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Analytical approach to the influence of motivation on the dynamics of heterogeneous employees and expected average costs of efficient work

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Abstract: *Motivating employees with different characteristics has a significant effect on company performance. This paper models the relationship between employer and heterogeneous employees working in pairs as a principal-agent problem. Every worker can encounter moral hazard with regard to the stimulation of the employer and the efficient work of co-workers. Employee behavior describes a reaction function based on which the equilibrium of appropriate pairs of employees and their overall effective performance is described. The employer determines the optimal stimulation that minimizes the expected average cost of effective work for each individual group of employees. The total expected average cost of efficient work of the entire company in the short run depends on the distribution of employees with different characteristics. How the attitude of employees towards work in the long run changes is described by replicative dynamics and shows that the stability of the employee population is achieved in two cases where the long-run total expected average cost of efficient work is differentiated by approximately eight percent. This paper describes a new conceptual framework for quantitative analysis of the effects of motivation on the short and long run financial results of an enterprise.*

Keywords: employee motivation; principal-agent problem; efficient work; replicator dynamics; expected average costs of efficient work

JEL Classification: C02, C73, D01

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Introduction

Leadership is one of the most responsible and difficult roles that a person can have. The firms that recognize the importance of leadership are and will be the most successful in the market. The challenges that leaders face today are how to motivate employees to work more efficiently and be more engaged and committed to the firm. New employee generations are different from the previous ones and leaders are required to constantly develop themselves and adapt to new approaches.

Many studies introduce the strategy of how to lead. The study George P. Allen et al. (2016) focuses on two roles of leadership: Servant and Transformational. According to the study, servant leadership focuses on supporting and developing the individuals, while transformational leadership focuses on inspiring followers to work towards a common goal. Even though they focus on leaders in academic pharmacy, it can be transferred to all other business areas.

V. Kumar et al. (2015) in the study on measures of benefits of employee engagement and Dick Finnegan (2014) in the study on how much employee engagement correlates with profitability both explained that higher employee engagement leads to higher profits and profitability.

Although it is better for a firm to have more employees with a high level of engagement, the question is whether there is a level of proportion of the number of highly engaged employees that no longer leads to greater efficiency. Due to the fact that highly engaged employees also have higher demands and expectations, they are harder to manage.

Likewise, expectations are that this type of employee loves innovative and creative jobs and will find it difficult to settle for some simple and repetitive jobs. It is also important to note that, especially in large corporations, there are many recurring jobs and it should be taken into consideration that firms need to have employees who will do such jobs as well. For that reason, it should be important to divide employees into different employee types such as those that can be seen in many studies (Luis E. Romero, 2016; Derek Heim, 2017; Britta Kallevang, 2018; Carloe Gaskell at al., 2016; Chong W. Kim, 2011).

Different types of employees possess different traits and skills, and for this reason, leaders need to tailor their approach to a particular type. This paper will analyze the behavior of certain types of employees, and how their behavior can affect the costs and final financial results of the firm. Employees are divided into three types: Enthusiast (highly engaged employee), Worker (moderately engaged employee), Parasite (no engagement)

After the introduction of these types, they will be observed in teams of two. The emphasis will be put on the fact that type Worker is present in the team and the other part of the team could be any of the three. The logic behind is the fact that in today's business, teamwork is considered as the crucial ingredient of a successful organization. The research studies (Shouvik Sanyal et al., 2018) revealed that teamwork, leadership, team trust and performance, appraisal and rewards have a significant and positive impact on the performance.

This is the reason why this paper starts with team observation and modeling of general "Worker " type utility function in combination with other employee types. In the analysis of the relationship between leaders and their employees (principal-agent modeling), a relationship between principle and agent with no distinction among agents can mostly be found in the studies (H. Scott Bierman et al., 1993; David Besanko et al., 1991; Miller G.J, 2005). There are no relevant researches that deep dive into a more detailed principle-agent relationship which include different agent types. In this paper, the analysis is focused on agents' differentiation (three employee types mentioned above) and therefore it is a novelty introduced by this paper.

The payments are calculated after the introduction of the utility function. Based on those calculations, an optimal working effort is calculated in order to maximize the utility of the employee. According to equilibrium, in a particular team, the employer determines an optimal stimulation to minimize the expected average cost by unit of effective work. Based on previous calculations, the dynamic of the proportions (probability) of being a specific employee type will be described. The main acknowledgment for this comes with the usage of replicator dynamics as the main analytic tool.

The stability of the stationary points is tested using the stability of nonlinear systems theorem (Simon ad Blume, 1994). This theorem proposes the usage of eigenvalues of Jacobi matrices of proportion (probability) for the three employee types, with the final proposition of where the system tends to in the long run. The usage of replicator dynamics and stability theorem is the novelty because other similar analyses in the area of leadership do not offer concrete quantification and this kind of modeling.

The contribution of this paper is reflected in the new analytical framework for the analysis of the impact of motivation on the work efficiency of heterogeneous employees who change their attitude towards work in the long run, which affects the overall expected average costs of efficient work. This complex quantitative analysis, which is brand new in the literature, combines the results of game theory and population dynamics and describes, in a measurable way, the short-run and long-run effects of stimulation measures on the labor force structure in the company. The economic effects are described in the long run by the relative difference of the total expected average costs of efficient work. Paper contribution also results from an analysis of the stability of possible long-run equilibria, a visual representation of movement to long-run equilibrium, and a clear description of the locus of labor distribution that results in relative savings in total expected average costs of effective work.

The paper starts with literature overview where current knowledge on the researched topic will be briefly presented. After this section, the methodologies used in the paper will be described in detail together with all the mathematical tools used for model development and analysis that can be found. The main part of the paper is the section where a model is designed step-by-step with regard to different scenarios. The scenarios are observed by teams of two members with a different combination of employee types. An optimal expected average cost of effective work will be observed in respect to proportion (probability) of every employee type in the population. In the end, in the conclusion part, the summary of the modeling results and analysis will be presented with the proposal of future research.

Literature overview

In the relationship between leader and followers, there are several ways of interaction that lead to different short-run and long-run outcomes. As mentioned above, this interaction is not simple and makes this kind of structure complex for the analysis. In many studies, the level of employee engagement is the number one priority that a leader should consider.

V. Kumar et al. (2015), in the study on measuring the benefits of employee engagement, defined the frame of employee engagement. They also developed a scale for engagement measurement together with the "engagement scorecard", and the strategy for maximizing it. The main result from the research is a positive correlation between the level of engagement in the firm and the financial results. More or less the same results can be found in Dick Finnegan's study about positive correlation between the level of engagement and firm profitability (2014).

The insufficiency of these studies lies in the term „level of engagement" and the fact that highly engaged employees are great but at the same time difficult to lead because of their needs. In many cases they are creative, innovative and do not like repetitive work. On the other hand, in many organizations, especially in large corporations, there are a lot of unattractive jobs that need to be done. They stated that higher level of engagement leads to higher profits, but do not have an argument for the level on which this profit can start decreasing. This is the point where it will be difficult to satisfy employee needs and their motivation will start decreasing which will lead to a decrease in the firm profit.

Leaders in organizations deal with different types of employees. These employees have a different engagement level. This imposes the main question of how to get more from this mix of employees. This study will take into account the results from the above-mentioned studies and take those with high engagement as main drivers and added value in terms of profit and efficiency. These employees will be one separate type. In addition to this type, two more types will be introduced and observed together for utility maximization. From the principle-agent (leader – follower) view, many studies show research on principle-agent relationship, but with no differentiation on agents. The added value of this paper will be agents' distinction in principle – agent model.

H. Scott Bierman et al. (1993), observed the principle-agent model through involuntary unemployment. In this study, they observed the principle – agent model without the distinction among agents and did not observe the dynamics of the population. They showed that voluntary unemployment is consistent with an equilibrium wage. Furthermore, to ensure higher level of effort, the firm pays a wage that is higher than the wage that would normally be considered based on supply – demand framework. The insufficiency of this study lies in no distinction of agent types and having only one is far from real situations in organizations, especially in large corporations.

In a more recent research, Katerina Stankova et al. (2016) introduced the Inverse Stackelberg Games versus Adverse-Selection Principal-Agent Model Theory in which again only the observation of principal and only one type of agent can be seen. Askar, S.S. (2018), argues on tripoly Stackelberg game model with one leader versus two followers. Her paper is focused on oligopoly market where three firms are competing and one of them is a leader. Although this article is not directly related to the principle-agent model in one firm, it does show an approach in principal-agent modeling (one leader and more different followers).

Different selection of employee types can be found in the literature. Aziz et al. (2010), divide workers in six main categories: “Workaholic”, “Positively engaged worker”, “Worker enthusiast”, “Relaxer”, “Disenchanted worker” and “Unengaged worker”. Each of the six mentioned groups has its own specific attributes in terms of: engagement at work, encouragement, and enjoyment of work. In his work, Luis E. Romero (2016) divides employees in three core groups: “Freeloader”, “The Worker”, “The Entrepreneur”, again having different attributes depending on engagement and motivation for the job. Heim (2017) worked on four types of employees: “Adventurer”, “Warrior”, “Guide”, “Diplomat”, and how to manage them. Again, he looks at them with regard to work attitude and interaction with colleagues. According to Kim (2011), employees are divided into three categories according to their key features: Necessities (directed to achieving a goal and a significant contribution to collective success), Commoners (do what they have to do without the desire for proactivity), Parasites (negative influences on group results and organizational climate).

Similar to these last-mentioned employee types will be used in this paper, because all other divisions can be reduced to these three groups. The common denominator of all divisions of employee types encountered in the literature is the level of employee engagement.

The literatures mentioned above, and many others are modelling principal – agent relationship with only one agent type. For the basic understanding it is good, but not enough for description of real and complex principal – agent interaction. The novelty of tis paper is introduction of different types agents and building a model that describes those complex interactions.

Methodology

In the principal – agent model, it is assumed that the employer (leader) is the principal who decides on a wage and investment in the development of an employee (agent). In this paper, there are three groups of agents who are motivated in a different way by wage and investment in development. "Necessities" are strongly motivated by investment in development due to their high level of engagement to the company. A "Parasite" is motivated by wages, while perceiving the development investment in a negative way because they feel strong disengagement to the company in any way. The primary motivating factor for "Commoners" is the wage, and having a low level of engagement, they take investment in the development as neutral.

Depending on the motivational factors affecting these groups differently, the agents decide on the invested effort and maximize expected payments. Each agent determines the share of time they will work effectively and takes the risk of the principal discovering that they are not doing enough and firing them for that reason. The uncertainty faced by an agent involves the methods of game theory analysis, whereby it is important to model attitude to risk. In the context of maximizing the agent's expected payment, it is also important to estimate the probability of being caught, depending on their invested effort. Mathematical methods of optimization are used to determine the agent reaction functions of particular groups. In the equilibrium, no players (leader or agents) have an incentive to change the decision they make. This is the basic property of the Nash equilibrium and will be used in this paper.

This study will show, using an evolutionary or replicator dynamics, that behavior of players in principal – agent game is coherent with their long-term goals. In the first step, armed with game theory tools, the utility function of type "Commoners" will be introduced. This function will consider the relationship with all other employee types observing the pairs "Commoners vs Other Types", including "Commoners vs Commoners". After the utility function definition, the expected utility will be calculated. Based on this, cost of the effective work and payment will be analyzed.

The principal optimum wages and development investment decisions take into account the current agent structure and total costs. Here, as the research methods, the above mathematical optimization methods are used as well. Based on the analysis, the resulting table will show payoffs for all possible pairs of defined employee types. Because of optimum principal decisions on wages and investment in development, the final agent payments for each individual group are determined in every moment and the average payment of the population as well, which depends on the employee structure. If the payment of an agent group is higher than the average payment, it is natural to expect that their relative frequency will increase because some agents in the group, where the agent's payment is lower than the average payment, will be transferred to that group. In each small unit of time, the relative change in the number of members of a group of agents describes the deviation of their average payment. Therefore, dynamic analysis of the population will be used as a research method that involves the study of differential equations and questions of stability of dynamic systems.

Methods for studying ordinary differential equations and population dynamics are described extensively in the work by Carl P. Simon et al. (1994). The basic theorems of equilibrium stability of the population in linear and nonlinear systems will be one of the main tools for studying equilibrium. Additional tools for studying the equilibrium will be the basic methods of seeking Nash equilibrium of pure and mixed strategies, which are systematically described in, Kopal et al. (2011). In addition to the usual methods of solving differential equations, the results of the Fundamental Theorem of Differential Equations that prove the existence and uniqueness of a differential equation solution will be used in the paper. Carl P. Simon et al. (1994), geometrically prove this theorem.

The fact is that if it is possible to draw a segment of a part of a curve f (t, y) at each interval (t, y) in the plane, we will get a clear picture of what the graph of the general solution of the equation looks like:

$$\dot{y} = f\left(t, y\right)$$

The sets of such segments are called the “Direction Field” of differential equation.

Furthermore, as the consolidation of the solution and the way that will follow how differential equation solution changes over time, the so-called “Phase Portrait” method for a given differential equation will be used. “Phase portrait” means that it is of interest to know where y(t) is increasing or decreasing and where the derivation ($\dot{y}$) is positive or negative respectively. To draw a “Phase Portrait” for the equation $\dot{y}$ = f (y), the first step is to find null – points from the f (y) and then check the function f(y) in each of the intervals between the nulls. If it is possible to draw the graph function f(y), denote the intervals where the graph lays above the x axis with the arrow to the right, otherwise the arrow to the left.

Using such a graph, stability of the system in the corresponding points will be analysed. Similar principle will be used in situations where non-linear systems exist, which is when the differential equation has two or more variables. For example, such two equations that are derivation of x and y equals:

$$\dot{x} = f(x, y)$$

$$\dot{y} = g(x, y)$$

Such a system represents the motion of the particles in the plane. The system states that when the particle is in the point (x, y) it will move so that the vector of acceleration ($\dot{x}$, $\dot{y}$) is a vector of the corresponding functions f and g:

$$(f(x, y), g\ (x, y))$$

It will be drawn as a vector beginning with (x, y) that points in the direction of the particle movement. For this reason, vector (f (x, y), g (x, y)) is used for describing the dynamics of this system, starting at point (x, y) for each point of the plane. Such a family of vectors will be called the Vector Field.

The geometric solution of the system is vector:

$$(\dot{x}, \dot{y}) = (f(x, y), g(x, y))$$

at each point (x, y). The corresponding geometric solution (x*(t), y*(t)), is the curve in the space, tangent on the vector field. Since the point (x, y) lies on the curve (x* (t), y* (t)), its vector ($\dot{x}$*(t), $\dot{y}$*(t)) should point in the direction of the vector (f(x,y),g(x,y)). A set of all such curves is called the "Phase Portrait" or "Phase Diagram" of the system in space. This method of geometric solution of differential equation will be used in the paper, in order to perform stability analysis. The approach is based on replicator dynamics equation, built for each option from the probability of choices and their outcome values. First, the mean value E_i for every employee type will be obtained using the probability p_i for every outcome coresponding to that employee type. The mean value M of all is obtained as:

$$M = \sum_i p_i * E_i$$

The fitness of each decision can be made in the form of differential equation (replicator dynamic theorem):

$$\dot{p}_i = p_i * (E_i - M) \tag{1}$$

Using (1), stationary points will be calculated. Using the "Phase Portrait" stability analysis on given points will be described. The same stability will be tested using stability of nonlinear system theorem (Simon and Blume, 1994). The method is based on eigenvalues of Jacobi matrices of probability (percentage) of employee type and gives result where the system will be in the long run. This is also a novelty in the field of leadership because other similar analysis that are in natural dynamic do not offer concrete quantification and do not observe stability in this way.

Modeling and analysis

As mentioned above this model includes three types of employee groups based on the level of engagement they have toward the organization. These three types are:

- Enthusiast – Highly engaged, creative and tends to do even more than expected. They are valuable to the organization.
- Worker – Low or medium engaged and tends to do what they have to do. They prefer more structure tasks and are valuable for the organization.

- Parasite – Not engaged and tends not to work with a focus on "not to be caught". They have little or no value for the organization.

In principal – agent modeling these three employee types will be three different types of agents. Suppose the principal pays reservation salary (w) to the agent. Agents can decide to shirk or not to shirk and decide how much effort to invest in particular work. On the other hand, the principal stimulates the worker with a certain amount (s) which depends on the level of effort the employee has invested. The level of effort of the employee ranges from the case that the worker does not work at all (0) to the case that the worker works constantly during working hours (1)

Based on agents' type description, initially it is supposed that "Enthusiast" works constantly during working hours which implies that level of effort is $e_1 = 1$. For "Parasite" initially it is assumed that they do not like to work and the effort is close to 0 ($e_3 = 0$). "Worker" level of effort is somewhere between those two extremes, $e_2 \in [0,1]$. The cases where agents work in teams of two will be observed. As a first step, the utility function of the agent "Worker" (u_2) will be defined taking into account teamwork.

If level of Effort of agent "Work" is e_2 and level of effort of any other agent type (including "Worker" is e, then the probability that the team works is given by:

$$\frac{e+e_2}{2}$$

In that case, the utility for the "Worker" will be defined as:

$$w(1-e_2)+s*\frac{e+e_2}{2}$$

where s is the stimulation factor with which the employer stimulates the worker for working.

This generates the cost for the employer for a unit of work. In team formation effective work is given by $e+e_2$. When multiplied by s, it generates stimulation cost for the employer $s*(e+e_2)$. For every team member it is a gain of $s*\frac{e+e_2}{2}$.

For the purpose of this research, increasing function is needed with respect to s and w, and decreasing with respect to e_2. The above-defined function meets all the mentioned conditions. In this situation, the stimulation s positively affects the utility and wage respectively. On the other hand, a higher level of work leads to a lower level of utility.

The probability that the teams does not work is given by:

$$1-\frac{e+e_2}{2}$$

In that case, the utility for the "Worker" will be defined as a salary, w.

To summarize, the above utility function u_2 can be defined as:

$$u_2 = \begin{cases} w(1-e_2)+s*\frac{e+e_2}{2} & \textit{with given probability } \frac{e+e_2}{2} \dagger \\ w & \textit{with given probability } 1-\frac{e+e_2}{2} \end{cases} \tag{2}$$

From (2) the expected utility for the “Worker” is given by:

$$Eu_2 = \frac{e+e_2}{2}*\left(w(1-e_2)+s*\frac{e+e_2}{2} \right)+\left(1-\frac{e+e_2}{2}\right)*w$$

which is equivalent to:

$$Eu_2 = w - w*e_2*\frac{e+e_2}{2}+s*(\frac{e+e_2}{2})^2 \tag{3a}$$

From (3a), it implies that:

$$Eu_2 = \frac{1}{4}*\left[4*w-s*e^2+2*(s-w)*e*e_2+(s-2*w)*{e_2}^2\right] \tag{3b}$$

Because the stimulation (s) will be measured by wages (w), without reducing generality it can be stated:

$$\mathrm{w=1} \tag{4}$$

This implies that (3b) can be written as:

$$Eu_2 = \frac{1}{4}*\left[4-s*e^2+2*(s-1)*e*e_2+(s-2)*{e_2}^2\right] \tag{5}$$

Depending on the value of s (5) can be quadratic or linear.

The next step is the maximization of (5) by e_2 taking into account $e_2 \in [0,1]$:

$$\max_{e_2} Eu_2$$

In case (5) is quadratic ($s \neq 2$) maximum can be found using apex, first order conditions of unconstrained optimization and the fact $e_2 \in [0,1]$:

$$\frac{dEu_2}{de_2} = 0$$

$$\frac{1}{4}*\left[2*(s-1)*e+2*(s-2)*e_2\right]=0\dagger\dagger \tag{6}$$

Because of (6) $\mathrm{Eu_2}$ has the apex at:

$$\mathrm{e_2} = \frac{\mathrm{s}-1}{2-\mathrm{s}}*\mathrm{e} \tag{7}$$

For the e_2 calculation the following cases should be considered:

a) If $s \leq 1$ then apex of quadratic function which describes expected "Worker" utility is to the left of the origin and function reaches maximum on the relevant interval [0,1] for:

$$e_2 = 0 \tag{8}$$

b) If $s \in (1,2)$ function is quadratic, two subcases should be considered:
$\frac{s-1}{2-s} \leq 1$ implies $s \leq \frac{3}{2}$. The graph of quadratic function that describes expected "Worker" utility is faced down. Apex of the function is in [0,1]. For that reason (see Figure 1) the maximum of the function is in:

$$e_2 = \frac{s-1}{2-s} * e \tag{9}$$

$$\frac{s-1}{2-s} > 1 \text{ implies } s > \frac{3}{2}.$$

In this case, if $e \leq \frac{2-s}{s-1}$ then apex of the same quadratic function is in [0,1] (see Figure 2) and graph of function is faced down. For that reason, the maximum of the function is in:

$$e_2 = \frac{s-1}{2-s} * e \tag{10}$$

If $e > \frac{2-s}{s-1}$ then apex of the function is to the right of 1 for that reason and fact that $e_2 \in [0,1]$ (see Figure 2)

$$e_2 = 1 \tag{11}$$

c) If s=2 then function that describes expected "Worker" utility is linear.
For that reason if $e > 0$ and the fact that $e_2 \in [0,1]$ implies that maximum is for:

$$e_2 = 1 \tag{12}$$

In case $e = 0$ (5) is constant and maximum is reached for every:

$$e_2 \in [0,1] \tag{13}$$

d) If s>2 then (5) is quadratic function faced up. Apex of the function is to the left of origin. For that reason maximum is reached for:

$$e_2 = 1 \tag{14}$$

Based on the above analysis, the reaction effort curve (Figure 1 and Figure 2) of the "Worker" (e_2) to the effort of other employee types (e) can be considered in order to gain a maximum expected utility.

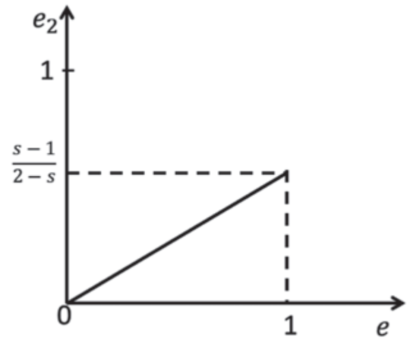

Figure 1: Reaction curve of the „Worker" effort to the effort of other employee type (Case: $s \in \left(1, \frac{3}{2}\right]$) – – author's original

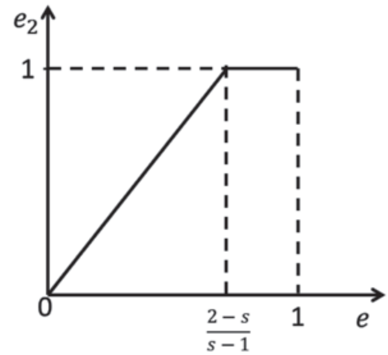

Figure 2: Reaction curve of the „Worker" effort to the effort of other employee type (Case: $s \in \left(\frac{3}{2}, 2\right]$) – author's original

Because of (2) and (4) Total Expected Costs for the firm in particular team:

$$TEC = \frac{e+e_2}{2} * \left[2 + 2 * s * \frac{e+e_2}{2}\right] + (1 - \frac{e+e_2}{2}) * 2$$

$$= 2 + s * \frac{(e+e_2)^2}{2} \tag{15}$$

From (15) Total Expected Payment for each player is given by:

$$TEP = \frac{TEC}{2} = 1 + s * \left(\frac{e+e_2}{2}\right)^2 \tag{16}$$

An effective work is given by:

$$EW = e + e_2 \tag{17}$$

Using (15) and (17) the average expected cost for the firm can be calculated by unit of effective work as:

$$AEC = \frac{TEC}{EW} = \frac{2}{e+e_2} + s*\frac{e+e_2}{2} \tag{18}$$

Using the previous results, from now on, different team scenarios will be introduced and minimum AEC with respect of s will be calculated. In the end of each scenario, total payment for the employee will be calculated.

SCENARIO 1

Team "Entusiast" vs "Worker" will be observed in this scenario. As mentioned above, the effort for "Entusiast" is given by $e = e_1 = 1$ because it is assumed that "Entusiast" makes maximum effort.

In this respect "Worker" effort is given by:

$$e_2 = \begin{cases} 0, & s \le 1 \\ \frac{s-1}{2-s}, & s \in \left(1, \frac{3}{2}\right] \\ 1, & s > \frac{3}{2} \end{cases}$$

Using this, AEC calculation is given by:

$$AEC = \begin{cases} 2 + \frac{s}{2}, & s \le 1;\ \text{because of } e_1 + e_2 = 1 \\ 2*(2-s) + \frac{s}{2*(2-s)}, & s \in \left(1, \frac{3}{2}\right];\ \text{because of } e_1 + e_2 = \frac{1}{2-s} \\ 1+s, & s > \frac{3}{2};\ \text{because of } e_1 + e_2 = 2 \end{cases}$$

For the calculation of minimum s in which AEC reaches its minimum, the situation when "Workers" do not work ($e_2 = 0$) should be excluded because the leader wants to stimulate them to work.

For $s \in \left(1, \frac{3}{2}\right]$ it can be calculated:

$$\frac{d^2AEC}{ds^2} = -2*(2-s)^{-3}*(-1) = \frac{2}{(2-s)^3} > 0$$

This implies that $\frac{d^2AEC}{ds^2} > 0$ for every $s \in \left(1, \frac{3}{2}\right]$. It means that in this area AEC is convex function.

For $s = \frac{3}{2}$ it can be calculated that $AEC = \frac{5}{2}$ for either the second or the third scenario of the above defined AEC function. Considering the fact that AEC is convex in the second scenario, it is clear that it reaches its minimum on the second scenario in the area of $s \in \left(1, \frac{3}{2}\right]$.

To find minimum, start with the first order condition:

$$\frac{dAEC}{ds} = -2 + \frac{1}{(2-s)^2} = 0$$

Stationary points are:

$$s = 2 \pm \frac{\sqrt{2}}{2}$$

Because $s \in \left(1, \frac{3}{2}\right]$ it should be taken $s = 2 - \frac{\sqrt{2}}{2}$.

It has been already calculated that $\frac{d^2 AEC}{ds^2} > 0$ for every $s \in \left(1, \frac{3}{2}\right]$.

It means $s = 2 - \frac{\sqrt{2}}{2} \approx 1.3$ is the value of stimulation at which AEC reaches its minimum.

Putting this value into the AEC the value of minimum AEC is calculated and given by:

$$AEC = 2 * \frac{\sqrt{2}}{2} + \frac{2 - \frac{\sqrt{2}}{2}}{\sqrt{2}} = 2 * \sqrt{2} - \frac{1}{2} \approx 2.33$$

For this s, the corresponding effort level for "Worker" can be calculated as:

$$e_2 = \frac{s-1}{2-s} = \frac{1 - \frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \sqrt{2} - 1$$

Based on this, the total expected payment is given by:

$$TEP = 1 + s * \left(\frac{e_1 + e_2}{2}\right)^2 = 1 + \left(2 - \frac{\sqrt{2}}{2}\right) * \frac{1}{2} = 2 - \frac{\sqrt{2}}{4} \approx 1.6$$

Total payment for the "Worker" using (3a) and (16)

$$TP = TEP - e_2 * \frac{e_1 + e_2}{2} = 1 + \frac{\sqrt{2}}{4} \approx 1.35$$

Effective work of the team is calculated by using (17):

$$EW = 1 + 0.41 = 1.41$$

SCENARIO 2

Team "Parasite" vs "Worker" will be observed in this scenario. As mentioned above, the effort for "Parasite" is given by $e = e_3 = 0$.

Observed "Worker" effort is given by:

$$e_2 = \begin{cases} 0, & s < 2 \\ [0,1], & s = 2 \\ 1, & s > 2 \end{cases}$$

It is clear that optimal s=2 and it can be taken $e_2 = 1$ because with a litle higher s, the "Workers" can be made to work with maximum effort ($e_2 = 1$.)

The observd measures are calculated as:

$$AEC = \frac{2}{e_3 + e_2} + s * \frac{e_3 + e_2}{2} = 2 + 2 * \frac{1}{2} = 3$$

$$TEP = 1 + s * \left(\frac{e_3 + e_2}{2}\right)^2 = 1 + 2 * \frac{1}{4} = \frac{3}{2}$$

$$TP = TEP - e_2 * \frac{e_3 + e_2}{2} = \frac{3}{2} - \frac{1}{2} = 1$$

Effective work of the team is calculated as:

$$EW = 0 + 1 = 1$$

SCENARIO 3

Team "Entusiast" vs "Entusiast" will be observed in this scenario. Each has level of effort 1. As they have level of effort 1, stimulation s=0.

Corresponding measures are: AEC=1; TEC=2; TEP=1; TP=1; EW=2

In this scenario the work does not decrease the utility of the employee, so TP=TEP.

SCENARIO 4

Team "Entusiast" vs "Parasite" is observed in this scenario. "Entusiast" has level of effort 1 and "Parasite" has level of effort close to 0. In this scenario, the leader does not have incentive for stimulation (s=0).

Corresponding measures are: AEC=2; TEC=2; TEP=1; TP=1; EW=1

In this scenario, the work does not decrease the utility of the employee type “Entusiast”. “Parasite” does not work unless they behave as a “Worker”, which is considered in the following scenarios. For all stated TEP=TP in this scenario.

SCENARIO 5

Team “Worker” vs “Worker” will be observed in this scenario. Before specific measures calculations, the level of effort for every “Worker” should be estimated. Figure 1 and Figure 2 show reaction curves for “Worker” against other types. In this scenario, “Worker” against “Worker” should be observed. In case of $s < \frac{3}{2}$ (Figure 3) both reaction curves (“Worker; “Worker”) intersect in 0. This case is not preferred because the firm wants “Worker” to work.

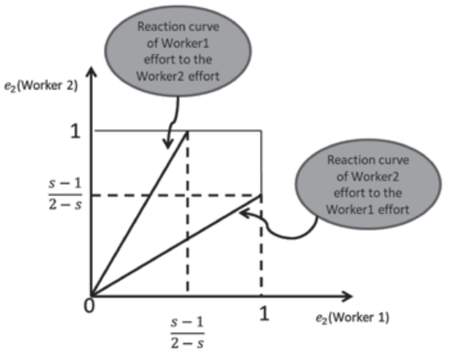

Figure 3: Reaction curves of the Worker1 and Worker 2 based on each other effort (Case: $s \in \left(1, \frac{3}{2}\right]$) – author’s original

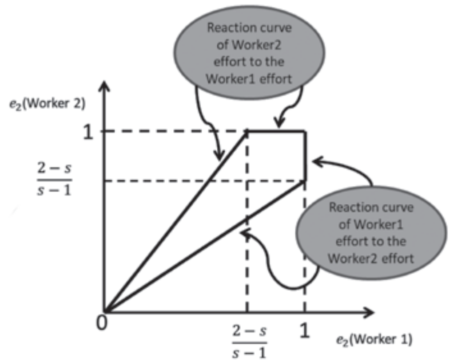

Figure 4: Reaction curves of the Worker1 and Worker 2 based on each other effort (Case: $s \in \left(\frac{3}{2}, 2\right]$) – author’s original

For $s > \frac{3}{2}$ (Figure 4) both reaction curves intersect at 0 and 1. In this case, between those two, the firm prefers “Worker” to work and $e_2 = 1$.

If $s = \frac{3}{2}$ Figure 4 becomes Figure 5 and both reaction curves intersect at the line in Figure 5. Both “Worker” employees are in equilibrium on this line. For that reason, “Worker” will prefer to have an effort of 1 because for this effort “Worker” will have maximum utility in comparison with all other equilibriums.

From the above analysis, it can be concluded that in this scenario $e_2 = 1$ for both employees.

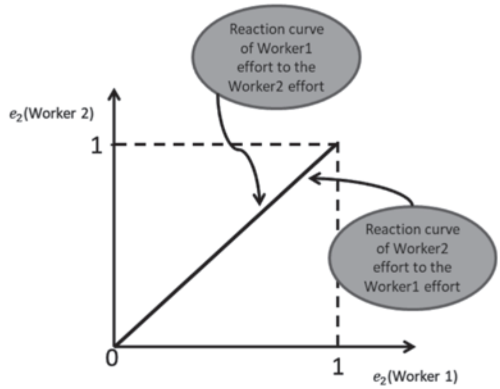

Figure 5: Reaction curves of the Worker1 and Worker 2 based on each other effort (s=1.5)) – author's original

Using above and (9) implies that $s = \frac{3}{2}$.

With this, other measures are calculated: $AEC = \frac{5}{2}$; $TEC = 5$; $TEP = \frac{5}{2}$; EW=2.

$$TP = TEP - e_2 * \frac{e_2 + e_2}{2} = \frac{5}{2} - 1 = \frac{3}{2}$$

SCENARIO 6

Team "Parasite" vs "Parasite" will be observed in this scenario. Each of them has level of effort close to 0. In this scenario, the leader does not have incentive for stimulation(s=0).

It will be assumed that in this case the team "Parasite" acts as a "Worker". In other case, the task that the leader gives them will not be finished because of level effort of 0. The "Parasite" does not want to be detected not working.

Corresponding measures are: $AEC = \frac{5}{2}$; $TEC = 5$; $TEP = \frac{5}{2}$; TP=$\frac{3}{2}$; EW=2.

Calculated results from all scenarios will be presented in four different payment matrix (tables).

Table 1 shows symmetric payment matrix in regard to Total Expected Payment for all team combinations.

Table 1: Payment matrix with TEP

	p_1	p_2	$p_3 = 1 - p_1 - p_2$
Employee Type	E	W	P
E	1	$2-\frac{\sqrt{2}}{4}$	1
W	$2-\frac{\sqrt{2}}{4}$	$\frac{5}{2}$	$\frac{3}{2}$
P	1	$\frac{3}{2}$	$\frac{5}{2}$

Table 2 shows payment matrix in regard to Total Payment for row players for all team combinations.

Table 2: Payment matrix with TP

	p_1	p_2	$p_3 = 1 - p_1 - p_2$
Employee Type	E	W	P
E	1	$2-\frac{\sqrt{2}}{4}$	1
W	$1+\frac{\sqrt{2}}{4}$	$\frac{3}{2}$	1
P	1	$\frac{3}{2}$	$\frac{3}{2}$

Table 3 shows payment matrix with two numbers for all team combinations. First numbers are Total Expected Costs while second numbers are effective work of the corresponding team.

Table 3: Payment matrix with TEC; Team effective work

	p_1	p_2	$p_3 = 1 - p_1 - p_2$
Employee Type	E	W	P
E	2; 2	$\left(4-\frac{\sqrt{2}}{2}\right);\sqrt{2}$	2; 1
W	$\left(4-\frac{\sqrt{2}}{2}\right);\sqrt{2}$	5; 2	3; 1
P	2; 1	3; 1	5; 2

Table 4 shows payment matrix with Average Expected Costs for the firm by unit of effective work.

Table 4: Payment matrix with AEC

	p_1	p_2	$p_3 = 1 - p_1 - p_2$
Employee Type	*E*	*W*	*P*
E	1	2.33	2
W	2.33	2.5	3
P	2	3	2.5

In all the above tables, the probability of being the specific employee type is marked for every employee:

- p_1 = percentage (probability) of being “Enthusiast” (E)
- p_2 = percentage (probability) of being “Worker” (W)
- $p_3 = 1 - p_1 - p_2$ = percentage (probability) of being “Parasite” (P)

Based on the above Table 1, the overall expected payment (E_i) for every employee type is given by:

Enthusiast: $$E_1 = p_1 * 1 + p_2 * \left(2 - \frac{\sqrt{2}}{4}\right) + (1 - p_1 - p_2) * 1 = 1 + p_2 * \left(1 - \frac{\sqrt{2}}{4}\right) \tag{19}$$

Worker: $$E_2 = p_1 * \left(1 + \frac{\sqrt{2}}{4}\right) + p_2 * \frac{3}{2} + (1 - p_1 - p_2) * 1 = 1 + p_1 * \frac{\sqrt{2}}{4} + p_2 * \frac{1}{2} \tag{20}$$

Parasite: $$E_3 = 1 * p_1 + \frac{3}{2} * p_2 + (1 - p_1 - p_2) * \frac{3}{2} = \frac{3}{2} - \frac{1}{2} * p_1 \tag{21}$$

Using (19), (20) and (21) overall expected payment is given by:

$$E_{TOTAL} = p_1 * E_1 + p_2 * E_2 + (1 - p_1 - p_2) * E_3$$

$$E_{TOTAL} = \frac{3}{2} - p_1 - \frac{1}{2} * p_2 + \frac{1}{2} * p_1^2 + \frac{1}{2} * p_2^2 + \frac{3}{2} * p_1 * p_2 \tag{22}$$

According to replicator dynamics, a strategy probability (percentage) growth rate is measured as a difference between the fitness of an observed player (E_i) and total fitness (E_{TOTAL}).

After those calculations, population dynamic will be observed using replicator dynamic equations:

$$\dot{p}_i = p_i (E_i - E_{TOTAL}) \qquad \text{for } i = 1, 2 \tag{23}$$

For each pair (p_1, p_2), a vector ($\dot{p}_1$, $\dot{p}_2$) indicates a direction in which probabilities change in case of the system of the above differential equations.

Calculations of right side of the equation are given by:

$$E_1 - E_{TOTAL} = \frac{1}{4} * \left[-2 + 4*p_1 + \left(6 - \sqrt{2}\right)*p_2 - 2*p_1^{\,2} - 2*p_2^{\,2} - 6*p_1*p_2 \right] \quad (24)$$

$$E_2 - E_{TOTAL} = \frac{1}{4} * \left[-2 + \left(\sqrt{2} + 4\right)*p_1 + 4*p_2 - 2*p_1^{\,2} - 2*p_2^{\,2} - 6*p_1*p_2 \right] \quad (25)$$

Using (23) and (24) the following equation is calculated:

$$\dot{p}_1 = p_1 * \frac{1}{4} * \left[-2 + 4*p_1 + \left(6 - \sqrt{2}\right)*p_2 - 2*p_1^{\,2} - 2*p_2^{\,2} - 6*p_1*p_2 \right] \quad (26)$$

Using above two isoclines ($\dot{p}_1 = 0$) should be observed:

$$p_1 = 0; \quad (27)$$

$$\frac{1}{4} * \left[-2 + 4*p_1 + \left(6 - \sqrt{2}\right)*p_2 - 2*p_1^{\,2} - 2*p_2^{\,2} - 6*p_1*p_2 \right] = 0; \quad (28)$$

Considering the fact that $p_i \in [0,1]$ for $i = 1,2$ those isoclines can be seen on Figure 6.

The same reasoning will be done for type "Workers" using (23) and (25):

$$\dot{p}_2 = p_2 * \frac{1}{4} * \left[-2 + \left(\sqrt{2} + 4\right)*p_1 + 4*p_2 - 2*p_1^{\,2} - 2*p_2^{\,2} - 6*p_1*p_2 \right] \quad (29)$$

Two isoclines ($\dot{p}_2 = 0$) should be observed:

$$p_2 = 0; \quad (30)$$

$$\frac{1}{4} * \left[-2 + \left(\sqrt{2} + 4\right)*p_1 + 4*p_2 - 2*p_1^{\,2} - 2*p_2^{\,2} - 6*p_1*p_2 \right] = 0; \quad (31)$$

Considering the fact that $p_i \in [0,1]$ for $i = 1,2$ those isoclines can be seen on Figure 7.

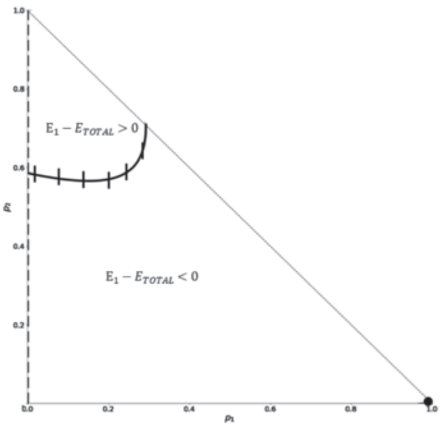

Figure 6: Isoclines for employe type „Enthusiast” ($\dot{p}_1$=0) – author's original

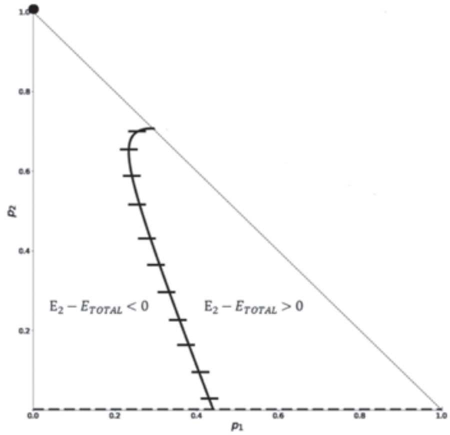

Figure 7: Isoclines for employe type „Worker” ($\dot{p}_2$=0) – author's original

Using sign analysis of $\dot{p}_1$ **and** $\dot{p}_2$ from Figure 6 and Figure 7, in the areas which jointly bound the isoclines, the direction of the change (orbits) in the probability (percentage) of "Enthusiast" and "Worker" (Figure 8 and Figure 9) can be determined.

The orbits describe how the proportions of certain types of employees in the total population change in the long run. This will be observed in more detail after stationary points are calculated.

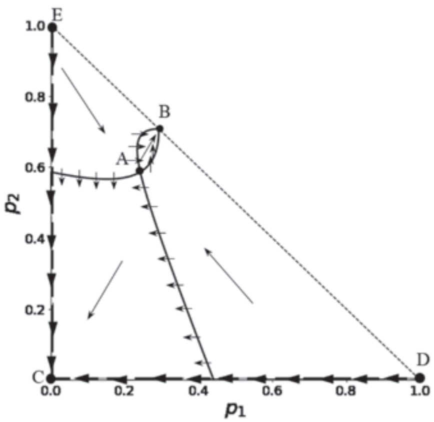

Figure 8: Vector field of „Enthusiast“ and „Worker efforts – author's original.

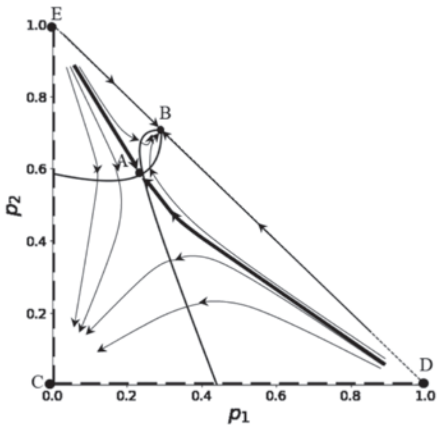

Figure 9: Orbits of „Enthusiast“ and „Worker efforts – author's original.

In Figure 8 and Figure 9, five intersection points A, B, C, D and E will be observed. Those five points are stationary points of this system. While C(0,0), D(1,0) and E(0,1) can be easily observed from the above graphs, the other two intersection points (A and B) from the figure should be calculated using (28) and (31) and the fact that in the intersection it should be:

$$2*p_2^2+(6*p_1-6+\sqrt{2})*p_2+2*p_1^2-4*p_1+2=2*p_2^2+(6*p_1-4)*p_2$$
$$+2*p_1^2-\left(\sqrt{2}+4\right)*p_1+2$$

From above it is implied:

$$\mathrm{p}_1=\left(\sqrt{2}-1\right)*\mathrm{p}_2$$

Putting this in (28) and (31) point A and B are calculated as:

$$\mathrm{A}\left(3*\sqrt{2}-4,2-\sqrt{2}\right),\quad \mathrm{B}\left(\frac{2-\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$$

It would be interesting to observe, using the designed model, what the total average expected cost of effective work in the company's employee population of N employees group (2N employees) is.

The overall expected cost and expected effective work of all employee group combinations in the population of 2N employees can be calculated using Table 3 as:

Total TEC =

$$\left(2*p_1^2+\left(4-\frac{\sqrt{2}}{2}\right)*2*p_1*p_2+2*2*p_1*p_3+5*p_2^2+3*2*p_2*p_3+5*p_3^2\right)*N$$

Total EW =

$$\left(2*p_1^2+\sqrt{2}*2*p_1*p_2+1*2*p_1*p_3+2*p_2^2+1*2*p_2*p_3+2*p_3^2\right)*N$$

From above, the total average expected cost of effective work for the population is:

$$\text{Total AEC}=\frac{\mathit{Total\,TEC}}{\mathit{Total\,EW}}=$$

$$=\frac{2*p_1^2+\left(4-\frac{\sqrt{2}}{2}\right)*2*p_1*p_2+4*p_1*p_3+5*p_2^2+6*p_2*p_3+5*p_3^2}{2*p_1^2+\sqrt{2}*2*p_1*p_2+2*p_1*p_3+2*p_2^2+2*p_2*p_3+2*p_3^2} \tag{32}$$

Using point A, the short-run total average expected costs of effective work in A can be calculated. In this point the proportions of every employee type are: $p_1 = 24.26\%, p_2 = 58.58\%, p_3 = 17.16\%$.

Putting those probabilities in the above formula (32), the total expected cost is equal 3.687 and total expected effective work of the population is 1.549, which gives 2.38 as the total expected average cost of effective work for the population.

Employees may at some point begin to behave unpredictably and some may deviate from their usual behavior. Observing point A in Figure 9 and the movements of the orbits, it is suspected that even a small change in employee behavior in the long run can take it further from point A to points B or C. From the figure, it can be observed that if the change is such that the shift is below prominent orbits (those which go directly to point A) then in the long run it leads to point C. If the change is such that the shift is above the prominent orbits then in the long run it leads to point B. These visually (geometrical) anticipated assumptions can be checked quantitatively by using Stability of nonlinear systems theorem (Simon and Blume, 1994). For the stability test, Jacobian matrix and eigenvalues should be calculated for all stationary points mentioned above.

First using (26) and (29) the following functions are defined:

$$f(p_1,p_2) = \dot{p}_1 = p_1 * \frac{1}{4} * \left[-2 + 4*p_1 + \left(6-\sqrt{2}\right)*p_2 - 2*p_1^2 - 2*p_2^2 - 6*p_1*p_2\right]$$

$$g(p_1,p_2) = \dot{p}_2 = p_2 * \frac{1}{4} * \left[-2 + \left(\sqrt{2}+4\right)*p_1 + 4*p_2 - 2*p_1^2 - 2*p_2^2 - 6*p_1*p_2\right]$$

For the Jacobian matrix determination, the partial derivatives of the above defined functions should be calculated. The calculations are given by:

$$\frac{\partial f}{\partial p_1} = -\frac{1}{2} + 2 * p_1 + \left(\frac{3}{2} - \frac{\sqrt{2}}{4}\right) * p_2 - \frac{1}{2} * p_2^2 - 3 * p_1 * p_2 \tag{33}$$

$$\frac{\partial f}{\partial p_2} = \left(\frac{3}{2} - \frac{\sqrt{2}}{4}\right) * p_1 - p_1 * p_2 - \frac{3}{2} * p_1^2 \tag{34}$$

$$\frac{\partial g}{\partial p_1} = \left(\frac{\sqrt{2}}{4} + 1\right) * p_2 - p_1 * p_2 - \frac{3}{2} * p_2^2 \tag{35}$$

$$\frac{\partial g}{\partial p_2} = -\frac{1}{2} + \left(\frac{\sqrt{2}}{4} + 1\right) * p_1 + 2 * p_2 - \frac{1}{2} * p_1^2 - \frac{3}{2} * p_2^2 - 3 * p_1 * p_2 \tag{36}$$

Jacobian is constructed from partial derivatives of the functions in a way that each row consists of partial derivatives of the function by all variables. In this case, two functions (f and g) and two variables (p_1 and p_2) exist and Jacobian looks like:

$$J(p_1,p_2)=\begin{bmatrix}\frac{\partial f}{\partial p_1}(p_1,p_2) & \frac{\partial f}{\partial p_2}(p_1,p_2)\\ \frac{\partial g}{\partial p_1}(p_1,p_2) & \frac{\partial g}{\partial p_2}(p_1,p_2)\end{bmatrix} \tag{37}$$

(37) is completely determined with (33), (34), (35) and (36).

From the stability analysis using Jacobian, the eigenvalues form the Jacobian matrix should be calculated. For this purpose, the defined determinant should be:

$$D\left(J(p_1,p_2),\lambda\right)=\begin{vmatrix}\frac{\partial f}{\partial p_1}(p_1,p_2)-\lambda & \frac{\partial f}{\partial p_2}(p_1,p_2)\\ \frac{\partial g}{\partial p_1}(p_1,p_2) & \frac{\partial g}{\partial p_2}(p_1,p_2)-\lambda\end{vmatrix}$$

To calculate eigenvalues above, the determinant should be calculated and equaled to 0. This expression is given by:

$$\left(\frac{\partial f}{\partial p_1}(p_1,p_2)-\lambda\right)*\left(\frac{\partial g}{\partial p_2}(p_1,p_2)-\lambda\right)-\frac{\partial f}{\partial p_2}(p_1,p_2)*\frac{\partial g}{\partial p_1}(p_1,p_2)=0 \tag{38}$$

Using (37) for each stationary point eigenvalues are calculated and the stability test will be performed according to the above-mentioned theorem about stability of nonlinear systems.

Based on the values of eigenvalues, the theorem proposes the stability of a corresponding stationary point. The following is an analysis of the stability of stationary points:

For point C Jacobian is: $J(0,0)=\begin{bmatrix}-\frac{1}{2} & 0\\ 0 & -\frac{1}{2}\end{bmatrix}$

Using (38) eigenvalues are calculated and given by: $\lambda_1=\lambda_2=-\frac{1}{2}$

As $\lambda_1<0$ and $\lambda_2<0$, according to theorem about stability of nonlinear systems this point is an asymptotic stable stationary point.

For point D Jacobian is: $J(1,0)=\begin{bmatrix}0 & -\frac{\sqrt{2}}{4}\\ 0 & \frac{\sqrt{2}}{4}\end{bmatrix}$

Using (38) eigenvalues are calculated and given by: $\lambda_1=0$ and $\lambda_2=\frac{\sqrt{2}}{4}$

As $\lambda_1 = 0$ and $\lambda_2 > 0$, according to theorem about stability of nonlinear systems this point is not a stable stationary point.

For point E Jacobian is: $J(0,1) = \begin{bmatrix} \frac{1}{2} - \frac{\sqrt{2}}{4} & 0 \\ \frac{\sqrt{2}}{4} - \frac{1}{2} & 0 \end{bmatrix}$

Using (38) eigenvalues are calculated and given by: $\lambda_1 = 0$ and $\lambda_2 = \frac{1}{2} - \frac{\sqrt{2}}{4} \approx 0.15$

As $\lambda_1 = 0$ and $\lambda_2 > 0$, according to theorem about stability of nonlinear systems this point is not a stable stationary point.

For point A Jacobian is: $J\left(3*\sqrt{2}-4, 2-\sqrt{2}\right) = \begin{bmatrix} 34-24*\sqrt{2} & -\frac{89}{2} + \frac{63*\sqrt{2}}{2} \\ \frac{13}{2} - \frac{9*\sqrt{2}}{2} & 17-12*\sqrt{2} \end{bmatrix}$

Using (38) eigenvalues are calculated and given by: $\lambda_1 = 14.92$ and $\lambda_2 = -14.92$

As $\lambda_1 > 0$ and $\lambda_2 > 0$, according to theorem about stability of nonlinear systems this point is not a stable stationary point.

For point B Jacobian is: $J\left(\frac{2-\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) = \begin{bmatrix} \frac{5}{2} - \frac{7}{4}*\sqrt{2} & -\frac{1}{4} + \frac{5\sqrt{2}}{4} \\ 0 & \frac{1}{4} - \frac{3\sqrt{2}}{4} \end{bmatrix}$

Using (38) eigenvalues are calculated and given by: $\lambda_1 = \frac{1-\sqrt{2}}{4} \approx -0.1$ and $\lambda_2 = \frac{1-3*\sqrt{2}}{4} \approx -0.81$

As $\lambda_1 < 0$ and $\lambda_2 < 0$, according to theorem about stability of nonlinear systems this point is an asymptotic stable stationary point.

From the above analysis, the conclusion is that point B and C are asymptotic stable and A, D, and E are not stable. Using the above analysis and Figure 9, long – run changes within the employees' population can be described.

Orbits (Figure 9) describe how the proportions (frequencies) of certain types of employees in the total population change in the long run. Of particular interest are the two orbits leading to the unstable point A because they divide space into two surfaces of attraction. If the initial probability distribution of employee types is below these two orbits, the long-run result of the replicator dynamics is described by point C. Then all employees behave like "Worker" and the expected average cost of effective work is 2.5. If the initial probability distribution of employee types is above these two prominent orbits, the long-run result of the replicator dynamics is described by point B where the proportion of "Enthusiast" is $1-\frac{\sqrt{2}}{2}$ and the proportion of "Worker" is $\frac{\sqrt{2}}{2}$. Using (32) it is obtained that the total expected cost in the population is equal to 4.035 and the total expected effective work is 1.757, which gives 2.296 as a total average cost of effective work for the whole population.

Comparing these results with the total average cost of effective work in point A that were calculated above, it can be seen that those short – run costs are less than in point C, but higher than in point B. So, in short – run it is preferred to be in A over C and B over A. But in the long run, dynamic of the population tends to go toward two stable points B and C.

Conclusion

In a complex business environment, employee motivation of different characteristics is a significant problem for the short and long run financial aspects of the company. At the same time, there is a lack of papers in the literature that quantitatively describe the relationship between the employer and heterogeneous employees who have the possibility of moral hazard. Starting from the practical problem of different employee types motivation, this paper uses game theory and replicator dynamics to model a new conceptual framework for analyzing how the stimulation system affects employee performance and expected average costs of effective work in the short and long run. The expected utility of a worker depends on wages, employer incentives, effective work, and the likelihood of being caught not working. This paper first describes Nash equilibrium of all possible employee pairs that depends on employer stimulation. Employees are divided into three types ("Enthusiast", "Worker" and "Parasite") who have different attitudes towards work. In particular, the reaction function of the "Worker" is analyzed and the amount of stimulation by which the employer minimizes the expected costs per unit of effective work for each pair of employees is determined. The relationship between employer and employees working in teams is modeled as a principal-agent problem. The total expected costs per unit of efficient work in the short run depend on the distribution of employees of different characteristics. The question arises as to the long-run distribution of employees and how much the total expected costs per unit of efficient work are. Replicator dynamics (evolutionary game theory), is used to analyze this problem. Isocline analysis derives stationary points whose stability is described using eigenvalues. The population dynamics of employees of different characteristics is graphically described by a vector field and orbits. Thus, combining game theory and population dynamics, this paper creates a new conceptual framework for quantitative analysis of the impact of motivation of heterogeneous employees on short and long run expected costs per unit of efficient work.

In addition to this model explaining the differences in the average cost of efficient work for different pairs of employees, it is shown that long-run stability of the employee population is achieved in two cases. In one case, the population consists of a "Parasite" who behaves like a "Worker" and the expected average cost of efficient work is 2.5 times higher than wages. The reason for having the cost 2.5 times higher than wages can be explained by the fact of having a moral hazard situation. In the second case, the population consists of approximately 29% "Enthusiast" and 71% "Worker" and the expected average cost of efficient work is approximately 2.3 times higher than the wage. As from the previous case, the moral hazard is the reason for having the cost 2.5 times higher than wages. The savings achieved compared to the previous case are described by a relative decrease in the expected average cost of efficient work by approximately 8%.

This paper shows how stimulation affects the effort of employees and the expected average cost of effective work in a complex dynamic environment in which the attitude of employees towards work is changing.

For future research, the problem is to design a combination of stimulation and rewards that would minimize the expected average cost of effective work in the long run.

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