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Risk Aversion and the Purchase of Risky Insurance

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Abstract

This paper examines the choice of an insurance contract when insurers might default on indemnity claims. In particular, we show that more-risk-averse preferences do not necessarily lead to the purchase of higher levels of insurance coverage in such situations. Our results are shown not to apply in situations where the insured receives a premium refund for nonperforming insurance policies.

1. INTRODUCTION

Insurance purchased at "fair" prices is usually considered a method of reducing a particular financial risk. However, this need not be the case when insurance policies have a possibility of not paying indemnities for their claims. If there exists a chance that the individual will not be indemnified following a loss claim, then the purchase of insurance also introduces a new risk, which is the risk of losing premium dollars as well as experiencing the loss. Since a higher degree of risk aversion implies a greater relative emphasis on downside risk, an increase in the level of risk aversion, *ceteris paribus*, will not necessarily lead to the purchase of a higher level of insurance coverage--a result which stands in marked contrast to the usual insurance literature. The purpose of this paper is to present this result more formally.

Types of situations in which a nonpayment of insurance claims is possible include the following:

(i) Insurer insolvency. In this case, the insurer has a probability of defaulting on claims.

(ii) Questionable perils and hazards. Although most claims are clearly valid or not, some occurrences are questionable within the legal scope of which losses are covered and which are not.

(iii) Contract conditions. Certain losses may not be covered under prescribed conditions. For example, there may be a waiting period before some loss is covered so that an occurrence during the waiting period is essentially uninsured.

Each of the above possibilities might be known by the insured and reflected in lower insurance premiums; but the insurance market still remains incomplete since these losses cannot generally be covered.

The model we use is essentially that of Doherty and Schlesinger (1986). Readers interested in a more complete development of the model are referred to this paper.

2. THE MODEL

Consider a risk-averse individual with preferences expressed by the von Neumann-Morgenstern utility function $U(W)$, $U'>0$, $U''<0$. We consider a simple three-state model in which no loss occurs with probability q_1, a loss that can be indemnified occurs with probability q_2, and a loss that cannot be indemnified occurs with probability q_3. The size of the loss itself is assumed to be the same in states 2 and 3. Thus, our model is a variation of the simple two-state, loss vs. no-loss model, modified to account for the chance of nonpayment. The model abstracts from any potential moral-hazard and/or adverse-selection problems, and the probabilities q_1, q_2 and q_3 are assumed to be fixed and known. We should also point out that states 2 and 3 are identical when no insurance coverage is purchased.

The consumer may protect against the loss by purchasing an insurance policy which promises to pay a fixed proportion α of the loss if it occurs. The complication is, of course, that there is a $q_3/(1-q_1)$ probability that the insurer does not pay on a loss claim.

The individual's expected utility following the purchase of insurance is

$$EU = q_1U(W_1) + q_2U(W_2) + q_3U(W_3) \tag{1}$$

where

$W_1 = A - P$

$W_2 = A - P - L + \alpha L$

$W_3 = A - P - L$

A = initial wealth level

P = insurance premium

L = magnitude of potential loss.

Insurance prices are assumed to be proportional to the expected insurance payment so that

$$P = m\alpha q_2 L \qquad \text{where } m \geq 1. \tag{2}$$

Note that insurance prices also reflect the probability of non-payment since they don't include q_3. Since we assume the premium to be given, we do not concern ourselves with the relationship between m and q_2 here.

The consumer's objective is to maximize expected utility (1) subject to the price schedule (2). The first-order condition is[1]

$$\frac{dEU}{d\alpha} = U'(W_2) - [BU'(W_1) + CU'(W_3)] = 0, \tag{3}$$

where

$$B = mq_1/(1-mq_2) \text{ and } C = mq_3/(1-mq_2).$$

It is straightforward from (3) to show that full coverage is purchased only when both $q_3 = 0$ (a zero chance of nonpayment) and $m = 1$

(actuarially-fair insurance prices). Otherwise, less than full coverage is purchased.[2]

3. INCREASED RISK AVERSION

We now turn our attention to the main focus of the paper, namely how insurance purchases are affected by the consumer's level of risk aversion. To this end, let utility function V represent uniformly more risk-averse preferences than U. By Kihlstrom and Mirman (1974), there exists a concave function g such that $V = g^\circ U$.

Consider now the marginal expected utility of V with respect to α, evaluated at the optimal insurance level under U, α^*.

$$\left.\frac{dEV}{d\alpha}\right|_{\alpha^*} = g'[U(W_2)]\ U'(W_2) - \{g'[U(W_1)]\ BU'(W_1) + g'[U(W_3)]\ CU'(W_3)\} \qquad (4)$$

We can assume, without loss of generality, that $g'[U(W_2)] = 1$. Thus, by the concavity of g,

$$g'[U(W_1)] < 1 < g'[U(W_3)]. \qquad (5)$$

Comparing (4) with (3), it follows from (5) that $dEV/d\alpha$ can be either positive or negative at α^*. Thus, the optimal level of insurance coverage can be either higher or lower with the more risk-averse utility. This result stands in contrast to the more definitive result that more coverage is always purchased (whenever possible) under more risk-averse preferences in models that don't allow for nonpayment of claims. Note that when nonpayment is impossible $q_3 = 0$ and thus comparing (4) with (3) it is trivial to show that more coverage is purchased under preferences V.

4. AN EXAMPLE

We consider the following example to demonstrate our point that it is possible for a more risk-averse individual to purchase less insurance coverage if there is a positive probability of not being indemnified for a loss. Let us suppose the following utility function

$$U(W) = -e^{-\beta W} \text{ with } \beta > 0. \tag{6}$$

This function has the property that the individual's absolute risk aversion as defined by Arrow and Pratt is given by the exponential coefficient β.

In our example, β will vary from 0.5 to 4.5. We further assume that $A = 3.43$, $L = 1.43$, $m = 1.3$ and $q_1 = 0.8$. This implies a thirty percent loading on the net-premium and a loss-probability of 0.2. We will consider three different cases: $q_3 = 0$, $q_3 = 0.02$, and $q_3 = 0.06$. The first case describes what standard insurance-economic theory normally assumes, that the insured will always be indemnified if a loss occurs. In the second and third case, a ten or thirty percent chance is respectively assumed that a loss occurs but the insurance company refuses to reimburse the individual.

Computer simulation results are plotted in Figure 1. The upper curve shows the optimal level of insurance coverage for the case where $q_3 = 0$. As is well known, α^* increases as risk aversion increases.

Insert Figure about here

The optimal level of coverage is 0.52 for $\beta = 0.5$ and increases to 0.95 as β grows to 4.5. The second and third curves correspond to the cases where there is a positive probability that the insurer will not indemnify the insured's loss-claim. When $q_3 = .02$, the optimal level of insurance coverage is increasing to a maximum of approximately 0.77 at $\beta = 1.75$ and decreases afterwards as absolute risk aversion. Similar results are seen to obtain for the case where $q_3 = .06$ as well.

5. "MONEY BACK GUARANTEES"

There is a provision in some actual life insurance contracts that suggests an interesting extension of our model. Under this provision, there is a return of the premium paid if the insured dies and the cause of death is excluded under the policy. Although we know of no similar provisions in property and liability insurance contracts, it is interesting to examine whether unambiguous comparative-static results hold (with respect to the degree of risk aversion) for the case where the premium is returned to the policyholder following the nonpayment of an insurance claim--a "money back guarantee" of sorts. In this case, the actuarially-based premium is given by

$$P = (m\alpha q_2 L)/(1-q_3). \qquad (7)$$

It is straightforward to show that

$$\frac{dEU}{d\alpha} \propto - mq_1U'(W_1) + [q_1 + (1-m)q_2]U'(W_2). \tag{8}$$

From the proportionality condition (8) we see that the sign of $dEU/d\alpha$ is independent of q_3. It follows trivially from (8) that full coverage is purchased whenever prices are actuarially fair (m=1) and partial coverage is purchased when prices are "loaded" (m>1), thus restoring the usual results of rational insurance purchasing (cf. Mossin (1968)). It is also straightforward to show from (8) that strictly-more-risk-averse preferences would lead to the purchase of a higher level of insurance coverage, unless of course m=1, in which case full coverage is purchased with both utility functions.[3]

6. CONCLUDING REMARKS

We have shown that increased risk aversion does not necessarily imply a higher level of insurance coverage when insurers can default on indemnity payments. However, we should caution against making too much of our results in a practical sense. Our previous example shows an inverse relationship between the degree of risk aversion and the level of insurance coverage only for levels of risk aversion that are much higher than levels thought to be empirically valid. The probabilities of nonpayment by insurers in the example seem to also be unrealistically high. We were unable to obtain an example with parameter values in more realistic ranges. Thus, while we have shown that it is theoretically possible to see such inverse relationships between insurance levels and risk aversion, we might not expect to observe such results very often as a matter of practice.

FOOTNOTES

[1] Second-order conditions are not trivially satisfied, but are assumed to hold. For a discussion of the complexities involved in the second-order conditions, the reader is referred to Schlesinger (1985).

[2] For a more complete analysis of insurance levels, see Doherty and Schlesinger (1986).

[3] This is accomplished by substituting $V = g \circ U$ for U in the right-hand-side of the proportionality condition (8) and differentiating. Evaluating $dEV/d\alpha$ at the optimal coverage level for U shows that coverage should be increased.

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Figure 1: Optimal Insurance Purchasing and Risk-Aversion