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Supplemental Appendices to Defensive Marketing Strategies: An Equilibrium Analysis Based on Decoupled Response Function Models

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Abstract

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DEFENSIVE MARKETING STRATEGIES: AN EQUILIBRIUM ANALYSIS BASED ON DECOUPLED RESPONSE FUNCTION MODELS

Lemma 1: All existing products adjacent to the attacker will lose some finite portion of their before entry unadjusted market demand to a viable attacker.

Proof: Viability of all products in the market dictates that the following conditions be satisfied (shown in Lane 1980, Appendix 1). If one or more of these conditions is not satisfied, then it is possible for a product to be dominated by its competitors and hence capture no part of the market (Lane (1980)).

Viability conditions:

Condition 1: $z_j\ (Y-p_j) > z_k\ (Y-p_k)$ for all $k > j$ (1)

Condition 2: $w_j\ (Y-p_j) < w_k\ (Y-p_k)$ for all $k > j$ (2)

Condition 3: for all i and k s.t. $i < j < k$, there exists a customer $\bar{\alpha}$ who is indifferent between products i and k,

that is $U_{\bar{\alpha}}(w_i, z_i, p_i) = U_{\bar{\alpha}}(w_k, z_k, p_k)$ (3)

which implies $U_{\bar{\alpha}}(w_j, z_j, p_j) > U_{\bar{\alpha}}(w_i, z_i, p_i)$ (4)

(with $z_j > z_k$, $w_j < w_k$, $f_j < f_k$ for all $j < k$)

Let the attacker [product (N+1)] enter the market and be viable after all competitive adjustments have been made. The new product (N+1) can be positioned either (i) between two existing products, or (ii) at either extreme. We consider each of these cases separately.

Case 1

For attack at this position to be viable, the attacker's price is governed by Equation (4) with $i = L$, $j = L + 1$ and k being the index for the attacker.

The Cobb-Douglas utility function form provides unique indifferent customers (1) α_L between brands L and L + 1 before entry, (2) $\hat{\alpha}_{L1}$ between brands L and (N+1), (3) $\hat{\alpha}_{L2}$ between (N+1) and (L+1). Clearly, $\hat{\alpha}_{L1} \neq \alpha_L \neq \hat{\alpha}_{L2}$ and Condition 3 of viability implies $\hat{\alpha}_{L1} < \alpha_L < \hat{\alpha}_{L2}$ proving the lemma.

Case 2

In this case, viability equations (1) and (2) dictate that the consumers whose taste parameters lie between the attacker and its closest edge of the taste distribution domain (either 0 to N+1, or N+1 to 1), prefer the attacker. This completes the proof of Lemma 1.

$\square$

Note that this loss in share by the defenders upon viable attack is in terms of unadjusted demand and does not consider the effects of advertising and distribution.

Lemma 2: Let p_i^*, $i = 1, 2, \ldots, N$ be the optimal Nash price equilibrium prices of N brands. Let β_i^*, $i = 1, 2, \ldots, N$ be their corresponding unadjusted market shares. Let another brand enter this market. In the ensuing equilibrium let $\hat{p}_i$, $i = 1, 2, \ldots, N$ be the optimal Nash equilibrium prices, and $\hat{\beta}_i$, $i = 1, 2, \ldots, N$ be the corresponding unadjusted market shares for the incumbent brands. For any existing brand j

$$\hat{p}_j \geq p_j^*$$

implies that

$$\hat{\beta}_j \geq \beta_j^*.$$

Proof:

The first order condition (for profit maximization) for any existing product j with respect to its price is given by

$$\frac{\partial \pi_j}{\partial p_j} = (\beta_j + (p_j - c)\frac{\partial \beta_j}{\partial p_j})\, MA(k_{aj})D(k_{dj}) = 0 \tag{5}$$

The pre-entry optimal price and market share for product j (p_j^* and β_j^* respectively) must satisfy

$$\beta_j^* + (p_j^* - c)\left.\frac{\partial \beta_j}{\partial p_j}\right|_{p_j^*} = 0 \tag{6}$$

Notice that this equation assumes that the prices of all the other brands are at their pre-entry Nash equilibrium levels.

We know (from Lane 1980, Equations (4)-(8)) that

$$\frac{\partial \beta_j}{\partial p_j} = \frac{1}{R}\ln(\frac{f_{j+1}}{f_{j-1}})(-\frac{1}{Y-p_j}) \qquad j=2,3,\ldots,N-1 \tag{7}$$

$$\frac{\partial \beta_1}{\partial p_1} = \frac{1}{\ln(\frac{f_2}{f_1})}(-\frac{1}{Y-p_1}) \tag{8}$$

$$\frac{\partial \beta_N}{\partial p_N} = \frac{1}{\ln(\frac{f_N}{f_{N-1}})}(-\frac{1}{Y-p_N}) \tag{9}$$

where $\quad R = \ln(\frac{f_{j+1}}{f_j}) \ln(\frac{f_j}{f_{j-1}}) \qquad j=2,3,\ldots,N-1 \qquad (10)$

$$f_j = \frac{w_j}{z_j}, \qquad j=1,2,\ldots,N \qquad (11)$$

and $\quad f_j > f_{j-1}, \qquad j=2,3,\ldots,N \qquad (12)$

Similarly the post-entry optimal price and market share for product j ($\hat{p}_j$ and $\hat{\beta}_j$ respectively) satisfy

$$\hat{\beta}_j + (\hat{p}_j - c) \left.\frac{\partial \beta_j}{\partial p_j}\right|_{\hat{p}_j} = 0 \qquad (13)$$

Note that this assumes that the prices of all the other brands are at their post-entry Nash equilibrium levels. $\hat{p}_j$ and $\hat{\beta}_j$ also satisfy Equations (7) to (9) with a total of (N+1) products in the market, the attacker denoted as brand (N+1), and for any incumbent brand j, j = 1, 2, ..., N, one of its immediate neighbors j - 1 or j + 1 could be brand N + 1.

From Equations (6) and (13),

$$\frac{\hat{\beta}_j}{\beta_j^*} = \frac{(\hat{p}_j - c)}{(p_j^* - c)} \frac{\left.\frac{\partial \beta_j}{\partial p_j}\right|_{\hat{p}_j}}{\left.\frac{\partial \beta_j}{\partial p_j}\right|_{p_j^*}} \qquad (14)$$

$$= \frac{\hat{k}_j}{k_j^*} \frac{(\hat{p}_1 - c)}{(p_1^* - c)} \frac{(Y - p_1^*)}{(Y - \hat{p}_1)} \qquad (15)$$

where $\hat{k}_j$ and k_j^* are parameters dependent on the position of brand j and its immediate neighbor(s) after and before attack, respectively.

To prove this lemma, we need to consider four cases of the relative positioning between brand j and the attacker. For any given incumbent product j, it can be:

Case A: It is not an immediate neighbor of (N+1). That is, its immediate neighbors remain unchanged after entry.

Case B: j = 1 and its left immediate neighbor is brand N + 1 after entry.

Case C: j = N and its right immediate neighbor is brand N + 1 after entry.

Case D: The attacker (N+1) is adjacent to j and to one other incumbent brand.

Case A:

$$\hat{k}_j = k_j^* = \frac{1}{R} \ln\left(\frac{f_{j+1}}{f_{j-1}}\right) \qquad j = 2, 3, \ldots, N - 1$$

$$\hat{k}_j = k_j^* = \frac{1}{\ln\left(\frac{f_2}{f_1}\right)} \qquad j = 1$$

and

$$\hat{k}_j = k_j^* = \frac{1}{\ln\left(\frac{f_N}{f_{N-1}}\right)} \qquad j = N$$

From Equation (15), if $\hat{p}_j > p_j^*$ it follows that $\hat{\beta}_j > \beta_j^*$ and if $\hat{p}_j = p_j^*$ then $\hat{\beta}_j = \beta_j^*$.

Case B:

$$\hat{k}_1 = \frac{\ln(\frac{f_2}{f_{N+1}})}{\ln(\frac{f_2}{f_1})\ \ln(\frac{f_1}{f_{N+1}})}$$

and $$k_1^* = \frac{1}{\ln(\frac{f_2}{f_1})} .$$

By assumption of the position of brand (N+1) to the left of brand 1, $f_{N+1} < f_1 < f_2$.

This implies that $\frac{\hat{k}_1}{k_1^*} > 1$.

From Equation (15), if $\hat{p}_1 \geq p_1^*$ it follows that $\hat{\beta}_1 > \beta_1^*$.

Case C:

$$\hat{k}_N = \frac{\ln(\frac{f_{N+1}}{f_{N-1}})}{\ln(\frac{f_{N+1}}{f_N})\ \ln(\frac{f_N}{f_{N-1}})}$$

and $$k_N^* = \frac{1}{\ln(\frac{f_N}{f_{N-1}})} .$$

By assumption of the position of brand (N+1) to the right of brand N, $f_{N-1} < f_N < f_{N+1}$.

This implies that $\frac{\hat{k}_N}{k_N^*} > 1$.

From Equation (15), if $\hat{p}_N \geq p_N^*$ then $\hat{\beta}_N > \beta_N^*$.

Case D: Since the attacker has two immediate neighbors j and j + 1, we consider:

i) Brand j, j=1,2,...,N-1

$$\hat{k}_j = \frac{\ln(\frac{f_{N+1}}{f_{j-1}})}{\ln(\frac{f_{N+1}}{f_j})\ \ln(\frac{f_j}{f_{j-1}})} \qquad j=2,3,\ldots,N-1$$

$$\hat{k}_j = \frac{1}{\ln(\frac{f_{N+1}}{f_j})} \qquad j=1$$

$$k_j^* = \frac{\ln(\frac{f_{j+1}}{f_{j-1}})}{\ln(\frac{f_{j+1}}{f_j})\ \ln(\frac{f_j}{f_{j-1}})} \qquad j=2,3,\ldots,N-1$$

$$k_j^* = \frac{1}{\ln(\frac{f_2}{f_j})} \qquad j=1$$

By assumption of the position of brand (N+1) between brands j and j + 1,

$$f_{j-1} < f_j < f_{N+1} < f_{j+1} \qquad j=2,3,\ldots,N-1$$

and $$f_j < f_{N+1} < f_{j+1} \qquad j=1$$

Therefore for j = 1,

$$\frac{\hat{k}_j}{k_j^*} > 1.$$

For j = 2,3,...,N-1

$$\frac{\hat{k}_j}{k_j^*} = \frac{\ln(\frac{f_{N+1}}{f_{j-1}})}{\ln(\frac{f_{N+1}}{f_j})} \cdot \frac{\ln(\frac{f_{j+1}}{f_j})}{\ln(\frac{f_{j+1}}{f_{j-1}})}$$

$$= \frac{\ln(\frac{f_{N+1}}{f_{j+1}}) + \ln(\frac{f_{j+1}}{f_{j-1}})}{\ln(\frac{f_{N+1}}{f_{j+1}}) + \ln(\frac{f_{j+1}}{f_j})} \cdot \frac{\ln(\frac{f_{j+1}}{f_j})}{\ln(\frac{f_{j+1}}{f_{j-1}})} .$$

Note that $\ln(\frac{f_{N+1}}{f_{j+1}}) < 0$, $\ln(\frac{f_{N+1}}{f_{j+1}}) + \ln(\frac{f_{j+1}}{f_j}) > 0$, and

$\ln(\frac{f_{j+1}}{f_{j-1}}) > \ln(\frac{f_{j+1}}{f_j}) > 0$.

Therefore, $\frac{\hat{k}_j}{k_j^*} > 1$.

From Equation (15), if $\hat{p}_j \geq p_j^*$ then $\hat{\beta}_j > \beta_j^*$.

ii) Brand j + 1, $\qquad j = 1,2,\ldots,N-1$

$$\hat{k}_{j+1} = \frac{\ln\left(\frac{f_{j+2}}{f_{N+1}}\right)}{\ln\left(\frac{f_{j+2}}{f_{j+1}}\right)\ln\left(\frac{f_{j+1}}{f_{N+1}}\right)} \qquad j=1,2,\ldots,N-2$$

$$\hat{k}_{j+1} = \frac{1}{\ln\left(\frac{f_{j+1}}{f_{N+1}}\right)} \qquad j = N-1$$

$$k^{*}_{j+1} = \frac{\ln\left(\frac{f_{j+2}}{f_{j}}\right)}{\ln\left(\frac{f_{j+2}}{f_{j+1}}\right)\ln\left(\frac{f_{j+1}}{f_{j}}\right)} \qquad j=1,2,\ldots,N-2$$

$$k^{*}_{j+1} = \frac{1}{\ln\left(\frac{f_{j+1}}{f_{j}}\right)} \qquad j=N-1.$$

By assumption of the position of brand (N+1) between brands j and j + 1

$$f_j < f_{N+1} < f_{j+1} < f_{j+2} \qquad j=1,2,\ldots,N-2$$

and $\quad f_j < f_{N+1} < f_{j+1} \qquad j=N-1.$

Therefore for $j = N-1$,

$$\frac{\hat{k}_{j+1}}{k^{*}_{j+1}} > 1,$$

and for $j=1,2,\ldots,N-2$

$$\frac{\hat{k}_{j+1}}{k^*_{j+1}} = \frac{\ln(\frac{f_{j+2}}{f_{N+1}})}{\ln(\frac{f_{j+1}}{f_{N+1}})} \cdot \frac{\ln(\frac{f_{j+1}}{f_j})}{\ln(\frac{f_{j+2}}{f_j})}$$

$$= \frac{\ln(\frac{f_{j+2}}{f_j}) + \ln(\frac{f_j}{f_{N+1}})}{\ln(\frac{f_{j+1}}{f_j}) + \ln(\frac{f_j}{f_{N+1}})} \cdot \frac{\ln(\frac{f_{j+1}}{f_j})}{\ln(\frac{f_{j+2}}{f_j})} .$$

Note that $\ln(\frac{f_j}{f_{N+1}}) < 0$, $\ln(\frac{f_{j+1}}{f_j}) + \ln(\frac{f_j}{f_{N+1}}) > 0$ and

$\ln(\frac{f_{j+2}}{f_j}) > \ln(\frac{f_{j+1}}{f_j})$.

Therefore, $\frac{\hat{k}_{j+1}}{k^*_{j+1}} > 1$.

From Equation (15), if $\hat{p}_j \geq p^*_j$ then $\hat{\beta}_j > \beta^*_j$.

This completes the proof of Lemma 2.

$\square$

<u>Lemma 3</u>: Holding all other prices fixed, the unadjusted market share of any existing product decreases (increases) with the decrease (increase) in the price of any of its immediate neighbors and decreases (increases) with an increase (decrease) in its own price, i.e.,

$$\frac{\partial \beta_j}{\partial p_{j-1}} > 0, \qquad j=2,\ldots,N \qquad (16)$$

$$\frac{\partial \beta_j}{\partial p_{j+1}} > 0, \qquad j=1,\ldots,N-1 \tag{17}$$

$$\frac{\partial \beta_j}{\partial p_j} < 0, \qquad j=1,\ldots,N \tag{18}$$

Proof: From Lane (1980) Equations (4) - (8)

$$\frac{\partial \beta_j}{\partial p_{j-1}} = \frac{1}{R} \ln\left(\frac{f_{j+1}}{f_j}\right) \frac{1}{Y-p_{j-1}} > 0, \qquad j=2,\ldots,N-1 \tag{19}$$

$$\frac{\partial \beta_j}{\partial p_{j+1}} = \frac{1}{R} \ln\left(\frac{f_j}{f_{j-1}}\right) \frac{1}{Y-p_{j+1}} > 0, \qquad j=2,\ldots,N-1 \tag{20}$$

$$\frac{\partial \beta_1}{\partial p_2} = \frac{1}{\ln\left(\frac{f_2}{f_1}\right)} \frac{1}{(Y-p_2)} > 0 \tag{21}$$

$$\frac{\partial \beta_N}{\partial p_{N-1}} = \frac{1}{\ln\left(\frac{f_N}{f_{N-1}}\right)} \frac{1}{(Y-p_{N-1})} > 0 \tag{22}$$

From Equations (7) - (9) we can see $\frac{\partial \beta_j}{\partial p_j} < 0$, $j=1,2,\ldots,N$.

This completes the proof of Lemma 3. $\square$

Theorem 1: The optimal defensive pricing, for any product in a N-product market in equilibrium and in which consumer tastes are uniformly distributed, is the reduction of price.

Proof: The positioning of the attacker can occur as per one of the following two cases: (1) between any two existing products L and L+1, or (2) at either extreme, i.e., to the left of product 1 or to the right of product N.

Case 1: Given the attacker's entry strategies as fixed, the defensive reactions of products 1 through L are independent of those of products L+1 through N.

For products 1 through L, the defensive price change part of Theorem 1 will be disproved only if any of the two following scenarios occur: (A) Product L's optimal price is increased or remains unchanged and (B) The prices of products L, L-1, ..., L-m+1 are decreased ($1 \leq m \leq L-1$) and product L-m's optimal price is increased or remains unchanged.

The change in share of any product j can be written as the sum of the changes in its share caused by the change in the strategy of each of its immediate neighbors and that caused by the change in its own strategy. More specifically,

$$d\beta_j = \frac{\partial \beta_j}{\partial p_j} dp_j + \frac{\partial \beta_j}{\partial p_{j-1}} dp_{j-1} + \gamma_j \quad \text{for } j=2,3,\ldots,N-1 \tag{23}$$

$$d\beta_1 = \frac{\partial \beta_1}{\partial p_1} dp_1 + \gamma_1 \quad \text{for } j=1 \tag{24}$$

where γ_j is the change in β_j due to a change in the strategy of product j+1,

and
$$\gamma_{j-1} = -\frac{\partial \beta_j}{\partial p_{j-1}} dp_{j-1}. \tag{25}$$

Scenario A:

Consider product L. After attack its right adjacent product is (N+1). Further, from Lemma 1, if (N+1) is viable then $\gamma_L < 0$.

Let the after entry price, $\hat{p}_L$ of product L be greater than or equal to its before entry price p_L^*, i.e., $dp_L = \hat{p}_L - p_L^* \geq 0$. Since $\hat{p}_L \geq p_L^*$, from Lemma 2 (Case D), $\hat{\beta}_L > \beta_L^*$.

Or $$d\beta_L = \hat{\beta}_L - \beta_L^* > 0.$$

In Equation (23) with j=L, we know that $d\beta_L > 0$, $dp_L \geq 0$, $\gamma_L < 0$ and from Lemma 3, $\frac{\partial \beta_L}{\partial p_{L-1}} > 0$ and $\frac{\partial \beta_L}{\partial p_L} < 0$.

Therefore $dp_{L-1} > 0$. Or, $\hat{p}_{L-1} > p_{L-1}^*$. It follows from Lemma 2 that

$$\hat{\beta}_{L-1} > \beta_{L-1}^*$$

or $$d\beta_{L-1} > 0.$$

From Equation (25), $\gamma_{L-1} < 0$. Therefore from Equation (23), with j=L-1, $dp_{L-2} > 0$.

Following this line of reasoning, for product 1, $dp_1 > 0$. Therefore, from Lemma 2, $d\beta_1 > 0$. But from Equation (25) $\gamma_1 < 0$. Therefore, Equation (24) is violated; thus, product L's price could not have increased.

<u>Scenario B</u>: For some $1 \leq m \leq L-1$,

$$\hat{p}_j < p_j^*, \qquad j=L-m+1,\ldots,L$$

and $$\hat{p}_{L-m} \geq p_{L-m}^*.$$

Since $\hat{p}_{L-m+1} < p_{L-m+1}^*$, i.e., $dp_{L-m+1} < 0$, and $\hat{p}_{L-m} \geq p_{L-m}^*$, it is implied that $\gamma_{L-m} < 0$. Since $\hat{p}_{L-m} \geq p_{L-m}^*$, i.e., $dp_{L-m} \geq 0$, Lemma 2 implies that $\hat{\beta}_{L-m} > \beta_{L-m}^*$, i.e., $d\beta_{L-m} > 0$.

In Equation (23) with $j=L-m$, $dp_{L-m} \geq 0$, $d\beta_{L-m} > 0$, $\gamma_{L-m} < 0$ implies that $dp_{L-m-1} > 0$, i.e., $\hat{p}_{L-m-1} > p^{*}_{L-m-1}$.

Continuing this, brand 1 must have increased price, and this creates the contradiction as in Case A.

Given that Scenarios A and B are impossible, the optimal prices of products 1,2,...,L must decrease.

Following the same logic it can be seen that the optimal prices of all the products L+1,L+2,...,N must decrease. This concludes the proof of decrease of optimal defensive prices when the attacker enters between any pair of existing products.

Case 2: In this case the attacker can either enter to the right of product N, or to the left of product 1. If it enters to the right of product N with a fixed entry strategy, this situation is analogous to considering the defensive reactions of products 1 through L in Case 1, proving that the optimal prices of all the N existing products must decrease in this situation.

Entry to the left of product 1 is analogous to considering the defensive reactions of products L+1 through N in Case 1, proving that the optimal prices of all the N existing products must decrease in this situation also.

This concludes the proof that the optimal defensive price strategy is the reduction of price by every existing product.

□

Theorem 2: For any existing product j under conditions of Theorem 1, its optimal advertising and/or distribution expenditures must decrease if the market size, M, does not increase. Otherwise, its advertising and/or distribution expenditures must increase.

The intuition behind this theorem is simple. Because the response functions are decoupled and concave, advertising and distribution will decrease if the revenue, $(p_j - c)\beta_j M$, decreases. Theorem 1 and Lemma 1 cause both $(p_j - c)$ and β_j to decrease hence revenue decreases. This argument and the formal proof follow that of Hauser and Shugan (1980, Theorem 7). Our contribution is to extend the results to the case where all defenders respond to the attacher <u>and one another</u> until an equilibrium is reached.

<u>Proof</u>: With respect to optimal defensive advertising and distribution strategies the following first order conditions have to be satisfied:

$$\frac{\partial \pi_j}{\partial k_{aj}} = (p_j - c)\ \beta_j M\ D(k_{dj})\ \frac{\partial A}{\partial k_{aj}} - 1 = 0 \tag{26}$$

$$\frac{\partial \pi_j}{\partial k_{dj}} = (p_j - c)\ \beta_j M\ A(k_{aj})\ \frac{\partial D}{\partial k_{dj}} - 1 = 0 \tag{27}$$

(Note that by choice of concave response functions the second order conditions for a maximum are satisfied, i.e., the Hessian is negative definite.) As shown above, upon attack, the optimal p_j must decrease (to, say, $\hat{p}_j^*$), as must the corresponding β_j (to $\hat{\beta}_j^*$).

<u>Case i</u>: If $(p_j - c)\ \beta_j M$ decreases after entry, then from (26), it follows that

$$\left.\frac{\partial \pi_j}{\partial k_{aj}}\right|_{(\hat{p}_j^*, k_{aj}^*, k_{dj}^*)} < 0 \tag{28}$$

$$\left.\frac{\partial \pi_j}{\partial k_{dj}}\right|_{(\hat{p}_j^*, k_{aj}^*, k_{dj}^*)} < 0 \tag{29}$$

where k_{aj}^* and k_{dj}^* are the optimal pre-entry advertising and distribution expenditures, respectively.

This implies that if we maintain pre-entry advertising (distribution) expenditures after attack, then the distribution (advertising) expenditures must necessarily decrease.

To analyze the effects of joint changes in advertising and distribution expenditures, we note that the total derivative of π_j is given by

$$\left. d\pi_j \right|_{(\hat{p}_j^*, k_{aj}^*, k_{dj}^*)} = \left.\frac{\partial \pi_j}{\partial k_{aj}}\right|_{(\hat{p}_j^*, k_{aj}^*, k_{dj}^*)} dk_{aj} + \left.\frac{\partial \pi_j}{\partial k_{dj}}\right|_{(\hat{p}_j^*, k_{aj}^*, k_{dj}^*)} dk_{dj} \tag{30}$$

From equations (28) and (29), we know that (k_{aj}^*, k_{dj}^*) is suboptimal. Therefore, we seek $(\hat{k}_{aj}^*, \hat{k}_{dj}^*)$ such that

$$\left. d\pi_j \right|_{(\hat{p}_j^*, k_{aj}^*, k_{dj}^*)} \geq 0. \tag{31}$$

This condition (31) cannot be satisfied if both $dk_{aj} > 0$ and $dk_{dj} > 0$, i.e., the post-entry expenditures on both advertising <u>and</u> distribution cannot be higher than their pre-entry levels.

<u>Case ii</u>: If M increases after entry and $(p_j - c)\ \beta\ M$ has increased, then from (26) and (27), it is obvious that the inequalities in (28 and (30)

are reversed. The same logic as in case i applies and the result follows.

$\square$

Other formulations of response functions would not vitiate Theorem 1. But Theorem 2 would have to incorporate additional conditions depending on the specific nature of the response functions. For example if $A(k_{ai}) = k_{ai}/\sum_i k_{ai}$ (us/everyone), then the optimal advertising and distribution expenditures upon attack, will have to be found by solving 2(N+1) equations in 2(N+1) unknowns for all N + 1 products as compared to 2 equations in 2 unknowns for each of the (N+1) products in response function models.

$\square$