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CHROMOSPHERICALLY ACTIVE STARS. VI. HD 136901 = UV CrB: A MASSIVE ELLIPSOIDAL K GIANT SINGLE-LINED SPECTROSCOPIC BINARY

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ABSTRACT

The variable star HD 136901 = UV CrB is a chromospherically active K2 III single-lined spectroscopic binary with an orbital period of 18.665 days. It has modest strength Ca H and K emission and ultraviolet features, while H α is a strong absorption feature containing little or no emission. The inclination of the system is 53° ± 12°. The v sin i of the primary is 42 ± 2 km s⁻¹, resulting in a minimum radius of $15.5 \pm 0.8 R_{\odot}$. When compared with the Roche lobe radius, this results in a mass ratio $\mathcal{M}_1/\mathcal{M}_2 \ge 2.9$. Additional constraints indicate that the secondary has a mass between 0.85 and 1.25 \mathcal{M}_{\odot} . Thus, the mass of the primary is at least 2.5 \mathcal{M}_{\odot} and probably is in the range 2.5-4 \mathcal{M}_{\odot} . Modeling the light variations due to ellipsoidal and reflection effects would give additional information on the nature of this system.

I. INTRODUCTION

Bidelman (1983), in his initial report of "early result" objective-prism discoveries in the northern sky, listed HD $136901 = \text{UV CrB} (\alpha = 15^{\text{h}} 20^{\text{m}} 16^{\text{s}} 3 \delta = 25^{\circ} 48' 07'', 1950,$ V = 7.20, K2 III) as having weak emission in his table of Ca II emission stars of G and K type. Heard (1956) reported a spectral type of K1 III and a radial-velocity range of 30 $km s^{-1}$ from nine spectroscopic observations. On the basis of five observations, Guetter (1980) listed it as a suspected variable star. Boyd, Genet, and Hall (1984) confirmed the light variations, finding a period of 9.63 days and an amplitude of 0.16 in V. Fekel, Moffett, and Henry (1986) found an orbital period of 18.68 days, a value about twice the photometric period. To reconcile the photometric and spectroscopic periods, they suggested that the light variations might be primarily due to the ellipsoidal shape of the K III component. Such a shape would be consistent with the star's large v $\sin i = 42 \pm 2$ km s⁻¹ and large minimum radius of about 16 R_o (Fekel, Moffett, and Henry 1986). Additional information is summarized by Strassmeier et al. (1988). In this paper, we present the spectroscopic observations for the orbital-period determination and discuss other properties of this chromospherically active system.

II. OBSERVATIONS AND REDUCTIONS

From 1983 to 1988, 33 high-dispersion spectroscopic observations were obtained at Kitt Peak National Observatory with the coudé feed telescope and coudé spectrograph system. Table I lists the detector and spectrograph combinations for the observations. All but one of these observations included the wavelength range 6390–6455 Å (Fig. 1). A single observation obtained on Julian Date 2447245 was centered at 6690 Å and included the lithium line at 6708 Å.

The vast majority of the radial velocities were determined relative to μ Her, for which we assumed a velocity of -15.6 km s⁻¹ (Wilson 1953). This value is 0.6 km s⁻¹ more nega-

tive than the velocity determined by Beavers and Eitter (1986). Although μ Her is not an International Astronomical Union (IAU) radial-velocity standard, the assumed velocity appears to differ from the IAU system by less than 1 km s⁻¹ (Beavers and Eitter 1986). Several radial velocities were determined relative to the IAU standards, β Vir or β Oph, which have radial velocities of 3.8 km s⁻¹ (Fekel 1981) and -12.0 km s⁻¹ (Pearce 1955), respectively. Two velocities were determined relative to HR 6469A, whose velocity for those observations is -1.2 km s⁻¹ (Fekel, unpublished). Details of the cross-correlation reduction procedure have been given by Fekel, Bopp, and Lacy (1978).

One ultraviolet spectroscopic observation was obtained with the short-wavelength primary (SWP) camera of the *International Ultraviolet Explorer (IUE)* satellite. This 80 min exposure, SWP 33467, obtained through the large ($10'' \times 20''$) aperture, has a resolution of about 6 Å and covers a wavelength region from 1100 to 2000 Å. The observation was absolutely calibrated with the standard computer software routines at the Regional Data Analysis Facility of the Goddard Space Flight Center.

III. ORBIT

Listed in Table II, along with the new high-dispersion observations from Kitt Peak National Observatory, are the nine radial velocities obtained at the David Dunlap Observatory (DDO) (Heard 1956). An adjustment of -3 km s^{-1} has been made to these published DDO velocities to put them approximately on our velocity system (see Griffin 1980 for additional information).

The period-finding program of Deeming (Bopp *et al.* 1970) was used to determine a preliminary period with the Kitt Peak observations. Preliminary orbital elements were determined with a slightly modified version of the computer program of Wolfe, Horak, and Storer (1967) which uses the

TABLE I. Detector-spectrograph combinations.

Detector	Dispersion (Å mm ⁻¹)	Resolution (Å)	Wavelength range (Å)	Source code
Texas Instruments CCD	7.6	0.23	90	KT1
Texas Instruments CCD	7.0	0.18	84	KT2

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FIG. 1. Portion of a CCD observation of HD 136901 compared with that of β Oph (K2 III), showing the line broadening of HD 136901.

Wilsing-Russell method. Final orbital elements were computed with a differential corrections program (Barker, Evans, and Laing 1967). The Kitt Peak observations alone gave a period of 18.670 ± 0.001 (m.e.), which then was revised by including the DDO velocities with a weight of 0.02, resulting in a period of 18.665 ± 0.002 . This period was fixed in the final orbital solution, which used only the Kitt Peak velocities. The eccentricity is 0.059 + 0.014, so, in accordance with the precepts of Lucy and Sweeney (1971), we have adopted the eccentric solution. Final orbital elements are listed in Table III. These orbital elements supersede the values given by Strassmeier et al. (1988). Because of the star's large projected rotational velocity of 42 km s⁻¹, the standard error of an observation of unit weight for HD 136901 is 1.1 km s⁻¹, about 50% greater than for orbital solutions of narrow-lined stars in this series. The observed radial velocities and the computed radial-velocity curve are plotted in Fig. 2.

TABLE II. Radial velocities of HD 136901.

HJD 2400000+	Phase	V (km s ⁻¹)	0-C (km s ⁻¹)	Source	Standard
32303.793	0.821	-3.4	1,.5	DDO	
32718.644	0.047	-24.6	11.2	DDO	
33061.710	0.427	-24.7	-7.3	DDO	
33423.724	0.822	-3.7	1.4		
33490.608	0.406	-19.2	0.9	DDO	
34111.846	0.842	6.2	13.5	DDO	
34482.803	0.564	-12.9	-10.0	DDO	
34504.803	0.742	2.7	1.6	DDO	
34525.713	0.862	-14.4	-4.5	DDO	
45594.601	0.888	-16.6	-3.1	KT1	μHer
45595.661	0.945	-24.7	-2.6	KT1	uHer
45721.005	0.661	1.9	-0.2	KT1	μHer
45783.885	0.029	-33.6	0.2	KT1	HR6469A
45784.940	0.086	-35.4	3.7	KT1	HR6469A
45811.842	0.527	-6.2	-0.0	KT1	βVir
45812.834	0.580	-1.2	0.4	KT1	ßVir
45813.843	0.634	1.4	0.1	KT1	µHer
45814.888	0.690	2.0	-0.3	KT1	μHer
45853.823	0.776	-0.9	0.0	KT1	μHer
45854.717	0.824	-6.0	-0.7	KT1	μHer
45855.766	0.880	-12.7	-0.3	KT1	uHer
45941.684	0.484	-9.6	1.1	KT1	βOph
46077.025	0.735	0.7	-0.7	KT1	μHer
46530.869	0.050	-37.9	-1.9	KT1	µHer
46531.856	0.103	-40.3	-0.2	KT1	µHer
46532.908	0.159	-41.0	0.4	KT1	uHer
46533.933	0.214	-40.4	-0.7	KT1	uHer
46534.916	0.267	-35.0	0.9	KT1	μHer
46583.799	0.886	-12.4	0.7	KT1	μHer
46584.872	0.943	-21.5	0.2	KT1	µHer

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HJD 2400000+	Phase	V (km s ⁻¹)	0-C (km s ⁻¹)	Source	Standard
46585.785	0,992	-30,3	-1.3	КТ1	uHer
46586.822	0.048	-34.3	1.5	KT1	uHer
46817.022	0.381	-23.6	-0.4	KT1	uHer
46866.960	0.056	-37.3	-0.6	KT1	μHer
46868.942	0.162	-39.5	1.8	KT1	µHer
46973.763	0.778	-1.4	-0.4	KT1	µHer
47244.882	0.307	-32.6	-2.6	KT2	μHer
47245.906	0.362	-27.7	-1.8	KT2	μHer
47247.911	0.470	-12.6	0.2	KT2	μHer
47248.920	0.524	-5.9	1.0	KT2	µHer
47308.840	0.734	1.6	0.0	KT2	μHer
47310.873	0.843	-4.6	2.4	KT2	µHer

IV. ACTIVE-CHROMOSPHERE CHARACTERISTICS

Although Heard (1956) noted the Ca II H and K emission of HD 185151 (Bopp *et al.* 1982), there is no mention of H and K emission for HD 136901. Thus, Bidelman (1983) was the first to report such emission.

In the most active RS CVn binaries, the H α line is an emission feature. But in the vast majority of RS CVn binaries, H α is an absorption feature, although it is typically weaker (that is, has a smaller equivalent width than in similar stars that are not chromospherically active (Smith and Bopp 1982)). Figure 14 of Fekel, Moffett, and Henry (1986) shows the H α line of HD 136901, which has an equivalent width of 1.21 Å. For comparison, the equivalent width of H α for β Gem, obtained with the same telescope and spectrograph-detector system, is 1.29 Å. Thus, the H α line of HD 136901 is, at most, weakly active.

The spectrum obtained with the *IUE* satellite's SWP camera (Fig. 3) is underexposed, but still shows ultraviolet emission features typical of chromospherically active stars. Table IV gives the emission-feature identifications and the measured observed fluxes. Following the procedure of Linsky *et al.* (1979), the observed fluxes have been converted into surface fluxes. Since the V - R color of chromospherically active stars has a color excess relative to inactive ones (Fekel, Moffett, and Henry 1986), we use the surface-brightness-(B - V) color relation of Barnes, Evans, and Moffett(1978)

TABLE III. Orbital elements.

$P = 18.6651 \pm 0.0013$ (m.e.) days.
$T = 2446585.94 \pm 0.67$ HJD.
$\gamma = -18.89 \pm 0.21 \mathrm{km s^{-1}}.$
$K_1 = 21.87 \pm 0.28 \text{ km s}^{-1}$.
$e = 0.059 \pm 0.014.$
$\omega_1 = 118.9 \pm 13.0.$
$a_1 \sin i = 5.60 \pm 0.073 \times 10^6$ km.
$f(m) = 0.0202 + 0.0008 M_{\odot}$
Standard error of an observation of unit weight = 1.1 km s^{-1} .

along with V = 7.20 and B - V = 1.24 (Guetter 1980). Simon and Fekel (1987) examined the dependence of ultraviolet chromospheric emission upon rotation among late-type stars. For HD 136901 the C IV surface-flux value of 5×10^4 ergs cm⁻² s⁻¹ for its assumed rotation period is below the least-square Gaussian-curve value of 1.7×10^5 ergs cm⁻² s⁻¹ shown in Fig. 3 of Simon and Fekel (1987) but still within the general range of values for such active-chromosphere stars.

V. DISCUSSION

As noted previously, Heard (1956) gives a spectral classification of K1 III. Bakos (1968) determined V = 7.21 and B - V = 1.25 from three observations. From five observations, Guetter (1980) found a V magnitude range of 7.20-7.29, as well as U - B = 1.09 and B - V = 1.24. These colors are consistent with a K2 III spectral type according to the color-spectral-type relation of Johnson (1966).



FIG. 2. Observations and computed radial-velocity curve for HD 136901. Dark circles are Kitt Peak observations, +'s are DDO observations. Phase zero is periastron passage.

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TABLE IV. Ultraviolet emission-line fluxes (ergs cm $^{-2}$ s $^{-1}$).

Ion	Wavelength (Å)	Observed flux $(\times 10^{-14})$	Surface flux $(\times 10^4)$
Nv	1240		
01	1305	19.9	5.4
Сп	1335	4.6	1.3
Si iv	1400	4.0	1.1
C IV	1550	18.4	5.0
He II	1640	6.4	1.7
Сі	1657	14.8	4.0
Si 11	1808,1817	14.7	4.0
Si III	1893	4.8	1.3

A spectrum of the lithium region of β Oph, a K2 III spectral-type standard (Morgan and Keenan 1973), was broadened to 42 km s⁻¹ and compared with HD 136901 (Fig. 4). The features of β Oph appear to be just slightly stronger. Since the line strengths of the many weak features in this region increase rapidly from K0 III to K2 III, we conclude that HD 136901 is not as early as K0 III but is a K2 III, a type consistent with the colors of the star.

Since the system is not double lined, it is more difficult to determine the properties of the system, such as the orbital inclination and the nature of the secondary. Yet, some limits can be determined with reasonable assumptions and the available information.

Most of the masses of the active stars in eclipsing RS CVn binaries range from 1.2 to 1.8 \mathscr{M}_{\odot} (Popper 1980). There are indications (Fekel 1988) that many chromospherically active giants may be somewhat more massive than these subgiants. Combining the value of the mass function with each assumed primary mass from 1.5 to 4.0 \mathscr{M}_{\odot} , we determine the corresponding minimum value of the secondary mass, which occurs when $i = 90^{\circ}$. These values are listed in Table V along with the mass of the secondary for $i = 65^{\circ}$ and 40°.

Constraints can be placed on the lowest possible inclination by comparing the observed value of $R_1 \sin i$ = 15.5 ± 0.8 R_{\odot} as determined from $v \sin i$ and the assumed synchronous rotation period, with the computed value of the mean minimum Roche lobe radius $R_L \sin i$, where



FIG. 3. *IUE* satellite low-resolution short-wavelength observation SWP 33467. The more prominent emission lines are identified. Ly α is of geocoronal origin. The feature marked by an \times is not real, but results from a particle hit.



FIG. 4. Portion of the spectra of the lithium region of HD 136901 and β Oph at a resolution of 0.18 Å. The wavelengths of Fe I at 6707.44 Å, Li I at 6707.81 Å, and 6708.10 VI are indicated. In the top spectrum, the solid line is HD 136901, while the dotted line is the broadened spectrum of β Oph. The middle spectrum is the unbroadened spectrum is the difference spectrum is the difference spectrum in the sense HD 136901 – β Oph (broadened, $v \sin i = 42$ km s⁻¹).

(Plavec 1968) $R_{\rm L} = (a_1 + a_2)$ (0.38 + 0.2 log $\mathcal{M}_1/\mathcal{M}_2$). The total semimajor axis is $a_1 + a_2$, and $\mathcal{M}_1/\mathcal{M}_2$ is the mass ratio. Since there is no photometric or spectroscopic evidence for mass loss, the actual radius should be less than the Roche lobe radius. Table VI lists for $i = 40^{\circ}$ the assumed primary mass, computed secondary mass, the resulting mass ratio, the total projected semimajor axis, and $R_{\rm L} \sin i$. Except for the 3.5 and 4 \mathcal{M}_{\odot} cases, the Roche lobe radius is smaller than the observationally determined radius, so the minimum inclination is slightly greater than 40° for those cases. Another important constraint is that the mass ratio $\mathcal{M}_1/\mathcal{M}_2$ must be at least 2.9 for $R_{\rm L} \sin i \ge 14.7R_{\odot}$, where $14.7R_{\odot}$ is the minimum radius of 15.5 R_{\odot} minus its uncertainty.

Finally, Strassmeier *et al.* (1989) confirm that the variability of HD 136901 results primarily from the ellipticity and reflection effects and not from starspots. The reflection effect has a full V amplitude of 0.03 mag. Thus, the secondary must be somewhat hotter than the primary, an important constraint on the secondary. So the secondary must have a G dwarf or earlier spectral type, or be a white dwarf or, perhaps, an OB subdwarf.

The ultraviolet spectrum was obtained primarily to look for a possible white dwarf or OB subdwarf companion. It showed no hot continuum due to such stellar remnants. Nor was there any evidence of a continuum of an A or early F main-sequence star, although a longer exposure would be necessary to completely rule out an F star. To satisfy these

TABLE V. Mass and inclination combinations.

\mathcal{M}_1	$\mathcal{M}_2(i=90^\circ)$	$\mathcal{M}_2(i=65^\circ)$	$\mathcal{M}_2(i=40^\circ)$
1.5	0.42	0.47	0.72
2.0	0.50	0.56	0.85
2.5	0.58	0.65	0.97
3.0	0.65	0.72	1.08
3.5	0.71	0.79	1.19
4.0	0.77	0.86	1.29

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TABLE VI. Projected Roche lobe radii for $i = 40^{\circ}$.

\mathcal{M}_1 (\mathcal{M}_{\odot})	\mathcal{M}_{2}	MJM2	$a \sin i$ (R_{\odot})	$R_{\rm L} \sin i$ (R_{\odot})
1.5	0.73	2.08	24.8	11.0
2.0	0.85	2.35	27.0	12.3
2.5	0.97	2.58	28.8	13.3
3.0	1.08	2.78	30.4	14.3
3.5	1.19	2.95	31.8	15.1
4.0	1.29	3.10	33.0	15.8

spectral-type considerations, the mass of the secondary must be between about 0.85 and 1.25 \mathcal{M}_{\odot} . The minimum mass ratio of 2.9 results in a minimum primary mass of 2.5 \mathcal{M}_{\odot} .

Since there is no photometric evidence for eclipses, $R_1 + R_2 < a \cos i$, where R_1 and R_2 are the radii of the primary and secondary, respectively, and a is the semimajor axis of the relative orbit. For a mass ratio of 2.9, the maximum value of the inclination is 65°.

The range of possible inclinations increases as the primary mass increases. For a $2.5M_{\odot}$ primary it has a single value of 45° while at $4M_{\odot}$ it ranges from 67° to 41°. We estimate the inclination as 53° ± 12°. Modeling the ellipsoidal light variations and the reflection effect would be very useful and provide further constraints on the system.

Assuming that the rotation axis of the primary and the inclination axis of the orbit are parallel, the radius of the

Fekel (1988) has detected lithium in a number of chromospherically active single K III stars. He suggested that these giants evolved from rapidly rotating B, A, and F stars. Such stars would not deplete their surface lithium abundances until they crossed the Hertzsprung gap and convection began. Since HD 136901 has probably evolved from a late B star, an observation of the lithium region was obtained at a resolution of 0.18 Å (Fig. 4). The broad feature at about the lithium wavelength has an equivalent width of 77 ± 5 mÅ. However, in early K giants, it is often partially blended except at very high resolution, with an Fe I line at 6708.44 Å. The equivalent width of the Fe I line is 61 mÅ in β Oph (K2 III). Thus, a substantial part of the equivalent width of HD 136901 is not due to lithium. Nevertheless, it is possible that a small amount of lithium is present.

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