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Exports, Production and Uncertainty

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Abstract

Using a mean variance portfolio optimization framework, this paper analyzes firm export and production decisions when returns to both the domestic and external market are not known with certainty. It suggests that level and price responses of exports should be a function of external return variance, offers a rationale for competitive firms to operate simultaneously in two markets, introduces risk as an impediment to PPP, shows that the efficacy of a depreciation and duration of J Curve depend on the credibility of the new exchange rate and suggests another contractionary effect of devaluations. It also introduces a second channel, the covariance of returns in each market, through which events in each market effect each other and which may alter the expected results of a relative price change.

I. Introduction

While the study of firm export behavior has largely proceeded by analogy to the non-stochastic market discrimination theory, a sizeable literature has emerged introducing uncertainty about export earnings as inhibiting exports.[1] Empirical studies by Abrams (1980), Hooper and Kohlahagen (1978), Cushman(1983, 1986, 1988), Kenen and Rodrick (1984, 1986) and Thursby and Thursby (1985, 1987) have found significant negative trade flow effects of real or nominal exchange risk using industrial country data sets in the floating rate period. Coes (1981) offers similar evidence on the reverse process for Brazil upon moving to a crawling peg and Paredes (1990) has established a relationship between exchange rate volatility and export elasticities in Latin America.

With exception of Coes, the supporting theoretical models, (Hooper and Kohlhagen, Katz, Paroush and Kahana(1982), and Paredes) have extended the production under uncertainty literature initiated by Sandmo to two markets with returns to the external market rendered uncertain by unpredictable exchange rate movements. While successful in deriving the export disincentive effects cited above, the assumption of a safe, certain domestic return is not easily defended and limits the potential insights of uncertainty based trade models. Theoretically, if exchange rate movements are at all passed through, domestic prices must share some of their variance. More concretely, it is impossible to contemplate the extreme fluctuations in output and prices in many LDC's during the 1980's and credit entrepreneurs with complete certainty about the domestic market.

This paper employs a simple mean variance portfolio optimization framework to examine risk averse firm production and export decisions when **both** domestic and foreign market "returns," prices adjusted for all transaction costs, are not known with certainty at the time that production and export commitments must be made. Maximizing expected profits therefore involves both choosing a level of output and an optimal

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[1] Goldstein and Khan's (1985) comprehensive survey of "Income and Price Effects in Foreign Trade" in the Handbook of International Economics includes no mention of efforts to integrate price uncertainty into the production and export decisions of the firm. There has, however, been extensive research introducing uncertainty into Walrasian general equilibrium models. See John Pomery (1984) for a survey of this literature.

"portfolio" of markets. In assuming a particular utility function and normal distribution of returns, the model is less general than Coes (1981) but gains by determining total output endogenously.

The model proves rich in insights into a variety of questions in international economics. While confirming earlier findings that the level and price responses of exports should be a declining function of expected return variance, it provides a rationale for competitive firms to operate in two markets, and introduces risk as an important impediment to the arbitrage process required for purchasing power parity to hold. Further, the efficacy of a depreciation, the degree of pass through to domestic prices and the duration of the J curve are shown to depend critically on the perceived credibility of the new exchange rate to the point that if thought unsustainable, [2] both exports *and total output* may decrease. Exchange risk thus provides another supply side explanation for the sometimes observed contractionary side effects of devaluations in LDCs (Faini and De Melo 1990, Edwards 1985, Cooper 1971).

The model also establishes a second channel, the covariance of internal and external price movements, through which events in one market affect another. In a standard non-stochastic market discrimination model markets interact through the cost function. An increase in price in one market will decrease output to the second market if marginal costs are upward sloping. A risk averse entrepreneur however, is seeking to minimize variance as well as maximize profits and will therefore consider the covariance between innovations in the two markets and will hold a "balanced" portfolio containing both. It follows that a completely integrated market where domestic prices are perfectly equated with the world price may lead to lower levels of production than if a producer was able to diversify away risk. This provides a rational both for export diversification strategies and even the import substitution strategies which sought as one objective, to insulate domestic economies from movements in world demand. Finally, the impact of devaluation and more generally stabilization programs becomes more complicated since their impact on the domestic market, through reduced absorption or increased uncertainty, feeds back into decisions to produce and export.

[2] The uncertainty could be about the nominal or real value. The 1987 Nigerian structural adjustment program provides an example of a 75% nominal devaluation that was strongly opposed by powerful segments of the population and was constantly in danger of being reversed. The inflationary consequences of depreciation in Mexico make the resulting real change far less than the nominal change would have suggested.

That firms should be risk averse is plausible particularly in the developing world. Managers are frequently owners with large shares of their wealth tied up in the enterprise. Complete capital markets are not available for non-managing shareholders to diversify away all risk so that the variability of the firm's profits, as well as its level, matters. Sandmo (1971) provides the reference work on a risk averse firm's behavior when the output decision must be taken, perhaps due to production lags, prior to the sales date when the market price becomes known. The firm's beliefs about the sales price can be summarized in an exogenous subjective probability distribution.[3] It therefore remains a price taker, but in a probabalistic sense. For any utility function displaying risk aversion, output levels in a competitive market will be below what they would be with price certainty, a result which Leland (1972) has extended to a more general formulation of random demand where firms have monopoly power. Coes(1977) proved Sandmo's intuition that a marginal increase in uncertainty will lead to a decrease in output.

If firms could direct output to either the domestic or foreign market *after* the foreign "returns", prices adjusted for all costs of transacting abroad, were revealed, there would be no "portfolio" dimension arising from the addition of a second market. That this is clearly not the case is demonstrated by the emergence in the industrialized world of institutions to cover exporter's risk. Forward exchange markets arise because most contracts are denominated in foreign currencies and it is unknown at the time of contracting what the exchange rate adjusted price in domestic currency will be. Export-Import banks insure against payment default by foreign contractors, effectively a unilateral reduction in the price. The drive for Free Trade Agreements with the US by Mexico and other Latin American countries can be partially understood as a way to eliminate foreign trade barriers that are known to change capriciously and on short notice. Finally, transportation costs may rise or fall dramatically with fuel prices or geopolitical tensions. In countries where risk reducing institutions do not exists, the producer must pre-commit output and then bear the uncertainty himself. It follows that with overall output and exports decided before the price is revealed, output to the domestic market is also constrained to be pre-committed. The problem becomes analogous to the problem of choosing

[3] Davidson's (1991) argument that modeling uncertainty as a subjective probability implies more knowledge than agents often have is a reasonable criticism of this approach, especially in cases of far reaching structural reform.

simultaneously a level of savings and a portfolio of assets in which to hold it.

The plan of the paper is as follows. The next section presents a general problem of a risk averse firm under uncertainty and confirms that a "marginal risk premium" drives a wedge between marginal cost and price, leaving output below the competitive level. Section III demonstrates the same without any approximations but assuming a negative exponential utility function and a normal distribution of prices. Section IV then introduces a second market, first assuming that marginal costs are constant so as to highlight covariance effects, then allowing increasing marginal costs show how covariance effects may reverse or reinforce the conventional cost effects.

II. Production Decisions Under Price Uncertainty

Following Sandmo, Leland and Coes, we model the firm's decision as

$$\max_{\{x\}} E\,[U(\pi)]. \tag{1}$$

where x is the level of output and π represents profits. It will be useful for future reference to recall that a second order Taylor expansion around mean profits yields that the expected utility of randomly varying profits is equal to the utility of expected profits minus a risk premium [4]

$$E[U(\pi)] = U[\pi^e - rp(\pi^e, x)] \tag{2}$$

where the bracketed term on the right is the certainty equivalent level of profits. More specifically

$$rp = -\frac{1}{2}\sigma_\pi^2 \frac{U''(\pi^e)}{U'(\pi^e)} \tag{3}$$

[4] See Copeland, Thomas and J Fred Weston, p. 88 for a complete derivation.

This is only an approximation, but is of heuristic value and will prove useful later on. If we define profits as

$$\pi = px - c(x) \quad where \quad p \sim (\mu, \sigma_p^2) \tag{4}$$

then it can be seen that

$$\max_{\{x\}} E[U(\pi)] = \max_{\{x\}} U[\mu x - c(x) - rp(\pi^e, x)] \tag{5}$$

or that

$$U_\pi[\mu - c_x - rp_x(\pi^e, x)] = 0 \tag{6}$$

and

$$U_\pi[\mu - rp_x(\pi^e, x)] = U_\pi c_x \tag{7}$$

$$\mu - rp_x(\pi^e, x) = c_x \tag{8}$$

The second term on the left is precisely the marginal change in the risk premium given an increase in production. While from equations (2) and (3) it is clear that its value will depend on the particular production function utilized, its sign will depend only on utility increasing with wealth but at a decreasing rate, the condition imposed by the assumption of risk aversion. Thus, a profit maximizing firm will set certainty equivalent marginal profits equal to zero which occurs at a point where marginal costs are below price. With increasing marginal costs, this implies that output is below the level arising when prices are known with certainty. Intuitively, we can see the possibility of an equilibrium in a competitive market with constant costs or even decreasing costs if the marginal risk premium rises faster than marginal costs fall.

III. Production with Constant Absolute Risk Aversion

The analysis becomes more tractable if we assume a specific function form for utility. The negative exponential utility function

$$U(\pi) = \alpha - \beta \exp(-\gamma \pi) \tag{9}$$

where gamma is the coefficient of risk aversion, has the property of constant absolute risk aversion (CARA) as profits increase. It is preferred to the popular quadratic function which predicts, counterintuitively, that as entrepreneurs make higher profits, they become more risk averse. If we assume that prices are distributed normally and hence their distribution can be summarized completely with the first and second moments, we can write

$$E[U(\pi)] = \alpha - \beta \exp[-\gamma \pi^{e} + \frac{\gamma^2}{2} \sigma_{\pi}^2] \tag{10}$$

The firm maximization problem becomes

$$\max_{\{x\}} E[U(\pi)] = \max_{\{x\}} \{\alpha - \beta \exp[-\gamma \pi^{e} + \frac{\gamma^2}{2} \sigma_{\pi}^2]\} \tag{11}$$

which is equivalent to

$$\max_{\{x\}} [\pi^{e} - \frac{\gamma}{2} \sigma_{\pi}^2] \tag{12}$$

or

$$\max_{\{x\}} [x\mu - c(x) - \frac{\gamma}{2} x^2 \sigma_{p}^2] \tag{13}$$

The first order condition yields

$$\mu - c_x - \gamma x \sigma_p^2 = 0 \tag{14}$$

The third term on the left has an intuitive interpretation as the marginal increase in risk premium with respect to output if we remember that

$$rp = \frac{1}{2}\sigma_\pi^2 \frac{U''(\pi)}{U'(\pi)} = \frac{1}{2}\sigma_\pi^2\gamma = \frac{1}{2}x^2\sigma_p^2\gamma \tag{15}$$

and so

$$\frac{\partial rp}{\partial x} = \gamma x \sigma_p^2 \tag{16}$$

Marginal certainty equivalent profit equals zero, confirming for the specific negative exponential utility function the results of Leland and Sandmo that output is below what it would be in a competitive case where rising marginal costs would equal price.

IV. The Market Allocation Decision

We now introduce the possibility of directing output to two or more markets that differ in distributions of "returns." We assume the return to a unit sold in the foreign market P_f is a function of the price in foreign currency, the exchange rate, tariffs levied domestically and externally, transport costs etc. Pd represents the price received in the domestic market. In sum

$$P_d \sim N(\mu_d,\sigma_d^2) \qquad P_f \sim N(\mu_f,\sigma_f^2) \tag{17}$$

The firm optimization problem becomes

$$\max_{\{x_d,x_f\}} x_d\mu_d+x_f\mu_f-\frac{\gamma}{2}(x_d^2\sigma_d^2+x_f^2\sigma_f^2+2x_dx_f\sigma_{df})-c(x) \tag{18}$$

It is important to note that the variance of the "portfolio" of markets depends on three things, the variance of each price, the covariance of the two prices, and the quantity of output dedicated to each market. In general, an increase in production to either market increases total variance, the cost of which must be weighed against the resulting increased profits. First order conditions yield

$$\mu_d-\gamma(x_d\sigma_d^2+x_f\sigma_{df})-c_x=0 \tag{19}$$

$$\mu_f-\gamma(x_f\sigma_f^2+x_d\sigma_{df})-c_x=0 \tag{20}$$

or

$$x_d=\frac{(\mu_d-c_x)}{\gamma\sigma_d^2}-\frac{\sigma_{df}}{\sigma_d^2}x_f \qquad x_f=\frac{(\mu_f-c_x)}{\gamma\sigma_f^2}-\frac{\sigma_{df}}{\sigma_f^2}x_d \tag{21}$$

It is helpful to incompletely solve for the quantity of output directed to each market.

$$x_d=\frac{\sigma_f^2(\mu_d-c_x)-\sigma_{df}(\mu_f-c_x)}{\gamma[\sigma_d^2\sigma_f^2-\sigma_{df}^2]} \qquad x_f=\frac{\sigma_d^2(\mu_f-c_x)-\sigma_{df}(\mu_d-c_x)}{\gamma[\sigma_d^2\sigma_f^2-\sigma_{df}^2]} \tag{22}$$

or

$$x_d = \frac{\left[\frac{(\mu_d - c_x)}{\sigma_d^2} - \frac{\rho(\mu_d - c_x)}{\sigma_f \sigma_d}\right]}{\gamma[1-\rho^2]} \qquad x_f = \frac{\left[\frac{(\mu_f - c_x)}{\sigma_f^2} - \frac{\rho(\mu_d - c_x)}{\sigma_d \sigma_f}\right]}{\gamma[1-\rho^2]} \tag{23}$$

where ρ is the correlation coefficient. Note that $\rho > 0$ raises the possibility of a negative quantity directed to each market which we will rule out by assumption. These expressions are still not closed form because both quantities depend on marginal costs which are a function of output in both markets. It is nevertheless a good place to begin understanding the two channels through which the markets can influence each other and analyze firm behavior when marginal costs are constant.

In both the monopolistic and competitive non-stochastic cases, the interaction between the two markets occurs through the cost function. In the competitive case, if prices rise to the external market, increased production will drive up marginal costs to both markets and above the price in the domestic market. All output will therefore be redirected the external market. The results are less extreme but similar in the monopolistic case, the rise in marginal cost will result in less being produced for the domestic market as output is reduced to push marginal revenue again equal to the higher marginal cost.

Equation (21) illustrates the second channel of interaction through the covariance of the two market prices and one which can overturn the results of the customary non-stochastic formulation. We will highlight this effect by eliminating the first channel of interaction and assume constant marginal costs.

Constant Marginal Costs

In the non-stochastic monopolistic case, constant marginal costs imply that demand changes in one market will have no effect on the other. Output can expand or contract and not affect costs, hence output in the other market. In the competitive non-stochastic case, there is no equilibrium. For the stochastic case, equations (19) (20) indicate that, as long as the marginal risk premium is a function of output, an equilibrium level of output exists since risk adjusted marginal revenue is downward sloping and will intersect a horizontal marginal cost curve. Equation (22) above can be thought of as relating the output directed to each market to the "net profits" from each market. Now, the only independence of decision arises from the covariance of the two market prices.

1. Constant Marginal Costs and No Price Covariance

Taking the extreme case of no covariance of returns from each market, equation (22) collapses to

$$x_d = \frac{\mu_d - c}{\gamma \sigma_d^2} \qquad x_f = \frac{\mu_f - c}{\gamma \sigma_f^2} \tag{24}$$

which is the special case that Paredes investigates. We can see that quantity directed to each market can be solved entirely independently of the other market, each a problem identical to that of the single good case in equation (14). The other market's characteristics enter neither through costs, nor through the joint variance.

Result 1a: *With constant costs and independent price movements, the decision of how much to produce for each market depends positively on the net return and negatively on the price variance in that market and the degree of risk aversion. Total output will unambiguously increase with increased output to either market.*

The result that an increased price, ceteris paribus, in either market unambiguously increases total output is in agreement with standard non-stochastic models. The introduction of variance as an argument in the utility function, however, alters the classical result.

Result 1b: *The price response of output in each market is a function of the degree of uncertainty in that market.*

It is this result that underlies Paredes' empirical work on Brazil, Chile and Peru where he finds that the supply price-elasticity point estimates for manufactured exports to be positively correlated with real exchange rate risk.

Result 1c: *If marginal risk premia differ in each market, differentials between domestic and foreign prices measured in the same currency may not be arbitraged away and purchasing power parity will not hold.*

In the non-stochastic case, a higher external price would lead to a redirection of output to the foreign market from the domestic driving the two prices towards equality. The arbitrage process only continues until risk adjusted returns are equal, however, and if the external market appears very uncertain, the deviations may be large.

Result 1d: *If a rise in price is accompanied by a perceived increase in the variance of that price, both the magnitude and direction of output change are indeterminate.*

If, for example, a large devaluation increases the expected price received in the external market, but the sustainability of this change is questioned, then the perceived variance of that price will also rise with the degree of producer risk aversion determining which effect dominates. If both the external return and the marginal risk premium rise equivalently there should be no reallocation of quantities sold and thus no upward pressure on domestic prices. The degree of pass-through, therefore, is a function of the perceived change in the variance of the external price as well. Export and import decisions may remain fundamentally unaltered causing the trade balance to remain in the hook of the J-curve until agents grow confident that the devaluation is permanent.

2. Price Covariance and Constant Costs

Dropping the assumption of independence of price movements in the two markets adds even greater indeterminacy to the results.[5] The second terms in equations (21) that distinguish them from line (24) imply that output to either market is a function of the quantity directed to the other.

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[5] The degree of covariance in returns is clearly not independent of firm decisions. If firms do not respond to a rise in external return by directing output away from the domestic market, domestic prices will not move. Yet, in the same way that a perfectly competitive firm "takes" or acts "conditioned on" prices despite the fact that in the aggregate, firms do affect them, or that a portfolio manager "takes" asset covariances despite an impact on them in the aggregate, we may, for present purposes, assume our firm to be a covariance taker.

Result 2a: *If return movements are positively correlated, an increase in production in one market increases the total variance and the marginal risk premium in* ***both*** *markets and thus decreases production to the other market. Alternatively, if prices covary negatively, an increase in the production for one market leads to increased production in the other market too through a decline in the marginal risk premium.*

The second sub-result seems counterintuitive simply because we are accustomed to a negative cross price elasticity unless there are increasing returns to scale. However, this is not a portfolio problem with "wealth" constant. An increase in production for the external market, can offset the increased variance arising from an increase in production for the domestic market and reduce the marginal risk premium, or uncertainty arising from production for the domestic market. With perfect negative correlation, a zero variance portfolio can be constructed eliminating any risk premium and making output to each market infinite.[6] In reality, arbitrage forces almost certainly cause domestic and export prices to covary positively so this may not be an interesting result.

It is precisely the positive correlation that yields the counterintuitive result when we consider total output, however. Combining both equations from line (23) we get that

$$\frac{\partial X}{\partial u_f} = \frac{\left[\frac{1}{\sigma_f^2} - \rho\right]}{\gamma\ (1-\rho^2)} \tag{25}$$

Result 2b: *Under constant costs, a positive covariance implies that a price rise in either market will have an indeterminate impact on total output. A negative covariance between prices implies that a price rise in either market will lead to an increase in total output.*

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[6] The deceptively similar result for perfect positive correlation is a result of the particular utility function chosen. $\sigma_{df} = \sigma_d \sigma_f$ implies from the first order conditions that

$$\frac{\mu_f}{\sigma_f} = \frac{\mu_d}{\sigma_d}$$

which implies that the numerator in equation 23 is also 0 leaving the result indeterminate. The problem arises because when $\rho = 1$, the variance-return locus is linear as is the negative exponential utility function. Tangency occurs when the above condition is met at which point there are an infinite number of solutions.

If prices are less than perfectly correlated, and if the covariance is greater than the variance of the other market, output will fall but a greater percent of it will be directed to the external market. This corresponds to a pure portfolio effect with no increase in "wealth." In the non-stochastic case we might expect a reallocation of output, but unless marginal costs were extremely steep, we would expect to see an increase in output. With a positive covariance, we are no longer guaranteed that a rise in the external return through a depreciation will have a stimulative impact on production.

This effect may be compounded when we recall that a depreciation differs from an individual external price rise because it affects the price of all tradeables and must affect domestic production and consumption decisions, reducing the output- absorption gap, if a trade deficit is to be eliminated. This is likely to add uncertainty about the strength of domestic demand and to increase the expected variance of the domestic price. This clearly could imply a fall in overall output, but equation (23) suggests that a negative impact on exports as well.

Finally, taking the derivative of output to each market, letting μ hat represent "net" returns (μ -c), with response to a change in the covariance yields

$$\frac{\partial x_d}{\partial \sigma_{df}} = \frac{[2\sigma_{df}(\sigma_f^2\hat{\mu}_d - \sigma_{df}\hat{\mu}_f) - \hat{\mu}_f(\sigma_d^2\sigma_f^2 - \sigma_{df}^2)]}{\gamma(\sigma_d^2\sigma_f^2 - \sigma_{df}^2)^2} \tag{26}$$

The denominator is always positive and since

$$\sigma_d^2\sigma_f^2 - \sigma_{df}^2 = \sigma_f^2\sigma_d^2(1-\rho^2) \tag{27}$$

which is invariably positive, then for all negative values of σ_{df} , output to the domestic market increases as the covariance decreases. For positive covariances, the condition

$$2\sigma_{df}(\sigma_f^2\hat{\mu}_d - \sigma_{df}\hat{\mu}_f) - \hat{\mu}_f(\sigma_d^2\sigma_f^2 - \sigma_{df}^2) < 0 \tag{28}$$

implies by equation (22) that

$$\hat{\mu}_f - 2\gamma\sigma_{df}x_d > 0 \tag{29}$$

Remembering that an entirely symmetrical set of equations exists for foreign directed output and using line (21) we get

$$x_d - \frac{\sigma_{df}}{\sigma_d^2}x_f > 0 \qquad x_f - \frac{\sigma_{df}}{\sigma_f^2}x_d > 0 \tag{30}$$

and that

$$x_d - \frac{\sigma_{df}}{\sigma_d^2}\frac{\sigma_{df}}{\sigma_f^2}x_d > 0 \qquad x_f - \frac{\sigma_{df}}{\sigma_f^2}\frac{\sigma_{df}}{\sigma_d^2}x_f > 0 \tag{31}$$

Combining the two yields that for a reduction in covariance to always increase output

$$x_d(1-\rho^2) > 0 \qquad x_f(1-\rho^2) > 0 \tag{32}$$

which will hold as long as the quantities of output produced are non-negative.

Result 2c: *A decrease in the covariance of returns leads unambiguously to an increased production to both markets.*

This implies that the worst case scenario is one where the domestic economy is wide open and PPP holds perfectly. From an economy wide point of view, some rational emerges for the import substitution regimes prevalent in Latin America after collapse of the world market in the 1930's. The creation of a domestic market insulated, and therefore exhibiting less positive covariance is a rational strategy from the point of view of expanding production.

3. Price covariance and Increasing Marginal Costs:

The introduction of increasing costs adds another feedback between markets and one that is most commonly modeled in the literature on market discrimination. If we assume quadratic costs in total output, the firm decision becomes

$$\underset{\{x_d, x_f\}}{MAX}\; x_d\mu_d + x_f\mu_f - \frac{\gamma}{2}(x_d^2\sigma_d^2 + x_f^2\sigma_f^2 + 2x_d x_f\sigma_{df}) - (x_d + x_f)^2 \tag{33}$$

and the first order conditions are

$$\mu_d - \gamma(x_d\sigma_d^2 + x_f\sigma_{df}) - 2(x_d + x_f) = 0 \tag{34}$$

$$\mu_f - \gamma(x_f\sigma_f^2 + x_d\sigma_{df}) - 2(x_d + x_f) = 0 \tag{35}$$

The resulting nightmarish expression for domestic production is

$$x_d = \frac{\mu_d(2 + \gamma\sigma_f^2) - \mu_f(2 + \gamma\sigma_{df})}{\gamma[(2\sigma_d^2 + 2\sigma_f^2 + \gamma\sigma_d^2\sigma_f^2) - \sigma_{df}(4 + \gamma\sigma_{df})]} \tag{36}$$

Two things differentiate this equation from (22) above. First, even in the absence of any covariance between prices, both the return and the variance of the alternate market price enter in determining the level of production for the domestic market:

$$x_d = \frac{\mu_d(2+\gamma\sigma_f^2)-2\mu_f}{\gamma(2\sigma_d^2+2\sigma_f^2+\gamma\sigma_d^2\sigma_f^2)} \tag{37}$$

The interaction comes through costs. An increased attractiveness of the external market that leads to increased production for that market increases costs at the margin and makes production of the home destined good less profitable and output contracts. This is in fact the more traditional result.

Result 3a: *Under increasing marginal costs, and no covariance in prices, an increase in return or decrease in variance of prices in one market must, ceteris paribus, lead to a decline in production in the alternate market.*

The redirection of output is not likely to be as extreme as in the perfectly competitive non-stochastic case. Deriving the cross price response yields

$$\frac{\partial x_d}{\partial \mu_f} = -\frac{2}{\gamma(2\sigma_d^2+2\sigma_f^2+\gamma\sigma_d^2\sigma_f^2)} \tag{38}$$

and

Result 3b: *The cross price elasticity will not be infinite if there is any uncertainty in the system.*

The vexing problem of unobserved extreme reallocations of output in response to small changes in relative price predicted by the competitive model is eliminated. Despite the fact that marginal costs are rising in both markets and price in only the external, the simultaneous reduction of the risk premium and output leads to a lower equilibrium output for the domestic market, but not a zero level. In a sense we may view this an extension of the "imperfect substitutes" model described by Goldstein and Khan where it is the markets, rather than the goods, that differ.

The increasing costs analogue to result 2a becomes

Result 3c: *Under increasing marginal costs and positive covariance, the decrease in production to one market brought on by an increase in production for the other market is compounded by the increased marginal risk premium. With a negative covariance of prices, an increase in price in one market increases output to the other only if the reduction in marginal risk premium exceeds the rise in marginal costs.*

Whether a devaluation increases or decreases output to the domestic market depends on production technology and the covariance of returns.

Total production is affected similarly as before although with an extra term and a more complicated denominator

$$\frac{\partial X}{\partial \mu_d} = \frac{\gamma\sigma_f^2 - 2 - \gamma\sigma_{df}}{\gamma[(\sigma_d^2 + 2\sigma_f^2 + \gamma\sigma_d^2\sigma_f^2) - \sigma_{df}(4 + \gamma\sigma_{df})]}$$

The covariance can be less than in the constant cost case in equation (25) to for the rise in price to cause a fall in output.

Conclusion

The introduction of price uncertainty into firm production and distribution decisions complicates several key relationships in international finance. First, in allocating their portfolio of output between the domestic and foreign market, firms will look at the risk adjusted return in each market, not simply the price. This impedes the arbitrage process required for PPP to hold, and implies that the efficacy of a depreciation, the degree of passthrough to the domestic prices, the duration of the J-curve, and even the direction of change of output depend critically on the perceived credibility of the new exchange rate. Second, an additional channel of interaction between markets emerges in the covariance of internal and external price movements. This implies that firms would choose to hold a diversified portfolio of domestic and foreign markets making sharp redirections of output with small changes in relative prices unlikely. Further, increases in domestic uncertainty must be incorporated in analyses of the impact of depreciations and stabilization policies on exports.

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