Uncertainty, Investments and Property Rights — A Simple Exposition

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Abstract

The paper explores the implication of uncertain property rights for investment decision and effort level. Based on the analysis of Dixit and Pindyck (1992) for investment under uncertain returns our major result further demonstrates that when uncertain property rights are present along with uncertain returns, the trigger for entry becomes higher than the trigger derived for the case of uncertain returns only. The analysis has implications for the elimination of entry barriers and the formation of private sectors in economic transitions.

Key-words. Investment, Property Rights, Uncertainty, Economic Transition.

1. Introduction

Recent revival of investment theory suggests that when uncertainty is introduced, a quite different picture emerges concerning entry and exit decisions of firms. The familiar long-run and short-run analysis implies immediate investment if price exceeds long-run average costs and suspension and exit if price falls below average variable cost. When return from investment is uncertain, firms' choices are widened, waiting acquires a value

because it renders flexibility for the investor. Immediate investment and en-
try kill the opportunity to wait. The value of waiting constitutes part and
parcel of the costs of investment. To induce investment and entry, the price
needs to be substantially higher than the long-run average cost. Likewise,
fall of price to average variable cost would not result in suspension and exit
unless the price falls further down below the variable cost. Thus the prize
zone for entry and exit under uncertain return becomes wider than the usual
Marshallian inaction zone.

　　Implicit in the new theory is the assumption of certain property
rights, i. e. a certain entitlement to the investment and returns from the in-
vestment. Evidence in the market economies, and signs since the economic
transformation in centrally planned economies in particular, seem to sug-
gest that uncertain property rights further complicate the investment deci-
sion of the firm. It is observed every now and then, that the incumbent
managers and employees of the state enterprises do little to improve the cur-
rent affairs of the enterprise, or delay the effort supply or investment when
facing the prospect of privatization or re-organisation. Worse more, even
maintenance of current assets is called into question. This is partly ex-
emplified in the dramatic falls of economic activity in some East European
countries and in the former USSR, as well as in the transition, after reunifica-
tion of Germany (Sinn, 1993). Historical evidence also indicates that the ef-
fects of uncertain property rights on investment and effort supply can not
be neglected. Evidence from mergers and take-overs in the market eco-
nomies, though mixed, are perhaps not less suggestive (Holmstrom and
Tirole, 1993). In spite of these, little formal analysis has been made on the
cases in which uncertain property rights loom large. Even the most recent
seminal book on the state of knowledge by Dixit and Pindyck (1994) seems

to be mute on this issue.

The purpose of this paper is to explore the implications of uncertain property rights for investment decision and effort level. First, as a benchmark, effects of uncertain return on investment under certain property rights are analyzed. Uncertain property rights are then introduced. The issue of optimal delineation and enforcement of property rights is indicated. This paper is organized as follows. Section 2 presents some recent advance in the theory of investment based on Dixit (1992). The option value of investment opportunity under uncertain returns is investigated and implications are drawn for optimal entry and exit decisions. Section 3 provides the optimal investment policy under the given conditions. The analysis in section 4 further demonstrates that when uncertain property rights are present along with uncertain returns, the trigger for entry becomes higher than the trigger derived by Dixit for the case of uncertain returns only. The analysis also has implications for the elimination of entry barriers of various kinds and the formation of private sectors in economic transitions. The likely behaviour of state-owned enterprises facing re-organisation is also indicated. We wish to shed lights on the appropriate ways of transforming state-owned enterpriese in transition, (Section 5). Section 6 concludes the paper.

2. Irreversibility, Uncertainty and Investment

This section discusses the timing and value of waiting under uncertain returns from investment; presents an example to facilitate analysis. Then we are concerned with effects of waiting in investment where an exogenous investment trigger is introduced. We are dealing with the optimal investment trigger. The benchmark model introduced is based on Dixit

(1992). Uncertain property rights are introduced in a very simple manner where a probabilistic fashion is added to the analysis, the implications of which for reform in property rights structure turn out to be great.

Most investment decisions exhibit three features. First, investment entails sunk cost, an expenditure that cannot be recouped if the action is reversed; second the economic environment is uncertain and information comes gradually. Finally, an investment opportunity does not usually disappear if not taken immediately. When these conditions are present, waiting has positive value. As long as the opportunity remains, a later decision may be a better one. Because investment is sunk cost, it is not always wise to take an imperfect action now and change it later except for the purpose of entry deterrence.

Of course, the value of waiting has to be set against the sacrifice of current profit. If current conditions are sufficiently favourable, one should eventually take the action that is optimal according to the current calculation, and not wait any longer. But the 'trigger' level of currently expected profit that makes it optimal to proceed exceeds the Marshallian normal return. Similarly, waiting has value when contemplating disinvestment. The Marshallian criterion of failing to cover average cost should not trigger abandonment; the correct point is a critical negative level of operating profit. An example may illustrate this.

Suppose a project can be launched by incurring a sunk cost K, and once launched, lasts for ever. Let R denote its flow of net operating profit per unit time.

This is where the uncertainty of the first type comes in. Future revenues are imperfectly predictable from the current observation. The probability distribution of future net revenues is determined by the present, but

the actual path remains uncertain. This probabilistic law of evolution of R can take many forms, but a simple specification is very useful and realistic. Suppose R follows a random walk whose steps are of equal proportions, that is, they form a geometric series. If the period of each step of R is very short, then the distribution of the logarithm of R_t at a future time t, given the initial R_o at time 0, is approximately normal. Then R is said to follow a proportional geometric Brownian motion. For simplicity, assume the trend rate of growth of R is zero.

To see the implications of uncertainty, we first consider the case of certainty and then we make the comparison with uncertainty. A risk neutral investor maximizes the expected present value of profits. Let future revenues be discounted at a positive rate, the opportunity cost of risk-free capital specified exogenously. Then given the current revenue R, the expected present value of the discounted future stream of revenues is R/ρ.

The usual criterion suggests investment when the project has positive expected net worth, i. e. $R/\rho > K$. M denotes the borderline level of the current revenue flow that makes one indifferent between investing and not investing. M is given by

$$M = \rho K \ldots \tag{1}$$

The textbook recommends investment if $M > \rho K$. Following Dixit, call M the"Marshallian investment trigger".

This criterion comes obviously from the case where the choice is between acting right now to get $R/\rho\text{-}K$ and not investing at all, which gets nothing. The underlying assumptions of Marshallian investment strategy, as noted by McDonald and Siegel (1986) is zero variance of the present value of future returns and costs or minus infinite expected rate of growth

of the present value. Actually when returns from the investment are not certain, the menu of choices is wider, and waiting for a period of time and reassessing the decision is possible. At the Marshallian trigger, waiting is better than either investing right away or not investing at all because returns are assured. When R is not deterministic, an alternative strategy can emerge: wait for a fixed interval of time, and observe the value of R, say R_1 at the end. If $R_1 > M$ invest at once, otherwise don't. If the return at the end of the fixed waiting time exceeds the Marshallian trigger, then the net worth of the investment must be positve at that time, and remains positive when discounted back. Therefore the proposed alternative strategy serves better than either investing at once or not investing at all, each of which yields zero when the current revenue is exactly at the Marshallian trigger. But waiting remains better than investing for initial values of R slightly in excess of M.

The upshot is that if the investor knows that R follows a random walk waiting for a certain amount of time may enable an investor to avoid the downward risk in revenues over the interval, while realizing the upward potential. This selective reduction in risk over time generates a postitive value of waiting. On the other side, the cost of waiting is the sacrifice of profit flow over the period of waiting. Hence, if the current net revenue flow reaches a sufficiently high level, it won't pay to wait any longer. There is still a critical or trigger level, say H, such that investment is optimal when current revenue exceeds it. But this H is larger than the Marshallian trigger M.

The precise argument can be illustrated by first considering the parameters that determine the difference between an exogenous H and the Marshallian trigger M. Endogenous H is considered in the next part of this
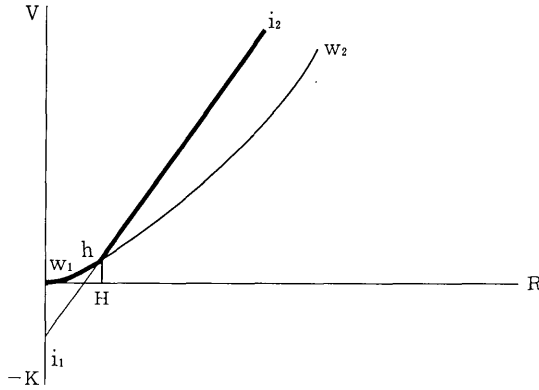
Figure 1 : The value of waiting and investing

section.

The upward-sloping straight line i_1 i_2 represents the value to be received from investment immediately; that is, R/ρ-K. If R is zero, then the project would lose K. Otherwise, the value of this function increases with slope $1/\rho$ as the return R increases.

Now consider how the expected return from this project changes if the rule is applied that investment will occur only if the expected return R exceeds a trigger H. If the trigger is surpassed, then the investment project takes place, and the return is given by the thickly drawn portion of the line i_1 i_2 above the point H, where R=H. If the expected return is equal to the trigger, then the firm would be indifferent between waiting and investing immediately. If the expected return is less than H, then wait. There is a positive probability that at some future time R will climb above H and generate a positive net worth. The firm rationally anticipates this so the net worth is positive even now. The value is merely the value of waiting, or that of the opportunity or "option" to invest at some future date.

The option value approaches zero if R is very low. Successively

higher current values of R raises the value of waiting increasingly rapidly. For R close to H but just below it, the probability of reaching H in the very near future approaches one, and the option value approaches the net worth of a live project at H. This is shown as the convex curve w_1 w_2 in Figure 1, starting from the origin and meeting the straight line i_1 i_2 at h. We should note that only the thickly drawn portion w_1h to the left of H gives the value of waiting; beyond h investment takes place, the option is exercised and the value of waiting is irrelevant. The overall value of the opportunity to invest is given by the thick curve w_1h and the thick line hi_2 taken together. The functional forms of these curves are derived by Dixit (1992) to be of

$$V(R) = BR^\beta \ \forall R \leq H(B > 0, \beta > 1);$$
$$\frac{R}{\rho} - K \ \forall R \geq H \ldots \tag{2}$$

The upper formula is the value of waiting given by the convex w_1 w_2 in Figure 1. The lower expression is the value of investing given by i_1 i_2 in Figure 1. The thickly drawn portions correspond respectively to the value of waiting or investing in its valid range, and the light portions show the continuation of the separate parts into the irrelevant regions. At H there is indifference, so the two expressions are equal.

Two terms in the first expression showing the value of waiting require explanation. The power β depends on the discount rate ρ and the volatility of the revenue, which is measured by the variance of σ^2 of the logarithm of R per unit time. Dixit has shown that

$$\beta = \frac{1}{2} \left[1 + \sqrt{1} + B\frac{\rho}{\sigma^2} \right] > 1 \ldots \tag{3}$$

B is a multiplicative constant. It is determined by the condition that the two expressions for net worth V(R) must be equal when R equals H.

Therefore

$$BR^\beta = \frac{H}{\rho} - K \dots \tag{4}$$

where R = H. To rephrase in a familiar way, we obtain

$$\frac{H}{\rho} = K + BH^\beta \dots \tag{5}$$

3. The Optimal Policy

In the previous example, the investment trigger H was assumed to be exogenously given. Now consider how the investment trigger should optimally be chosen. If the trigger value H is increased slightly above its value in Figure 1, that shifts the junction point h between the thickly drawn curve and the line to the right. This can be accomplished by raising the whole curve $w_1 w_2$ if the risk-less discount rate is constant which corresponds to raising B in the upper formula. Alternative to Dixit who allows $w_1 w_2$ to shift, the tangency can also be accomplished by shifting $i_1 i_2$ downward which corresponds to raising the discount rate. It is noted that many firms in practice use this simpler procedure. They adjust the discount rate rather than adjusting the present value. The appropriate rates of discount that investors should use are derived in what follows.

With respect to the shift conceived by Dixit, to maximize value, such shift should be pushed as far as possible, that is, until the graph of the value of waiting-the curve given by BR^β-becomes tangential to that of the straight-line return of investing immediately: R/ρ-K. The optimal investment trigger H is determined by equating the slopes of the two formulas. This is often referred to as "smooth pasting condition". It is obtained by differentiating the two expressions in equation (2) with respect to R, evaluate the

derivatives at H, this gives

$$\beta BH^{\beta-1} = \frac{1}{\rho}\cdots \tag{6}$$

Use equations (5) and (6) to solve for H and eliminate B. The optimal H is given by

$$H = \frac{\beta}{\beta-1}\rho K \cdots \tag{7}$$

Remember the Marshallian investment trigger M is to invest when $M = \rho K$. In terms of equation (7), this holds when β/β- 1 approached one, i. e. β approaches infinity. This is so if the variance is zero. Under certainty, the two triggers coincide. Therefore we have a very simple relation between the Marshallian trigger under certainty and the optimal trigger under uncertainty. Since β is greater than one, we must have $H > M$ as shown in Figure 2.

Alternatively, we can express the optimal trigger in a way that parallels the Marshallian formula. Define a new discount that allows for the value of waiting induced by the uncertainty. For this we need $H = \rho' K$ or

$$\rho' = \frac{\beta}{\beta-1}\rho \cdots \tag{9}$$

To avoid confusion, the corrected discount rate derived by Dixit is henceforth called Dixitian rate of discount. As the next section shows still another discount rate can be derived for the case of uncertain property rights. Both Dixitian rate and the rate that is to be derived are subjective rates used by investors in evaluating investment projects under uncertainty. The formulation here can be applied to develop some estimates of the potential difference between the optimal trigger and the Marshallian trigger. A numerical illustration is given in Sec. 5.

We may at this point have an intuitive understanding of (5). Instead of correcting the discount rate for the value of waiting, we could correct the cost of investment. Immediate action has an opportunity cost, namely loss of option to wait. This is valued at BR^β and we must add it to the actual cost of investment to obtain the full cost of immediate action. Then such action is justified when the benefit exceeds the full cost. This happens when the current revenue R reaches the trigger H.
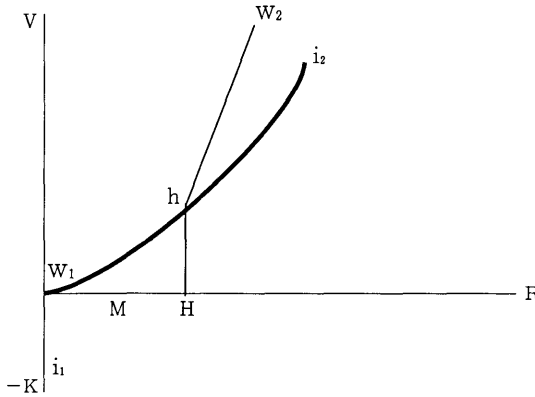

Figure 2 : The optimal investment trigger

That the opportunity to invest is an option can at least be dated back to Irving Fisher (1930). For the real project under consideration, the exercise price is the sunk cost K. If the option is exercised, the firm would acquire ownership of the investment that pays a dividend stream of discounted expected present value. It is little surprise that option pricing techniques can be used to analyze how and when to exercise the investment opportunity. Pindyck (1991) provides the details.

4 . Uncertain Property Rights, Costs of Investment and Entry Deterrence

The foregoing analysis takes property right certainty for granted, i. e. once the investment is made, the investor is assured of owning it and the right to the dividend streams, if any. This is so if ownership right is honoured and enforced at zero costs or the costs of enforcement are born by an outside party, be it government, individuals or group of individuals. As far as our analysis is concerned, it is a limiting case. A general case is of property rights that are uncertain to some degree. This is perhaps of more importance and significance if the cost of ensuring ownership right is not negligible.

If the investor is unsure whether to obtain the ownership rights or not, before and after making the investment, we say that uncertain property rights are present. Then what determines the option value of the investment opportunity? Is the value of the option in the presence of uncertain property rights influenced by the degree of property rights uncertainty in addition to the variance of return? It may be that under some circumstances, the investor has no incentives to invest since he has to make sure he will acquire the ownership of the investment. Worse more, the investor may have incentives to give away the option to invest if the option has negative value. If it does have negative value, no-one would exercise the option. If granted the option, he would be willing to pay a sum of money to get rid of the option. Likewise, if the return is high enough to offset the effects of ownership uncertainty, the option value of investment becomes higher. Then it may be worthwhile for the investor to incur some expenses in oder to secure ownership rights. The actions of the investor may take many forms ranging from

bribery in the case of legal barriers to entry to rent-seeking in the case of regulation. It is clear that the costs of investment must now include the resources spent on securing the ownership right.

To simplify the illustration, suppose uncertain property rights take a probabilistic form. There is a probability $P(0 <P< 1)$ that the realised revenues can not be secured by the investor due to a variety of reasons: expropriation, taxation and regulation by the state; theft by individuals; natural and artificial disasters such as earthquakes and wars. ρ is like a tax rate in the case of taxation. Since a rational investor anticipates these possibilities before making the commitment, the amount being expropriated constitutes part and parcel of the costs of investment. Suppose for simplicity the "take-away" as proportional to revenues R/ρ. The firm should invest if and only if

$$\frac{R}{\rho} > K + BR^\beta + P\frac{R}{\rho}$$

Equalisation and (6) give us yet another investment trigger at which the firm is indifferent between investing right now and waiting. Denote this new trigger H'. It is evident that

$$H' = \frac{1}{1-P}\frac{\beta}{\beta-1}\rho K > H \dots \tag{10}$$

Expressed in a familiar form as before, we obtain still

$$\rho'' = \frac{1}{1-P}\frac{\beta}{\beta-1}\rho = \frac{1}{1-P}\rho' > \rho \dots \tag{11}$$

another discount rate denoted ρ''. It is clear that this rate is higher than the Dixitian rate of discount. The intuition behind this discrepancy is that when property rights are uncertain, future returns are more heavily discounted. In other words, the discrepancy acts as a risk premium for uncertain proper-

ty rights which is not diversifiable by the investor.

Two limiting cases of intuition are to be noted: $\rho = 0$ corresponds to property rights certainty over investment and returns, we then have H'=H; $\rho = 1$ corresponds to a complete lack of property rights. By this we mean that when investment is made, both the sunk cost and returns are bound not to be obtained by the investor. Then we have H' approaching infinity. In other words, investment will never take place, if P approaches one. With P approaching one, discount rate ρ''goes to infinity. Future returns are most heavily discounted, i. e. future returns do not count in the decision concerning investment. Interpreted this way, anticipated taxation, regulation or theft influence behaviour of rational investors while unanticipated disturbances may not have the same effects. In general, if $0 < P < 1$, then H'>H. When property rights uncertainty is present, the cost of investment is raised, a higher trigger is thus needed to induce investment. Therefore, uncertain property rights are more damaging to investment and innovation than if only uncertain returns are present. The presence of property rights uncertainty deters investment akin to other barriers of entry and deserve attention. The assertion that individuals will delimitate property rights as part of their maximizing process is only part of the truth(Barzel 1989). There are circumstances under which individual maximization do not lead to maximization of total products. One glaring case is the misallocation of resources due to redirection of incentives induced by uncertain property rights. Rentseeking, monopoly and tariffs are obvious examples. Even if one wishes to define and enforce property rights, the optimal delimitation of property rights may never be obtained due to the costs of doing that.

We thus arrive at a chain of trigger levels of investment and the equivalent rates of discount for investment.

$$H' > H > M; \ \rho'' > \rho' > \rho \dots \tag{12}$$

A Marshallian trigger of investment applies under double certainties about property rights and returns, while the Dixitian trigger H applies under uncertainty about returns. Our derivation of trigger H' seems most realistic of all: it applies under double uncertainties about returns and the ownership rights on investment and return. It is of prime importance for practical investment and public policy. We do not often consider uncertain property rights except under certain specific circumstances.

The forgoing analysis, based upon individual rationality, provides an alternative explanation of why business in underdeveloped countries is often difficult. This is so not because of dishonesty but of uncertain property rights. The analysis also rationalizes an age-old faith in the security of property. It may imply that "without establishing initial property rights there can be no market transactions to transfer and combine them (Coase, 1960). If only the common laws arbitrate property rights, all the court needs to do is to understand the economic consequence of their decisions and should make the delimitation possible without creating too much uncertainty about the legal position itself. A variety of arbitrators of property rights exist and common laws and private laws are two of them although it is true in all cases that the optimal criterion is to assign property rights to those who can use them most productively and with incentives that lead them to do so (Coase, 1992).

Further discussion of property rights enforcement is too far afield. A brief remark may be appropriate. In the absence of public honour of property, individuals will set out to secure ownership rights only if expected returns are sufficiently larger. In other words, the trigger for investment

under uncertain property rights will be higher than the trigger under uncertain returns. The former trigger may seem excessive. But the excess is nothing but the costs incurred in securing and protecting ownership rights. It is part of the costs of investment. Such costs are often dubbed as transactions costs. In a large exchange economy, private enforcement of some property rights can be costly due to diseconomies of scale. Public enforcement of some property rights might reap the economies of scale, therefore reducing private costs of investment, Shavell (1991). This by no means implies that public actions are always desirable. Subdivision, combination and exchange of property rights are constantly taking place, not all of which are enforced formally. Numerous "invisible institutions" and "non-contractual arrangements" are also available such as customs, conventions, religions and even fashions. Nevertheless it remains true that if there were no collective actions to delineate and enforce property rights or the public actions did the job poorly, the growth potential would be seriously curtailed. That is why well-governed societies tend to have larger potential to growth than badly governed societies other things being equal. But if the costs of delimitating and enforcing property rights are zero, optimal allocation of resources can always be attained.

5. Implications for Privatisation

        Given the costs of delineating, acquiring, combining and enforcing property rights, different criteria of assigning previously unrecognized property rights must have consequences for efficiency. They are signs to suggest that the drastic fall of economic activity in a number of transitional economies may be partly caused by inappropriate criteria of assigning property rights to the parties involved, for instance the lack of attention to in-

̈cumbent managers and employees despite the repeated assertion of the failure of the labour-managed firm, Pejovich (1992). The optimal assignment of previously unrecognized property rights warrants separate studies, however.

To illustrate the magnitude of the difference between H and M, or equivalently the difference between the modified discount rate and the risk-free discount rate, suppose the variance of the return is 0.04, if the risk-free discount rate is 5 % per year, β is 2.15 and the Dixitian discount rate amounts to 9.35%. Thus the current revenues have to rise to nearly double the level that ensures a positive net worth before waiting ceases to be optimal. With the introduction of uncertain property rights, the corresponding figures will be higher than the Dixitian investment trigger and discount rate.

To have a general sense of how the underlying parameters affecting β and the Dixitian discount rate, note the definition of β given earlier that a lower risk-free discount rate or a higher standard deviation of the revenue yields a lower β. In turn, a smaller β means a larger factor $\beta/(\beta-1)$ and therefore the longer it is optimal to wait. It is intuitively evident that when the future is less heavily discounted, the value of waiting for more information goes up. It is also intuitive that greater uncertainty means a higher value of waiting. If in the numerical example we raise the variance to 0.15 while keeping the risk-free discount rate at 5 %, then $\beta = 1.43$, the Dixitian discount rate rises to 16.6% and H is 3.32 times M.

Two limiting cases are worth mentioning. If the future is very heavily discounted or very certain, then β goes to infinity and $\beta/(\beta-1)$ goes to one. Option values become unimportant in this limit and the usual Marshallian criterion applies. In the opposite extreme, as the risk-less dis-

count rate goes to zero or the variance of the return goes to infinity, β goes to one and $\beta/(\beta-1)$ goes to infinity. The Marshallian criterion becomes misleading.

6. Extensions and Qualifications

The above example assumes that the option to invest, assigned to one firm or individual lasts for ever. It may be conceivable that the investment opportunity is perceived by one entity only. In practice, one has to consider more realistic situations.

First, there may be more than one entity who perceives the opportunity so that there is a competition for the option to invest as is usually the case in the field of competition. This competition may in effect shorten the maturity date of the option. Then the option value declines as its maturity date approaches, if the value of the investment does not change. Further analysis of this aspect may enable us to have a better understanding of how competition works.

Secondly, when there are several potential investors, their information may differ. If firms are under different institutional settings as in transitional economies, this will surely occur because there are not only differences in information, but also in the ability and incentives to act upon the opportunity. Potential private entrants, if they have the same access to finance, may discern investment opportunity quicker than their state-owned counterparts who face the prospect of privatisation. The value of the investment opportunity for the two types of investors may well differ. This could be one reason why a private sector in the transitional economy may quickly emerge.

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