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Comparing stress distribution, strain, and total deformation in contact surface for the three types of rack and pinion

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ABSTRACT

Used helical gear in many engineering applications as in rack and pinion. This paper shows a study of three types of rack and pinion, these types used helical gear, the first type is single gear, the second type is double gear, and the third is herringbone gear. The research explained the distribution of stresses, strains, and elastic deformation in the contact area between all types of the rack and pinion. Some important conclusions were presented for the purpose to can be used in design such as choose a high-strength material for gear with the specified helix angle, and selected a larger face width in the gear shape is preferred, total deformation varies from one type of rack pinion gears to another. The results show, total deformation varies from one type of rack pinion gears to another, but It seems that the single helical gear elastic deformation is high value and the rack-pinion (herringbone gears). The best of the three types in this research is according to the values of the distribution of stresses, strains, and elastic deformations.

Keywords: Rack and pinion, Double helical gear, Herringbone gear, Stress distribution, Total deformation.

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1. Introduction

A rack and pinion gear system converts linear into rotational motion. There are three sets such as single helical gear; between the two parts of a double helical gear is a tiny relief gap. As a result, the left-hand helix's teeth do not come into contact with the right-hand helix's teeth. A herringbone gear, on the other hand, does not have this space, thus teeth with the left-hand helix rub against teeth with the right-hand helix. Double helical gear is typically smaller than the comparable herringbone gear and is ideally suited for high shock and vibration applications, [1]. Patil and et al, (2014) [2], the contact stresses between the helical gear pairs have been studied in this research under static conditions, the stresses were calculated using the Lagrange multiplier technique between the contacting pairs. Cao and et al (2016) [3], using finite element analysis, the benefits and drawbacks of the suggested gear drives, as well as the gear stress distribution and contact deformation, were examined. The suggested gear drives can lower contact stress by 23.8 percent to 35.3 percent or 23.2 percent to 29.5 percent Von-Mises contact stress, according to the results. Paliwal and et al, (2018) [4], designed pinion and rack assembly and carried out total deformation and Von- Mises Analysis. Analysis of meshing of rack and pinion is done by using ANSYS while keeping the environmental temperature at 22°C. A load of 100 N is applied for time 5min 44sec. In the total deformation test, the maximum value found is $1.9387X10^{-5}$metre and in Equivalent Von-Mises Stress the maximum value found is $1.6702X10^{7}$Pa respectively. Babu and et al, (2021) [5], the major purpose of this research is to create a rack and All-terrain vehicle (ATV) pinion steering system and minimize its weight by optimizing the topology. For structural analysis, ANSYS is utilized, and the stress and deformation distribution is shown. The natural frequencies are calculated using ANSYS vibrational analysis and compared to the system's operational frequencies. The aim of this research is to theoretically study the stress distribution, contact stress, and total deformation of three contact surfaces for types of rack and pinion single, herringbone gears, and double gears.

2. Double helical gear

Double helical gears are a form of helical gear that has two helical faces adjacent to one another separated by a gap. Each face has identical but opposite helix angles. Utilizing a double helical gear set reduces thrust loads and enables increased tooth overlap and smoother performance. Double helical gears are widely used in enclosed gear drives, much as helical gears are. Helical gears are often preferred for applications requiring high torque. Helical gears are favored for quiet operation and function, such as in-vehicle applications, because to their quiet and smooth performance. Helical gears are useful in a wide variety of applications; nevertheless, the following are a few instances when they are superior. Helical gears are used in a variety of sectors, including fertilizer, printing, and earthmoving. Helical gears are used in the steel, rolling mill, section rolling mill, electricity, and port industries. Helical gears are used in the textile, plastic, and food industries, as well as conveyors, elevators, blowers, compressors, the oil industry, and cutters [6].

3. Herringbone gear

Herringbone gears are similar to double helical gears, except that the gap between the two helical faces is eliminated. Herringbone gears are frequently smaller than double helical gears, making them ideal for situations involving significant shock and vibration. Herringbone gearing is rarely used due to its difficulty of manufacture and high cost.

4. Theoretical design equations

It is defined that the material of pinion is 16MnCr5 carburizing and quenching, the hardness of tooth surface is 56-62hrc, and that of the rack is 45 steel after quenching and tempering, the hardness of tooth surface is 50HRC. It is defined that the pinion tooth surface is the contact surface, the rack tooth surface is the target surface, the mesh surface is the soft body contact analysis, and the two contact bodies have approximate stiffness. When creating contact pairs, ANSYS can automatically ensure that the external normal direction of the contact element is face-to-face. The degree of freedom relationship between two contact bodies is analyzed, and the boundary conditions of contact are determined. The gear rack constraint is applied so that the gear has only the degree of freedom of rotation around its axis of rotation, and the rack has only the degree of freedom of movement along its axis, [7].

The design parameters of the rack and pinion are very important in terms of efficiency and safety. It is better to choose the same metal from which each is made rack and pinion, therefore, the choice of metal should have high mechanical properties. Table 1 demonstrates the characteristics taken into account while designing a rack and pinion gear in this work. Fig.1 displays the tooth profile of a typical rack tooth and its mating gear in full depth. It has a h_a= 1m addendum and a dedendum of h_f 1.25m. A "stub" tooth is one that is shorter than the entire length of the tooth, while a "high" depth tooth is one that is longer than the entire length of the tooth, [8].

Table1. Parameters are taken into account when designing a helical pinion gear

Property	Value
Density	7850 kg/m^3
Thermal Expansion Coefficient	1.2×10^{-5} $1/^oC$
Elastic Modulus	200×10^9 N/m^2
Poisson's Ratio	0.3
Bulk Modulus	166.66×10^9 N/m^2
Shear Modulus	79.92×10^9 N/m^2
Tensile Yield Strength	250×10^6 N/m^2
Compressive Yield Strength	250×10^6 N/m^2
Tensile Ultimate Strength	460×10^6 N/m^2
Applied Force	981 N
The number of pinion teeth (n_p)	8
The number of rack teeth (n_R)	15
Helix angle (ψ)	23°
Pressure angle (α)	20°

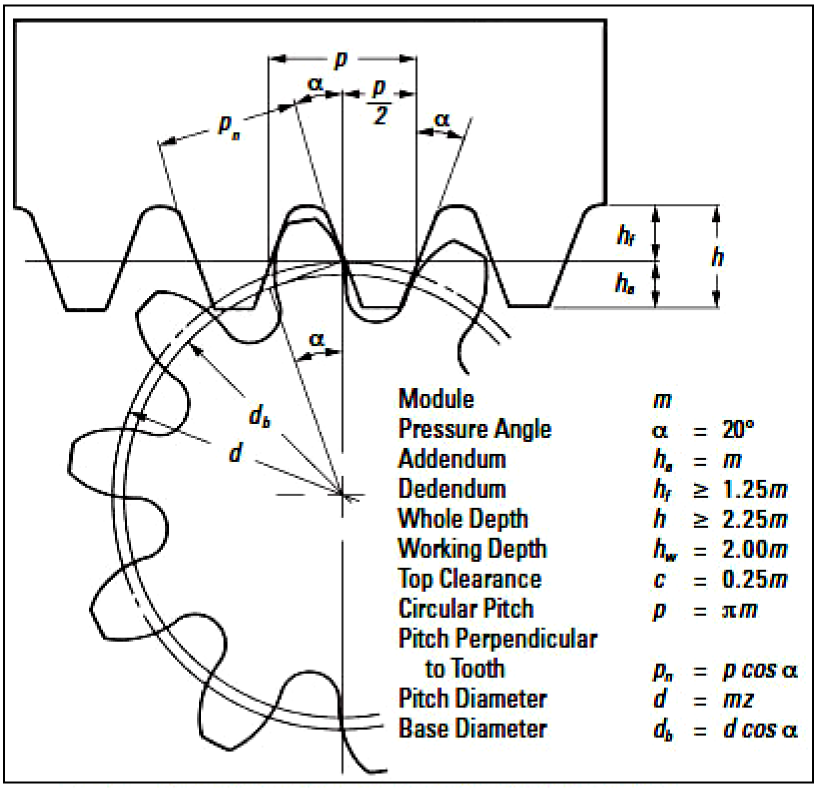

Figure 1.The tooth profile of rack and pinion [8]

The following equations are suitable for the design, [9]

Computation of teeth diameter (D_p)

Module from the series of conventional modules was chosen.

$$D_p = n_p \,.\, m_a \tag{1}$$

Where: n_p is No. of teeth of pinion and m_a is Module equal PCD/no. of teeth.

Torque Computation (M_t)

$$M_t = F_t \,.\, D_p \tag{2}$$

Where: F_t is tangential load, N

Pitch axis velocity computation (V)

The velocity of the pitch line may be determined using

$$V = \frac{\pi \,.\, D_P \,.\, N_P}{60} \tag{3}$$

$$n_f = \frac{n_p}{\cos^3 \psi}$$

$$y_f = \frac{0.666}{n_f}$$

Where: N_p is pinion speed, n_f is the final speed , y_f *is* the center of gravity of sector area and ψ is helix angle.

Calculation of the maximum permissible stress (σ_{all})

The amount of stress that may be tolerated can be estimated using the formula below.

$$\sigma_{all} = \sigma_o \left[\frac{3}{3+V}\right], \qquad for\ V \le 10m/\sec. \tag{4}$$

Determining the stress of endurance (σ_o)

$$\sigma_o = \frac{\sigma_u}{3} \tag{5}$$

Where: σ_u is the ultimate stress N/m^2.

The Lewis equation may be used to determine the actual produced stress.

$$\sigma_{ind} = \frac{2.M_t}{m^3.k\pi^2.y_p.\cos\psi} \tag{6}$$

Checking of the stresses σ_{all} and σ_{ind}

If $\sigma_{all} > \sigma_{ind}$, design that has been satisfactory (7)

If this is not the case, continue compute by expanding the module until it satisfies the condition.

The face width of helical gear is calculated (b)

Helical gear has a variable face width computed using the following formula:

$$b_{\min} = k_{red}\ .\ \pi\ .\ m_n \tag{8}$$

$$k_{red} = k_{\max}.\frac{\sigma_{ind}}{\sigma_{all}} \tag{9}$$

It is vital to examine the dynamic effect after defining the design from a strength standpoint.

Checking the dynamic load

The transmitted load can be computed using the following formula:

$$F_t = \frac{2M_t}{D_p} \tag{10}$$

Kinetic load computation (F_d)

$$F_d = F_t + \frac{21.V(b.\cos^2\psi + F_t)\cos\psi}{21.V + \sqrt{(b.C.\cos^2\psi + F_1)}} \tag{11}$$

Where: C is Clearance, mm.

Calculation of Limiting Endurance Load (F_o)

$$F_o = S_o.b.y_p\pi.m.\cos\psi \tag{12}$$

Where: F_o is the limiting endurance load.

Limit wear load calculation (F_w)

$$F_w = \frac{D_p.b.K.Q}{\cos^2 \psi} \tag{13}$$

$$K = \frac{S_{es}^2 .\sin \varphi_n}{1.4} . \left[\frac{2}{E} \right] \tag{14}$$

The criterion that must be met in order to pass the dynamic check is F_o ,F_w>F_d, [10] . If this is not the case, continue calculate by increasing the module until it satisfies the requirement.

5. Using the AGMA equation

The rack and pinion gear was subjected to contact stress analysis. Pitting, a failure due to surface fatigue caused by repeated significant levels of contact stress occurring on the gear tooth surface when a set of teeth is transferring one of the most common gear tooth problems is power, [11].

The formula for contact stress: (σ_c)

$$\sigma_c = C_p . \sqrt{\frac{F_t}{b.d.I} (\frac{\cos\psi}{0.95.CR}).K_v .K_o (0.93.K_m)} \tag{15}$$

Where: K_o is the factor of overload, K_v is the factor for movement and K_m is the factor affecting load distribution.

The equation for the elastic coefficient factor (C_p)

$$C_p . = 0.564 \sqrt{\frac{1}{\frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2}}} \tag{16}$$

Geometric factor (I)

It is calculated as:

$$I = \frac{\sin \varphi . \cos\varphi}{2} . \frac{i}{i+1} \tag{17}$$

Where: ϕ is pressure angle

Speed ratio (i)

It is calculated as:

$$i = \frac{n_1}{n_2}$$

The equation for the contact ratio (CR) is as follows

$$CR = \left[\frac{\sqrt{r_o^2 - r_R^2} + \dfrac{h_{ar} - r_p \sin\phi}{\sin\phi}}{\pi . m_n \cos\phi} \right] \quad (18)$$

Failure occurs when the principle stress in a complex system achieves in simple tension, the maximum stress at the elastic limit, according to the principle stress theory.
Where: m_n is normal module m_n = m x cos 35
considering standard value on higher side= 8 mm

The principal stresses (σ_1 and σ_2)
It is calculated as:

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{\left(\sigma_x - \sigma_y\right)^2 + 4\tau_{xy}^2} \quad (19)$$

For any criteria for yield, the term "effective stress (σ_v)" is used to describe the amount of stress, It is a result of the applied stresses. The applied stress condition produces yielding when the magnitude of σ_v reaches a critical value, indicating that it has now reached an degree level efficiency. The equation below is used to determine the Von-Mises stress.
Where: σ_x, σ_y and τ_{xy} are stresses in x and y directions and shear stress in xy direction respectively.

The Von-Mises stress (σ_v)

$$\sigma_v = \frac{1}{\sqrt{2}}\left[\left(\sigma_1 + \sigma_2\right)^2 + \left(\sigma_2 + \sigma_3\right)^2 + \left(\sigma_3 + \sigma_1\right)^2\right]^{0.5} \quad (20)$$

Where: σ_1, σ_2 and σ_3 are the principles stress in the three directions of the axes is orthogonal.

6. Stress analysis of herringbone gear

To theoretically determine the bending stresses induced on gear teeth that cause tooth failure, [12] . The AGMA bending stress (σ_b) equation:

$$\sigma_b = \frac{F_t}{b.m_n . j} K_v . K_o .(0.93 K_m) \quad (21)$$

7. Dimensions of rack and pinion gears for three types

The dimensions selected for the study are types of rack and pinion shows in Figs.2,3 and 4 the three projections, these Figures were drawn by Solid Work. Table 1 showing the dimensions of the rack and pinion. Figure 5 show the external load support on the rack and pinion 100 kg. The three types differ in the performance of their work as well as the complexities of manufacturing, and therefore accuracy is a very important factor in bearing the fatigue load. There is a scientific fact that fatigue load is a major cause of mechanical vibration generation.

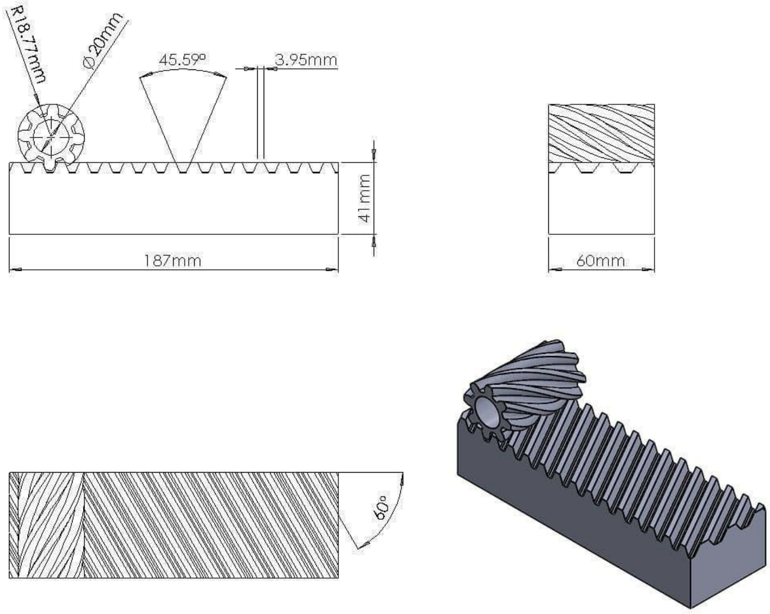

Figure 2. Shows the three projections and the final structure of the single helical gear (pinion) and rack

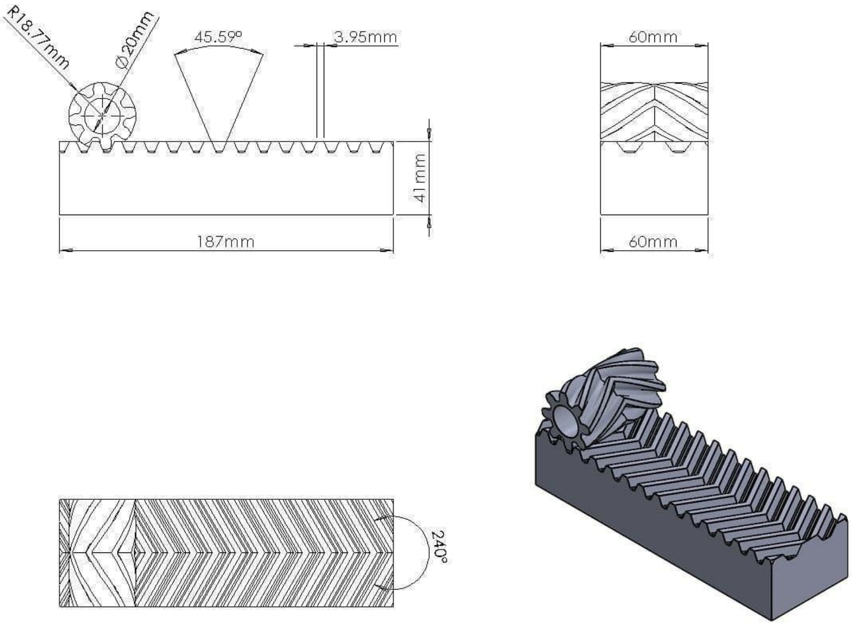

Figure 3. Shows the three projections and the final structure of the herringbone gear (pinion) and rack

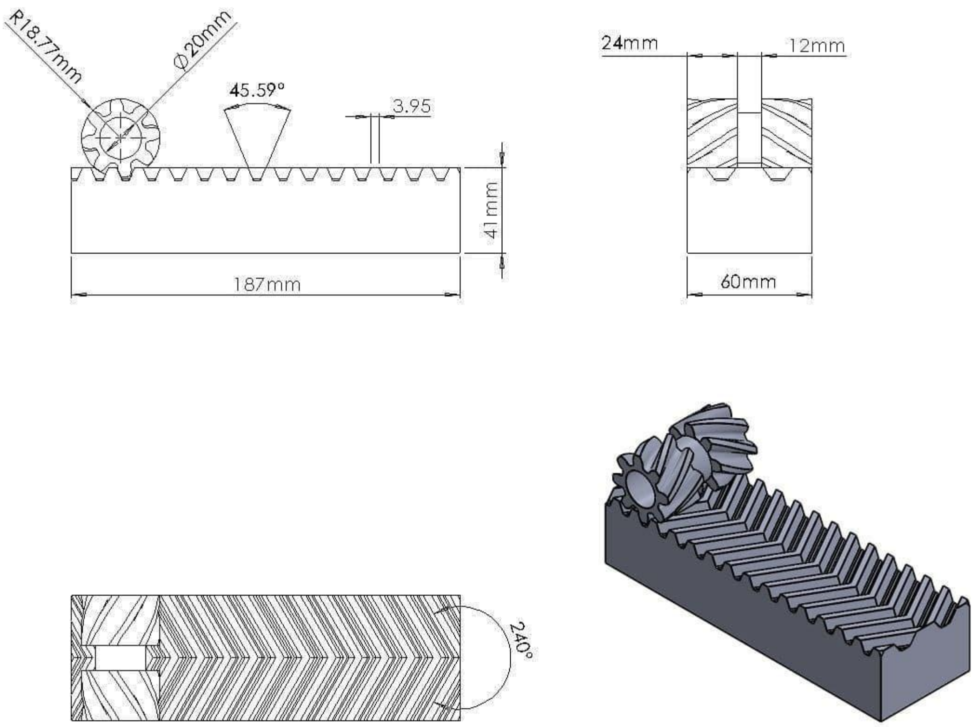

Figure 4. Shows the three projections and the final structure of double helical gear (pinion) and rack

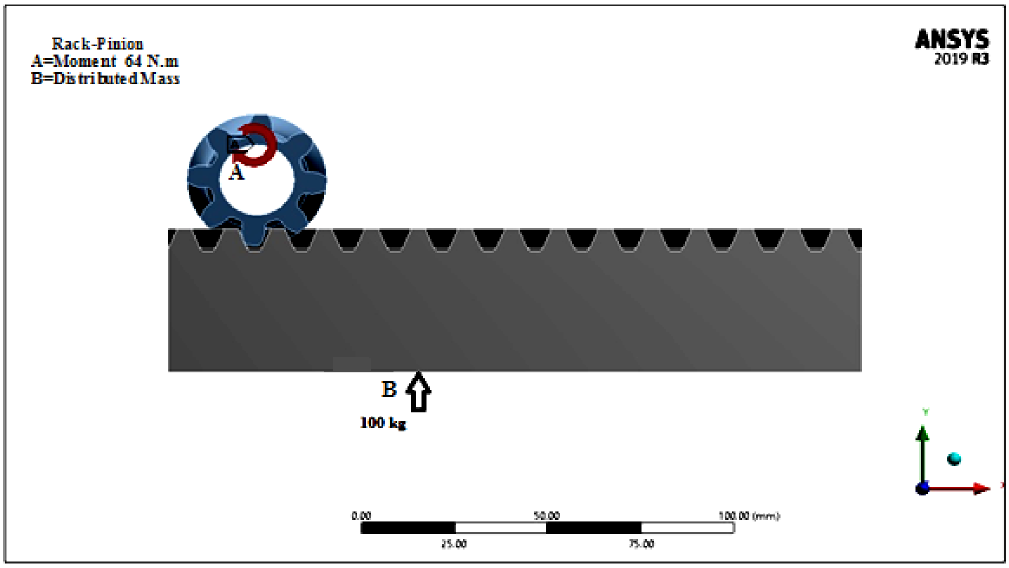

Figure 5. Shows the side view and the external loaded mass

8. Journal bearing used for support helical pinion gear

A multi-layer journal bearing surface can be gathered so that the respective layers will have an interference fit. This will make the inner element with compressive residual stresses and the outer element's tensile residual stresses heating and shrinking on' exterior elements with a bore diameter slightly smaller than that of the corresponding surface of the inner component, or matching taperness on the internal and external surfaces of the outer and inner elements, respectively, and pressing the elements against one another, [13] and [14].

9. Results and discussions

The modulus of elasticity for the material each is made of rack and pinion, it has an effect on performance. The rack width sets the width of the pinion. The distribution of stresses in the contact area between rack and pinion, when an external force is applied 981 N, is of great importance because if the stresses are high, it will affect the wear of the contact area, time over, a high value of wear rate will occur that leads to a decrease in performance efficiency. Fig. 6 shows the distribution stresses by the equivalent stress analysis type equivalent (Von-Mises) stress of a single helical gear of both the rack and pinion type, The greatest level of stress is 6.234 Mpa. note that the maximum stress occurs in a very small area of the contact surfaces. Figure 7 shows the distribution of stresses for double helical gear, the amount of pressure distribution will decrease so that the maximum distributor stress will be 0.6981 Mpa. Figure 8 shows the distribution stress of the herringbone gears for rack and pinion under external load about 981N, which is about 0.3691 Mpa. The reason for changing the values in the Figs.6,7 and 8 is the distribution of stresses axially on the surface of the transverse for each type, and the stresses can be reduced by increasing the area in the direction of rack width.

Figures 9, 10, and 11 show the strains that occurred at the contact surface between the three types of rack-pinion, the maximum value of strain on the surface of rack and pinion type single helical gear is about 0.00256, the values of strains for rack-pinion (double helical gear) and of the rack - pinion (herringbone gears) are about 0.00011, and 8.375×10^{-5} respectively, the scientific fact that the strains are directly proportional to the stresses. Elastic deformation is of great importance in determining the preferred mechanical properties in the selection of the rack and pinion type used safely as shown in Figures 12, 13, and 14 are about 0.000866 mm, 0.001238 mm, and 0.000866 mm respectively with types of rack-pinion gears.

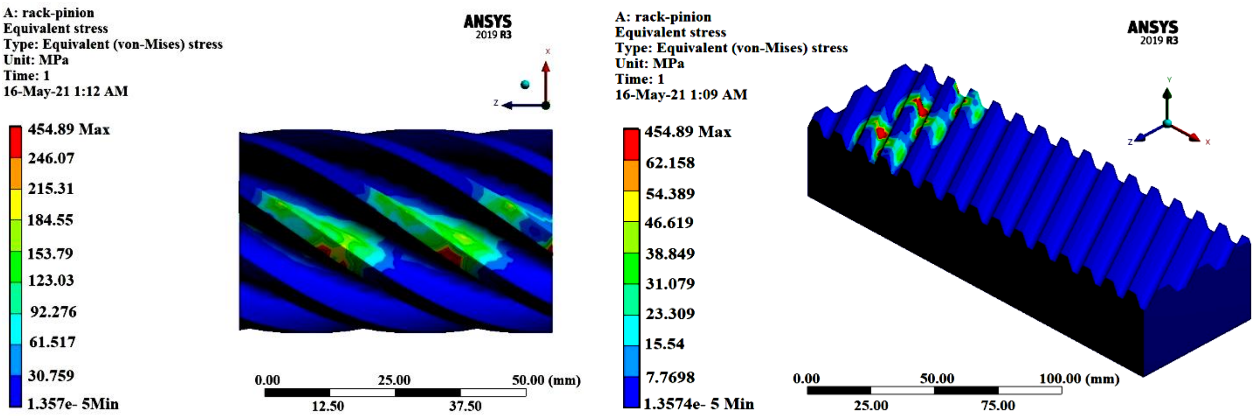

Figure 6. The distribution stresses of rack-pinion (single helical gear).

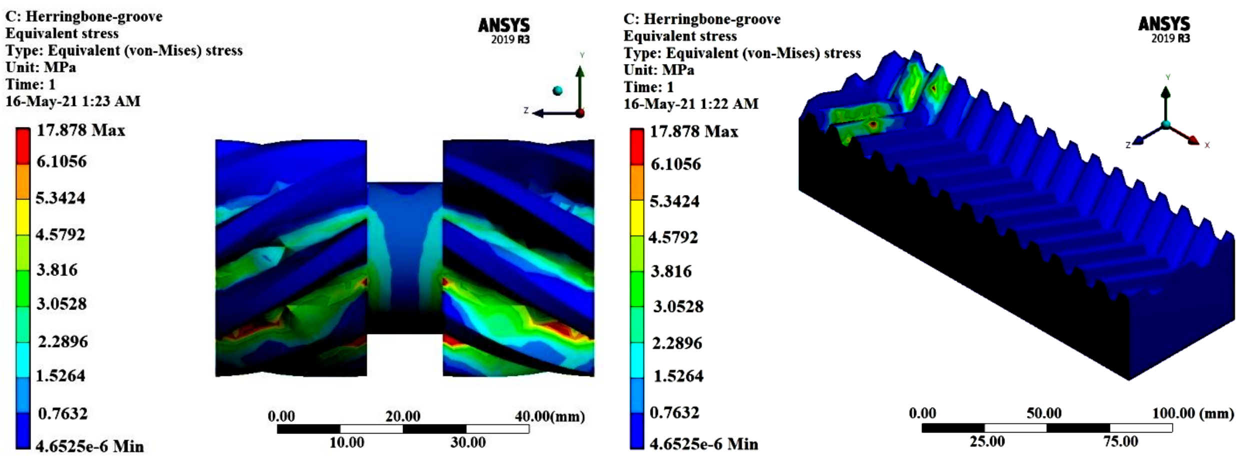

Figure 7.The distribution stresses of rack-pinion **(**double helical gear)

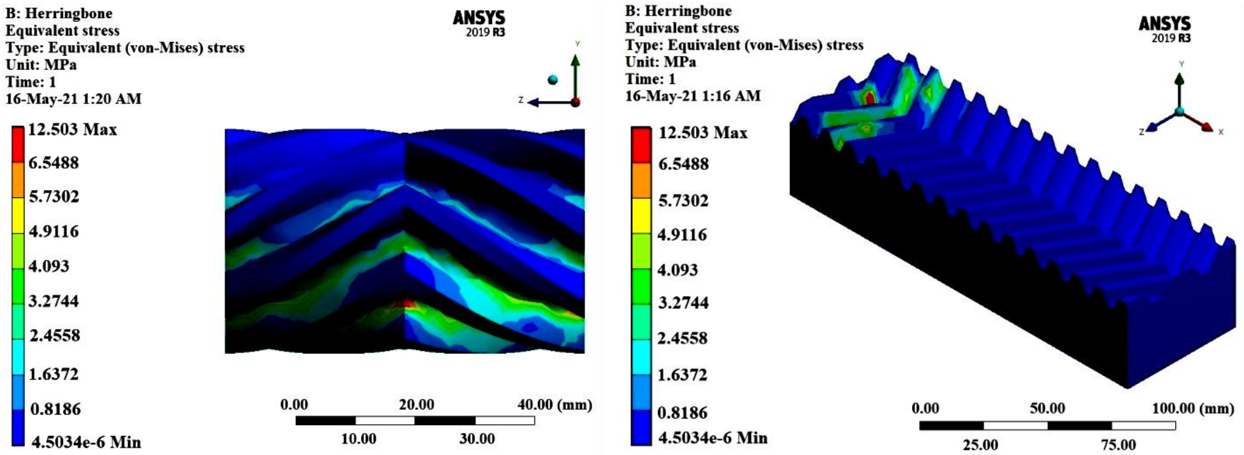

Figure 8.The distribution stresses of rack - pinion (herringbone gears)

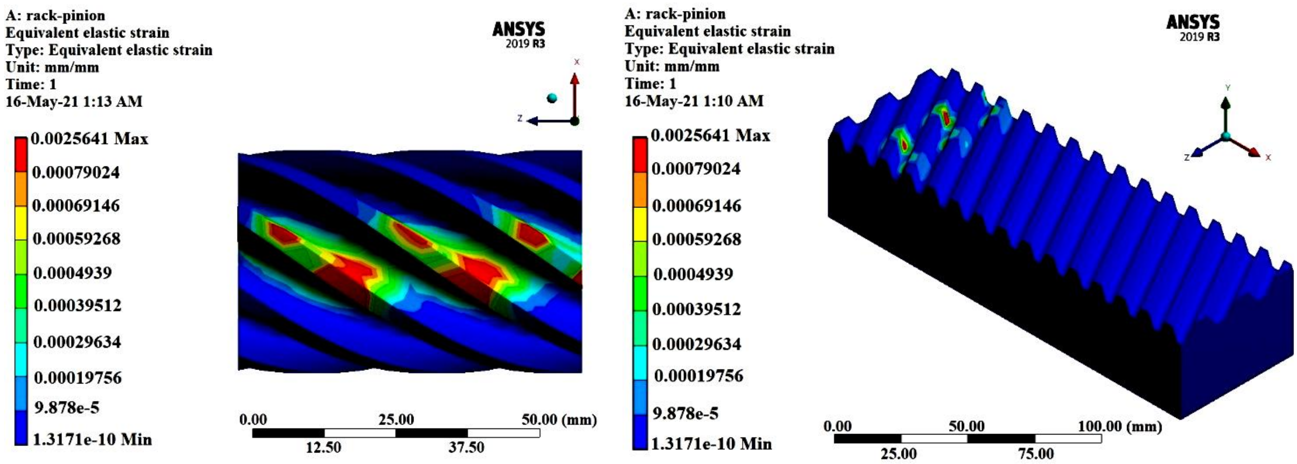

Figure 9.The strain of rack-pinion (single helical gear)

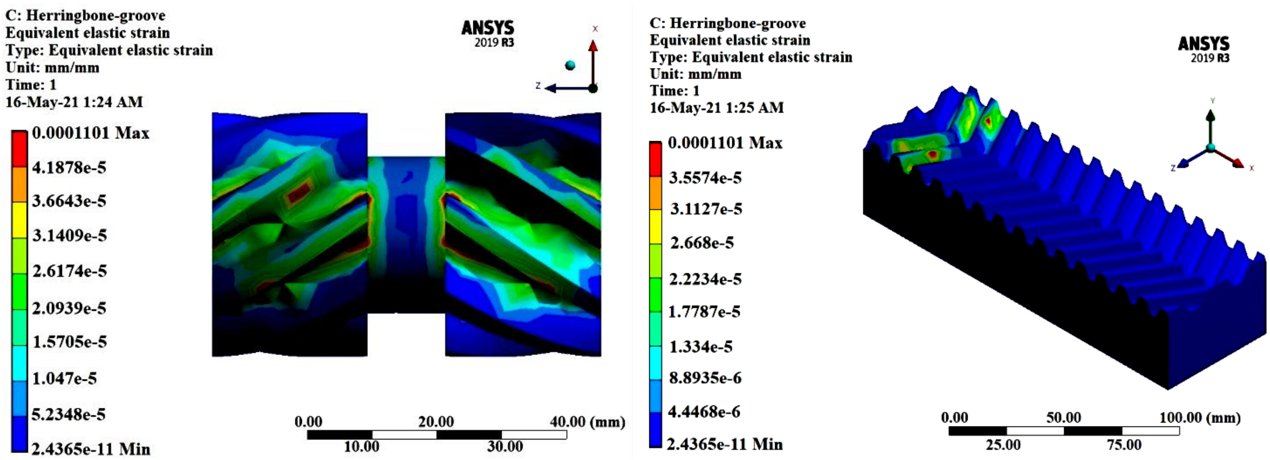

Figure 10. The strains of rack-pinion (double helical gears)

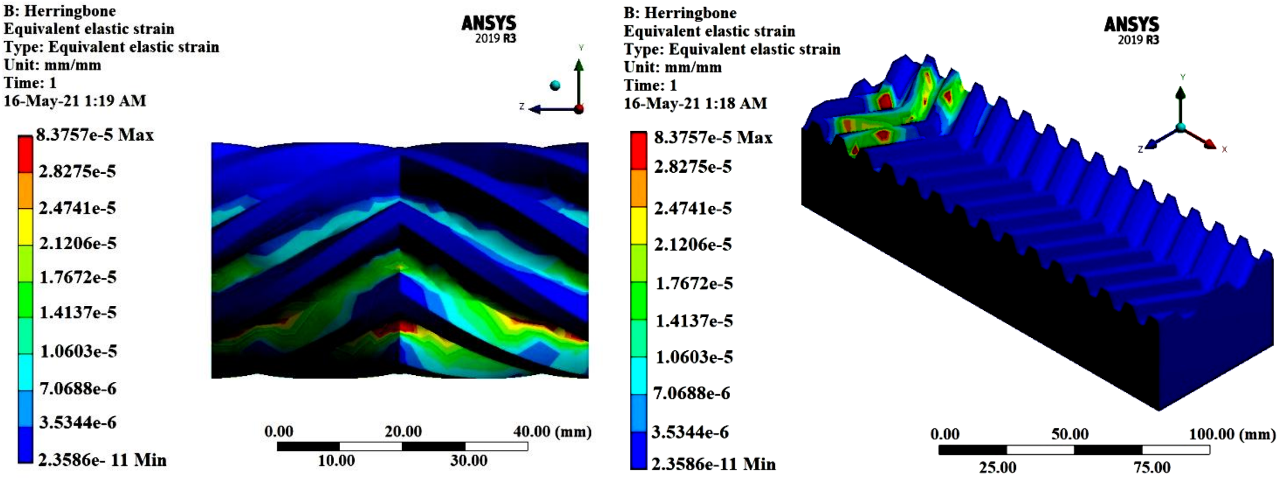

Figure 11.The strains of rack- pinion (herringbone gears)

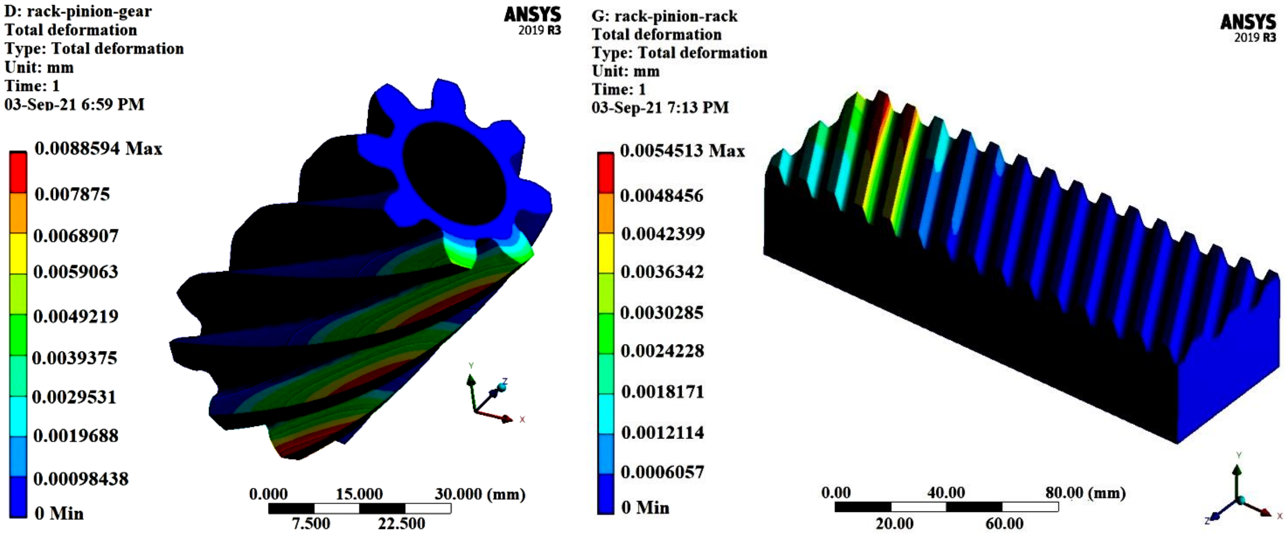

Figure 12.The total elastic deformation of rack-pinion (single helical gear)

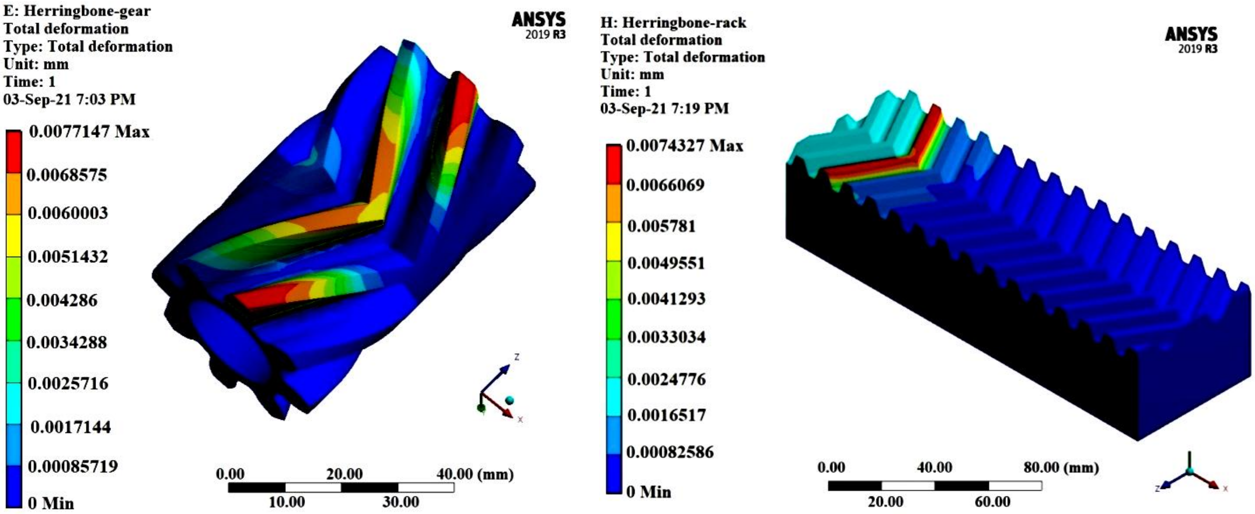

Figure 13.The total elastic deformation of rack-pinion (double helical gears)

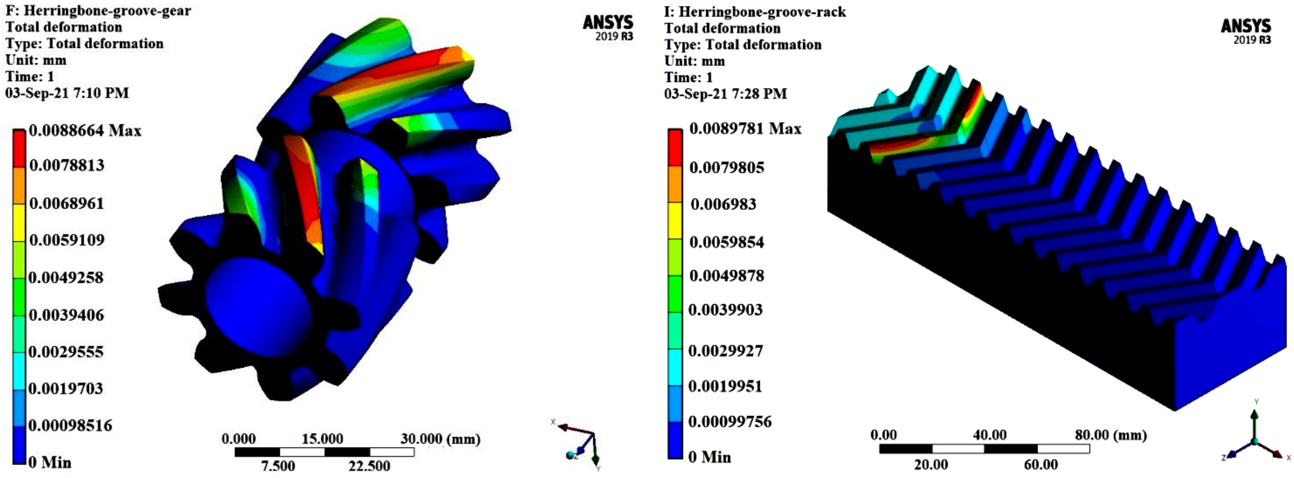

Figure 14 .The total elastic deformation of rack-pinion (herringbone gears)

10. Conclusions

By reviewing the results of this research and comparing the three types of rack and pinion gears, the following conclusions were reached:

- The design requirement is to choose a high-strength material for gear with the specified helix angle, and a larger face width in the gear shape is preferred.
- 2-Total deformation varies from one type of rack pinion gears to another, but it seems that the single helical gear elastic deformation is high value.
- The rack-pinion (herringbone gears) the best of the three types in this research is according to the values of the distribution of stresses, strains, and elastic deformations, which results were obtained by used Ansys program.

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