$$\frac{P}{C_2} \frac{dC_2}{dP} = \frac{dX_2}{dP^*} \frac{P^*}{X_2} \frac{X_2}{C_2} - \frac{dE^*_2}{dP^*} \frac{P^*}{E^*_2} \frac{E^*_2}{C_2} \quad (18)$$

Rewriting in terms of elasticity, it follows

$$\xi_2 = e_2 \cdot \frac{X_2}{C_2} - \varepsilon^*_2 \cdot \frac{E^*_2}{C_2} \quad (19)$$

Assuming that the home country imports the second commodity, this implies

$$\frac{d\xi_2}{d\varepsilon^*_2} > 0 \quad (20)$$

Next, we now check the relation between m and the elasticity of supply of X_2 with respect to P^* . Solving equation (12) for X_1 as a function of X_2 , P^* and substituting the result into equation (10) yields the expression for X_2 as a function of P^* ,

$$X_2^2 \left(\frac{P^*m^3 - P^{*2}m^2 - m^2 - 4mP^* + 4P^{*2} + 4}{m^2 - 4mP^* + 4P^{*2}} \right) = T^2 \quad (21)$$

Denoting the expression in parentheses by D for simplicity, differentiating equation (21) totally by P^* , and rearranging terms yields the elasticity of supply of X_2 with respect to P^* ,

$$e'_2 \equiv -\frac{1}{X_2} \frac{dX_2}{dP^*} = \frac{1}{2} \cdot \frac{1}{D} \cdot \frac{dD}{dP^*} \quad (22)$$

Reforming the differentiation and simplifying the result yields the elasticity formula

$$e'_2 = \frac{1}{2} \cdot \frac{(m+2)(m^3 - 2m^2 - 4m + 8)}{(2P^* - m)(P^*m^3 - P^{*2}m^2 - m^2 - 4mP^* + 4P^{*2} + 4)} \quad (23)$$

whence, substituting $P^* = 1$, the elasticity of supply of X_2 at the free trade point is

$$(e'_2)_f = \frac{1}{2} \cdot \frac{2+m}{2-m} \quad (24)$$

From this it follows that as m decreases from $+2$ to 0 , $(e'_2)_f$ falls from ∞ to $\frac{1}{2}$.

V

In the foregoing analysis using a simplified model of two-country, two-commodity, we suggest the framework for the examination of the gain from trade by exploiting monopoly power in international trade, i. e. a fiscal policy. However, unfortunately, the model used in this article could not permit the drawing of any firm conclusion about it. It will be possible to provide a clear and plausible calculation about the welfare effectiveness of optimal consumption taxes in the case when the model can be more sophisticated. Generally speaking, it is necessary to attempt to present various general-equilibrium models of international trade, introducing monopolistic elements. This article is only one step to present the framework to the welfare effects of exercising monopoly power in international trade. The studies on the setting monopolistic situations in the standard model of international-trade theory appear to be just started. Anyway, it is left for the future to do comprehensive research on the comparative welfare observation of possible policies exploiting monopoly power in international trade.

Appendix

The formula of the optimal-consumption tax rate:

We will make a direct derivation of the tax rate. As already stated in Section II, three conditions are satisfied at the optimal equilibrium. That is,

$$P = MRS \quad (1)'$$

$$P^* = MRT \quad (2)'$$

$$E^*_1 + P^*E^*_2 = 0 \quad (3)'$$

We can derive from equation (1)'

$$-\frac{dC_1}{dC_2} = (1+t)P^* \quad (4)'$$

and recalling the definition C_i ($i=1, 2$), this equation can be re-written as,

$$-\frac{dX_1 - dE^*_1}{dX_2 - dE^*_2} = (1+t)P^* \quad (5)'$$

Substituting the total derivation of equation (3)', and the relation $P^* = (-dX_1 / dX_2)$, given by equation (2)', into equation (5)' yields

$$t = \frac{E^*_2 dP^*}{P^* dE^*_2 - P^* dX_2} \quad (6)'$$

In terms of elasticity, it follows that

$$t = \frac{1}{\varepsilon^*_2 - e_2 \eta_2} \quad (7)'$$

ε^*_2 : the foreign(total) price elasticity of exports,

e_2 : the price elasticity of the production of importables,

and

η_2 : X_2/E^*_2 .

Thus, it is optimal for the home country to impose a consumption tax on its importables with such rate of tax. This formula is an alternative expression of one derived by Friedlaender and Vandendorpe.⁹ (21, 12, 1974)

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