Image Reconstruction of Objects with Time Reversal and Equivalent Fields

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Abstract − **Most inversion schemes for reconstructing electrical parameter distributions of unknown objects assume the explicit knowledge of incident fields illuminating them. We have proposed an inversion approach based on the field equivalence principle without requiring the knowledge of incident fields. The approach, however, requires measurement of both electric and magnetic tangential components of total field on the observation surface. In this paper, a new inversion scheme for the reconstruction from measured data of only electric tangential components is presented using time reversal and equivalent fields. Numerical reconstruction examples show the validity of the proposed scheme.**

1   INTRODUCTION

Inverse scattering problems have been attracted many researchers because of a lot of potential applications such as non-destructive testing, medical diagnostics, geophysical exploration, etc. A variety of inversion techniques [1, 2] have been proposed to estimate the electrical parameter distributions of an unknown object from measurements of scattered field data made on a surface surrounding the object. Most methods assume the knowledge of incident fields in the domain of interest containing the object as well as on the measurement surface, while it is know that total field data measured on a closed surface is sufficient for reconstructing an object [3]. We have proposed an alternative approach based on the field equivalence principle using only measured total field data [4]. The approach requires measuring both electric and magnetic total fields in the same way as the reference [3]. It was shown that only electric field data is enough for reconstruction if two measurements are made with and without an object [5].

Time reversal imaging methods have also received considerable attention since they yield accurate location estimate of point scatterers. Up to now, most time reversal methods have focused on detection and localization of point-like or small targets [6, 7] and not dealt with estimation of electrical parameter profile of scatterers. We have proposed a time-reversal imaging which can reconstruct electrical parameter distributions of unknown objects [8]. This imaging technique requires the knowledge of incident field. In our previous work [8], a primary source as well as an unknown scatterer is located in the region enclosed by a measurement surface. Placing the primary source external to the measurement surface in the same manner as [4] yields reconstruction method without the information of the incident field produced by the primary source [9].

In this paper, we show that by combing the inversion method based on the field equivalence principle [4] with the time reversal imaging method [9] the electrical parameter distributions of an inhomogeneous object can be reconstructed using only total electric field measured data. If an antenna is close to an object, the radiated field from the antenna, i.e. the incident field illuminating the object may be different from a radiated field without the object since the antenna characteristics are changed by the object. The proposed method requires only one measurement under the existence of an object so that we can avoid the uncertainty of the incident field.

2   FORMULATION

Let us consider a scattering problem by an inhomogeneous object embedded in a homogeneous background medium with the relative permittivity ε_b, permeability μ_b, and conductivity σ_b as shown in Fig. 1(a). The short pulsed wave is generated by an electric current source $\mathbf{J}(\mathbf{r}, t)$, where $\mathbf{r} = (x, y, z)$. We assume that the transmitter source is turned on at time $t = 0$ and there is no electromagnetic fields before time $t = 0$. Then, the total electromagnetic fields $\mathbf{v}(\mathbf{r}, t)$ are the solution of the following problem:

$$L\, \mathbf{v}(\mathbf{r}, t) = \mathbf{j}(\mathbf{r}, t) \quad \mathbf{r} \in \text{whole region}, t \in (0, \infty)$$
$$\text{initial condition} \quad \mathbf{v}(\mathbf{r}, 0) = \mathbf{0} \tag{1}$$

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where $\mathbf{v}=[E_x,\ E_y,\ E_z,\ \eta H_x,\ \eta H_y,\ \eta H_z]^t$, $\mathbf{j}=[\ \eta J_x,\ \eta J_y,\ \eta J_z,\ 0,\ 0,\ 0]^t$. The superscript 't' indicates 'transposed'. $\eta(=\sqrt{\mu_0/\varepsilon_0})$ is the characteristic impedance of free space. The partial differential operator L is defined by

(a)

(b)

(c)

Figure 1: (a) Original problem. (b) equivalent problem with PEC and (c) equivalent interior boundary value problem.

$$L \equiv \overline{A}\frac{\partial}{\partial x}+\overline{B}\frac{\partial}{\partial y}+\overline{C}\frac{\partial}{\partial z}-\overline{F}\frac{\partial}{\partial(ct)}-\overline{G} \qquad (2)$$

where $c(=1/\sqrt{\varepsilon_0\mu_0})$ is the speed of light in free space, $\overline{A}$, $\overline{B}$, $\overline{C}$ are 6×6 constant matrices. The matrix $\overline{F}$ depends on the relative permittivity ε_r, and the relative permeability μ_r, while the matrix $\overline{G}$ depends on the conductivity σ of the medium (refer to [4] for more details).
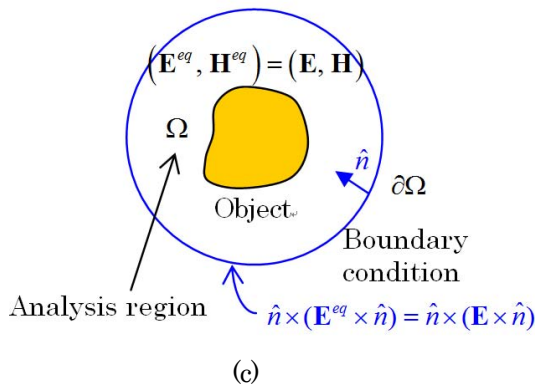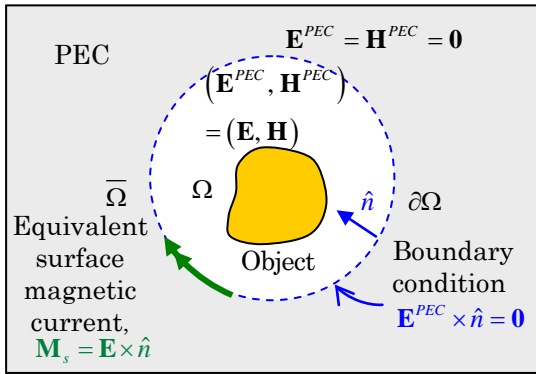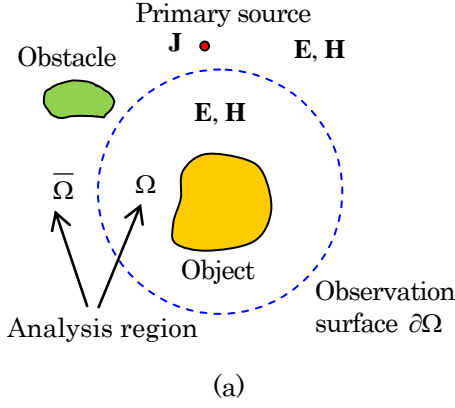
2.1 Equivalent field

Let us consider two regions, denoted by Ω and $\overline{\Omega}$, and separated by an observation surface $\partial\Omega$. The impressed source $\mathbf{J}$ is assumed to be in the region $\overline{\Omega}$ while the scattering object in the region Ω. Following the same procedure using the field equivalence principle as in [4, 5], we can construct an equivalent problem for the total field in the interior region Ω surrounded by a perfect electric conductor (PEC) as shown in Fig. 1(b). The surface magnetic current $\mathbf{M}_s = \mathbf{E}\times\hat{n}$ on the boundary $\partial\Omega$ which is given by the tangential component of the original total electric field $\mathbf{E}$ on $\partial\Omega$ faithfully produces the total field $\mathbf{v}^{PEC}$ identical to the original field $\mathbf{v}$. The vector $\hat{n}$ is a unit inward normal to $\partial\Omega$. This problem further reduces to the following interior initial-boundary value problem whose solution $\mathbf{v}^{eq}$ is identical to that $\mathbf{v}$ of the original problem (1) in the interior region (see Fig. 1(c)):

$$L\,\mathbf{v}^{eq}(\mathbf{r},t)=\mathbf{0}, \qquad \mathbf{r}\in\Omega,\, t\in(0,\infty)$$

initial condition : $\qquad \mathbf{v}^{eq}(\mathbf{r},0)=\mathbf{0},\,\mathbf{r}\in\Omega \qquad (3)$

boundary condition :

$$\hat{n}\times\left(\mathbf{E}^{eq}(\mathbf{r},t)\times\hat{n}\right)=\hat{n}\times\left(\mathbf{E}(\mathbf{r},t)\times\hat{n}\right),\,\mathbf{r}\in\partial\Omega,\, t\in(0,\infty)$$

Note that, the boundary value produces the same total fields as does the primary source $\mathbf{J}$ only if a material of the interior region Ω is the same one of the original problem (1), i.e., the same object is placed in the boundary value problem:

$$\mathbf{v}^{eq}(\mathbf{r},t)=\mathbf{v}(\mathbf{r},t), \qquad \mathbf{r}\in\Omega,\quad t\in(0,\infty) \qquad (4)$$

2.2 Time-reversed field

Suppose time-varying data of tangential component of total electric field $\mathbf{E}(\mathbf{r},t),\mathbf{r}\in\partial\Omega$ is recoded on the observation surface $\partial\Omega$ during a time interval $[0,T]$. In this finite time interval measurement, the relation (4) holds only in the time interval. The recorded data is time-reversed as a boundary value under the assumption that the measurement duration time T is long enough for the fields in the interior region Ω

to be negligibly small after the time T. Then, we have another equivalent problem for the total field in Ω:

$$L\,\mathbf{v}^{tr}(\mathbf{r},t)=\mathbf{0}, \qquad \mathbf{r}\in\Omega, \quad t\in(0,T)$$

final condition: $\qquad \mathbf{v}^{tr}(\mathbf{r},T)=\mathbf{0}, \quad \mathbf{r}\in\Omega$ (5)

boundary condition:

$$\hat{n}\times\left(\mathbf{E}^{tr}(\mathbf{r},t)\times\hat{n}\right)=\hat{n}\times\left(\mathbf{E}(\mathbf{r},t)\times\hat{n}\right), \mathbf{r}\in\partial\Omega, t\in(0,T)$$

The time-reversed field $\mathbf{v}^{tr}$ is calculated backward in time from $t=T$ to 0. The boundary value faithfully produces the same total field in the interior region Ω as does the primary source. This is the case only if a material of the interior region Ω is the same one of the original problem (1).

2.3 Inverse problem

The time-domain inverse scattering problem under consideration here is the estimation of $\varepsilon_r(\mathbf{r})$, $\mu_r(\mathbf{r})$, and $\sigma(\mathbf{r})$ with the knowledge of the tangential components of the original total electric field $\mathbf{E}(\mathbf{r},t)$ measured on the surface $\partial\Omega$ but without the explicit knowledge of the incident field. When the estimated electrical parameters are identical with the true ones, both the equivalent field and time-reversed field are identical to the original field, while they are different from each other for incorrectly estimated electrical parameters. Based on this observation, we formulate the inverse problem as an optimization problem in which the following cost functional needs to be minimized:

$$Q(\mathbf{p})=\sum_{n=1}^{N}\int_0^{cT}\int_\Omega\left|\mathbf{v}_n^{eq}(\mathbf{p};\mathbf{r},t)-\mathbf{v}_n^{tr}(\mathbf{p};\mathbf{r},t)\right|^2 d\mathbf{r}d(ct) \quad (6)$$

where $\mathbf{p}=\left[\varepsilon_r(\mathbf{r}), \mu_r(\mathbf{r}), \eta\sigma(\mathbf{r})\right]$ is an electrical parameter vector of the object. N successive illuminations are assumed to probe the object. The vectors $\mathbf{v}_n^{eq}(\mathbf{p};\mathbf{r},t)$ and $\mathbf{v}_n^{tr}(\mathbf{p};\mathbf{r},t)$ are, respectively, the total fields of the equivalent and the time reversal problems for an estimated parameter $\mathbf{p}$ due to the n*th* illumination. Note that the boundary value $\hat{n}\times\left(\mathbf{E}_n(\mathbf{r},t)\times\hat{n}\right)$ is given by the total electric field data $\mathbf{E}_n(\mathbf{r},t)$ recoded on $\partial\Omega$ for the object with true material parameters $\mathbf{p}^{true}=(\varepsilon_r^{true}, \mu_r^{true}, \eta\sigma)$ due to the n*th* illumination.

3 NUMERICAL RESULTS

In order to confirm the effectiveness of the proposed method, let us examine, for simplicity's sake, a two dimensional detection problem of retrieving the location and size of two unknown
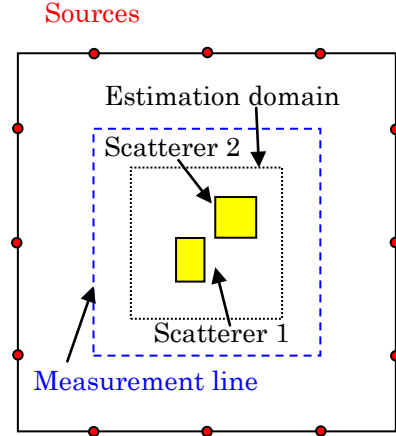

Figure 2: Detection of two scatterers.

scatterers in an estimation domain as shown in Fig. 2. The estimation domain containing the scatterers is a square of $2\lambda\times2\lambda$ whose center is located at the origin $(x, y)=(0, 0)$. Here, λ is the wavelength in free space corresponding to the highest frequency contained in the primary source (i.e., the frequency at which the amplitude spectrum of the source pulse becomes 5% of its maximum value):

$$\mathbf{j}_n=I(t)\delta(\mathbf{r}-\mathbf{r}_n)\hat{z} \qquad (7)$$

where $\mathbf{r}=\mathbf{r}_n$ is the location of the nth primary source and the time factor

$$I(t)=\frac{d^3}{dt^3}\exp\left[-\alpha^2\left(t-\tau\right)^2\right] \qquad (8)$$

and where $\tau=\beta\Delta t$, $\alpha=5.6/\tau$ and $\beta=132$ with the time step size $\Delta t=0.98(\Delta x/c\sqrt{2})$ and the cell size $\Delta x=\Delta y=\lambda/10\sqrt{4}$. Scatterer 1 is $0.3\lambda\times0.6\lambda$ square of $\varepsilon_r=2.0$ with its center $(-0.5\lambda, -0.5\lambda)$, and scatterer 2 is $0.4\lambda\times0.4\lambda$ square of $\varepsilon_r=2.0$ with its center $(0.5\lambda, 0.5\lambda)$. The measurements are performed along the periphery $\partial\Omega$ of the square of side 3λ. The primary sources are located λ away from the measurement line $\partial\Omega$.

The dielectric constant of the scatters are assumed to be known, while the center location (x_{c1}, y_{c1}) and the size $l_{x1}\times l_{y1}$ of the scatterer 1 and the center location (x_{c2}, y_{c2}) and the size $l_{x2}\times l_{y2}$ of the scatterer 2 are sought for. A genetic algorithm [10] is used to minimize the cost functional $Q(\mathbf{p})$. For the use of 12 successive illuminations, the estimated values of the characterizing parameters after 200 generations are shown in Table 1. They are averaged values of 20 trials. The successful estimation demonstrates the

effectiveness of the proposed approach. Interestingly, for a single illumination, satisfactory results were also obtained.

4   CONCLUSION

An inversion method without explicit information of incident fields for the reconstruction of the electrical properties of an unknown object from total electric and magnetic field data, previously developed based on the field equivalence principle, has been extended to the reconstruction from only total electric field data by combining a time reversal technique with the field equivalence principle. One principal advantage of the proposed method is that there is no need to model accurately the transmitting antenna to know the incident field radiated from the antenna. The mutual coupling between the antenna and the object of interest, which is remarkable when it is in the near field such as medical imaging and subsurface sensing, can also be taken into account since the approach does not use explicitly the information of incident fields and the measured total field data contains implicitly the mutual coupling effect. In two dimensional numerical examples satisfactory reconstructions have been obtained for a single illumination as well as multi illuminations, which indicate the capability of detecting dielectric objects by the proposed method.

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Table 1: The parameters of scatterers estimated by genetic algorithm

Estimate	Scatterer 1		Scatterer 2	
	Location	*Size*	*Location*	*Size*
True	(-0.50λ, -0.50λ)	0.30λ × 0.60λ	(0.50λ, 0.50λ)	0.40λ × 0.40λ
Estimated	(-0.49λ, 0.50λ)	0.33λ × 0.56λ	(0.48λ, 0.52λ)	0.39λ × 0.42λ