Very Low Interest Rate Policy under Imperfect Capital Mobility

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Abstract

After the global financial crisis, very low interest rate policy, which includes a zero interest rate policy (ZIRP) or a quantitative easing policy (QEP), spread throughout the United States, Japan, and Europe. International capital movements did not respond to returns sufficiently, because international investors had become more sensitive to risks during the financial crisis. In this paper, we construct a two-country model with imperfect capital mobility and the difference of the price-adjustment speed between countries. We characterize the optimal response of monetary policies to an asymmetric productivity shock by conducting numerical analyses. There were three primary results of our analysis. First, there exists a policy objective trade-off between output stability and optimal allocation of resources. The slightly tight monetary policy was found to be optimal, in the sense of the marginal cost equalization of the two objectives. Second, the higher the sensitivity of international capital mobility to a difference of returns, the smaller the interest cut necessary. Third, if capital is completely immobile between sectors, the policy rule insisted by Aoki (2001) is the optimal response.

Keywords: Global Economic Crisis, Monetary Economics, Monetary and Fiscal Policies

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1 Introduction

In Japan, the extremely lower interest rate policy, which includes zero interest rate policy (ZIRP), has been continued since the latter half of the 1990s. This event inspires the economists to explain the effects of the monetary policy under the zero lower bound of the nominal interest rates. Krugman (1998) pointed out that the equilibrium real rate of interest (or the natural rate of interest) is negative in the late of 1990s in Japan. Then, other economists developed some theories on the effectiveness of the extremely lower interest rate policy without capital accumulation[*1]. Some researchers criticize those theories for the dynamic inefficiency. Recent researches are developed including the factor of capital accumulation[*2].

Some of senior officials in the Bank of Japan (BOJ) and some economists are afraid that the current stance of monetary policy is excessively easy, as those theoretic analyses on the monetary policy under very low interest rate are progressed. In the beginning stage of the zero interest rate policy, some policymakers including the then-BOJ governor Hayami and some economists shared the idea that the deflation with relative-price changes was not always harmful to macroeconomic conditions because of the price reduction in some sectors with high productivity growth. The deflation became more serious after the ZIRP was abandoned in August 11th, 2000, the idea of the " good deflation " lost favor with the public.

On the other hand, a lot of researches focused on the supply-side of the economy, because the stagnation has lasted for the most period of the 1990s.

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*1 Eggertsson and Woodford (2003), Jung et al. (2006), and so on analyze the optimal monetary policy under the zero lower bound of the nominal interest rates.

*2 For example, the researches are of Woodford (2003, Chapter 5), Woodford (2005), Takamura et al. (2005), and so on.

Especially, many economists and commentators shared the idea that the economic system which had supported the rapid growth in the post World War II era caused the economic inefficiency as the economy matured. For example, Peek and Rosengren (2003) show that the Japanese financial system and banking supervision have promoted for commercial banks to accommodate credits to the inefficiently operated firms, and prevented restructuring the corporate sector. The " structural reconstruction " by the former Koizumi administration is supposed to be followed that perspective.

We construct a two-country model to investigate the changes in the real exchange rate and the international allocation of resources, for the background mentioned above. In particular, we suppose the incompleteness of the international capital mobility in the two-country model, and then investigate the optimal responses of monetary policy to the relative productivity shock between the countries.

This paper is constructed as followings. In the section 2, we construct a two-sector model with incomplete capital mobility between sectors. In the section 3, the structural equations are derived by the log-linear approximation of the model constructed in the section 2. Then, we derive the objective function of the monetary authority, and specify the optimal responses of monetary policy to the shock that needs some relative-price change. We conclude this analysis in the section 5.

2 The Model

In this section, we explain the structure of our model. The main assumptions of the model are as follows. First, there are many firms in the economy and each firm supplies differentiated goods. The firms are divided into the home

(H) and foreign (F) country. The firms in the foreign country are assumed to be able to adjust the price of their products any time in response to changes in the demand and supply condition. On the other hand, a longer period of time is required for the firms in the home country to adjust the prices of their products[*3].

Second, we assume incomplete capital mobility between the countries. Each firm produces goods with labor and capital stock. Labor is a firm-specific factor. Capital stocks can be rented in the rental market. The integration of the rental market occurs between home and foreign countries. The arbitrage is incomplete between the two countries, because of the mobile cost of capital.

We investigate the behavior of the agents in the factor and goods markets in the following subsections. For this purpose, we first depict the behavior of the agents in the factor markets. We then investigate the effects of imperfect capital mobility on the markets. Finally, we examine the demand and supply factors in the goods market.

2.1 Factor markets and capital mobility

Capital should move from a country to another to achieve the optimal capital allocation internationally. In our model, however, the optimal allocation is not realized because capital mobility is incomplete.

The setting of our model is as follows. First, the firms that produce differentiated goods are represented by a continuum of $[0,1]$. The firms of a continuum of $[0,\gamma)$ belong to the home country, while those of a continuum of $[\gamma,1]$ belong to the foreign country. Second, each firm receives a

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*3 This set of assumptions is the same as those developed by Aoki (2001). In this investigation, however, we remove the assumption of fixed capital to investigate sectoral capital mobility.

specific amount of capital stock K that they use to produce goods or rents for other firms in the rental market[*4]. The capital stock completely depreciates in one period. We define the aggregate function of the capital stock as

$$K=\int_{0}^{1} k_t(i)\,di, \tag{1}$$

$$K_{Ht}=\frac{1}{\gamma}\int_{0}^{\gamma} k_t(i)\,di, \quad \text{and} \quad K_{Ft}=\frac{1}{1-\gamma}\int_{\gamma}^{1} k_t(i)\,di,$$

where $k_t(i)$ represents the amount of the capital stock that firm i uses in the production. K_{Ht} and K_{Ft} represent the aggregate capital stock in the home (H) and foreign (F) countries, respectively.

We shall now explain the rental markets of the capital stock. There are two rental markets; each corresponding to the home and foreign countries. The arbitrage is complete in each national rental market. The national rental prices ρ_{Ht} (in the home country) and ρ_{Ft} (in the foreign country) are then formed in each market. Arbitrage is possible internationally. Capital moves from the low rental price country to the high rental price country, but the arbitrage is incomplete. We formulate this relationship as

$$\frac{K_{Ht}}{K_{Ft}}=\mu(\rho_{Ht}/\rho_{Ft}), \tag{2}$$

where the derivative of this function meets, $\mu(1)=1$, and $\mu\,(1)=\varepsilon_{\mu}$[*5].

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*4 Recently, some research investigations about capital accumulation applied an optimal monetary policy analysis. Takamura et al. (2005) develops an optimal monetary policy analysis with capital accumulation and assumes that firms can rent their capital stock in the rental market. On the other hand, Woodford (2005) supposes that capital stock is the firm-specific factor. The assumption of imperfect capital mobility in this paper is the intermediary between the perfectly integrated rental market, as in Takamura et al. (2005), and the firm-specific capital stock as in Woodford (2005), though we eliminate capital accumulation to focus our analysis on the effects of capital mobility.

*5 Casas (1984) formulates the factor mobility to meet the constant elasticity of substitution. Our functional form is consistent with the formulation of Casas (1984).

The parameter ε_μ represents the sensitivity of capital mobility to the difference in the national rental prices at the symmetric steady state equilibrium $K_H = K_F = K$. The magnitude of the parameter ε_μ indicates the smoothness of capital mobility.

Next, we shall investigate the factor demand of each firm. Each firm uses capital stock and labor, which is the firm specific factor. The production function of the firm is represented as

$$y_t(i) = k_t(i) \cdot f(A_t(i) h_t(i) / k_t(i)), \tag{3}$$

where $h_t(i)$ is labor employed by firm i. $A_t(i)$ represents the country-specific total factor productivity shock. As a result of the cost minimization problem of the firm, which is subject to the restriction of the production function, as Equation (3), factor demand becomes

$$s_t(i) = \frac{w_t(i)}{A_t(i) \cdot f'(A_t(i) h_t(i) / k_t(i))},$$

$$\rho_t(i) = s_t(i) \left[f(A_t(i) h_t(i) / k_t(i)) - \frac{A_t(i) h_t(i)}{k_t(i)} \cdot f'(A_t(i) h_t(i) / k_t(i)) \right],$$

where $s_t(i)$ is the Lagrange multiplier of the production function. $s_t(i)$ is assumed to be the marginal cost of production, because the multiplier represents the marginal effect of the change in the products on the production cost. $\rho_t(i)$, which is the rental price of the capital stock, is derived by the equalization of the factor price ratio with the marginal rate of technical substitution.

We define household i's life-time utility as

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(C_t^i; \xi_t) - v(h_t(i); \zeta_t(i))] \tag{4}$$

to derive the labor supply of the household i to the firm i. C_t^i and $h_t(i)$

represent the consumption and labor supply, respectively. ξ_t is the aggregate demand shock and $\zeta_t(i)$ is the country-specific labor supply shock. As a result, the labor supply condition of the household i is derived as

$$w_t(i) = v_h(h_t(i);\zeta_t(i))/\lambda_t^i, \tag{5}$$

where $\lambda_t^i = u(C_t^i;\xi_t)$ is the marginal utility of consumption and $w_t(i)$ is the real wage rate.

Substituting Equation (5) into the factor demand conditions mentioned previously yields

$$s_t(i) = \frac{v_h(f^{-1}(y_t(i)/k_t(i))k_t(i)/A_t(i);\zeta_t(i))}{A_t(i)\cdot f'(A_t(i)h_t(i)/k_t(i))\cdot\lambda_t^i}, \tag{6}$$

and

$$\rho_t(i) = \frac{v_h(f^{-1}(y_t(i)/k_t(i))k_t(i)/A_t(i);\zeta_t(i))}{A_t(i)\cdot\lambda_t^i} f^{-1}(y_t(i)/k_t(i))[\phi_h(y_t(i)/k_t(i))-1], \tag{7}$$

where $f^{-1}(\cdot)$ is the inverse function of the production function, and

$$\phi_h(y_t(i)/k_t(i)) = \frac{f(f^{-1}(y_t(i)/k_t(i)))}{f^{-1}(y_t(i)/k_t(i))f'(f^{-1}(y_t(i)/k_t(i)))}.$$

Equation (7) is derived from the equalizing condition of the factor price ratio with a marginal rate of technical substitution of $\rho_t(i)$.

2.2 Demand for the goods

Before it is revealed to which country each household belongs, the households purchase insurance by which the idiosyncratic risks are averted. This implies that the condition of the complete market is met[*6]. In addition, if the initial wealth of all households is equal, they would face the common intertemporal resource allocation problem. Therefore, the Euler condition of the

*6 This assumption is the same as that of Aoki (2001).

inter-temporal consumption substitution implies that

$$E_t\{Q_{t,t+1}\Pi_{t+1}\}=\beta E_t\{\lambda_{t+1}/\lambda_t\} \tag{8}$$

where $Q_{t,t+1}$ represents the stochastic nominal discount factor from the period t to period $t+1$ and satisfies $1+i_t=(E_t\{Q_{t,t+1}\})^{-1}$. P_t is the general price level and $\Pi_t=P_t/P_{t-1}$ represents the gross rate of general price inflation. In Equation (8), the superscript i of λ_t is ignored, because all types of households plan the same pattern of consumption by the ex ante completeness of the market.

To determine the consumption demand for each of the differentiated goods, we define the functional form of the consumption basket C_t as the constant elasticity of substitution (CES) type. The elasticity of substitution between the countries is unity. The elasticity of the substitution among the differentiated goods in each country is $\theta>1$. The price index, which corresponds to the consumption basket defined above, is derived as

$$P_t=P_{Ht}^{\gamma}P_{Ft}^{1-\gamma}, \tag{9}$$

$$P_{Ht}=\left[\frac{1}{\gamma}\int_0^{\gamma}p_t(j)^{1-\theta}dj\right]^{\frac{1}{1-\theta}}, \quad P_{Ht}=\left[\frac{1}{1-\gamma}\int_{\gamma}^{1}p_t(j)^{1-\theta}dj\right]^{\frac{1}{1-\theta}},$$

where P_{Ht} and P_{Ft} are the average prices of the home and the foreign countries, respectively, and $p_t(j)$ is the price of the differentiated goods produced by the firm j.

The government's consumption basket G_t is defined in the same manner as for C_t. The government collects corporate income tax at the rate τ (if the parameter is negative, it is a transfer to the firms) to finance its expenditures.

The series of assumptions mentioned above implies the demand for each category of differentiated goods as

$$y_t(i) = (p_t/P_{Ht})^{-\theta} x_{Ht}^{-1} Y_t, \qquad \text{for } i \in [0, \gamma), \tag{10}$$

$$y_t(i) = (p_t/P_{Ft})^{-\theta} x_{Ft}^{-1} Y_t, \qquad \text{for } i \in [\gamma, 1], \tag{11}$$

where $Y_t = C_t + G_t$, and $x_{Ht} = P_{Ht}/P_t$, $x_{Ft} = P_{Ft}/P_t$ represent the real exchange rates of the home and foreign countries, respectively. They are defined as the real exchange rates of each country.

2.3 Supply of the goods

Being subject to the demand for its products, each firm maximizes the after-tax profit. Thus, the problem of the representative firm in the foreign country is described as

$$\max_{p_t(i)} (1-\tau) \frac{p_t(i)}{P_{Ft}} y_t(i) - w_t(i) h_t(i) - \rho_{Ft} k_t(i),$$

$$\text{s.t.} \quad y_t(i) = \left(\frac{p_t(i)}{P_{Ft}} \right)^{-\theta} x_{Ft}^{-1} Y_t, \qquad \text{for } i \in [\gamma, 1].$$

Solving this problem and using Equations (3), (5), and (6), we obtain the profit-maximizing condition of the firm as

$$\frac{p_t(i)}{P_{Ft}} x_{Ft} = (1-\Theta)^{-1} s_t(i), \qquad \text{for } i \in [\gamma, 1], \tag{12}$$

where $1-\Theta = (1-\tau)(\theta-1)/\theta$. Equation (12) implies that the representative firm sets the price of its products following the mark-up principle at the rate $(1-\Theta)^{-1}$.

In the home country, it is assumed that in each period, a fraction $1-\alpha$ of firms get to set a new price, while the remaining α must continue to sell at their previously posted prices[*7]. The firms that get to set new prices are

*7 This assumption follows Calvo (1983). It is also adopted by Rotemberg and Woodford (1997), Woodford (2003), and others.

chosen randomly each period, with each having an equal probability of being chosen. The firms must count the case that they will not get to change their prices for a while. As a result, the price set by the representative firm satisfies the profit-maximizing condition that

$$\sum_{j=0}^{\infty} \alpha^{j} E_t \left\{ Q^{r}_{t,t+1} \frac{p_t(i)}{P_{Ht+j}} x_{Ht+j} \bar{y}_{t+j}(i) \right\} = (1-\Theta)^{-1} \sum_{j=0}^{\infty} \alpha^{j} E_t \{ Q^{r}_{t,t+1} \bar{s}_{t+j}(i) \bar{y}_{t+j}(i) \},$$

$$\text{for } i \in [0, \gamma), \tag{13}$$

where $Q^{r}_{t,t+1}$ is the stochastic real discount factor and $\bar{y}_{t+j}(i)$, $\bar{s}_{t+j}(i)$ are demand for the firm's products and the marginal cost of production at the price $p_t(i)$, which is set in the period t.

2.4 Deterministic steady state

Here we suppose the symmetric steady state in which no exogenous stochastic shocks and price changes occur. In this equilibrium, the following equations are satisfied:

$$k_t(i) = \bar{K}, \quad \text{for } \forall i, t,$$

$$x_F = x_H = 1,$$

$$\bar{i} = (1-\beta)\beta^{-1}.$$

As a result, the supply condition in common with both countries is

$$(1-\Theta) u_C(\cdot) = \frac{v_h(f^{-1}(\cdot)K;0)}{f'(f^{-1}(\cdot))}.$$

This equation can then be rearranged as

$$(1-\Theta)\, Y u_C(\cdot) = \phi_h h v_h(\cdot) \tag{14}$$

by using the following equations:

$$Y = f(f^{-1}(\cdot))K,\ h = f^{-1}(\cdot)K.$$

3 Log-Linear System

In this section, we log-linearize the developed model to obtain the system of structural equations. For this purpose, we first examine the factor market. Next we investigate the goods market, obtain the aggregate demand and supply, and investigate the equilibrium that would be achieved if all prices are flexible. These steps are conducted to obtain the natural level in which the Pareto optimal allocation is achieved. Using the previously calculated results, we obtain the structural equations as the difference between the log-linearized system and the natural level.

3.1 Factor markets

We log-linearize the equations associated with the demand and supply conditions in the factor market. To accomplish this, we first log-linearize Equation (7), which represents the optimal substitution between the factors of production in order to obtain

$$\hat{\rho}_t(i) = \rho_y \hat{y}_t(i) - \rho_k \hat{k}_t(i) - \omega q_t(i) - \hat{\lambda}_t, \tag{15}$$

where $q_t(i)$ represents the supply shock, defined as

$$q_t(i) = \omega^{-1}\left[(1+\nu)\hat{A}_t(i) - \frac{v_{h\zeta}(\cdot)}{v_{hh}(\cdot)}\hat{\zeta}_t(i)\right].$$

The supply shock $q_t(i)$ indicates the change in the marginal cost when the supply condition is modified by the changes in productivity or the preference. Our assumption of the rental markets of the capital stock implies that

$$\hat{\rho}_t(i)=\begin{cases}\hat{\rho}_{Ht} & \text{for} \quad i\in[0,\gamma),\\ \hat{\rho}_{Ft} & \text{for} \quad i\in[\gamma,1].\end{cases}$$

Equation (15) implies that the rental price of capital in each market would decline if the positive supply shock occurs. Using the fact that $\hat{Y}_{Ht}=\hat{Y}_t+\frac{1-\gamma}{\gamma}\hat{x}_{Ft}$ and $\hat{Y}_{Ft}=\hat{Y}_t-\hat{x}_{Ft}$, we can take the national aggregation on Equation (15) to obtain the national rental prices as

$$\hat{\rho}_{Ht}=\rho_y\left[\hat{Y}_t+\frac{1-\gamma}{\gamma}\hat{x}_{Ft}\right]-\rho_k\hat{K}_{Ht}-\omega q_{Ht}-\hat{\lambda}_t, \tag{16}$$

$$\hat{\rho}_{Ft}=\rho_y\left[\hat{Y}_t-\hat{x}_{Ft}\right]-\rho_k\hat{K}_{Ft}-\omega q_{Ft}-\hat{\lambda}_t. \tag{17}$$

We log-linearize Equation (2), which represents international capital mobility in order to obtain

$$\hat{K}_{Ht}-\hat{K}_{Ft}=\varepsilon_\mu[\hat{\rho}_{Ht}-\hat{\rho}_{Ft}],$$

where ε_μ is the parameter associated with the sensitivity of capital mobility to the international difference in the rental price. The larger the value of this parameter ε_μ, the smoother the international capital mobility is. International capital mobility does not occur if the parameter approximates to zero, and then the international difference of rental prices remains. Substituting the equation of the definition of the parameter ε_μ with Equations (16) and (17) implies that

$$\hat{K}_{Ht}-\hat{K}_{Ft}=\frac{1}{\gamma}\frac{\varepsilon_\mu\rho_y}{1+\varepsilon_\mu\rho_y}\hat{x}_{Ft}+\frac{\varepsilon_\mu\omega}{1+\varepsilon_\mu\rho_k}(q_{Ft}-q_{Ht}). \tag{18}$$

This equation implies that capital mobility depends on the real exchange rate, and the asymmetric supply shock, $q_{Ft}-q_{Ht}$.

3.2 Goods market

We investigate the structure of aggregate demand and aggregate supply in the goods market. First, on the demand side component, the definition of λ_t implies that

$$\hat{\lambda}_t=\sigma^{-1}\left[\hat{Y}_t-g_t\right], \tag{19}$$

where the demand shock for the aggregate economy is defined as

$$g_t=\hat{G}_t-\frac{u_{C\xi}(Y;0)}{u_{CC}(Y;0)}\hat{\xi}_t,$$

where $\hat{G}_t=dG_t/Y$ and $\hat{\xi}_t=d\xi_t/Y$, respectively.

Equation (8), which represents the Euler condition of consumption, is log-linearized as

$$E_t\hat{\lambda}_{t+1}-\hat{\lambda}_t=-\hat{i}_t+E_t\pi_{t+1}. \tag{20}$$

Equation (9) implies that

$$\pi_t=\pi_{Ht}+\frac{1-\gamma}{\gamma}\Delta\hat{x}_{Ft}. \tag{21}$$

This implies that general price inflation π_t can be divided into core inflation, π_{Ht}, and the change in the real exchange rate, $\Delta\hat{x}_{Ft}$.

Log-linearizing Equations (10) and (11) yields

$$\hat{y}_t(i)=\hat{Y}_t+\frac{1-\gamma}{\gamma}\hat{x}_{Ft}-\theta\tilde{p}_{Ht}(i),\quad \text{for}\quad i\in[0,\gamma), \tag{22}$$
$$\hat{y}_t(i)=\hat{Y}_t-\hat{x}_{Ft}-\theta\tilde{p}_{Ft}(i),\quad \text{for}\quad i\in[\gamma,1], \tag{23}$$

where $\tilde{p}_{Ht}(i)=\log(p_t(i)/P_{Ht})$ and $\tilde{p}_{Ft}(i)=\log(p_t(i)/P_{Ft})$.

Next, we investigate the supply side of the economy. Log-linearizing Equation (6), which represents the marginal cost of production, yields

$$\hat{s}_t(i) = \omega\hat{y}_t(i) - (\omega - \nu)\hat{k}_t(i) - \omega q_t(i) - \hat{\lambda}_t. \tag{24}$$

The third term on the right-hand side of Equation (24) represents the supply shock; the positive supply shock reduces the marginal cost of production.

In the foreign country, substituting Equations (15), (17), (23), and (24) into the log-linearized version of Equation (12) yields

$$\tilde{p}_{Ft}(i) = \left(1 + \frac{(\rho_y - \omega)\nu\theta}{\rho_k}\right)^{-1}\left[(\omega + \sigma^{-1})\hat{Y}_t - (1+\omega)\hat{x}_{Ft} - (\omega - \nu)\hat{K}_{Ft} - \omega q_{Ft} - \sigma^{-1}g_t\right].$$

The prices the firms in the foreign country determine are the same because the firm index number i is not seen in the right-hand side of this equation. Thus, $\tilde{p}_{Ft}(i) = 0 \ \ \forall i \in [\gamma, 1]$ is satisfied and we obtain the supply condition of the firms in the foreign country as

$$\hat{x}_{Ft} = (1+\omega)^{-1}\left[(\omega + \sigma^{-1})\hat{Y}_t - (\omega - \nu)\hat{K}_{Ft} - \omega q_{Ft} - \sigma^{-1}g_t\right]. \tag{25}$$

Here, we derive the supply condition of the representative firm in the home country. The definition of the expected demand for the products in the period $t+j$ implies

$$E_t\hat{y}_{t+j}(i) = E_t\left\{\hat{Y}_{t+j} - \hat{x}_{Ht+j} - \theta\left(\tilde{p}_{Ht}(i) - \sum_{l=1}^{j}\pi_{Ht+l}\right)\right\}.$$

Substituting this equation along with Equations (15), (16), and (24) into the log-linearized version of Equation (13), which represents the supply condition of the representative firm, yields the inflation dynamics of the home country. Substituting the supply condition in the foreign country, Equation (25), into the inflation dynamics obtained above yields the supply condition of the representative firm in the home country as

$$\pi_{Ht} = \gamma^{-1}\kappa\left[(\omega + \sigma^{-1})\hat{Y}_t - \omega q_t - \sigma^{-1}g_t\right] + \beta E_t\pi_{Ht+1}, \tag{26}$$

where $\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\left(1+\frac{(\rho_y-\omega)\nu\theta}{\rho_k}\right)^{-1}$.

3.3 Flexible price equilibrium

In this subsection, we describe the equilibrium achieved when it is supposed that all prices including the home country are flexible. The values of the endogenous variables, derived from this equilibrium, are referred to as the natural level of the variables. The flexible price equilibrium of the supply condition in the foreign country is straightforwardly derived from Equation (25) as

$$\hat{x}^n_{Ft} = (1+\omega)^{-1}\left[(\omega+\sigma^{-1})\hat{Y}^n_t - (\omega-\nu)\hat{K}^n_{Ft} - \omega q_{Ft} - \sigma^{-1} g_t\right]. \tag{27}$$

Referring to the supply condition of the foreign country, Equation (27), the flexible price equilibrium of the supply condition in the home country is obtained as

$$-\frac{1-\gamma}{\gamma}\hat{x}^n_{Ft} = (1+\omega)^{-1}\left[(\omega+\sigma^{-1})\hat{Y}^n_t - (\omega-\nu)\hat{K}^n_{Ht} - \omega q_{Ht} - \sigma^{-1} g_t\right]. \tag{28}$$

The definition of the aggregate function of the capital stock implies

$$\gamma\hat{K}^n_{Ht} + (1-\gamma)\hat{K}^n_{Ft} = 0, \tag{29}$$

and rearranging Equation (18), derived from the capital mobility condition, we obtain

$$\hat{K}^n_{Ht} - \hat{K}^n_{Ft} = \frac{1}{\gamma}\frac{\varepsilon_\mu\rho_y}{1+\varepsilon_\mu\rho_k}\hat{x}^n_{Ft} + \frac{\varepsilon_\mu\omega}{1+\varepsilon_\mu\rho_k}(q_{Ft} - q_{Ht}). \tag{30}$$

Next, rearranging the conditions mentioned above, we indicate the natural values as the combination of the supply and demand shocks. Taking the weighted average of Equations (27) and (28) and rearranging the results by using Equation (29), we obtain the natural level of output as the linear combination of the demand and supply shocks, that is,

$$\omega q_t + \sigma^{-1} g_t = (\omega + \sigma^{-1}) \hat{Y}_t^n. \tag{31}$$

By substituting this equation into Equation (26), we obtain the core-inflation dynamics or the " new Keynesian Phillips curve " of the home country as

$$\pi_{Ht} = \gamma^{-1} \kappa (\omega + \sigma^{-1}) \tilde{Y}_t + \beta E_t \pi_{Ht+1}, \tag{32}$$

where $\tilde{Y}_t = \hat{Y}_t - \hat{Y}_t^n$ represents the world GDP gap.

Subtracting Equation (28) from Equation (27), we derive

$$\hat{x}_{Ft}^n = -\frac{\gamma}{1+\omega} \left[(\omega - \nu) \left(\hat{K}_{Ft}^n - \hat{K}_{Ht}^n \right) + \omega (q_{Ft} - q_{Ht}) \right], \tag{33}$$

and substituting Equation (30) into Equation (33) to eliminate $\hat{K}_{Ft}^n - \hat{K}_{Ht}^n$, we obtain

$$\gamma \omega \psi_2 (q_{Ft} - q_{Ht}) = \psi_1 \hat{x}_{Ft}^n, \tag{34}$$

where $\psi_1 > 0$ and $\psi_2 < 0$. Equation (34) implies that the natural level of the real exchange rate depends on the asymmetric supply shocks between the countries. Substituting the loglinearized version of Equations (1) and (18) into Equation (25) to eliminate, and rearranging the result using Equations (31) and (34), we obtain

$$\hat{x}_{Ft} - \hat{x}_{Ft}^n = \psi_1^{-1} (\omega + \sigma^{-1}) \tilde{Y}_t. \tag{35}$$

This equation implies that the real exchange rate gap is proportionate to the output gap.

Equations (19), (20), (21), and (35) imply

$$\tilde{Y}_t = E_t\tilde{Y}_{t+1} - \sigma\eta^{-1}\left[i_t E_t \pi_{Ht+1} - r_t^n - \frac{1-\gamma}{\gamma} E_t \Delta \hat{x}_{Ft+1}^n \right], \tag{36}$$

where $\eta = 1 + \frac{1-\gamma}{\gamma}\psi_1^{-1}(1+\omega\sigma)$. This is the dynamic IS equation of this model.

Therefore, we can obtain the system of the structural equations in our model, which contains Equation (32), which represents the " new Keynesian Phillips curve " in the home country; Equation (35), which implies the relationship between the real exchange rate gap and the output gap; and Equation (36), which represents the aggregate demand condition, the so-called dynamic IS equation. Placing the interest rate rule of the monetary policy into this system, we obtain the equilibrium path of our model.

4 Optimal Monetary Policy under Imperfect Capital Mobility

In this section, we describe the optimal monetary policy under imperfect capital mobility between the countries. For this purpose, in the first subsection, we derive a welfare criterion from the representative agent's utility function. Next, the asymmetric shock used in our analysis is specified. In the last subsection, we derive the optimal response of monetary policy to the asymmetric shock under imperfect capital mobility and examine a numerical analysis to confirm the implications of the results of our analysis.

4.1 Social welfare criterion based on the utility function

The welfare criterion based on the agent's utility function is defined as

$$W_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u(C_t; \xi_t) - \int_0^1 v(h_t(i); \zeta_t(i)) d_i \right\} . \tag{37}$$

Following Woodford (2003, Ch. 6), the second-order Taylor expansion is applied to Equation (37). Aggregating the result with respect to the individuals, we can derive the welfare criterion. We accomplish this by first applying the Taylor expansion to the term $u(C_t;\xi_t)$ as shown below:

$$u(C_t;\xi_t)=u(Y_t-G_t;\xi_t)$$

$$\approx u_C(\cdot)dY_t+\frac{1}{2}\left[u_{CC}(\cdot)dY_t^2-2u_{CC}(\cdot)dY_t\,dG_t+2u_{C\xi}(\cdot)dY_t\,d\xi_t\right]+\text{t.i.p.}+O\left(\|\xi_t\|^3\right)$$

$$=\frac{1}{2}Yu_C(\cdot)\left\{(1-\sigma^{-1})\hat{Y}_t^2+2\sigma^{-1}g_t\hat{Y}_t+2\hat{Y}_t\right\}+\text{t.i.p.}+O\left(\|\xi_t\|^3\right). \quad (38)$$

On the other hand, we take the Taylor expansion to the term as follows:

$$v(h_t(i);\zeta_t(i))=v\left(f^{-1}(y_t(i)/k_t(i))k_t(i)/A_t(i)\right)$$

$$\approx\frac{1}{2}\phi_h h v_h(\cdot)\ \left\{(1+\omega)\hat{y}_t(i)^2+\frac{\phi_h-1}{\phi_h}(\rho_k-1)\hat{k}_t(i)^2-2(\omega-\nu)\hat{y}_t(i)\hat{k}_t(i)\right.$$

$$\left.-2\omega q_t(i)\left[\hat{y}_t(i)-\frac{\phi_h-1}{\phi_h}\hat{k}_t(i)\right]+2\left[\hat{y}_t(i)-\frac{\phi_h-1}{\phi_h}\hat{k}_t(i)\right]\right\}$$

$$+\text{t.i.p.}+O\left(\|\xi_t\|^3\right). \quad (39)$$

Aggregating Equation (39) for the agents and substituting the results, along with substituting Equation (38) into Equation (37), gives

$$W_0=-Yu_C(\cdot)E_0\sum_{t=0}^{\infty}\beta^t L_t+\text{t.i.p.}+O\left(\|\xi_t\|^3\right),$$

where L_t is the loss function as

$$L_t=\frac{1}{2}\left[(\omega+\sigma^{-1})(\tilde{Y}_t-\bar{Y}_t^*)^2+\gamma\theta\kappa^{-1}\pi_{Ht}^2+\frac{1-\gamma}{\gamma}\tilde{\psi}_1(\hat{x}_{Ft}-\Psi\hat{x}_{Ft}^n)^2\right], \quad (40)$$

where $\bar{Y}_t^*$ is the efficient level of output, and the parameters are

$$\Psi=\frac{\psi_1\tilde{\psi}_2}{\tilde{\psi}_1\psi_2},\ \ \tilde{\psi}_1=\psi_1-\frac{(\omega-\nu)\varepsilon_\mu\rho_y}{(1+\varepsilon_\mu\rho_k)^2},\ \ \tilde{\psi}_2=\psi_2+\frac{(\omega-\nu)\varepsilon_\mu}{(1+\varepsilon_\mu\rho_k)^2}.$$

The third term of Equation (40) represents the "real exchange rate gap." Although the term implies that the deviation of $\hat{x}_{Ft}$ from the natural level $\hat{x}_{Ft}^n$ has an effect on the social welfare, the coefficient of the natural level of the real exchange rate Ψ depends on the smoothness of the capital movement ε_μ. If the capital movement is impossible internationally, that is, $\varepsilon_\mu=0$, then $\psi_1=\tilde{\psi}_1$, $\psi_2=\tilde{\psi}_2$ would be satisfied, and therefore, the parameter satisfies $\Psi=1$. In this case, the third term of Equation (40) reduces to the normal real exchange rate gap. If international capital mobility is perfect, that is, $\varepsilon_\mu\to\infty$, the coefficient of the natural level of the real exchange rate would satisfies $\Psi=1$.

In case the value of the parameter ε_μ is finitely positive, the value of the coefficient Ψ is smaller than one. Figure 1 illustrates the result of the calculation of the coefficient Ψ to the different values of the parameter ε_μ. The coefficient Ψ is the smallest value 0.45 at $\varepsilon_\mu=5$, and approximates to unity as the parameter ε_μ increases.

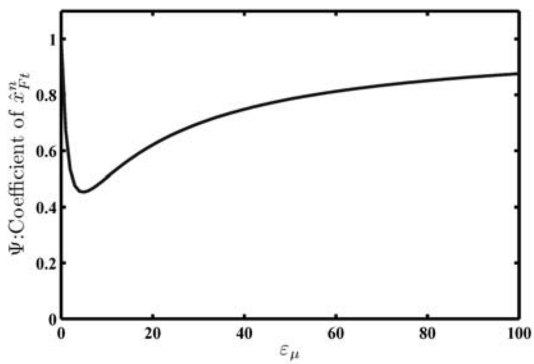

Fig. 1 The Coefficient of the Natural Level of the Real Exchange Rate in the Loss Function

4.2 Real exchange rate adjustments under imperfect capital mobility

In this subsection, we examine how much price adjustment is required to respond to the asymmetric supply shock under imperfect capital mobility. We can indicate the adjustment of the natural level of the real exchange rate as $\hat{x}^n_{Ft}$.

The exogenous shocks r^n_t and $\hat{x}^n_{Ft}$ are in the dynamic IS equation (36). These are the linear combinations of the underlying shocks such that

$$r^n_t = \frac{\omega\sigma^{-1}}{\omega+\sigma^{-1}} E_t\{\gamma\Delta q_{Ht+1} + (1-\gamma)\Delta\hat{x}_{Ft+1} - \Delta g_{t+1}\} + (1-\beta)\beta^{-1}, \quad (41)$$

$$E_t\Delta\hat{x}^n_{Ft+1} = \gamma\omega\psi_1^{-1}\psi_2 E_t\{\Delta q_{Ft+1} - \Delta q_{Ht+1}\}. \quad (42)$$

For the purpose of our analysis, we focus on the asymmetric shocks on the productivity. First, the definitions of the supply shocks are

$$q_{Ht} = \omega^{-1}\left[(1+\nu)\hat{A}_{Ht} - \frac{v_{h\varepsilon}(\cdot)}{v_{hh}(\cdot)}\hat{\zeta}_{Ht}\right],$$

$$q_{Ft} = \omega^{-1}\left[(1+\nu)\hat{A}_{Ft} - \frac{v_{h\varepsilon}(\cdot)}{v_{hh}(\cdot)}\hat{\zeta}_{Ft}\right].$$

For simplicity, we focus on the productivity shocks $\hat{A}_{Ht}$ and $\hat{A}_{Ft}$. Equation (42) is then rearranged by using the abovementioned equations as

$$E_t\Delta\hat{x}^n_{Ft+1} = \gamma(1+\nu)\psi_1^{-1}\psi_2 E_t\left\{\Delta\hat{A}_{Ft+1} - \Delta\hat{A}_{Ht+1}\right\}.$$

Thus, the shocks, which require the real exchange rate adjustment, are caused by the international difference in the productivity growth rates. Through the process such that the productivity difference is transformed to the real exchange rate shock $\hat{x}^n_{Ft}$, the smoothness of the capital movement ε_μ has an effect on the parameters ψ_1 and ψ_2. Figure 2 depicts the effect of the asymmetric shock $\hat{A}_{Ht}-\hat{A}_{Ft}$ on the natural level of the real exchange rate $\hat{x}_{Ft}^{n}$, with respect to the value of the parameter ε_{μ}. This figure shows that the effect of the constant asymmetric (productivity) shock on the natural level of the real exchange rate is decreasing as the smoothness of the capital movement ε_{μ} increases.

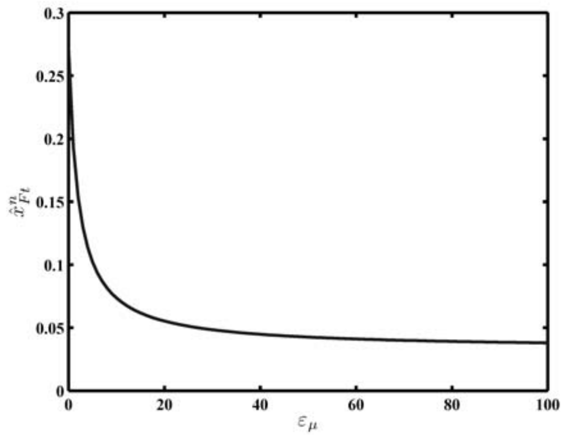

Fig. 2 The Effect of the Asymmetric Productivity Shock on the Natural Level of Real Exchange Rate

The adjustment mechanism from the asymmetric productivity shock to the natural level of the real exchange rate is as follows. One can ensure that the technological change in our model is the Harrod neutral from the production function defined by Equation (3). In this type of production function, the growth of the total factor productivity tends to reduce the rental price of the capital. This is confirmed in Equations (16) and (17).

Suppose that the productivity in the home country is improved. This shock reduces the rental price in the home country in comparison with the one in the foreign country. Then, as in Equation (18), the capital stock moves from the home country to the foreign country. Thus, the world economy is adjusted in response to the asymmetric productivity shock by the real exchange rate changes and the capital movements. The magnitude of the capital movements is defined by the parameter ε_{μ}. Equation (34) means that the range of the real exchange rate adjustment in response to the asymmetric productivity shock depends on the sensitivity of the capital

Table. **1** The Basic Parameters in the Numerical Analysis

α	0.66	β	0.99	γ	0.50	σ^{-1}	0.16
θ	7.67	ω_p	0.33	ν	0.11	ϕ_h	6.50

Table. **2** The Parameters of the Shocks in the Numerical Analysis

	Initial value of shocks	AR parameter of shocks
$\hat{A}_{Ht}$	0.5	0.8
$\hat{A}_{Ft}$	−0.5	0.8

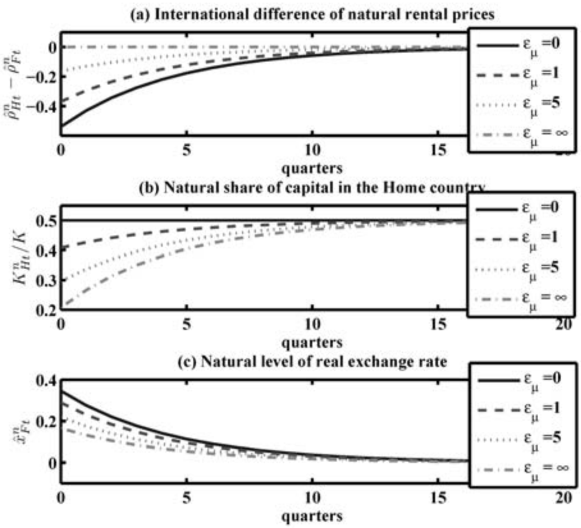

Fig. **3** Adjustment Process to the Asymmetric Shock mobility ε_μ through the parameters ψ_1 and ψ_2.

Figure 3 illustrates the adjustment process mentioned above under the set of parameters shown in Tables 1 and 2. In case the asymmetric productivity shock increases the international difference in the rental prices, following Equation (18), the capital stock moves to adjust the rental price difference. Panel (a) shows the rental prices and Panel (b) shows the national share of the capital stock in Figure 3.

In case $\varepsilon_\mu = 0$, without capital movements, the rental price adjustment depends only on the autonomic decrease in the relative supply shock $q_{Ht} - q_{Ft}$ and the real exchange rate changes. As a result, the adjustment is very slow.

In case the rental markets are integrated completely, $\varepsilon_\mu \to \infty$, the rental price difference diminishes quickly, as shown in Panel (a) of Figure 3. At the same time, massive capital movement occurs as shown in Panel (b). In case of imperfect capital mobility, the result is intermediate between the two cases mentioned above.

4.3 Optimal monetary policy under imperfect capital mobility

In this subsection, we specify the optimal responses of the monetary policy to the asymmetric productivity shock under imperfect capital mobility and investigate the effect of the monetary policy on social welfare. For simplicity, we adopt the following assumptions. First, the inefficiency associated with the monopolistic power of the producers can be removed by the tax or subsidy, that is, $\bar{Y}_t^* = 0$ is held. Second, we ignore the lower bound of the nominal interest rate for a while.

The loss function defined in Equation (40) is the welfare criterion in our analysis. The monetary authority minimizes this welfare loss by subjecting it to the restrictions, including the core-inflation dynamics shown in Equation

(32), the real exchange rate gap shown in Equation (35), and the dynamic IS equation shown in Equation (36).

The supposed first-best solution is that all the terms in the loss function are zero, that is, the output gap $\tilde{Y}_t$, the domestic inflation in home country π_{Ht}, and the real exchange rate gap in the loss function $\hat{x}_{Ft}-\Psi\hat{x}^n_{Ft}$ are zero. However, stabilization of the output gap $\tilde{Y}_t$ and the real exchange rate gap $\hat{x}_{Ft}-\Psi\hat{x}^n_{Ft}$ are generally inconsistent. The first-best solution is achieved only in case of no capital movement $\varepsilon_\mu=0$ or the case of perfect capital mobility $\varepsilon_\mu\to\infty$.

Under the assumption of discretionary monetary policy, the central bank re-optimizes in each period. Being subject to Equations (32), (35), and (36), the monetary authority minimizes the loss function represented as Equation (40). The first-order conditions of this optimization problem are

$$(\omega+\sigma^{-1})\tilde{Y}_t-\gamma^{-1}\kappa(\omega+\sigma^{-1})\phi_{\pi t}-\psi_1^{-1}(\omega+\sigma^{-1})\phi_{xt}=0, \tag{43}$$

$$\gamma\theta\kappa^{-1}\pi_{Ht}+\phi_{\pi t}=0, \tag{44}$$

$$(\gamma^{-1}-1)\tilde{\psi}_1(\hat{x}_{Ft}-\Psi\hat{x}^n_{Ft})+\phi_{xt}=0, \tag{45}$$

where $\phi_{\pi t}$ and ϕ_{xt} are the Lagrange multipliers associated with Equations (32) and (35), respectively.

Eliminating the Lagrange multipliers $\phi_{\pi t}, \phi_{xt}$ and the real exchange rate $\hat{x}_{Ft}$ in Equations (35), (43), (44), and (45), we obtain

$$\tilde{\eta}\tilde{Y}_t+\theta\pi_{Ht}=(\gamma^{-1}-1)\left(\frac{\tilde{\psi}_1}{\psi_1}-\frac{\tilde{\psi}_2}{\psi_2}\right)\hat{x}^n_{Ft}, \tag{46}$$

whete $\tilde{\eta}=1+(\gamma^{-1}-1)\frac{\tilde{\psi}_1}{\psi_1^2}(\omega+\sigma^{-1})$. Substituting Equation (32) into Equation (46) to eliminate $\tilde{Y}_t$, we obtain

$$E_t\pi_{Ht+1}=B\pi_{Ht}+\Gamma\hat{x}^n_{Ft}, \tag{47}$$

where

$$B=\beta^{-1}[1+\gamma^{-1}\theta\kappa(\omega+\sigma^{-1})],\quad \Gamma=\beta^{-1}\gamma^{-1}(\gamma^{-1}-1)\,\kappa\,(\omega+\sigma^{-1})\tilde{\eta}^{-1}\left(\frac{\tilde{\psi}_1}{\psi_1}-\frac{\tilde{\psi}_2}{\psi_2}\right).$$

Given the path of the exogenous shock $\hat{x}^{n}_{Ft}$, we can derive the unique solution of domestic inflation in the home country π_{Ht}, which is described in Equation (47) as

$$\pi_{Ht}=-E_t\sum_{j=0}^{\infty}B^{-j-1}\,\Gamma\hat{x}^{n}_{Ft+j}. \tag{48}$$

We can then obtain $\tilde{Y}_t$, $\hat{x}_{Ft}$, i_t from Equations (46), (35), and (36), respectively.

Now, in case $\varepsilon_\mu=0$ or $\varepsilon_\mu\to\infty$, $\Gamma=0$ is satisfied, and we can derive

$$\pi_{Ht}=0,\quad \forall t,$$

and then we can obtain $\tilde{Y}_t=0$, $\hat{x}_{Ft}=\hat{x}^{n}_{Ft}$ for any period. The first-best solution

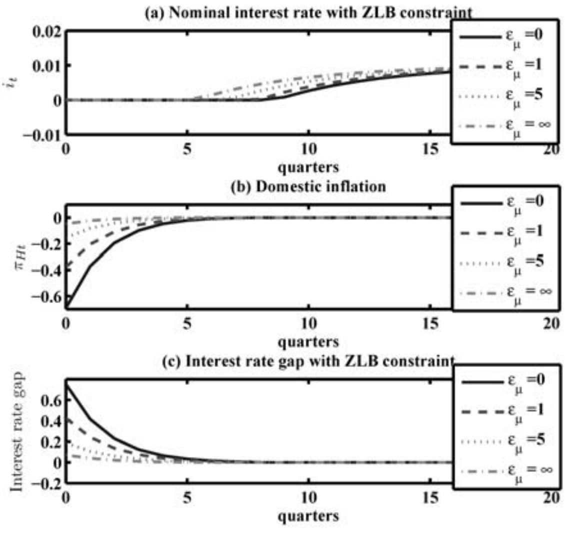

Fig. 4 The Optimal Responses of the Monetary Policy

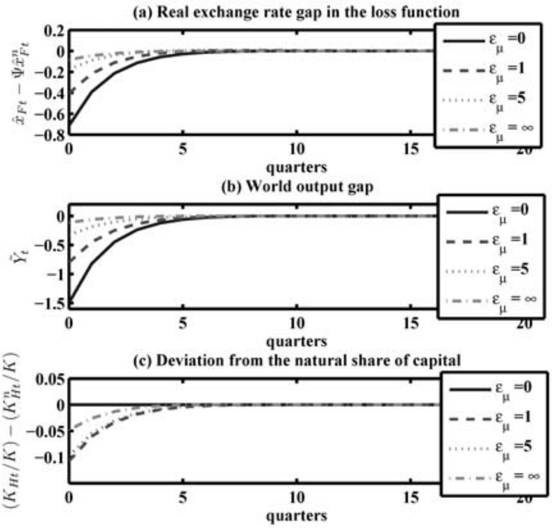

Fig. 5 The Dynamics of the Selected Variables in the Optimal Monetary Policy

without welfare loss is obtained because the parameter $\Psi=1$ is satisfied.

Figures 4 and 5 show the dynamics of the optimal responses of monetary policy, including the case that the value of ε_μ is positively finite. Panels (a) and (b) in Figure 5 show that the output gap and the real exchange rate gap are negative, because the trade-off between the output gap stabilization and the real exchange rate gap stabilization arise. The interest rate gap in Equation (36), $i_t - E_t \pi_{Ht+1} - r_t^n - (\gamma^{-1} - 1) E_t \Delta \hat{x}_{Ft+1}^n$, is positive, as shown in Panel (c) in Figure 4. In the case of imperfect capital mobility, the monetary policy is tightened for softening the real exchange rate change. In this case, the rental price of the foreign country $\hat{\rho}_{Ft}$ is relatively higher in Equation (17). As a result, the capital movement from the home to the foreign country is accelerated and the welfare loss is averted by the substitution of the factor.

Panel (a) in Figure 4 shows that the larger the smoothness of the capital movement ε_{μ}, the narrower the reduction in the nominal interest rate, as the optimal monetary policy responds to the asymmetric shock because the adjustment to the asymmetric shock is partly attributed to the capital movement. In case of perfect capital mobility, the nominal interest rate is not negative.

5 Concluding Remarks

In this paper, we examine the optimal monetary policy under imperfect capital mobility. First, we construct the two-country model with different adjustment speeds of prices and imperfect capital mobility between the countries. Second, we derive the welfare criterion on the basis of the agents' utility function. Third, we specify the exogenous asymmetric shock process to require the real exchange rate adjustment. Finally, the optimal monetary policy responses to the asymmetric shock are specified as the paths of the nominal interest rates under different smoothness of capital mobility. In addition, the optimal paths are compared with each other.

The main results of our analysis are as follows. First, in the cases with no capital mobility or perfect capital mobility, the optimal paths without welfare losses can be achieved unless the lower bound of the nominal interest rates is bound. The former case is the same as that developed by Aoki (2001). Second, the higher the sensitivity of capital mobility to the difference of the returns, the smaller the reduction in the interest rate necessary as the optimal response to the asymmetric shock. This result is consistent with the capital accumulation research (e.g., Takamura *et al.*, 2005). Third, in the case of imperfect capital mobility, the trade-off between the minimization of the output gap and the smooth capital movements occurs. It is easily confirmed in the loss function in the welfare criterion to be impossible to meet at the same time of the term of the output gap and that of the real exchange rate gap. More formally, the real exchange rate adjustments needed to reduce the output gap increase the cost of the capital movements because the capital movements depend on not only the difference in the supply shock between the countries but also the real exchange rate of the goods. Therefore, the optimal response of the monetary policy should balance the cost of the output gap with the cost of the movement of the capital.

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