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The Activity of Abstraction in Physical Chemistry Problem Solving and Instruction

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THE ACTIVITY OF ABSTRACTION IN PHYSICAL CHEMISTRY PROBLEM SOLVING AND INSTRUCTION

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ABSTRACT

THE ACTIVITY OF ABSTRACTION IN PHYSICAL CHEMISTRY PROBLEM SOLVING AND INSTRUCTION

August 2021

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Productive problem solving, concept construction, and sense making occur through the core process of abstraction. Although the capacity for domain-general abstraction is developed at a young age, the role of abstraction in increasingly complex and disciplinary environments, such as those encountered in undergraduate STEM education, is not well understood. Undergraduate physical chemistry relies particularly heavily on abstraction because it uses many overlapping and imperfect mathematical models to represent and interpret phenomena occurring on multiple scales. To reconcile these models, extract meaning from them, and recognize when to apply them in problem solving requires processes of abstraction. This dissertation aims to develop a framework that can be used to make abstraction in physical chemistry visible to better understand how undergraduate physical chemistry students navigate these processes and abstract in problem solving scenarios.

Using an activity theoretical lens, this dissertation has three aims: (1) to operationalize abstraction as a series of epistemic actions, and to use this operationalization to investigate (2) what motivates and influences whether abstraction is realized in the moment, and (3) the role abstraction plays in physical chemistry instruction. First, problem solving teaching interviews with individuals and pairs (n=18) on thermodynamics and kinetics topics are analyzed using a constant comparative approach. The resulting Epistemic Actions of Abstraction framework characterizes eight epistemic actions along two dimensions: increasing abstractness relative to the context (*concretizing*, *manipulating*, *restructuring*, and *generalizing*) and nature of the object the action operates on (*conceptual* or *symbolic*). These teaching interviews are then inductively analyzed to identify what sparks abstraction and the influence of interaction on abstraction. Three types of needs (*task-directed*, *situational-insufficient*, and *situational-emergent*), and three major themes (*framing*, *interviewer intervention*, and *peer interaction*) are found. Finally, a multiple-case study of physical chemistry instructors (n=2) at two different institutions is conducted to investigate how and why instructors model abstraction. Analysis of classroom video and video-stimulated recall interviews yields two identified roles abstraction plays in physical chemistry instruction: developing mathematical tools grounded in conceptual understanding, and developing conceptual understanding grounded in mathematics. Implications for research and teaching are discussed.

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DEDICATION

To the women in my family, who passed their strength onto me.

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LIST OF ABBREVIATIONS

ABBREVIATION	TERM
CC	Conceptual concretizing
CM	Conceptual manipulating / conceptual manipulation
CR	Conceptual restructuring
CG	Conceptual generalizing / conceptual generalization
PC1	Physical chemistry I
SC	Symbolic concretizing
SM	Symbolic manipulating / symbolic manipulation
SR	Symbolic restructuring
SG	Symbolic generalizing / symbolic generalization
VSRI	Video stimulated recall interview
ZPD	Zone of proximal development

CHAPTER 1

INTRODUCTION

"In the events however, that human beings perceive reality as dense, impenetrable, and enveloping, it is indispensable to proceed with the investigation by means of abstraction. This method does not involve reducing the concrete to the abstract (which would signify the negation of its dialectical nature), but rather maintaining both elements as opposites which interrelate dialectically in the act of reflection. This dialectical movement of thought is exemplified perfectly in the analysis of a concrete existential, "coded" situation. Its "decoding" requires moving from the abstract to the concrete; this requires moving from the part to the whole and then returning to the parts; this in turn requires that the Subject recognize himself in the object (the coded concrete existential situation) and recognize the object as a situation in which he finds himself, together with other Subjects. If the decoding is well done, this movement of flux and reflux from the abstract to the concrete […] leads to the supersedence of the abstraction by *the critical perception of the concrete, which has already ceased to be a dense, impenetrable reality."* (Freire, 1970, p. 105)

"[Abstraction is] the special separation, the singling out of what is common and its juxtaposition to the particular." (Davydov, 1972/1990, p. 7)

The notion of the abstract and of abstraction has preoccupied humans since the days of Aristotle. Making sense of what it means to develop an idea or concept that is not grounded in concrete, everyday reality has been pursued by philosophers and scientists. These abstractions have also made up the core of much of science education, as we ask students to comprehend ideas for which they have no simple physical referents, from concepts primarily defined mathematically (e.g., wavefunctions) to ones they cannot observe and that are expressed through increasingly complex models (e.g., atoms). At the same time, science is often grounded in empirical investigations and observations. These empirical data are interpreted through scientific models and abstractions, which in turn refine and shape the same models. As noted in the quotes above, both the concrete (observations of physical phenomena) and the abstract (models and concepts) have to be maintained as opposites (Freire, 1970/2017), and the abstract has to be juxtaposed to the particular (Davydov, 1972/1990).

Understanding the role abstraction plays in learning science is complex, as it requires us to parse several ideas, such as what it means for something to *be* abstract (what makes an idea more or less concrete?) and what it means *to abstract* (how does one develop abstract ideas? How does one remove particulars?). To make this more complicated, what we call "abstraction" often refers to slightly different concepts, such as capacity to develop categories gained by children during development (Hahn & Chater, 1998; Piaget, 1964), the process by which one develops theoretical knowledge, such as scientific knowledge (Davydov, 1972), and the concepts (abstractions) themselves (Mitchelmore & White, 2007; Ozmantar & Monaghan, 2007).

A brief review of abstraction

For the purposes of this dissertation, it is most fruitful to focus this brief overview on two concepts related to abstraction. In a study on chemistry reasoning, Weinrich & Sevian

(2017) focused on what it means for something to be considered "abstract" and what is the process of "abstraction." These two facets are the focus of this introduction because learning in chemistry involves students making sense of abstract concepts through abstracting processes. However, to understand how these processes occur, it is important to understand what makes a concept more or less abstract, and what an abstracting process actually entails.

How abstract an entity is can be captured by the idea of "abstractness." Abstractness may characterize the *quality* of an idea or object. We consider the general category of *dog* more abstract than the family German Shepherd, because *dog* refers to a general category that can be described by a minimum set of common features, whereas Bo, the family dog, is a specific, concrete referent with features that do not necessarily describe dogs beyond him (Inhelder & Piaget, 1958; Piaget, 1964). Similarly, abstractness can be used to describe the characteristics of scientific concepts, such as those encountered in chemistry. *Experiment* is an abstract idea, *synthesis* is more specific yet still abstract, and *the Grignard reaction I ran last Thursday* is a specific referent to a concrete event that occurred at some point in time. Recent work further problematizes the distinction between concrete and abstract, by suggesting that concepts be characterized on two axes: abstract-concrete, and generic-specific (Bolognesi et al., 2020).

Characterizing the abstractness of scientific concepts becomes more complicated as we consider the relationship between *what* is considered abstract, and *who* does the considering. Some researchers, such as systemic functional linguists, believe that pieces of knowledge are objects of study that have the property of semantic gravity, which describes how removed knowledge is from concrete, observable reality (Maton, 2013). From this perspective, abstractness is a characteristic of the piece of knowledge, not of how that piece of knowledge is perceived (Maton, 2014). Others consider abstractness to be an emergent property of the relationship between the knower and the situation, and that whether a concept is abstract or concrete depends on how it is being used (Barsalou et al., 2018). This raises several questions pertinent to what makes a scientific concept abstract. Is a molecule as abstract to a general chemistry student who has only encountered the idea in a textbook, as it is to a biophysicist who does single molecule force spectroscopy? Is an electric field as abstract to an introductory physics student as it is to their professor, who has spent years refining their understanding and populating it with details? What if the context is changed? Is the concept of atom more abstract in a philosophy class than it is in a chemistry class, where it is imbued with specific meaning?

This role of context becomes more complicated when it is applied to the process by which abstract concepts are reified—e.g., how knowers *acquire* these concepts. The idea of "context" has two meanings when applied to a process of abstraction. Context encompasses the sociocultural environment in which an abstraction process takes place (for example, a student *abstracting* the concept of atom in a chemistry class or in a philosophy class). However, context also refers to the aspects of a concept that are grounded in concrete reality and are removed through a process of decontextualization, the idea that abstracting involves generating an object that "stands above." For the former, context refers to a set of external factors, and for the latter, context refers to the properties that are excised when one abstracts (Hershkowitz et al., 2001). In fact, in a recent review on the use of the concept of abstraction in mathematics education, Scheiner & Pinto (2016) identified decontextualization as one of the major controversies of opinion. For example, Noss, Hoyles, and Pozzi (2002) suggest instead that abstraction is situated:

> "The idea of abstraction as a conceptualization or a piece of knowledge lying in a separate realm from action, tools, language, or any external referential sign system is an important ingredient of mathematical discourse. An abstraction gains its power and applicability from the fact that it is distinct from the situation that gave rise to it; it lies inside a system with its own objects and its own rules for transforming them. This self-contained aspect of formal mathematical abstraction is central to its utility. Situated abstraction does not seek to challenge that utility but questions whether mathematical abstractions can always be separated from the context of their construction or application. On the contrary, we suggest that what is seen as "noise" in the traditional view of abstraction may be, in fact, central to its meaning." (Noss et al., 2002, p. 207)

Situated abstraction views context as central to the meaning of an abstraction and of a concept. Van Oers (1998; 2001) similarly argued that decontextualization is not an appropriate basis for conceptualizing abstraction, because it removes the individual and the knower (to whom context is relative), and that context is part of what makes a concept rich. This also aligns with the work reviewed above, that the abstractness of a concept may be relative to a knower. Noss and Hoyles (1996) theorized abstraction as a process of connection rather than of ascension—that is, that abstraction is a process of developing increasingly complex and connected pieces of knowledge, rather than a process of detaching details hierarchically. As concepts become more connected, they become more imbued with meaning, and thus become more concrete. These newly consolidated ideas can then be used as objects in further abstraction (Tabach et al., 2017).

Scheiner (2016) summed up these different ideas around abstraction as two major trends in how abstraction has been framed in mathematics education literature. Abstraction-from-actions approaches “assume[s] that learners first learn processes and procedures for solving problems in a particular domain and later extract domain-specific concepts through reflection on actions on known objects” (p. 168). That is, abstraction-from-actions approaches view abstraction as a process of extracting details to make more generalizable concepts (a process of “ascension”). Abstraction-from-objects approaches “assume that learners are first faced with specific objects that fall under a particular concept and acquire the meaningful components of the concept through studying the underlying mathematical structure of the objects” (p.171). That is, abstraction-from-object approaches view abstraction as way to make increasingly complex concepts and make the abstract concrete by populating it with details (a process of “connection”).

To illustrate how these may be relevant in chemistry, consider a student learning about periodic trends. An abstraction-from-actions approach to learning about periodic trends may occur when a student works through multiple examples and starts to recognize generalizable patterns about the relationship between atomic number and atomic radius. An abstraction-from-objects approach, on the other hand, may involve figuring out periodic trends by deeply studying the periodic table and the relationship between atomic structure and placement on the periodic table. Scheiner suggests that these two types of abstraction represent equally important processes that occur during learning and suggests that rather than choosing one or the other, research focuses on abstraction as a dynamic process of sense making, which may involve ascension or connection.

This brief review of the literature suggests multiple things: first, the abstractness of concepts cannot be separated from the learner. Second, both abstractness and abstraction are dynamic processes that shift as a knower interacts with the world and acquires knowledge. Third, the way in which the knower interacts with the world is important to understanding abstraction. When trying to understand how students abstract at the undergraduate level, it may be most useful to consider abstraction as a dynamic process that is context-dependent and contextually situated through which a learner makes sense by making connections between concepts (Scheiner & Pinto, 2016).

How abstraction has been framed in chemistry education literature

Most of the work on abstraction reviewed above has drawn from literature outside of chemistry education, such as cognitive science, linguistics, and mathematics education. However, because abstraction is a contextually-dependent, and thus domain-specific, process, it is important to examine how it has been studied in the field of chemistry education until this point. Previous work examining abstraction in chemistry has largely focused on how students reason with and generate abstract representations in problem solving and toward conceptual learning (e.g., Corradi et al., 2014; Domin & Bodner, 2012; Graulich et al., 2012; M. Strickland et al., 2010). For example, Strickland and collaborators (2010) found that students largely reasoned with the surface features of representations such as diagrams, and postulated that as representations become more abstract, learners reason with the representation rather than the underlying concept (Schwartz, 1995). For example, in Strickland's study, students were asked to explain electron movement in an organic mechanism. One participant's reasoning focused completely on the features of the representation and suggested that moving the arrows was the reason for removing a leaving group, rather than reasoning about the underlying structural properties that cause electron movement, which is represented by the arrows. In a study of graduate organic chemistry, Domin and Bodner (2012) found that abstract representations were more utile for problem solving when they were one of many representations students drew on; otherwise, highly abstract representations could hinder understanding. To characterize the abstractness of concepts in chemistry, Santos and Mortimer (2019) drew on literature in physics and on Johnstone's triangle to develop a taxonomy that characterizes the abstractness of representations in chemistry (Jiménez et al., 2016; Johnstone, 1991).

A smaller set of literature has focused on abstraction as a process. Work done by this research group conceptualized abstraction as a process of representation mapping (Sevian et al., 2015; Weinrich & Sevian, 2017), which builds largely on abstraction-from-actions approaches. Representation mapping was developed to reconcile competing traditions in cognitive science that reasoning can be distinguished between rule-based and similarity-based processes (Hahn & Chater, 1998). This approach proposed that reasoning can be characterized on two axes: the *abstractness* of a mental representation, and how *strictly matched* the mental representation is with one's prior knowledge. Weinrich and Sevian (2017) used this framework to examine abstraction in organic chemistry problem solving and found that students trended toward higher abstractness over the course of the semester, and that students who proposed more plausible mechanisms had more representational flexibility. That is, these students tended to use different types of abstracting while solving problems, while students who tended to propose implausible mechanisms used strict matching and representations with low levels of abstractness. Frey and collaborators (2017) also looked at the relationship between abstracting and success in chemistry. They classified learners into two types based on how they developed concepts: through rote memorization (exemplar learners) or through abstraction (abstraction learners). They found that abstraction learners had some advantages in performance in general and organic chemistry in both exams (McDaniel et al., 2018) and problem solving (Frey et al., 2020).

This brief review of the chemistry education literature shows that most work done in chemistry has focused on how students work with and understand abstract representations. However, less work has examined abstraction processes, and much of this work has tended to view it as a static capacity, rather than a dynamic and situated process. Furthermore, most of the literature reviewed focuses on organic chemistry rather than other subfields. However, abstraction may play an important role in other chemistry courses as well. Sevian and collaborators (2015) identified thermodynamics (typically the first semester of physical chemistry) as a course in the undergraduate chemistry curriculum where there may exist an "abstraction threshold, at which point a typical student's innate capacity for abstraction is not matched to the complexity of the problems being posed" (Sevian, et al., 2015, p. 444).

Focus on physical chemistry

The reasons why the first semester of physical chemistry (PC1) was identified as a potential abstraction threshold course are plenty. Students and instructors perceive the concepts in physical chemistry as challenging because they are highly abstract (Bain et al., 2014; Sözbilir, 2004). This is compounded by the fact that these concepts are often expressed and defined mathematically, and that shifting between mathematical and conceptual sense making is often challenging for learners (Becker & Towns, 2012). However, instructors view understanding the connection between math and concepts as one of the major goals of physical chemistry instruction (Bain et al., 2014; Fox & Roehrig, 2015; Mack & Towns, 2016). Furthermore, PC1 is one of the more advanced courses in the undergraduate chemistry sequence, and as ideas and concepts become more complex, there is often more utility for abstract representations to express these ideas (Domin & Bodner, 2012).

Therefore, the problem of practice that guides this dissertation is that of how students are expected to build and apply knowledge in undergraduate physical chemistry. A national survey of physical chemistry instructors (Fox & Roehrig, 2015) found that a key aspect of physical chemistry courses is problem solving, and that problem solving is a common tactic to support students' development of conceptual knowledge. In organic chemistry, there is evidence that students do not necessarily do this, and that there is a disconnect between what professors expect students to take from problem solving, and what students actually do take away (Caspari et al., 2018), however, this has not been studied in physical chemistry. To examine this disconnect, this dissertation will examine both the role abstraction plays in student problem solving in thermodynamics and kinetics as well as the role abstraction plays in instruction.

Activity theory and sociocultural theory as a theoretical lens

The overarching theoretical framework that guides this dissertation is activity theory. First, as motivated in the brief review of abstraction, abstraction is a socially mediated, dynamic process that occurs during sense making. It is thus important to use a theoretical framework that can capture this, as well as capture how context and actions feed into each other. Second, and more practically, activity theory provides a structure to investigate and develop a conceptual framework to study abstraction in chemistry problem solving by identifying three important aspects: the actions, the need, and the conditions (Lantolf, 2000; van Aalsvoort, 2004).

Activity theory is a theoretical framework for organizing human actions that is derived from sociocultural theory (Leont'ev, 1978). Sociocultural theory views learning as something that happens in the social sphere rather than in the mind of the individual. Knowledge is constructed through dialogic interaction and is situated in sociocultural and sociohistorical contexts (Vygotsky, 1978). Activity theory is grounded in sociocultural epistemologies and is interested in how knowledge and activities are contextualized in lived realities, and how knowledge and actions are mediated through interaction.

In activity theory, the unit of human work is an "activity." An activity is motivated by a need (culturally constructed or biological), which becomes a motive when it is directed at a certain object and is realized in goal-directed actions. The actions that can be taken depend on the objective or external conditions, such as cultural or geographic context. These three levels—action, motivation, and condition—make up an activity, which can only be directly observed by others at the level of condition (Lantolf, 2000). This can be illustrated by the example of a chemist running a column. In the laboratory (*condition*), the chemist can be observed preparing a column and collecting aliquots. These acts are considered *actions* because they are goal-directed; they are performed to separate a mixture. The goal of each action is determined by an overall *motivation*; here, the motive is to purify the substance. The motive and the action-goals cannot be observed directly, because they are both internally motivated (the chemist herself has reasons to purify the substance) and sociohistorically situated (the history of column chromatography and the way the chemist was trained influence her decision to use it in this moment). However, action and motive both *inform* what is observed at the level of condition (what the chemist is physically doing) and *are informed by* the conditions (the fact she is working in a synthetic chemistry lab in the 21st century). By studying these three levels, the following questions can be answered: “What is taking place? Why is this taking place? How is what is taking place informed and constrained by external conditions?” Because action, motive, and condition inform and constrain each other, the activity has to be considered in light of all three.

Overview of this dissertation work

This brief overview of the literature demonstrates the two broad problems that motivate this work. First is a problem of practice: there is a disconnect between instructors and students in physical chemistry that may be bridged through a better understanding of abstraction. Second is a problem of theory: the work that has been done in chemistry does not provide a working definition of abstraction that aligns with our understanding that emerges from current literature in adjacent fields. In this work, the problem of theory is addressed first by operationalizing abstraction through activity theory. Chapters 2 and 3 report the development of a conceptual framework, the Epistemic Actions of Abstraction in Physical Chemistry, through a constant comparative process. To develop this conceptual framework, teaching interviews designed to elicit abstraction were conducted, and the data were analyzed through cycles of comparison with literature to refine a working definition of abstraction. This framework was then applied to address the problem of practice from the perspective of student problem solving (Chapter 4) and instructor teaching (Chapter 5). Chapter 4 reports the exploratory analysis of problem-solving interviews in two physical chemistry contexts, kinetics and thermodynamics, to examine what elicits abstraction during problem solving as well as what features of interaction constrain or sustain abstraction. Chapter 5 reports a multiple case study of two physical chemistry instructors and the role abstraction played during their teaching.

The structure of these studies is guided by first-generation activity theory. First-generation activity theory conceptualizes activities as having three parts. An activity is constituted of actions (1), which occur in response to a need (2) under certain conditions (3) that determine the specifics of the activity. The chapters in this dissertation roughly map to these three features of an activity. Chapter 3 reports the types of actions that can be used to identify the abstraction activity. Chapter 4 examines how a need for abstraction might manifest during problem solving, and the features of interaction that influence how abstraction activities are realized. Chapter 5 looks at two different chemistry classrooms to begin to characterize the conditions under which domain-specific abstraction processes are fostered. Implications for instruction and research on physical chemistry teaching and learning, as well as the potential applicability of these findings to work beyond physical chemistry, are discussed.

CHAPTER 2

DEVELOPMENT OF THE EPISTEMIC ACTIONS OF ABSTRACTION IN PHYSICAL CHEMISTRY PROBLEM SOLVING FRAMEWORK

Introduction and Motivation

Developing deep disciplinary understanding of physical chemistry concepts is challenging. Both students and instructors have called for pedagogical approaches that foster deeper conceptual understanding in physical chemistry (Bain et al., 2014; Sözbilir, 2004). A common instructional approach in physical chemistry to foster and assess student conceptual understanding is problem solving (Mack & Towns, 2016). When students solve problems in physical chemistry, they may be tasked with applying their knowledge to novel situations and making connections between the different concepts encountered in the problem task. Through this connection making, students may further develop and refine their conceptual understanding of the topic at hand.

One approach researchers have taken to investigate how students develop disciplinary conceptual understanding from problem solving is by studying how they abstract—that is, how students extract salient details, make connections, and match to exemplar cases (Sevian et al., 2015). Studies in developmental psychology (Inhelder & Piaget, 1958) suggest that abstraction underlies the development of concepts and generalizations, particularly when learners are still developing their cognitive capacity for abstract thinking at young ages. However, even once learners have developed a capacity for domain-general abstract thinking, they still may not necessarily be able to apply this capacity to increasingly complex and abstract concepts, such as those encountered in learning complex disciplinary material in undergraduate chemistry (Davydov, 1972/1990). Furthermore, studies conducted in different science courses have found that not only is abstraction an important process for learners, but abstraction itself manifests in domain-specific ways due to the nature of the knowledge, for example in physics or in organic chemistry (Jiménez et al., 2016; Santos & Mortimer, 2019; Weinrich & Sevian, 2017).

In physical chemistry, we hypothesize that these domain-specific ways include how students reason conceptually and mathematically (Bain et al., 2018). Bridging these conceptual and mathematical ways of reasoning has been shown to be a major barrier to developing disciplinary understanding in physical chemistry (Sözbilir, 2004), because even when students can make sense of the mathematics and the concepts separately from each other, bringing these two things together to explain a system can be challenging (Becker & Towns, 2012). Additionally, although physical chemistry instructors typically use problem solving to foster student conceptual understanding, these problems are often algorithmic and can be solved through rote application of equations rather than engagement with the underlying concepts (Fox & Roehrig, 2015). This disconnect means that the contexts instructors design to bridge conceptual and mathematical understandings often fail to do so—that is, they do not elicit abstraction.

To better understand how this challenge specifically manifests in physical chemistry, this chapter presents the development of a domain-specific framework to operationalize abstraction in problem solving. This framework identifies two dimensions to actions taken in physical chemistry problem solving: the nature of the knowledge the problem solver draws upon (mathematical or conceptual) and how far removed (how abstract) the knowledge constructed in-the-moment is from the established problem space. These two dimensions are used to define two types of abstraction in physical chemistry: horizontal and vertical abstraction.

Research Question

This study grew out of an earlier project in the Sevian research group examining how abstraction manifests differently in engineering and chemistry domains, particularly in organic chemistry (Sevian et al., 2015; Weinrich & Sevian, 2017). The analytic framework for that study (representation mapping) focused on how abstractly students represented the problem space and their prior knowledge, as well as how they matched the problem space and their relevant prior knowledge. In preliminary work applying this framework to physical chemistry, we found that although this approach was fruitful, it fully captured neither the differences in the abstractness that emerged from the use of mathematical and conceptual chemistry resources, nor how abstractness seemed to change over the course of a single problem-solving episode. Some studies in mathematics education analyze abstraction through the epistemic actions students make while problem solving (e.g., Williams, 2007; Halverscheid, 2008; Tabach et al., 2017). Combining this approach with theoretical aspects from the representation mapping framework, we designed a study to develop a domain-specific understanding of abstraction in physical chemistry.

The overarching research question that guided this study was: How does abstraction occur in students' reasoning while they are solving complex physical chemistry problems?

To motivate the domain-specific (physical chemistry) aspects of the framework, we first reviewed the literature on problem solving in physical chemistry. We then looked at prior literature on abstraction in chemistry and mathematics problem solving to provide a working definition of abstraction.

Literature Review

Problem Solving

Jonassen (2010) conceptualizes problem solving as twofold: the mental representation a problem solver constructs of what the problem is (the problem space), and the actions the problem solver takes to manipulate and transform the problem space toward a solution state. The problem space encompasses all of the possible paths a problem solver can take to reach a solution state. When problem solvers encounter a problem space, they use their previous experience and knowledge to transform the problem text into an internal representation (Reimann and Chi, 1989). For example, novice problem solvers often cue on surface features to decide what paths are possible, whereas experts use deep domain knowledge to classify problems (Chi et al., 1981). This process of identifying relevant information and restructuring the problem text to be something that can be solved using the problem solver's resources at hand is one of the first steps in problem solving (Bodner & McMillen, 1986). That is, one of the first things a problem solver does when they encounter a problem is to define the problem space by classifying the problem and relevant prior knowledge.

Problem Solving in Physical Chemistry

Problem solving approaches in physical chemistry have been studied in a variety of problem types and content contexts, such as chemical kinetics (Rodriguez et al., 2018), and free energy (Tsaparlis, 2005). Problem solving in undergraduate physical chemistry involves a mix of high level conceptual and mathematical reasoning, (Rodriguez et al., 2018), which can be challenging for students in part because the content covered in physical chemistry tends to be removed from concrete references to real systems (Sözbilir, 2004).

Bridging these two types of reasoning has also been identified as a specific challenge for students, particular during problem solving. For example, Becker and Towns (2012) showed that students working on Maxwell relation problems could successfully interpret the physical and mathematical meaning of a partial differential (e.g., (∂V/dT) P) separately; however, they struggle to apply this understanding to a problem. Similarly, Rodriguez and collaborators studied the epistemic games students use when solving kinetics problems. They found that students tended to compartmentalize their mathematics and chemistry knowledge, and often had trouble simultaneously navigating these two different sets of conceptual resources, even when they had otherwise productive problem solving approaches. (Rodriguez et al., 2020).

Because of the importance of problem solving in physical chemistry instruction and the difficulties students face in bridging conceptual and mathematical understanding, much of the literature on problem solving in physical chemistry has examined students' understanding of mathematical and conceptual knowledge. The disconnect between conceptual and mathematical reasoning has also been extensively documented in physics problem solving, where expert-like problem solving requires coherence between conceptual and mathematical reasoning. (e.g., Kuo et al., 2013, 2019; Niss, 2017).

Abstraction in Chemistry and Mathematics Problem Solving

A capacity for abstraction is an important skill for chemistry learning. As a process, abstraction involves extracting salient details, recognizing and developing generalities, and recognizing and applying conceptual meaning to symbols (e.g., abstractions). In general chemistry, Frey, Cahill and McDaniel (2017) found that students who learned concepts through abstraction, e.g., through the extraction of salient points, outperformed students who learned through rote memorization of examples. In organic chemistry, Weinrich and Sevian (2017) and Domin and Bodner (2012) found that students who were more flexible in their use of abstraction while solving organic chemistry problems tended to present more plausible solutions. In their study of organic mechanism problem solving, Weinrich and Sevian (2017) identified four domain-specific indicators of abstractness: (1) the extent to which students drew on information explicit (low) or implicit (high) in the problem, (2) whether students focused on the sequential order of mechanistic steps (low) or the explanation behind the steps (high), (3) whether students focused on structure (low) or function (high), and (4) the extent to which their representations were specific (low) or general (high). This work operationalized the act of abstracting as a function of how students constructed mental representations of the problem space and relevant prior knowledge, and the relative abstractness of these representations (Hahn & Chater, 1998; Sevian et al., 2015).

Santos and Mortimer (2019) built on previous work in middle school science (Jiménez et al., 2016) to propose four possible levels of "abstractness" that can be used to characterize chemistry knowledge and representations. Using Legitimation Code Theory (Maton, 2013), they proposed that chemistry knowledge can be categorized in two dimensions. First, chemistry knowledge can be characterized by semantic gravity, or how removed from physical reality a piece of knowledge is, into four levels: description, explanation, generalization, and abstraction. Second, building on Johnstone's triangle (Johnstone, 1991), chemistry knowledge can be characterized into four levels of semantic density, or how much information is encoded into a given representation: macroscopic or phenomenological (the least amount of information encoded), conceptual macroscopic, conceptual submicroscopic, and symbolic (the most amount of information encoded).

These works have largely focused on the conceptual aspects of abstraction or worked to identify domain-specific attributes in other sub disciplines, e.g., in organic chemistry. However, physical chemistry as a sub discipline is unique because it draws not only on highly abstract physical concepts, such as entropy, (Sözbilir, 2004) it also utilizes highly abstract mathematics, such as partial differentials. Mathematics and physics education literature have been found to be very fruitful in interpreting student reasoning, particularly in physical chemistry (e.g., Bain, Rodriguez and Towns, 2019). Thus, due to the mathematical nature of physical chemistry problem solving, we also draw heavily on literature published in mathematics education. In mathematics education, studies have focused on abstraction as the construction of mathematical knowledge. There are several traditions in mathematics education for investigating abstraction, which can largely be differentiated into two categories: abstraction-from-actions and abstraction-from-objects (Scheiner, 2016).

In abstraction-from-action approaches, abstraction is conceptualized as how learners learn procedures and extract mathematical meaning through reflecting on how these procedures are applied to an object. An object is defined as a mathematical concept that a learner can perform transformations on, such as an equation (Dubinsky & McDonald, 2001). An example of this would be a student who learns a procedure to solve a certain type of problem, and then after solving many of these types of problem, reflects on the similarities between them and recognizes a mathematical concept that underpins the similarities (e.g., Sfard, 1991; White and Mitchelmore, 2010). This reflective abstraction comes primarily from Piaget's theories on empirical abstraction, in which an individual extracts meaning from their encounters with the world (Piaget, 1964).

Abstraction-from-object approaches (structural abstraction) takes an opposite stance. If abstraction-from-action occurs when a learner reflects on the similarities between many different types of similar procedures, abstraction-from-object occurs when a learner takes a single object and places it in different contexts to extract the essence of that object, which gives it meaning. (Davydov, 1972) For example, a learner might be trying to develop the concept of "atom." By considering what an atom is in different contexts, such as the Bohr model of an atom or an atom as a part of a molecule, they may come to better understand its nature. In mathematics education, this type of abstraction involves trying to deeply understand the mathematical structure of an object. One mathematical tradition that utilizes abstraction-from-objects is the Dutch tradition of Realistic Mathematics Education (RME), which examines models that bridge the concrete (experiential knowledge) and the abstract (mathematical knowledge) (van den Heuvel-Panhuizen, 2003). RME identifies two ways in which learners may develop mathematical knowledge: horizontal mathematization, or translating between experiential and physical reality and a mathematical object; and vertical mathematization, or the reorganization and consolidation of previous mathematical concepts into a single mathematical construct. Some researchers define abstraction as the process of vertical mathematization (e.g., Hershkowitz, Schwarz and Dreyfus, 2001).

Instead of trying to reconcile these two approaches into a single definition of abstraction, Scheiner (2016) suggests that both are valid and reflect two different kinds of learners in mathematics. Reflective learners extract meaning from a mathematical object by working with it in order to formalize their understanding of it; that is, how they "abstract from actions." Structural learners primarily learn mathematics by considering it in light of their previous experiences and giving meaning to an object; that is, they "abstract from objects." Scheiner also proposes a third type of learner, who uses a hybrid of reflective and structural abstraction. These "reflectural" (reflective + structural) learners both extract meaning from and give meaning to objects through abstraction.

Although much of the work done on abstraction in mathematics uses a constructivist (e.g., knowledge is constructed within the mind of the individual) theoretical lens, some studies have also conceptualized abstraction using sociocultural theory (e.g., knowledge is mediated through socially and culturally constructed tools). Abstraction in Context (e.g., Hershkowitz, Schwarz and Dreyfus, 2001; Tabach et al., 2017) is a framework that conceptualizes abstraction as "a process that takes place in a complex that incorporates tasks, tools, and other artifacts; the personal histories of participants; and the social and physical settings" (Hershkowitz, Schwarz and Dreyfus, 2001, p. 204). That is, how a student abstracts and what tools they use to do so depend on the context in which the abstraction is taking place as well as the historical context of the students.

Theoretical and Conceptual Framework

Our orientation to abstraction draws primarily on the work from Scheiner (2016) and the Abstraction in Context framework (Hershkowitz et al., 2001). From Scheiner, we take the stance that abstraction may involve both extracting meaning from and giving meaning to an object. "Object" is defined as a concept that a learner can perform transformations on, such as an equation (Dubinsky & McDonald, 2001).Therefore, abstraction is how a learner makes sense of something like an equation or a concept, either extracting meaning from it to develop a generalized concept, or giving meaning to it to recognize generalizable features.

Unlike in Scheiner's theory, we do not categorize learners primarily using reflective, structural, and reflectural abstraction. Rather, we acknowledge that learners may use different types of abstraction processes depending on various contextual factors, including their personal histories and the context of the problem solving. To focus on abstraction as a contextualized process, rather than a capacity, two things need to be defined in our framework: what learners do *in the moment* (which we will characterized through epistemic actions), and the context in which they are working (which we will characterize through the problem space).

Abstraction as Epistemic Actions (What Learners Do)

To capture abstraction as something that is sociohistorically situated, a sociocultural theoretical framework was used as the overarching framework for the study: first-generation activity theory (Davydov, 1972; Hershkowitz et al., 2001; Leont'ev, 1978). Through activity theory, abstraction is characterized as an activity: practical human work that uses tools and occurs within a particular context in response to a need. Activity is constituted by a series of actions that an actor does that work toward resolving the need. For example, an example of an activity in a chemistry lab may be synthesizing a product. A chemist synthesizes a product toward a goal, for example developing a new pharmaceutical to treat cancer (the need). She may use procedures (tools) to design the synthesis, which she carries out in discrete steps (actions): mix the reactants, run the synthesis, separate and purify the product, characterize the results. This all occurs in a context: the methods she uses depend on what is available to her in the lab, such as tools and the physical space (her current sociocultural context), and what has been previously published and is considered to be the standard in drug discovery (sociohistorical context). The individual actions the chemist takes (mixing the reactant, etc.) all work toward resolving the need for a new product, and thus constitute one activity: drug synthesis.

Similarly, we view abstraction as a mental activity—an activity that is carried out using mental tools and actions. For example, while solving a problem in chemistry, a student may be faced with an equation they do not understand (the need). To understand it, they may need to give meaning to the equation by abstracting (the activity). They may use concepts (mental tools) they already know to define the different variables in the equation and figure out the meaning by reasoning about how the variables relate. How they do this depends on things like what concepts and problem solving approaches they learn in their chemistry class and in previous classes (sociohistorical context), and how they view the problem at hand and how they personally relate to the problem (current sociocultural context).

The individual actions the learner takes while abstracting are epistemic actions: goal-mediated mental actions learners take when they are constructing knowledge (Pontecorvo & Girardet, 1993). Previous work in mathematics education has identified the four epistemic actions that constitute abstraction as recognizing, building-with, constructing, consolidating. That is, when a learner abstracts in math, they may (1) recognize a previously learned schema, (2) use that schema to make sense of what has been given in the problem (building-with), and (3) construct a piece of knowledge, e.g., by recognizing a deep structure (Hershkowitz et al., 2001; Tabach et al., 2017). However, these actions rely on a definition of abstraction as "vertical mathematization," which may not be true in a different domain, such as physical chemistry.

In order to characterize abstraction in physical chemistry, the goal of this study is to identify the types of epistemic actions students take that give meaning to or extract meaning from objects in the problem space—that is, the epistemic actions that constitute abstraction in physical chemistry problem solving.

Abstractness as a Function of the Problem Space (The Context)

The conditions under which an activity occurs guide the specifics of the activity. For this study, the condition is problem solving during teaching interviews. In problem solving, part of what determines the knowledge that is relevant to use in a given situation and what path the problem solver may take is the problem space. The problem space is defined by how the problem solver interprets the task that is being given to them.

For example, a student in physical chemistry may be tasked with solving a problem that requires calculating the entropy of a system based on the possible orientations of the molecule (e.g., that relies on a statistical mechanical view of entropy). This student may cue on the notion of "entropy" and try to apply the Second Law to calculate an entropy change (e.g., an approach that utilizes a thermodynamic view of entropy), because that is the type of entropy problem they are most familiar with. If the student cues on the idea of possible orientations and probability, they may use the Boltzmann equation. This initial view of the problem and the prior knowledge cued change the pieces of information that the student cues on (Chi et al., 1981), as well as the subsequent actions the student takes toward solving the problem.

Abstractness (a quality that describes how removed something is from the context at hand) has been previously related to the idea of problem space. In this view, abstractness in problem solving is relative to the knowledge the problem solver cues on as relevant. The further removed from the initial problem space, the more abstract it is. For example, Domin and Bodner (2012) used this definition to investigate the abstractness of mental representations in graduate organic problem solving. They considered a mental representation to be more abstract when it was more strongly associated with the prior knowledge the learner had that was not directly relevant to the problem context. This suggests that what is abstract during problem solving depends on how the problem solver interprets the problem. Going back to the physical chemistry example, this means that for the thermodynamics student, it would be abstract to think about entropy as being related to microstates, because that is not how they interpret the problem. For the statistical mechanics student, it would be abstract to think about entropy as being related to the heat transfer of a system.

Epistemic Actions and Abstractness

To connect activity theory and the idea of problem space together, we consider that the problem task is part of what defines the current sociocultural context. The problem task, which is the problem as it is written, will be interpreted by students according to their mental tools, what they have previously learned in their courses, and their personal histories. This interpretation of the problem as written is what we call the problem space.

We assume that the problem space prescribes the actions a student takes during problem solving, and not just the knowledge that is relevant to solving the problem. That is, some actions may be cued by the problem space (less abstract), whereas others may be drawn from previous experiences (more abstract). That is, we assume that epistemic actions have the characteristic of abstractness. We thus looked for actions using the follow definition, revised from Domin and Bodner (2012): “Abstractness reflects the degree to which students’ actions rely primarily on the representations and paths explicitly cued by the problem space.” That is, actions are more abstract if the student does something that is not explicitly cued by the problem space, and less abstract if the student seems to be following a prescribed path. This definition guided our development of the conceptual framework. Table 2.1 provides an overview of the key terms in the framework.

Important Theoretical Terms	Definition used in this study
Object	Something a learner can perform transformations on (e.g., an equation)
Problem space	How a problem solver understands what a problem is about (their mental representation of the problem and the allowable problem solving paths)
Abstractness	the degree to which students' actions rely primarily on the representations and paths explicitly cued by the problem space
Abstraction	The process of extracting meaning from and giving meaning to an object
Epistemic action	Goal-mediated mental action

Table 2.1. Overview of common terms used in this study.

Methods

Data Collection

Students enrolled in physical chemistry 1 (an upper-level undergraduate course) were recruited with IRB approval from a physical chemistry (thermodynamics and kinetics) course in Spring 2018 at a highly diverse public institution in the Northeastern United States (IRB # 2013-010). With the professor's permission, participants were offered 10 extra credit points as a token of appreciation (equivalent to one homework assignment). To assign pairs, the 18 study participants were asked to identify classmates with whom they would feel comfortable working (n pairs = 9). Participants were interviewed twice, once during the unit on entropy (in early part of semester) and once during the unit on chemical kinetics (near the end of the semester). They were randomly assigned to participate in one interview individually and in one as a pair, such that there were an equal number of participants interviewing as pairs and individuals each round (26 total interviews). Participants used a LiveScribe pen; in pair interviews were asked to share a pen and pad of paper as they solve the problem. They were audio-recorded as they solved the problems and encouraged to talk through their solution aloud. I conducted the interviews and took field notes on salient gestures and interpersonal dynamics that were not captured by the audio recordings.

For the purpose of the study at hand, analysis focused on the data collected during the second round of interviews (kinetics).

Design of Interviews

Examples of abstraction in action are necessary for characterizing abstraction as an activity. Consequently, we designed the interview situation to facilitate the emergence of abstraction and to promote the student framing the interview as a sense-making interaction (Russ et al., 2012). In designing these interviews, three facets were considered: the relationship between the interviewer and the participant, the relationship between the participant and the task, and the relationship between participants. The methods sections of papers that investigate abstraction processes were examined to identify common features of these designs and to parse out the rationale.

Interviewer-participant: Studies that investigate abstraction and related processes tend to use a qualitative method, in which participants are recorded while solving a problem. One fruitful method used in mathematics education for eliciting abstraction is teaching interviews. (Kapon & diSessa, 2012). Teaching interviews have been used in studies of abstraction (Hershkowitz et al., 2001), because they allow the interviewer to probe deeply what a participant is thinking, and to scaffold connections participants are capable of making but may not immediately notice (Broman et al., 2018; Caspari & Graulich, 2019). This facilitated observation of the co-construction of knowledge during problem solving and the emergence of abstraction through this process.

Participant-task: Second, the problems participants solved consisted of seemingly disparate parts that had to be pieced together conceptually for participants to succeed in solving the problems. Although several expert solutions to the problem exist, participants generally did not have the prior knowledge to be able to access these solutions (e.g., the experience with ordinary differential equations to recognize the inflection point or the experience with autocatalysis to quickly map the problem to a two-step reaction). That is, there is a gap between the solution state, and the students' most likely problem space. To bridge these two, the student would be expected to abstract. These problems related to the course content the participants were learning at the time, and they were multi-disciplinary and covered content beyond the scope of what students had covered in the course so far, so participants could not rely on memorized procedures to solve the problem.

Participant-participant: Finally, we conducted both pair and individual interviews, because literature suggests that students working in pairs may solve problems at a higher level of abstractness than they do working alone (Dreyfus et al., 2001; Schwartz, 1995). There are several possible reasons for this. First, the two members of the pair may bring unique resources to the problem that they can build on jointly as shared assets, or they may need to develop an abstract representation that incorporates aspects of both of their representations to ease communication (Schwartz, 1995). Other reason may be that one of the members of the pair can behave as a more expert peer and support the other member to abstract (Dreyfus et al., 2001). Thus, when designing interviews to build theory about abstraction, it may be useful to include pair interviews to maximize the diversity of types out interactions that may lead to abstraction during problem solving. At the same time, the interaction dynamics in pair interviews are often complex, and pairs do not always work together productively even if they are both successful students (Barron, 2000, 2003; Sohr et al., 2018). Therefore, the interview schedules were designed such that each participant participated in one of the two interviews individual, and in one as a pair.

Consequently, we designed teaching interview situations (interviewer-participant), in which participants were solving problems (participant-task) either in pairs or individually (participant-participant) (see overview in Table 2.2).

Interview Design Facet	Theoretical Consideration	Data Collection Mechanism
Participant-Interviewer	More knowledgeable other can scaffold students' knowledge building and connection-making, supporting their abstraction	Teaching interview
Participant-Task	Productive problem solving in p-chem requires conceptual reasoning; however, many traditional p-chem tasks encourage algorithmic approaches, even when that is not the instructor's intention (Rodriguez, et al., 2018; Fox & Roehrig, 2015). To elicit abstraction, a task should be designed that promotes both conceptual and mathematical engagement and that allows students to bring different resources.	A novel, difficult task that requires problem solving beyond rote application of equations
Participant-Participant	Students in groups reason often reason more abstractly than alone (Schwartz, 1995; Dreyfus, Hershkowitz, Schwarz, 2001)	Each participant interviewed twice: once in a pair, once individually

Table 2.2. Overview of the interview design.

The first round of interviews (entropy) served as a pilot for the teaching interview approach, in order to better understand how to facilitate the emergence of abstraction during an interview context. During the interviews, I noticed that certain aspects of the design constrained abstraction: for example, providing the participants with an equation sheet, how certain probing questions were phrased, and the nature of the problem. These insights guided the second round of interviewing, resulting in data that were richer and thus more fruitful for theory building. These data (from the kinetics interview) were the primary source of data for developing the epistemic actions framework.

Interview Task.

The interview instruments were taken from the course textbook (Atkins & de Paula, 2014). In the entropy problem, participants were asked to solve for the residual entropy of a DNA molecule, in which the order of the binucleotides was random and they were given the length of the DNA ladder. In the kinetics problem, participants were given the rates of change of three populations (susceptibles, infectives, and removed class) and tasked with finding the ratio a/r that controlled the disease spread (see Figure 2.1 for problem texts). The problem was based on the epidemiological SIR model, which models disease spread in a constant population as a set of three coupled differential equations that depend on the effective transmission rate (r) and the removal rate (a) (Kermack et al., 1927).

Although the problems were closed-ended (there is one correct answer), there were multiple possible ways to solve the problem (see Appendix 1 for model solutions). At the time the interviews were conducted, participants had just begun learning about entropy and about chemical kinetics in their physical chemistry course. They were expected to be able to recognize the formalism dx/dt as an equation for the change in x over time (e.g., a rate) and to have used this formalism in the context of chemical kinetics. Participants were not necessarily familiar with the SIR model beforehand and had not taken an ordinary differential equations course.

Interview 1: Entropy

Warm-up question: What happens when a hot stone is dropped into cold water?

Novel question: An average human DNA molecule has 5x108 binucleotides (rungs on the DNA ladder) of four different kinds. If each rung were a random choice of one of these four possibilities, what would be the residual entropy associated with this typical DNA molecule?

Interview 2: Kinetics

Warm-up question: You've been asked to improve the rate of a chemical reaction. How will you do this? What might you need to know about the system?

Novel question: Many biological and biochemical processes are catalyzed by the presence of the product (this process is called autocatalysis). In the SIR model of the spread and decline of infectious diseases the population is divided into three classes: the 'susceptibles,' S, who can catch the disease; the 'infectives,' I, who have the disease and can transmit it; and the 'removed class,' R, who have either had the disease and recovered, are dead, are immune, or are isolated. The model mechanism for this process implies the following rate laws:

$$\frac{dS}{dt} = -rSI \qquad \frac{dI}{dt} = rSI - aI \qquad \frac{dR}{dt} = aI$$

Find the conditions on the ratio a/r that decide whether the disease will spread (an epidemic) or die out. What do you think a and r mean in this biological system?

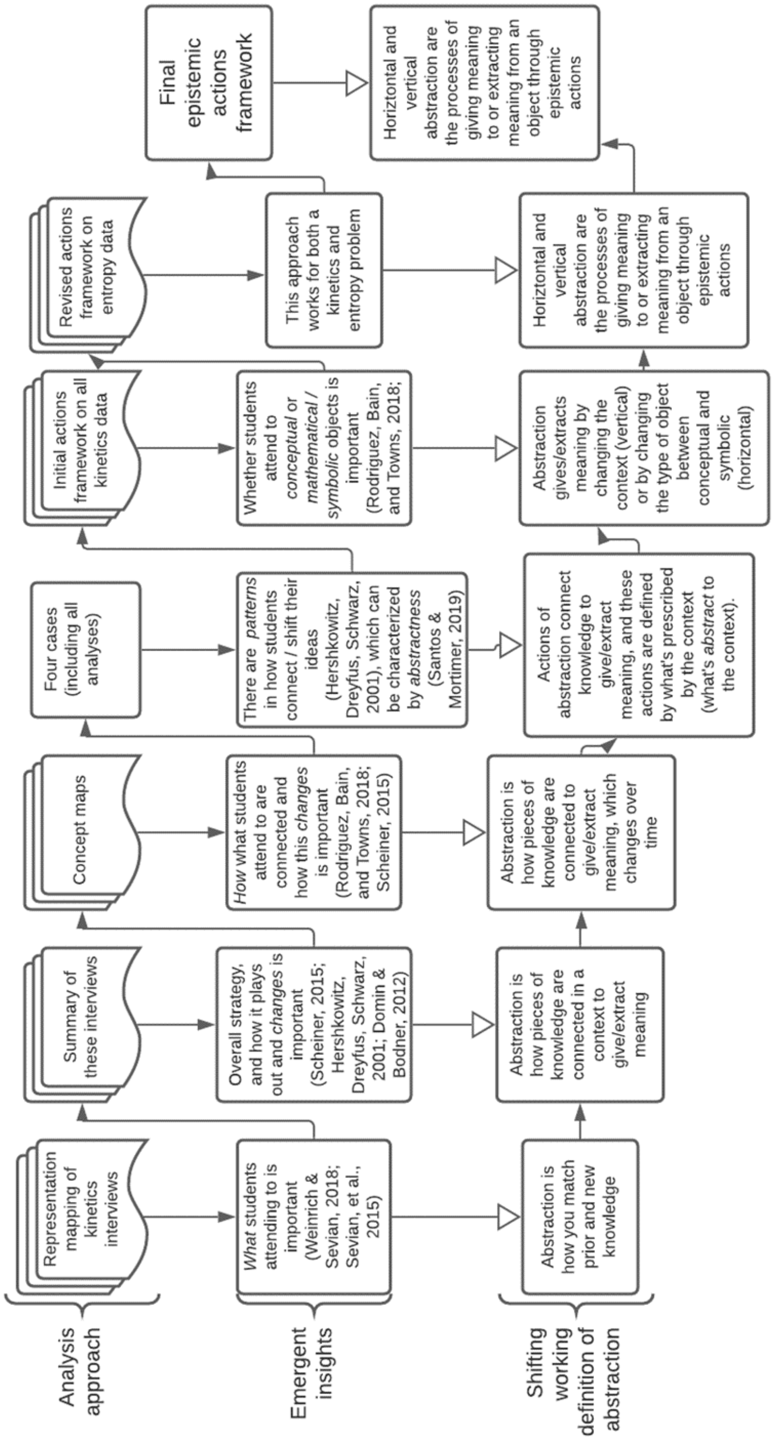
Figure 2.1. Text of problems used in interviews 1 and 2. Both novel questions were adapted from the course textbook Atkins, P., & de Paula, J. (2014). *Atkins' Physical Chemistry* (10th ed). Oxford University Press, with permission from Oxford University Press. Reproduced with permission of the Licensor through PLSclear. See Appendix 2 for the equation sheet provided with the entropy task.

Constant Comparative Approach to Develop the Framework

To develop our conceptual framework, we used an constant comparative approach, iterating between theory and data (Glaser & Strauss, 1967). The kinetics interviews yielded richer data, so initial analysis focused on the kinetics interview data. Once the framework was developed, it was used to code the entropy data to test whether the framework was applicable beyond the initial dataset used to develop it (see Figure 2.2).

To develop the initial actions framework, each transcript was read through in its entirety while listening to the audio and looking at the written student work, to capture intonation and written referents. Then, a representation mapping framework was applied (Hahn & Chater, 1998; Sevian et al., 2015) to parse participants' new instance representations (how they interpret the problem task and combine their stored knowledge with the problem text) and use of stored knowledge. A narrative summary was written of participants' problem solving, and then insights from this narrative summary were used to segment each transcript into 5-10 turn long chunks, in which a coherent idea was being developed or a particular strategy was used. These smaller coherent chunks were used to develop modified concept maps, which allowed us to visualize the connections participants made to advance their ideas forward.

Figure 2.2. Overview of the constant comparative approach. This approach was used to develop the Epistemic Actions of Abstraction framework.



Each node in the concept map consisted of a coherent idea identified through representation mapping (Hahn and Chater, 1998), and the nodes were connected based on participant seemed to be connecting them in their problem solving (Novak & Cañas, 2006). A connection was made between two ideas if one led to the other, e.g., a particular equation being manipulated to lead to another one. A second way that a connection was made was when a participant would refer back to an idea developed earlier to support their current reasoning. The maps showed the course of problem solving through the ways in which the ideas are used and developed (Rodriguez et al., 2018). The relevant sources of knowledge (e.g., mathematics, chemistry) were also identified to understand how students drew on and coordinated knowledge from different knowledge sources, which has been identified as a form of abstraction (Scheiner, 2016). These maps were constructed using CmapTools (Cañas et al., 2004) (see Figure 2.3a).

Level	Coding of Response	Definition and Example
5	Abstraction / Construction (1,2)	Student constructs generalized principle or knowledge that moves beyond specific case **Example:** student uses knowledge generated in problem to rederive / rediscover a physical principle or law
4	Mapping? Schematization? Building-with? (1)	Student restructures the information in a novel way that moves beyond the specific problem space / context
3	Interpretation? (2)	Student interprets the information given in the problem with information outside of the given problem space
2	Manipulation (3)	Student manipulates the variables in the problem without moving beyond the bounds of the problem (uses referents, variables, embedded mathematical procedures in problem) **Example:** plug-and-chug, algebraic manipulation of variables
1	Concretization (2,3)	Student reframes the problem to make it more concrete **Example:** plugging in numbers for variables

Table 2.3. Initial coding scheme for actions coding. These codes were drawn from both literature and data (represented by the superscripts). **(1)** from Hershkowitz, Dreyfus & Schwarz (2001); **(2)** from Santos & Mortimer (2019); **(3)** primarily from data

In combination with the transcripts, we used the concept maps to develop the actions, using a preliminary version of the scheme drawn from two previously developed frameworks that capture the epistemic actions of abstraction (Hershkowitz et al., 2001) and levels of abstraction for chemistry knowledge (Santos & Mortimer, 2019) (see Table 2.3). The unit of analysis for actions were several nodes within the concept map, in which a particular strategy was being played out, which corresponded to around 1-3 turns in the transcript. Combining these two approaches helped to flesh out the definition of action, because the concept map made it clear how the students connected and developed ideas over multiple steps. Common patterns were identified in how connections were being made, which became the basis of the 4 types of actions (see Figure 2.3) that ended up in the final framework: concretizing, manipulating, restructuring, and generalizing.

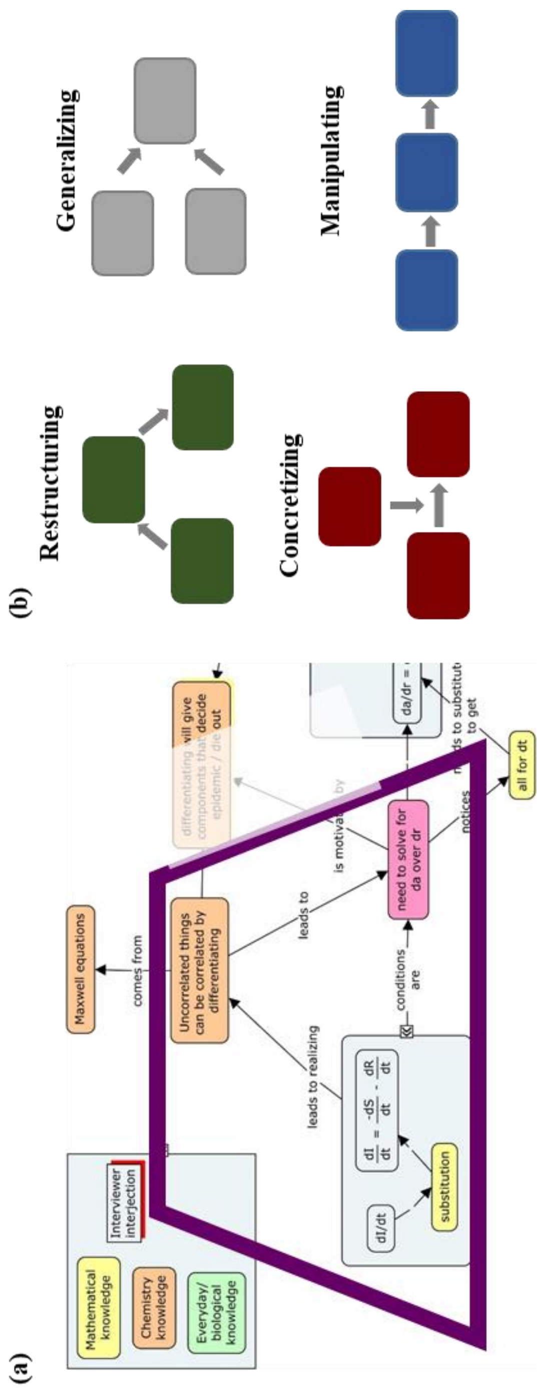


Figure 2.3. Example of the concept mapping approach and the abstracted structure of the four epistemic actions. **(a)** Example of a restructuring action drawn from Philip's kinetics interview. **(b)** The generalized structure of each of the 4 finalized epistemic actions

Four transcripts in the kinetics data were identified that were the strongest examples of these initial definitions of the actions. Each of these transcripts were used to develop a solid exemplar of each of these actions with initial definitions. Then, these actions were used to code the rest of the kinetics data, which served to refine the definitions of the actions. One major outcome of this refinement process was the characterization of two types of actions: conceptual (in which the action was operating on meanings and conceptual ideas) and symbolic (in which the action was operating on symbols or mathematics). The refined actions framework was then used to analyze the entropy data, which served to validate that the framework could be used to analyze problem solving that involved a different content area (thermodynamics rather than kinetics).

The analysis and development process were iterative; through each stage of analysis, theoretical notions of abstraction were refined and tested. At each stage, we also performed constant comparison with the literature to see if our developing understanding aligned with previously published works, and to see how those works further refined our emerging definitions and codebook (Glaser & Strauss, 1967). This process ended once we had developed a codebook (the actions) that could be reliably applied to the data and developed a working definition of abstraction that captured both these data-driven insights and insights from literature.

Trustworthiness of Findings

Each iteration of data analysis was staged by inter-rater reliability processes with researchers outside of the project. At each stage of developing an operational definition of abstraction and integrating a new stage of analysis, I met with a group of 8-10 researchers and discussed the interpretation of the data and the alignment of the analysis to theory. These discussions and feedback were incorporated into the evolving framework. These reliability measures enabled us to develop construct validity via consistent operationalization of the analysis (Dalgety, Coll, & Jones, 2003).

After developing a finalized version of the framework, inter-rater reliability was conducted. An external researcher coded 20% of the data (both kinetics and thermodynamics data) using the finalized codebook, and we discussed our coding until we reached 100% agreement. The results of this process and the finalized framework are presented in Chapter 3 of this thesis.

CHAPTER 3

THE EPISTEMIC ACTIONS OF ABSTRACTION IN PHYSICAL CHEMISTRY PROBLEM SOLVING FRAMEWORK

Introduction

This chapter introduces the findings from the constant comparative process described in Chapter 2. Through this constant comparison approach and codebook refinement, we identified eight epistemic actions that students took while solving problems related to kinetics and to entropy. These actions could be distinguished by common patterns in how representations in problem solving were connected in the concept maps. Because the kinetics interviews were used to develop the actions, exemplars are primarily shown from that subset of the data.

Actions

Eight actions were identified based on two dimensions: the abstractness of the action (vertical dimension) and type of resources the actions related to (horizontal dimension) (see Table 3.1 for an overview). Abstractness was operationalized as the extent to which the action transformed the problem space by utilizing resources that were not within the students' initial representation of the problem.

Symbol	Action	Description
	Symbolic concretizing	Putting a mathematical constraint on the problem space
	Conceptual concretizing	Putting a conceptual constraint on the problem space
	Symbolic manipulation	Advancing toward a solution/understanding by thinking by working with math or variables in a procedural way
	Conceptual manipulation	Advancing toward a solution/understanding by thinking through meanings and connections between conceptual aspects in a procedural way
	Symbolic restructuring	Transforming mathematical relationships to represent something that they were familiar with from another context, applying those constraints and meanings (focuses on changing the representation and mathematical relationships)
	Conceptual restructuring	Reimagining the meaning of the mathematics, which transformed the relationship between the math and the variables it stood for (focuses on the changing the meaning and conceptual underpinnings)
	Symbolic generalization	Connecting ideas and meanings to produce new mathematical relationships within the problem space
	Conceptual generalization	Connecting ideas and meanings to develop a new concept/idea within the problem space

Table 3.1. Overview of each of the actions, including the symbol used to represent them in text.

In the vertical dimension (abstractness of the action), we identified four types of action: concretizing (making the problem space more concrete by constraining it), manipulating (performing procedural changes within the problem space), restructuring (transforming the problem space by integrating other forms of knowledge), and generalizing (generating a new representation or relationship in the problem space) (Karch & Sevian, 2020). Compared to the initial abstractness of the problem space, restructuring and generalizing were considered actions that made the problem space more abstract. In a restructuring move, a student uses prior knowledge to redefine the constraints of the problem space—that is, they abstract away from the problem space to generate a transformed problem space. In generalizing, the student draws on sense making done during the problem solving as well as prior knowledge to generate a new understanding—that is, they abstract a new relationship. Manipulating is at the same level of abstractness, because the student follows the procedures and problem solving paths dictated by the problem space at hand, and concretizing was a lower level of abstractness, because the student makes the problem space more concrete.

In the horizontal dimension (type of resource), we identified two types of resources: conceptual and symbolic. We characterized an action as conceptual or symbolic based on the object that the action was performed on; that is, if the action (e.g., a manipulating action) was being performed on a concept or meaning, such as a description of the infectives, it was characterized as a conceptual action. If it was performed on a symbolic or mathematical representation, such as an equation or a graph it was characterized as symbolic.

The n value listed with each action lists the number of participants who performed this epistemic action (see Table 3.2). These values are included to provide additional context about how often they occurred in the data; however, because actions are contextually dependent, we do not make generalized claims about the relationship between the frequency of actions and the nature of the actions in physical chemistry problem solving.

Action	Conceptual Actions				Symbolic Actions			
	# Occurrences		# Participants		# Occurrences		# Participants	
	Kinetics	Entropy	Kinetics	Entropy	Kinetics	Entropy	Kinetics	Entropy
Concretizing	58	30	16	13	18	5	7	4
Manipulating	73	79	16	16	62	53	17	14
Restructuring	24	29	14	16	13	3	9	3
Generalizing	10	2	6	3	1	0	1	0

Table 3.2. Frequency counts for each type of action. Counts are included for conceptual and symbolic action, and number of participants who used that action during problem solving.

Concretizing.

Concretizing resulted in the student placing more constraints on the problem space to make it less abstract. In our dataset, this primarily occurred in one of two ways: (1) the student assigning a value to the variable (symbolic concretizing) or (2) the student assigning a meaning to the variable (conceptual concretizing). When a student concretized, they applied an external constraint to the problem, and then made meaning of the problem using the applied constraint.

Symbolic concretizing: In symbolic concretizing, a constraint was put on the problem by assigning a mathematical value to the variables, which allowed the participants to work with numbers rather than the variables in the abstract. In the kinetics problem, no values were provided in the problem. Students who symbolically concretized had to invent a situation that would allow them to assign numerical meanings to the values. A pair of participants, Akeyo and Jamila, invented a hypothetical population in which the disease spread was occurring:

Interviewer:	And so how does what you just came up with relate to whether the disease will spread or die out?
Jamila:	That's a good question.
Akeyo:	So since we just said that *r*, the constant, is depending on the number of people were infective. So [[10s]] if you plug in like random numbers. So if you said that *S* is 5 and *I* is 10, that's 50, negative 2. Negative 100. It can't be negative 100 people. [[7s]] So let's say in a population we have 300 people.
Jamila:	How will you know if a disease will die out?
Akeyo:	No, but see, if you think about it— Let's say if the city has like 300 people, right?
Jamila:	Hmm hmm.

Akeyo: You say 25 are—

Jamila: Susceptible.

Akeyo: I can't say that word. And then—

Jamila: Ten are infected or something.

Akeyo: Okay, 10, 11, 25. And then 250. That's the only way I can do that math. 250 are, yeah, removed, isolated, and whatever. Right?

In this excerpt, Akeyo and Jamila had been struggling to make sense of the problem after an extended period of unproductive mathematical manipulation they called "magic math," in which they had set up differential expressions for the two parameters, *a* and *r*. The interviewer asked them to elaborate on how their mathematical solution related to the original problem task of determining the spread and decline of the disease, which they struggled to answer. Connecting their work back to the idea of disease spread cued them to reframe the variables more specifically as representing populations. They concretized the problem by inventing a real-world situation with numbers they could work with, rather than trying to make sense of the variables in the abstract. Akeyo suggested a hypothetical situation where the populations belonged to a city with a specific number of people, and assigned numbers to each of the three populations. Using these numbers, they could modulate how the populations were changing according to the rate equations given in the problem. By symbolically concretizing, they put constraints on the problem space that allowed them to think productively about what the equations represented (e.g., the change in numbers of people in a disease event).

▲ *Conceptual concretizing:* Conceptual concretizing, similar to symbolic concretizing, resulted in the students putting more constraints around the problem space. A common example of conceptual concretizing was when participants defined or assigned a meaning to a particular variable, and then using that definition or meaning to make sense of the problem. For example, when Philip tried to make sense of the problem task, he first tried to define the parameters a and r:

Philip:	So I'm not exactly sure. The whole thing is a. The a is what's getting me stuck. Cause I would say it's the amount of people, but that wouldn't make sense because then there wouldn't be any susceptibles.
Interviewer:	Where do you notice a on these equations?
Philip:	So it's only for the infective and the removed class. So it's people that can actually be infected. Oh, wait, susceptible, sorry, they catch the disease. So it's people who have the disease and/or have recovered, died, immune, or isolated. So a equals have disease. Okay.

One aspect of the problem prompt was to figure out a potential meaning for a and r. Philip had previously defined t as time and r as rate, and was trying to make sense of what a could mean in the context of the problem. This action is a concretizing move, because he used those definitions as constraints to facilitate further sense making and had not arrived to them through any incorporation of outside knowledge.

Manipulating.

Manipulating actions are actions that are part of the set of possible paths explicitly cued by the students' problem space. When a participant manipulated, they used the pieces of information cued by the problem space and applied a strategy or procedure to move toward a solution.

Symbolic manipulation: Symbolic manipulation, or mathematical manipulation, occurred when students moved and worked with variables in procedural ways. Students used a range of manipulation strategies: rearranging the equations to solve for a and r separately then dividing to return the ratio, integrating the differential equations, taking the derivative of the equations, and a substitution strategy. A key aspect of manipulation was that it involved the resources and symbols that were explicit in the problem space—both what was explicit in the text (e.g., the equations) and what was cued by the participant's framing of the problem space.

> Chao: I thought about first that, if we want to calculate a divided by r, it should be something related with a and r, right? So here has an r, here has an a. I tried to just substitute to this equation, because this, you can, is an r and a. But what I substituted is only can [?] an issue with R and an I, like here, I, it's only R, S, and I. It's not a and r. So, but if I try to substitute again, it will be back. It will be back to here, and not meaning anything. Meaningless. So maybe some some some things I didn't find out from these questions. But.

In this example, Chao had noticed that dS/dt and dR/dt could be substituted into the dI/dt equation. His goal had been to manipulate the equations to try to algebraically solve for the ratio a/r, but his substitution strategy meant that the resultant equation did not include a or r, which were required by the problem as written (see Figure 3.1 for student work). Here, he applied several algebraic strategies (substitution, division) to transform the equations in a procedural way.

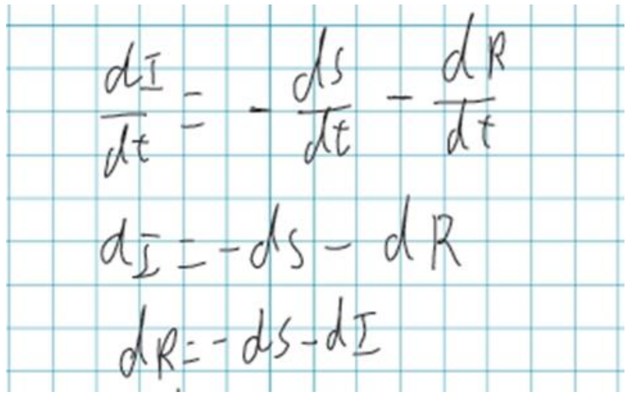


Figure 3.1. Chao's written work for his substitution strategy for the kinetics problem.

Because the problem space is a result of how the student interprets the problem, manipulating actions may vary in how mathematically complex they are. For example, some students cued on the partial derivatives and tried to apply calculus techniques (integration or derivation), whereas others noticed that the equations could be substituted for each other (e.g., Chao), whereas still others tried to algebraically isolate a and r.

▲ *Conceptual manipulation*: Conceptual manipulation involved participants focusing on the concepts or meanings of representations explicit in the problem space; for example, the definitions of S, I, and R as susceptibles, infectives or removed class. When a participant conceptually manipulated, they advanced toward a solution state by reasoning through the meanings of conceptual aspects of the problem in a procedural way. One common way this manifested was by participants substituting the meanings of variables in for the symbol and talking through the mathematical transformation aloud. Another way this emerged is illustrated by this example from the entropy interview. Alex is trying to figure out how the different parts of the problem text (the length of the DNA, the idea of entropy, and the idea of DNA) connect:

Alex:	Yeah. but I know that, I know that, I don't know if this is the rungs or whatever, so I know that *A* binds with *T* but the *T* binds with *A*, but the order, I'm not sure if it has to do with stability.
Interviewer:	Mmm
Alex:	So let's say we have *AT* next to a *GC*, it might not have to do with stability, but if you're saying randomizing and it still has its double helix shape and stuff like that and it still has its double helix structure, this might not apply. But if it's randomized and it has something to do with stability and it has an effect on the stability of the DNA structure, it might have associated with the residual entropy.

Here Alex is trying to reason through how all of these aspects are connected. Each of the ideas he brings up (e.g., “AT”, “CG”, “stability”, “order”) are ideas he had already introduced to the problem space. He is trying to figure out how they connect to each other but not through the framework of some prior piece of knowledge. Instead, it seems to be a sort of one-to-one mapping: reasoning through how random connects to stability and to helix. This kind of reasoning was considered a conceptual manipulation, because the participant is taking the concepts in the problem space and trying to manipulate them to figure out what they mean in terms of each other. However, it is “procedural” because it is not introducing new meaning or constraints.

Restructuring.

When restructuring happened, the student combined their own knowledge with what was given in the problem to change the constraints of the problem or generate new representations in a way that was mediated by the cued prior knowledge. This resulted in transforming the problem space, i.e., changing the possible paths and cued resources and changing how the student interprets the problem task. When a student restructures, they are abstracting away from their initial problem space by drawing on knowledge they did not initially see as relevant to the problem at hand. Because these restructuring moves draw on prior knowledge, they also populate the problem with the understandings associated with that piece of prior knowledge, giving participants more things to use for sense making and cued those other resources (e.g.—Maxwell relations, chemical reaction). Restructuring may result in a student refining their understanding of the task.

Symbolic restructuring: When a participant symbolically restructured, they transformed the equations to represent something that they were familiar with from another context, usually changing it mathematically. For example, when Philip tried to find the ratio a/r, he developed a strategy to solve for da/dr:

Philip: So I think would be *da* over *dr*.

Interviewer: Okay. Why do you think that? What made you think of that?

Philip: Um, because the um, you'd have to take the derivative of it, I think. I'm not sure. It's just kind of a little guess to try to get me thinking as to how I can figure that out. But that means— But they're all for *dt*. So you'd have to have this equal to something. You'd have to, so you'd have to take the derivative of like say that, but you want it in terms of *r*, little *r*. So let me see. So this is off of a wing.

[…]

Philip: So I decided to differentiate it because it made me think of like the Maxwell equations for p-chem. It made me think of how you have formulas that you can, you can make them, you can change them after you differentiate them, because then you can pretty much combine like terms that aren't pretty much defined as like correlating. You can kind of like, like with Maxwell equations, it's like, I can't think of one off the top of my head, but it's one that equals another, and then if you're, if you find one of them inside an equation, you then know if you have to get it in specific terms of say *N* or *G* or pressure or temperature, you

> then know you can plug that in to be able to figure it out to find out the answer, like the, yeah, the fundamental equations.

In this example, Philip cued on two aspects of the problem that invoked a similarity to Maxwell relations: the need to “combine terms that aren’t pretty much defined as like correlating,” and the presence of differential expressions. Maxwell relations are the set of derived partial differential expressions that relate the four thermodynamic potentials (internal energy, Helmholtz energy, enthalpy, and Gibbs’ free energy) with mechanical and thermal variables like pressure and entropy. Earlier in the semester, the course instructor had introduced Maxwell’s relations as the “fundamental equations,” which could allow the students to make relationships between different thermodynamic variables. Here, Philip restructured the equations symbolically by using the chain rule to differentiate the expression by d/dr.

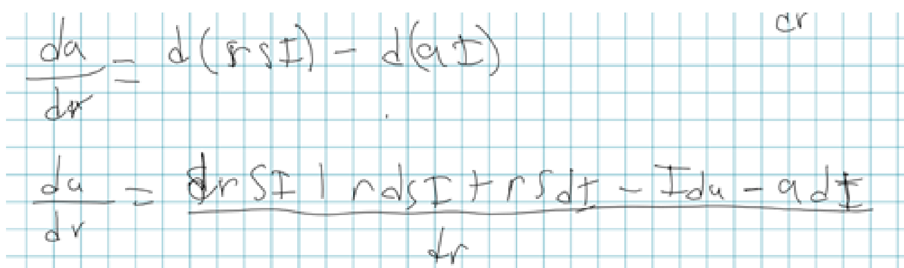


Figure 3.2. Philip’s written work in the kinetics interview depicting his “Maxwell relations” approach.

By cueing on the familiar feature of derivatives, Philip moved away from the problem space to reason about Maxwell’s relations, and then restructured the problem space to be one that could be solved using the “Maxwell relations” approach (see Figure 3.2 for student work). This changed the possible actions Philip could take to include taking the derivative of

a and r. It also transformed the relationship the variables had with each other to be "uncorrelated things that could be correlated."

▲ *Conceptual restructuring:* In contrast to symbolic restructuring, which involved changing the mathematical relationships the variables had to each other, conceptual restructuring integrated prior knowledge to transform the meanings of the variables in the problem. Unlike conceptual concretizing, in which a meaning was assigned to a variable as a way to concretize the problem space, conceptually restructuring resulted in reimagining the meaning of the mathematics, which transformed the relationship between the math and the variables it stood for.

The interview task had a biological context (disease spread) and many of the participants were biology or biochemistry majors, which may explain why conceptual restructuring in this dataset tended to occur when participants integrated formal biological and biochemical knowledge to make sense of the relationships between an epidemic and the equations. For example, one participant struggled to understand what the equations meant, and conceptualized the meaning of the equations to be that they each represented the number of the populations (that dS/dt, dI/dt, and dR/dt were conceptually equivalent to S, I, and R). However, he soon expressed that he was stuck and did not know how to move forward with the problem. The interviewer and participant spent several minutes discussing what the participant, a biochemistry major, knew about disease spread, before the interviewer asked him to bridge these two aspects (their biological knowledge and the model in the problem), to try to elicit abstraction:

Interviewer: Okay. So if we think about that in terms of these three classes of people, how do those two models work together? So not necessarily thinking about the math, but just thinking about infected people, people who can catch it and people who can't catch it. How does that work with this model of disease spread that you were just telling me about?

Joshua: Well susceptible people have a higher chance of getting the disease, so it would require like less transmission to get to them, I guess, than an average person, it would require more interactions with people who are infected, so I guess that's why it's infected people times the— Well I guess, you know what? The derivative doesn't necessarily mean like that, like *S* doesn't have to equal— *dS* over *dt* doesn't have to equal like *S*. So it's like the number of susceptible people times the number of infected people. So this ratio actually could be the number of people, number, the increasing number of people becoming, or reducing the number of people becoming less susceptible, because, like what am I saying? So susceptible people times infected people times a variable that's negative. So it's reducing the susceptibili-, the number of people being susceptible over time is decreasing, because of the negative number, I think, just because they're becoming infected as the number of people that are infected increase. There are like the higher chance that they can be— I guess, the transmission can happen. I don't know.

Prompted by the question from the interviewer, Joshua moved away from the problem space to think about how disease spread occurs in general in terms of transmission and contact. He then applied this understanding back to the problem space to restructure and reinterpret the conceptual meanings of the derivatives as representing this new idea of transmission over time. Although this understanding is closer to the canonical interpretation of the equations, this was a restructuring move for Joshua, because he had previously defined the rate equations as equivalent to the population variables.

Generalizing.

Generalizing actions used prior knowledge in conjunction with the problem space to generate a new object within the problem space, such as an equation or an idea. This was considered to be more abstract than restructuring, because generalizing actions resulted in a new object that could be acted on within the problem space, but that stands apart from the problem space, whereas restructuring actions were grounded in the transformation of the problem space. Generalizing may involve coordinating different types of reasoning, such as conceptual reasoning and mathematical reasoning; however, we differentiated conceptual and symbolic forms of the action based on the nature of the object the action generated (e.g., a new mathematical relationship or a novel conceptual idea).

■ *Symbolic generalization:* Symbolic generalizing actions result in the generation of a mathematical or representational relationship that was more abstract than the problem space and that could not be accessed through manipulating without integrating prior knowledge beyond what was cued in the problem space. Symbolic generalization only very infrequently in our dataset. During a pair interview, one participant (Hoa) engaged in symbolic generalization when she proposed an inequality to model the relationship between a and r. Prior to this point in the problem solving, the pair had gone through two concretizing cycles, in which they plugged in different numbers for the three rates and looked at how these changed the relationships between the three rate equations. They found that the sign of dI/dt depended on whether dR/dt or dS/dt was larger, and were trying to make sense of this finding:

Avni:	I think that makes more sense, because the disease will spread less, and more people, I mean less people will be affected by it, and maybe like people over here, they already had the immunity when the disease is like spreading. They are making their immunity in their body.
Interviewer:	Okay. That could definitely be what's happening.
Hoa:	So like it's telling us what condition. So I think it really depends on *rSI*, I guess, because if you look at this— [[10]] Because if *rSI* is greater than *aI*, which is right here, that means that the rate is larger. What does it mean when the rate is larger?
Avni:	That it will spread more.
Hoa:	Spread more? Do you think that's right? Do you think that's a good statement? ((directed toward her partner)) And then *rSI* is less. This is 2, and 1. This is 1 right here. Do you get what I'm saying or no? So this will die out. What do you think? I guess this is the condition. What do you think?
Avni:	So in order to be, in order to increase this one—
Hoa:	Yeah, so this one would be like 3, and then this would like 1, so *rSI* is greater.

In this example, Hoa and Avni connected their conceptual reasoning to the mathematical formalism of a negative or positive rate. They realized that the whether the rate of change of the susceptibles or the removed class was larger influenced the sign of the rate of change of the infectives, which determined whether the disease spread. They then performed a symbolic generalization move and abstracted away from their concrete examples to set up two inequalities: that $rSI > aI$ when the disease is spreading, and $rSI < aI$ when the disease is dying out. It is important to note that the interviewer utterance ("That could definitely be what's happening") was a deliberate move as part of the teaching interview design, as the goal of the interviewer was to support the emergence of abstraction, not to probe how participants solve problems without interference.

▲ *Conceptual generalization:* Conceptual generalization moves resulted in defining an idea that emerged from the problem space but can be viewed as separate from (more general than) the problem space. For example, a conceptual generalization move may result in extracting the idea of independent parameters from the data, or making a generalization about disease spread that emerged from examining the rate laws. Conceptual generalization had a fairly low frequency in our findings. One participant who engaged in conceptual generalization, Chao, tried to make sense of the meanings of the parameters *a* and *r*. This moment occurred at the end of the interview, after he had spent a considerable amount of time conceptually and symbolically manipulating the equations:

Chao: *a* and *r*, mmm, by the time [you released]. It maybe means— Ah, and also. Hey maybe it's only the constant, how the, how new things can, how stable for these things catch disease, because it's relatable if it's remove and catch, right? And the *R* should be the [?] of disease, and this one also how catch it, and, I don't know. *R* is too complex.

Interviewer: I actually don't know what *a* and *r* are, so that's—

Chao: Okay, so I thought—

Interviewer: It's definitely what your thoughts are.

Chao: I don't know.

Interviewer: But you think that *r* has something to do with how people catch the disease?

Chao: No no, I said *a*. *a* is how stable, how stable the things catch disease, catch some things, because if it's relatable, it's catch, and is remove, right? This means, where maybe *a* is less, it's easy to remove. When *a* is high, it's hard to remove.

Here, Chao is trying to figure out the meanings of *a* and *r*. Unlike the participants who assigned meanings to *a* and *r* in order to reduce the concreteness of the problem space, Chao is trying to coordinate different lines of reasoning about the mathematical nature of the parameters (“constant”) and prior reasoning about how the disease progresses to figure out what the variables may stand for. That is, Chao was trying to pull meaning out of the problem, rather than imposing meaning on it (a conceptual concretizing move).

Role of the actions in problem solving.

Drawing from our theoretical framework and looking across the data, we found that there were two roles actions could play as part of problem solving: transforming the problem space or moving towards a solution state (Jonassen, 2010). Actions that transformed the problem space changed the nature of what was considered “allowable” paths or information within the problem space. Actions that moved toward a solution state resulted in an object that could be further acted on to lead to an answer to the problem. Concretizing and restructuring actions transformed the problem space, while manipulating and generalizing actions moved toward the solution state.

We also found that defining the actions relative to the problem space provided further insight into the role the epistemic actions played in problem solving. Concretizing actions transformed the problem space by constraining it, for example to reduce the complexity of the unknown variables in the problem or to ground it in the real world. Restructuring actions transformed the problem space by abstracting away from the problem space to consider other pieces of knowledge that could be relevant to the problem solving, and mediating the transformation of the problem space through those pieces of knowledge. The result of both a concretizing action and a restructuring action was a transformed problem space that was either less abstract (concretizing) or more abstract (restructuring).

Manipulating and generalizing actions moved toward a solution state. Manipulating actions consisted of procedural transformations cued by the problem space (for example, a series of algebraic transformations). Generalizing actions consisted of integrating different streams of information to generate a new relationship or object in the problem space (for example, combining prior knowledge with ideas within the problem space to realize a mathematical relationship). The result of both a manipulating action and a generalizing action was a new object within the problem space, that was either at the same level of abstractness as the problem space (manipulating) or more abstract (generalizing).

Vertical and Horizontal Abstraction

Coding for actions based on the relationship of the objects to the problem space and the types of knowledge being leveraged allowed us to see how the problem solving was progressing from moment to moment. According to Activity Theory, an activity is a sequence of several actions. Characterizing the epistemic actions along these two dimensions (abstractness and conceptual-symbolic) revealed two types of abstraction activity: horizontal abstraction, in which participants navigated between symbolic and conceptual forms of an action at the same level of abstractness (e.g., symbolically and then conceptually manipulating); and vertical abstraction, in which participants navigated between actions at different levels of abstractness (e.g., manipulating then restructuring) (see Figure 3.3). These forms parallel horizontal and vertical mathematization from the Dutch Realistic Mathematics Education (RME) literature, in which horizontal mathematization is the movement between physical reality and a mathematical object, and vertical mathematization is the reorganization of relationships between mathematical objects (Drijvers, 2003; van den Heuvel-Panhuizen, 2003). Below, we illustrate how these forms of abstraction manifested in the data and the roles they played in problem solving.

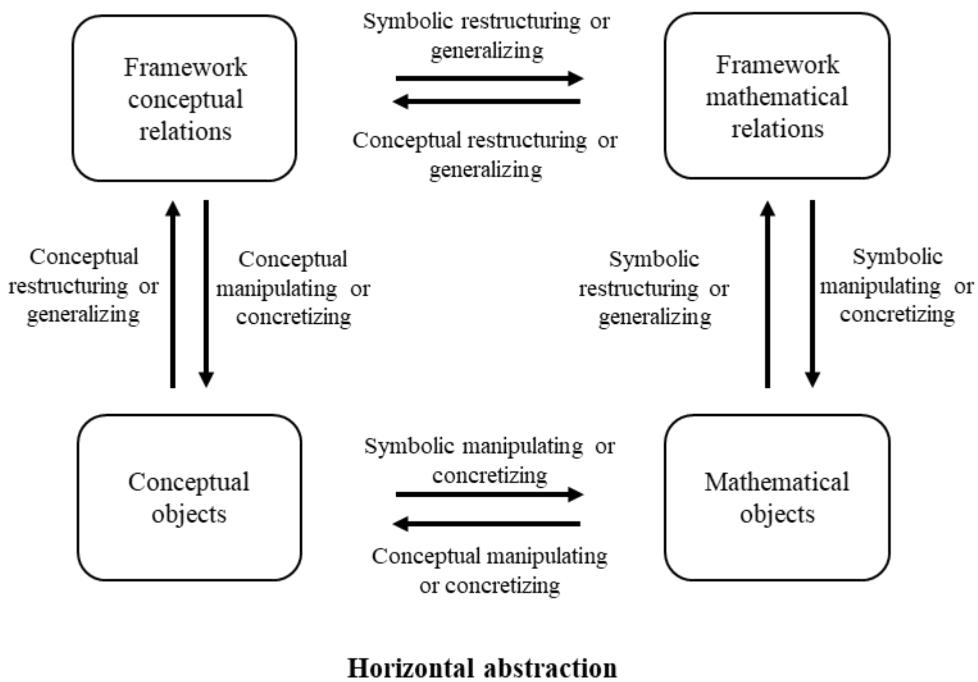


Figure 3.3. Depiction of vertical and horizontal abstraction. Figure shows how actions move between objects and frameworks, and how actions are related to the two forms of abstraction (vertical and horizontal).

Horizontal Abstraction.

In our conceptual framework, we define abstraction as extracting meaning from or giving meaning to an object within the problem space. Horizontal abstraction is a process by which a learner gives meaning to or extracts meaning from an object by navigating between a conceptual and symbolic understanding of the same object (e.g., a particular equation). This is horizontal because the learner stays at the same level of abstractness with reference to the problem space (e.g., conceptual followed by symbolic concretizing). It is abstraction because the learner moves between sources of knowledge to make sense of and give meaning to an object—e.g., the learner is abstracting away from one context (the mathematics) to give meaning to the object in another context (the conceptual meaning).

Horizontal abstraction occurred at all levels of abstractness, although the most common in this dataset was at the level of manipulation. Depending on the direction (conceptual to symbolic, or symbolic to conceptual), horizontal abstraction played two different roles in problem solving.

From symbolic to conceptual, horizontal abstracting involves making conceptual sense of a mathematical object. For example, at the level of manipulation, participants often engaged in several cycles of symbolic, then conceptual, manipulating. This process could be very superficial, for example, participants performing an algebraic manipulation, restating the change in terms of the variable names, and then repeating. This cycling may have been to reinforce conceptual meanings for the equations as participants tried to make sense of the mathematics. However, it could also bring new understanding, as participants could solve for an equation (symbolic manipulation) and then make sense about its meaning (conceptual manipulation).

From conceptual to symbolic, horizontal abstraction could also lead to new mathematical relationships through a mathematizing process. For example, Gabrielle and Fatima spent much of the problem-solving time grappling with the meaning of the variables and trying to reconcile what the problem task had to do with what they learned in their physical chemistry class. After a probing question from the interviewer, they conceptually restructured the problem space, shifting from an interpretation of the rate laws as "population" to "change over time." They then tried to reconcile this restructured meaning with their previous understanding of the disease spread. Gabrielle conceptually reasoned through how the populations would change over time (conceptual manipulation), and then sketched out this change as two graphs: a bell curve for the die out condition that showed the population of infectives increasing, peaking, then dying off; and a straight line that showed the infectives population linearly increasing for the epidemic condition (see Figure 3.4):

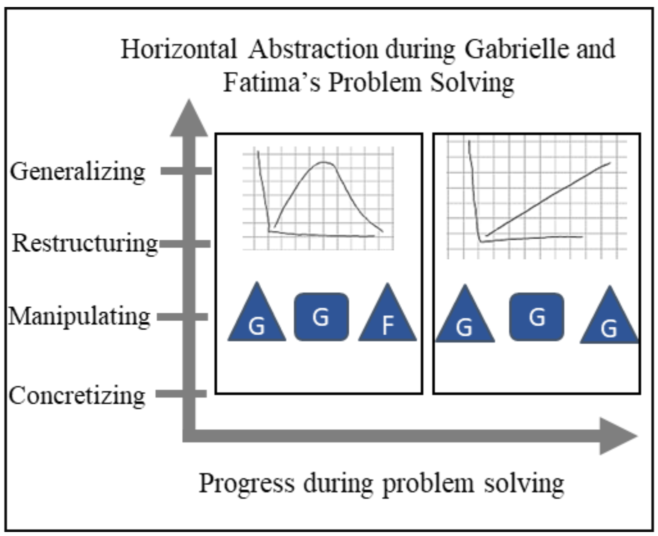


Figure 3.4. Gabrielle's and Fatima's actions during the kinetics problem. Diagram indicates who spoke when (shown by the letter) and the type of action (triangle is conceptual, square is symbolic). Above the actions are pictures of the graphs they drew while speaking.

Gabrielle:	▲So the number of people when the infection first start is small, and then over time, the amount of infected increases, and eventually if it dies off,
Gabrielle:	■ it will basically be like a bell ((draws first graph)), if like—
Fatima:	▲ You're saying we'd have a peak?
Gabrielle:	▲Yeah, and then it'll just die off. But if it, and this will basically mean that it dies off, but if it's an epidemic

Gabrielle: ■ ((draws second graph)), it'll just increase

Gabrielle: ▲until everyone's dead, and then everyone will be in the removed group, because everyone's dead.

This horizontal abstraction occurs at the level of manipulation, because the participants are not changing the problem space. Rather, Gabrielle mathematized the change in populations by mapping the conceptual manipulation directly into a graph. Reasoning about the change conceptually served as a basis for the graph Gabrielle sketches, as she draws exactly what she describes (a population that starts small, increases, then dies off as a bell curve). Her partner also seems to take up this graphical thinking, as indicated by how she talks about both the representation and the change in the population ("you're saying we'd have a peak?").

Vertical Abstraction.

Vertical abstraction involved participants taking actions in problem solving that moved them between different levels of abstractness (e.g., manipulating to restructuring). In Realistic Mathematics Education, vertical mathematization involves the reorganization of mathematical knowledge to create new structures (van den Heuvel-Panhuizen, 2003). Here, we consider vertical abstraction to be the process of reorganizing conceptual or mathematical knowledge within the problem space to create new relationships. Following our definition of abstraction, vertical abstraction can be a process of giving meaning to or extracting meaning from an object in the problem space. For example, when a problem solver restructures the problem space, they change the relationships between the objects in the problem space by recontextualizing the problem within the meaning space of another piece of knowledge. This may give new meaning to one of the objects, such as an equation. When a problem solver generalizes, they extract meaning by identifying and generating new relationships and objects. These processes are vertical, because the problem solver shifts the level of abstractness with reference to the problem space. They are abstraction because the learner moves away from the original problem space to make sense of the problem—e.g., the problem solver abstracts away from the context of disease spread (the initial problem space) to the context of chemical reactions (a restructuring move), which changes the problem space and the relationships between the variables.

We found that vertical abstraction played two major roles in moving participants forward in problem solving in our data set. The first function of vertical abstraction was to change the problem space, either through concretizing or restructuring, which allowed the participant to make forward progress in solving the problem, e.g., through manipulating. Concretizing-Manipulating allowed participants to place a constraint on the problem space, which could be useful because it reduced the number of variables the participant had to consider (see Philip's work in the conceptual concretizing section), or because it gave the participants concrete numbers to work with (in the case of symbolic concretizing). Restructuring-manipulating allowed the participant to change the problem space to resemble a more familiar example, which could change the problem solving paths that are allowed or reveal new relationships between variables. The second function of vertical abstraction was to generate a new object that could be manipulated or that moved the participant toward a solution state through generalizing.

To consider both types of vertical abstraction, let us revisit the example from Symbolic Generalization section, and in particular what led up to this moment. After plugging in a set of numbers for each of the rates, they decide to go through a second round of concretizing:

Hoa: ▲ I think it depends on this [the –*rSI* term], because what if it wasn't 3, you know? What if this wasn't 3?

Interviewer: Try it with a different set of numbers.

Hoa: ■ Yeah, cause what if it was like, let's say this was 4, right, and then this was like 3 minus 4, so this would equal negative 1, and then this would equal positive 1. So this increase—

In the first line, Hoa conceptually restructures ("I think it depends on [the –*rSI* term]" to point out that one of the terms in the rate laws seems to be important in figuring out the relationship and proposes a new symbolic concretizing move ("What if this wasn't 3?"). These two actions give the pair a way to manipulate the variables in a way that reveals new relationships, because they can compare the two sets of "conditions" that they have imposed on the rate laws. This was not possible before identifying the key term (through restructuring) and creating manipulable numbers (through concretizing). They reason through this comparison through a series of back-and-forth conceptual manipulations:

Avni: ▲ This will decrease. Does the decrease mean it will not spread that much?

Hoa: ▲ This means it won't spread that much, yeah. So we want this number to—

Avni: ▲ I'm not sure how to put it, because if it doesn't spread, how can we have like more people that have chance to get the disease?

Here they connect the rates decreasing with the spread of the disease. They then have an extended conversation with the interviewer reconciling the fact that changing *dR/dt* to 4 from their original value of 1 (see Figure 3.5) means that their *dI/dt* value also changed from positive 2 to negative 1. Avni then tries to conceptually reason about why increasing the rate of removal also makes the rate of spread lower:

Avni: ▲ I think that makes more sense, because the disease will spread less, and more people, I mean less people will be affected by it, and maybe like people over here, they already had the immunity when the disease is like spreading. They are making their immunity in their body.

Interviewer: Okay. That could definitely be what's happening.

This finally leads to Hoa's symbolic generalization, where she notices that this change, which they had previously modelled through concretizing, can be represented in a general form:

Hoa: ▲ So like it's telling us what condition. So I think it really depends on *rSI*, I guess, because if you look at this— [[10]]

Hoa: ■ Because if *rSI* is greater than *aI*, which is right here, that means that the rate is larger. What does it mean when the rate is larger?

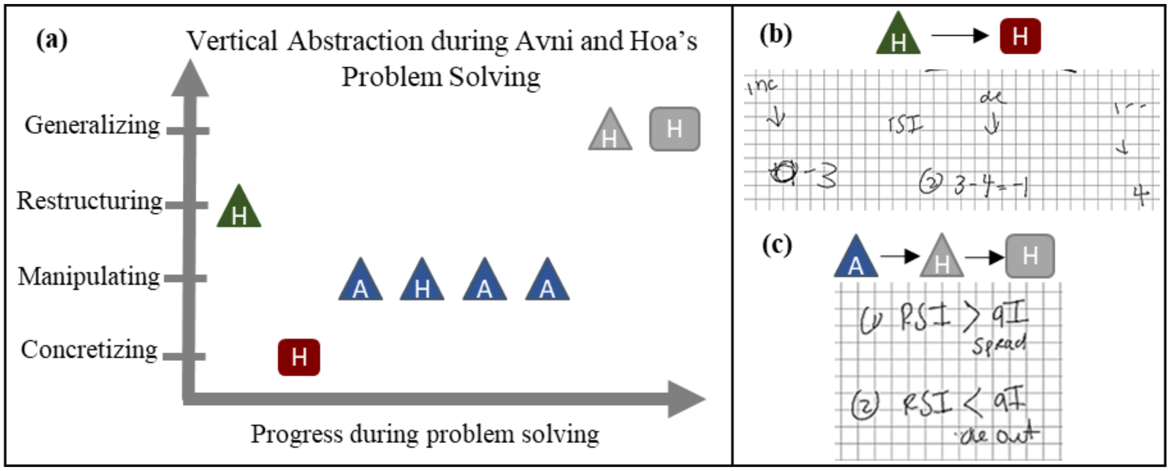


Figure 3.5. Epistemic actions and work during Avni and Hoa's kinetics problem solving. **(a)** Diagram shows how Avni and Hoa progressed in problem solving over several minutes. Triangles represent conceptual actions and squares represent symbolic actions. **(b)** Student work shows values plugged in for each rate. **(c)** Student work shows the inequalities generated to represent disease spread and die out.

The pair moves through several levels of vertical abstractness as they make sense of how to solve the problem. Both their restructuring and concretizing moves facilitate further problem solving by changing the problem space in such a way that they could compare the rates of change of the populations under different conditions. These actions seem to have enabled the comparison, because the relationship between the three equations did not seem to be clear to the participants when they represented it abstractly as variables. The manipulations that they performed as a result allowed them to identify a more general relationship, which precipitated their generalizing move, in which they generated the inequalities. These inequalities served as objects that they further manipulated after this section to solve for the ratio *a/r*, which was the goal of the problem.

Combined usage of vertical and horizontal abstraction.

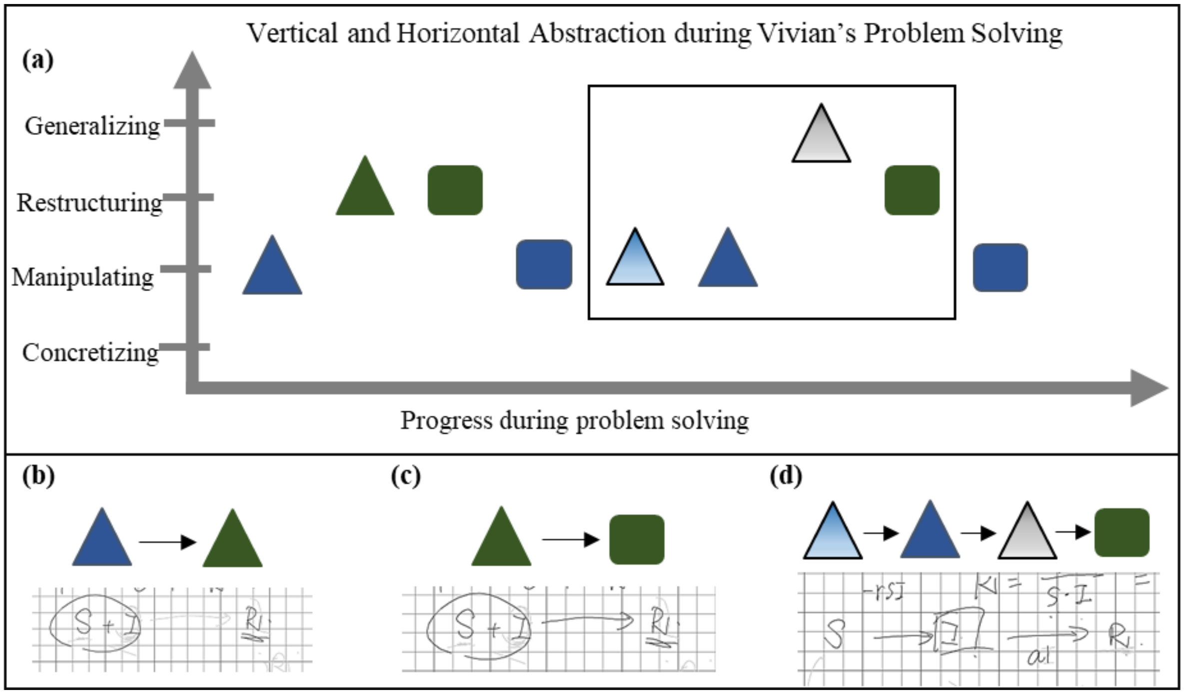


Figure 3.6. Vivian's epistemic actions during the kinetics problem. *Top.* **(a)** Diagram shows how Vivian progressed in problem solving over several minutes. The section in the box represents the interviewer-scaffolded discussion, and icons with gradient fills are interviewer interventions. Triangles represent conceptual actions and squares represent symbolic actions. *Bottom.* Diagram shows how Vivian's representation of the relationship between the variables progressed through **(b)** vertical, **(c)** horizontal, and **(d)** diagonal abstraction.

In general, horizontal and vertical abstraction worked together during problem solving, as participants refined their understanding of the problem over time. Vivian's problem solving is presented as an exemplar (see Figure 3.6). Vivian performed several manipulations that led her to surprising results—for example, when she tried to find the total population by adding the three equations together, she found that they added up to zero. This led her to rethink her assumptions about what the equations represented, which led to a conceptual manipulation action in which she considered how the disease spread:

Vivian: ▲ So I think when the times goes by, they probably have least people they can catch of the disease, because they will be either already catch the disease, or they won't do anything. So when we have more people catch the disease, the *S* should be decrease. I mean, for me it's *r* and *a*, one would be negative and one would be— no, wait.

This conceptual manipulation move solidified her understanding of how the three variables (*S*, *I*, and *R*) represented different populations, and set her up for a conceptual restructuring move. She then vertically abstracted by changing the problem space to represent the population as have two groups (*S*+*I* and *R*). She took the different elements she brought into the problem space through her previous actions (a conceptual understanding of the problem and the idea of change over time) and reorganized the relationships to recognize that there was one total, unchanging population:

Vivian: ▲ Okay, probably what I'd do is plus *I*, and compare with *R*, because *R* is either people die or recover, and here is the people going to catch the disease. Or they can increase the number, and here would be decrease the number. So if I can compare the relationship between them, so I guess I can know what's happening there. [[long pause]] Yeah. I don't really have any idea how to solve that.

This conceptual restructuring, that there are two groups of people that change over time, led to horizontally abstracting through symbolic restructuring, in which she represented the change over time as if it were a chemical reaction (see Figure 3.6b):

Interviewer: Okay. So what were you just thinking about?

Vivian: ■ [[long pause, ~30sec before adding arrow to relation written before]] [[long pause, ~15sec]] I'm thinking these people, they can catch the disease, and these people, they have disease or they can transmit that. At the end, they will all end up with the *R*. So.

Interviewer: The disease totally spreads to the entire population, right?

Vivian: ■ Hmm hmm, so they will be all to the *R* or recover. They'll also go to *R*. Then the rate law would be [long pause, ~20sec], if we're using the rate in chemistry. [[silence while writing]]

She then uses her restructured problem space ($\boldsymbol{S} + \boldsymbol{I} \rightarrow \boldsymbol{R}$) and immediately tests it with a manipulating move, in which she tries to apply the equation for finding the rate constant of a chemical equation. Through a scaffolded discussion with the interviewer in which they conceptually manipulated to understand how the disease spread occurred, she revises her restructured problem space to be a two- step chemical equation through a symbolic restructuring move:

Interviewer: Okay. So if the number of *I* is increasing, then the disease will spread.

Vivian: Yeah.

Interviewer: And what about if the number of *I* is decreasing?

Vivian: ▲ I mean, they still have changed the spread out, but compared with *I*'s larger [the] possible, it's less.

Interviewer: △ Okay. So then does the spread of the disease— In the spread of the disease, which of these three populations does it depend most on? Does it depend most on I, or does it also depend on the other two? In your opinion, from what you've been, how you think about the problem.

Vivian: ■ Okay, right now I'm thinking, just as we say *S*, they can become an *I*, and *I* can become *R*. So *I* would be the intermediate.

This example demonstrates three important features of abstraction. First, previous actions set the stage for abstracting moves (e.g., the work she did to develop the initial relationship $\boldsymbol{S} + \boldsymbol{I} \rightarrow \boldsymbol{R}$ was crucial for setting up the problem space to make $\boldsymbol{S} \rightarrow \boldsymbol{I} \rightarrow \boldsymbol{R}$ possible. Second, horizontal and vertical abstraction worked in tandem to shift and deepen her understanding of the problem space as she made sense of the problem. Third, she engaged in "diagonal" abstraction during the discussion with the interviewer. After she conceptually manipulates, the interviewer scaffolds through a conceptual generalizing action when she asks which population group the spread depends on the most. Vivian then directly shifts to a new symbolic restructuring move, shifting across different levels of vertical abstractness as well as shifting from conceptual to symbolic directly.

Discussion

Bridging between a physical interpretation and the mathematical meaning of an expression can often be challenging in physical chemistry (Becker & Towns, 2012). Several studies have suggested that students may silo their conceptual and mathematical resources in physical chemistry, and that some may “blend” them to generate a new object (Bain et al., 2018; Rodriguez et al., 2020). Similarly, we found that students faced barriers in bridging the conceptual and mathematical aspects of the problems they solved for this study. For example, some students made rich sense of the conceptual aspects of the problem through vertical abstraction, but were unsure how to connect it to the equations through horizontal abstraction.

Developing a disciplinary approach to operationalizing abstraction is key to understanding how students apply and appropriate knowledge during problem solving. In this study, we used a constant comparative approach to build from literature that operationalized abstraction in other domains (e.g., mathematics) to propose a set of epistemic actions in kinetics and thermodynamics problem solving. These actions led to two major outcomes. First, they provide a way to operationalize and identify abstraction processes during problem solving. Second, they led to the development of a framework that captures abstraction processes in physical chemistry specifically on two dimensions: horizontal abstraction and vertical abstraction (see Figure 3.3).

In developing this framework, we also refined the definitions of abstractness and abstracting presented in previous work by this group (Sevian et al., 2015; Weinrich & Sevian, 2017). Weinrich and Sevian (2017) used representation mapping to operationalize abstractness (noun) as “the degree of abstraction (non-concrete references, additional symbol systems, or underlying relations) present in a way a person imagines a problem” (p. 171). In this work, we refined this definition to specify that what is non-concrete references depends on how the person imagines the problem. That is, what is abstract is relative to the problem space, rather than something that can be defined absolutely (i.e., that something is inherently abstract or inherently concrete). Shifting this definition of abstractness to be something contextual and relative also required a shift in what it means to abstract (verb). Weinrich and Sevian (2017) defined abstracting on two dimensions: how strictly a problem solver matches their new instance and prior knowledge representations, and how abstract those representations are. This work builds off of this definition to consider abstracting to be related to (1) how a problem solver uses their prior knowledge to make sense of the problem at hand and (2) how their approaches relate to how they initially view the problem. This shift also represents a change from a cognitive view of abstraction to a more sociocultural perspective of abstraction: a shift from abstraction as a static capacity dependent on internal representations, to abstraction as something contextually dependent and constituting dynamic actions. This leads to our first claim:

Claim 1. In physical chemistry problem solving, epistemic actions can be characterized by how abstract they are relative to the problem space. This suggests that abstractness is both contextual to the problem space and dynamic as the problem space shifts. Our study demonstrates that it is possible to characterize actions at 4 different levels of abstractness, based on the degree to which students integrated knowledge from outside of the problem space (Domin & Bodner, 2012).

Defining the abstractness relative to the problem space implies two important things: first, abstractness is contextual, because what is abstract is determined by the context; and second, abstractness is dynamic, because the problem space can shift and be transformed throughout the course of problem solving. That is, what may be considered abstract at the beginning of problem solving may become concrete through the process of solving the problem, as the problem solver grapples with identifying relevant pieces of knowledge and resources and testing hypotheses.

To illustrate these points, consider two examples of successful problem solving in which the participants reached the correct solution state ($a/r > S$), but took very different paths to get there. Vivian restructured the problem space to be modelled as a 2-step chemical reaction with an intermediate ($S \rightarrow I \rightarrow R$). After this restructuring move, she recognized that dS/dt was the rate of formation of the intermediate and that dR/dt was the rate of consumption of the intermediate. Restructuring the problem space facilitated this realization, because identifying the rates of individual reaction steps is a common activity in physical chemistry. She then performed another common chemistry activity, cued by the restructuring of the problem solving: comparing the rates to see which is larger by setting up an inequality. This manipulating move allowed her to see that for the disease to spread, dS/dt had to be larger than dR/dt, and she was able to perform a second manipulating move to solve for a/r.

In contrast, Hoa and Avni approached their problem solving through a symbolic concretizing move. Similar to Akeyo and Jamila (see Symbolic Concretizing section), the pair substituted in concrete numbers for each of the population groups. They came up with several different sets of numbers to generate hypothetical situations with concrete rates of change that had a positive dI/dt and a negative dI/dt. Building off these concrete situations, they conceptually reasoned about how these different situations represented a disease spreading or dying out. To generate the relationship $a/r > S$, they had to integrate these multiple lines of reasoning and generalize a symbolic relationship (see Symbolic Generalization section).

Both sets of participants started from the same prompt and arrived at the same solution state; however, they took very different paths to get there. Through multiple cycles of Restructuring-Manipulating and Conceptual Restructuring-Symbolic Restructuring, Vivian generated and consolidated an abstract relationship between the variables as $S \rightarrow I \rightarrow R$. Generating this relationship allowed her to manipulate it as an object in the problem space. That is, although the relationship between the variables started as something abstract, it became a concrete element of the problem space, because she transformed the problem space to a model (chemical reaction) that explicitly included rules that governed that relationship.

Hoa and Avni, on the other hand, went through multiple cycles of Concretizing-Manipulating, testing different values to see how they changed the conceptual relationship of the variables. The expression $a/r > S$ was not something they could generate through manipulation, because it was not immediately cued by how they had transformed the problem space. Instead, they had to abstract away from their hypothetical situations to generate the possible relationship by recognizing a way to mathematically represent their conceptual reasoning. We can see from this example that the resources the students cued determined how abstract their actions were (abstractness is contextual), and that the actions they took changed the possible future sets of actions they could take (abstractness is dynamic). Comparing the resources these participants cued on leads us to Claim 2:

Claim 2. How a participant interprets a problem and how they select relevant pieces of prior knowledge influences how they abstract and whether this abstraction is productive for problem solving. However, this seems to be more complicated than just the use of prior knowledge. Prior literature has suggested that abstraction is generally a "better" approach to solve problems and learn concepts (Frey et al., 2017; Sevian et al., 2015). However, we found that abstraction does not necessarily mean more correct problem solving; rather, abstraction can lead to canonically incorrect solutions, and this is determined in part by how prior knowledge is leveraged during problem solving.

To examine this, we will compare exemplar cases of productive and unproductive abstraction. To contextualize student responses, it is important to remember that they were asked to solve these problems at the beginning of the unit; that is, before they had received extensive instruction about kinetics.

Both Vivian and the pair Avni and Hoa successfully solved the problem because they cued on appropriate pieces of prior knowledge. Vivian recognized that it was possible to model the three populations as chemical reactants and represent their changes over time using a chemical equation formalism, which was the first step in the textbook solution. She used her chemistry domain knowledge to recognize the deep relational features of the problem. Hoa and Avni, on the other hand, cued on prior knowledge about how disease spread. They reasoned through how the populations would change if there were specific numbers of people in each group (S, I, and R), and how different sets of numbers could lead to conceptually different situations, depending on whether the number of infectives was decreasing (disease dying out) or increasing (disease becoming epidemic). They cued not on specific domain knowledge, but rather on an understanding of the physical phenomena represented by the system of equations.

When participants cued on inappropriate prior knowledge, abstracting moves were not productive. For example, Philip restructured the problem as one that could be solved using a "Maxwell's relations" approach, by taking the derivative of the two parameters, a and r, to construct a relationship between "uncorrelated variables." He cued on the superficial similarity between Maxwell relations and the rate laws (the presence of derivatives) to restructure the problem. However, this superficial similarity did not reflect a conceptual similarity (the potential to substitute natural variables based on the first and second laws of thermodynamics vs. the rate of change of populations over time) (Chi et al., 1981). His application of this relationship at that point in the problem solving process potentially suggests that he had previously generalized a rule that the presence of derivatives means that he can "correlate uncorrelated things." Several participants mentioned this similarity to the "fundamental equations" (how the instructor referred to the Maxwell relations in class).

All three of these groups abstracted, however, only two of them reached the correct solution. Similarly, there were several participants who conceptually reasoned about the problems very deeply, often through vertical conceptual abstraction, but hit a barrier when they were asked to then apply this conceptual reasoning to the mathematics through horizontal abstraction, similar to the findings of Becker and Towns (2012). This suggests two things. First, there may be a barrier to abstraction that needs to be more fully investigated to understand how and why students abstract during problem solving. Second, attention needs to be paid to how students interpret problems and what they take away from class to understand how to support their abstraction, to support the use of "appropriate" domain knowledge in abstraction.

Conclusions and Implications

In this chapter, we present a framework to study and operationalize abstraction in physical chemistry through the epistemic actions students take during problem solving. Chapter 2 reported a constant comparative approach to characterize actions along two dimensions: conceptual vs symbolic and degree of abstractness, which allowed us to identify two types of abstraction in physical chemistry: horizontal abstraction and vertical abstraction.

The framework expands the existing and limited literature on problem solving in physical chemistry. It provides a lens through which student actions during problem solving can be viewed, and which can co-exist with existing problem solving frameworks that examine problem solving at a larger grain size, such as epistemic games (Rodriguez et al., 2020; Sevian & Couture, 2018; Tuminaro & Redish, 2007). It also presents a novel way of understanding abstraction and abstractness as contextual. Here, abstraction is defined in relation to a problem space; that is, the context in which the abstraction is occurring determines what resources are considered abstract (implicit) and which are considered concrete (explicit). This definition can be applied beyond physical chemistry, to understand and investigate how student problem solving is cued by the type of problem at hand.

Implications for research.

The Epistemic Actions framework was developed by interpreting data in kinetics and thermodynamics problem solving. This work provides a novel tool researchers can use in tandem with other frameworks to investigate student problem solving in physical chemistry, in particular to investigate how students leverage prior knowledge and make sense of problem tasks. It also provides an operational tool to identify and characterize abstraction in problem solving, which we previously mentioned is critical in learning physical chemistry, but is as of yet understudied.

This framework was also developed from a wide range of literature and tested in two contexts. Because it draws on such a wide base of knowledge, and because it was fruitful in analyzing data from different types of problems, we speculate that this framework can be applied beyond physical chemistry. There are two dimensions to the framework: a domain-general dimension, which is represented by the types of actions and their levels of relative abstractness (the vertical abstraction dimension); and a domain-specific dimension, which is represented by the integration of conceptual and symbolic forms of each action (the horizontal abstraction dimension). The 4 domain-general actions (concretizing, manipulating, restructuring, and generalizing) could be adapted to other disciplines to investigate abstraction in other areas of chemistry, science, and beyond.

Finally, the Epistemic Actions framework may also have utility for investigating a broad range of meaning making in physical chemistry, not just in problem solving. The RBC-C, a similar framework in mathematics education and an inspiration for this work, has been used in a range of contexts, including individual and group problem solving and documenting collective meaning making in math classes, and has been fruitfully combined with other theoretical frameworks (Dreyfus et al., 2001; Hershkowitz et al., 2007; Özçakir Sümen, 2019; Tabach et al., 2020).

Implications for practice.

There is intriguing theoretical work that suggests that deliberately modelling abstraction practices can support students' fluency in making sense of and navigating abstract representations (Blackie, 2014; Maton, 2013). Often, chemistry instructors model how to problem solve, and implicitly assume that students share their mental representation of the problem and prior knowledge; however, there is evidence from organic chemistry that this is not necessarily true (Caspari et al., 2018). This study provides a set of actions that students may engage in while problem solving, and thus provides a framework for instructors in physical chemistry to (1) be more intentional in how they model abstraction and (2) more carefully attend to how students make sense of complex mathematical and conceptual objects.

To illustrate this, imagine a physical chemistry professor who wants to emphasize that calculating an entropy change from the Second Law depends on the nature of the system at hand. The professor may start from an expression of the Second Law and define the system they are working in (concretizing)—for example, the isothermal expansion of a gas. They may then vertically abstract to talk through what kind of boundaries this system has (conceptual restructuring), and how those can be represented mathematically (symbolic restructuring). While solving the equation for entropy change of an isothermal expansion, the professor may engage in horizontal abstraction to reinforce how each calculation step corresponds to something meaningful about the system. By being deliberate in showing how the physical system and mathematical representations relate (horizontal abstraction), and how these relationships change the expression of the second law (vertical abstraction), the professor can model the reasoning they expect from their students during this type of problem solving.

Furthermore, there exists a discrepancy between instructional goals and assessment practices in physical chemistry in the United States—the majority of instructors report that their assessment goals for students are conceptual understanding, yet their assessments primarily consist of mathematical problem solving (Fox & Roehrig, 2015). This gap may be bridged by either intentionally modelling horizontal or vertical abstraction. The professor may intentionally design opportunities for horizontal and vertical abstraction into problems. For horizontal abstraction, the professor may ask students to think about the connections between concept and mathematics and deepen their understanding of an object. For vertical abstraction, they can design problems that require students to integrate and test different lines of reasoning and build toward concept construction.

Limitations

As a study that sought to build grounded theory, the analyses were based on an interview task that specifically sought to elicit abstraction. In more conventional physical chemistry tasks, abstraction may serve different purposes. For example, the task students were provided in the interviews required them to think about an unfamiliar context (disease spread), so abstraction activities often served the purpose of trying to understand what the problem was tasking them to do. In more familiar problem contexts, abstraction may serve a different role—for example, restructuring to try to figure out what equation may be the most appropriate for the problem at hand.

Furthermore, although we report the frequencies of each action to contextualize the data, these frequencies are deeply situated in the study context and do not necessarily suggest generalizable findings. For example, we observed that conceptual actions were much more frequent than symbolic tasks; however, the task had a biological context and several variables participants found to be ambiguous, which resulted in a higher frequency of conceptual than of symbolic actions as participants attempted to make sense of the variables' meanings. As we note above, abstraction is contextual, so frequencies of actions are likely to be related to the nature of the problem task. The frequency of the epistemic actions was also based on what participants vocalize. It is possible that participants made connections that they did not vocalize, and thus that we could not code for, or that were implicit. This may explain diagonal abstraction, when participants moved directly between a conceptual and symbolic move at two different levels of abstractness. It is possible that these participants had a "blended" understanding; that is, that their conceptual understanding of an object was tied to its mathematical representation, and so they moved through a blended framework of relations (Bain et al., 2018). It is also possible that they did the intermediate step (e.g., translating between conceptual and symbolic) in their heads. To better understand this phenomenon, further research is needed.

CHAPTER 4

WHY DO STUDENTS ABSTRACT? AN EXPLORATORY INVESTIGATION OF THE INFLUENCE OF DIALOGIC INTERACTIONS ON THE EMERGENCE OF ABSTRACTION IN THERMODYNAMICS AND KINETICS PROBLEM SOLVING

Introduction

Motivation.

The expectation for abstraction is often implicit and goes unfulfilled in the teaching and learning of physical chemistry. For example, *horizontal abstraction*, which involves bridging between mathematical and conceptual understandings of an object, has been extensively documented as challenging for students in physical chemistry (e.g., Becker & Towns, 2012; Rodriguez et al., 2020). Mathematical and algorithmic problems are often assigned by physical chemistry professors as a means to develop conceptual understanding (Fox & Roehrig, 2015; Mack & Towns, 2016); however, this can lead to a disconnect between instructors and students. Translating this phenomenon into the language of the

Epistemic Actions of Abstraction framework, professors expect students to horizontally abstract to build conceptual understanding. Furthermore, *vertical abstraction*, which involves extracting salient details and comparing cases to make generalizations, is known to be an important process for problem solving and conceptual learning in general and organic chemistry (Domin & Bodner, 2012; Frey et al., 2017; Weinrich & Sevian, 2017) and has been hypothesized to be particularly challenging in undergraduate thermodynamics (Sevian et al., 2015). To understand how to better foster these processes in physical chemistry courses, it is important to examine what influences how and whether abstraction occurs.

In Chapter 3, I present the Epistemic Actions Framework as a method for making abstraction during physical chemistry visible (see Appendix 3 for the codebook that emerged from the results of Chapter 3). However, observing abstraction is only the first step toward making sense of why abstraction occurs. In this chapter, I conduct an exploratory analysis of the teaching interviews collected in Chapters 2 and 3 toward two ends. First, I address a problem of theory and flesh out what it means for abstraction to be an activity in an activity theoretical sense. To this end, I examine what excites abstraction by analyzing what constitutes a need for abstraction, and better operationalize what constitutes an abstraction activity. Second, I address a problem of practice and conduct an exploratory inductive analysis of the dialogic interactions between three salient parts of the activity system (the interviewer or more knowledgeable other, the problem solver(s), and the task itself) to better understand what sustains and constrains abstraction.

The purpose of this study is not to extract some objective truth about the nature of abstraction, the existence of which is precluded by an activity theoretical orientation that posits abstraction as dynamic and contextual. Rather, I seek to understand how abstraction is mediated through important interactions as they occur in a clinical interview setting to understand how supportive interactions can be transferred back to the classroom.

Interactional Influences on Problem Solving

Although many factors influence problem solving, this brief literature review focuses on those that arise from interactions among different aspects of the activity system that can be directly observed during an interview setting, e.g., the interactions between the participant and their peers, the interviewer, and the task (see Figure 4.1). As described in Chapter 2, the teaching interviews were designed based on these three types of salient interactions (interviewer-participant, participant-participant, and participant-task). Thus in this literature review I will provide a brief overview of relevant literature under these three umbrellas.

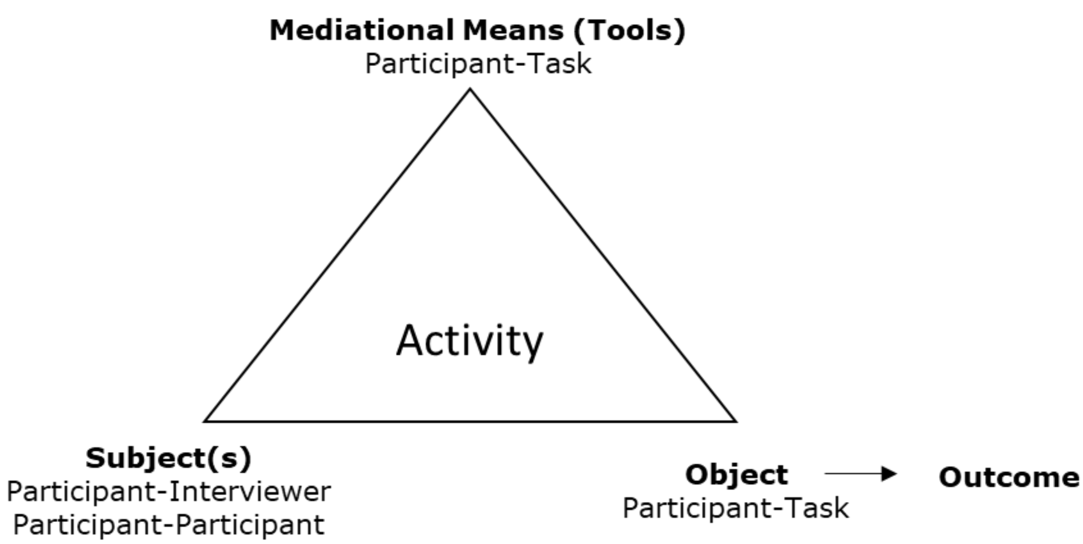


Figure 4.1. Depiction of the first generation activity cycle with the three types of interactions mapped onto it.

Interviewer-Participant. In qualitative research in the last twenty years, there has been increasing awareness of the role of the interviewer as a co-constructor of the interaction with the participant and of the interview itself as an interactional object (diSessa, 2007; Potter & Hepburn, 2005).Qualitative data in social science research are often collected through semi-structured interviews. There are many types of semi-structured interviews, such as clinical or cognitive interviews, but at their core, a semi-structured interview involves a researcher trying to collect data about a phenomenon using a protocol, and improvising in response to participants' answers to dig deeper into a phenomenon.

These responsive interactions between an interviewer and a participant can be intentionally leveraged and designed for to influence how an interview will proceed. This study, for example, used a clinical teaching interview design, which deliberately positions the interviewer as a more knowledgeable other whose role in the interview is to support and challenge students' meaning making to better understand the boundaries of what a participant knows and chooses to leverage during the problem solving session (Kapon, 2016; Kapon & diSessa, 2012). Intentional support of student meaning making has been implemented in studies designed to scaffold students' model-based reasoning (Broman et al., 2018) and mechanistic reasoning (Caspari & Graulich, 2019) in problem solving scenarios.

However, an interviewer can influence student meaning making, even when the interview is not intentionally designed for the interviewer to shift students' thinking. Russ, Lee & Sherin (2012) examined how participants' frames during interviews designed to assess conceptual physics knowledge were influenced by their interaction with the interviewer. Unlike in the studies discussed above, the interviewers in Russ and collaborators' study did not set out to shift how students were thinking through interaction. The authors found that there were three common frames students would enter during these interviews (the inquiry frame, the oral examination frame, and the expert interview frame), which influenced how the participant approached the task. They found that epistemic cues from the interviewer cued participants to enter different frames. For example, in one instance they found that the interviewer cued a student to shift from an “oral examination” to “expert interview” frame by reassuring the participant that they did not need to provide the “right answer” or “right word.”

Participant-Participant. As collaboration has become more important in the 21st century workplace, group work and has been increasingly integrated into education at all levels (Graesser, et al., 2015). In chemistry, collaborative problem solving has been shown to have positive effects on cognitive outcomes such as the ability to apply knowledge to new contexts (Stockwell, Stockwell, & Jiang, 2017), deeper conceptual understanding developed through communication with peers (Mahalingam, Schaefer, & Morlino, 2008), and improved problem-solving strategies that transfer to individual contexts after group work (Cooper, et al., 2008).

However, the influence of collaborative problem solving on learning is not so simple. When students collaborate, they have to navigate complex relational and task-specific factors, such as defining joint goals, that determine whether the problem solving is productive (Barron, 2000, 2003). For example, Sohr and collaborators (2018) found that groups manage socioemotional tensions by introducing conversational “escape hatches,” which shift the focus of the conversation even when conceptual resolution has not been reached. Langer-Osuna (2016) found that how students in dyadic mathematical problem solving took up each other's ideas based on perceived and constructed authority, the perception of which was influenced by interaction with the teacher.

The role of peer interaction on abstraction has been shown to be similarly complex. In middle school mathematics, Schwartz (1995) found that dyads produce more abstract representations on average than individuals, in part because developing abstract representations allowed the two members of the dyad to develop a shared representation. However, there is constrasting evidence about the types of peer interaction that foster successful abstraction. Hatano and Inagaki (1991) found that horizontal interactions (i.e., where peers are at similar levels of competence) are more likely to result in the construction of new knowledge, because the absence of a power dichotomy allows peers to challenge and advance each other's ideas. A case study by Dreyfus, Hershkowitz and Schwarz (2001), however, found that the presence of a more competent peer facilitated constructive (abstracting) processes, because the pseudo expert-novice relationship allowed the less competent peer to explore the boundaries of what they can only do with help (zone of proximal development). This suggests that abstraction facilitated in group problem solving is more complex than just the levels of competence of the members in the group and interactions may need to be attended to.

Participant-Task. How problem solvers approach and interpret problems has been the focus of a large amount of literature (Bodner & Herron, 2003; Bodner & McMillen, 1986; Domin & Bodner, 2012; Niss, 2017; Rodriguez et al., 2018; Sevian & Couture, 2018; Tuminaro & Redish, 2007; Weinrich & Sevian, 2017). In particular, a growing focus in physical chemistry education has been how students navigate the use of different epistemological resources and problem solving approaches (Bain et al., 2018; Rodriguez et al., 2018, 2020). For example, Rodriguez and collaborators (2020) investigated the epistemological commitments physical chemistry students use when approaching mathematically dense problems, in order to better understand how to subvert the compartmentalization of mathematical knowledge. This built on work in physics education, which developed the blended processing framework, an approach to understand how mathematical formalisms, such as equations, can be used as conceptual tools (Kuo et al., 2013, 2019).

The other critical aspect of the Epistemic Actions Framework is how problem solving is defined. I draw on Jonassen's (2010) definition of problem solving as having two components: the problem space and what the problem solver does to manipulate and transform the problem space toward a solution state. This suggests that the interaction between the participant and how they initially encounter the problem is vital for understanding how they will solve it, because the problem space consists of the problem solver's interpretation of the problem and its constraints (Chi et al., 1981). For example, one salient influence on problem solving is an individual's epistemological framing. Framing is one's answer to the question, "what is going on here?" and determines what resources they identify as relevant to solving a problem (Hammer et al., 2005). In physical chemistry, Rodriguez and collaborators (2020) found that students often stayed in a single frame while solving kinetics problems, which influenced the types of epistemic games they played while solving these problems.

This brief literature review is meant to convey the work that most strongly informed the interpretation of the data in this exploratory study. This leads to the two research questions:

(RQ1) How does a need for abstraction manifest during problem solving?

(RQ2) What in-the-moment interactions sustain or constrain abstraction during physical chemistry problem solving?

Activity Theory

This work conceptualizes abstraction as an activity in a Leontievian sense (Hershkowitz et al., 2001; Leont'ev, 1978). An activity is human work that is excited by a need and that takes place under certain conditions. Abstraction can be described by its activity structure—the structure that describes the relationship between what is done (the action), why it is done (the motive), and the context in which it is done (the condition) (see Figure 4.2). According to activity theory, these three things (action, motive, and condition) are the three parts that describe an activity. Chapter 3 introduced the 8 actions that I identified to identify abstraction in physical chemistry. In this chapter, the need and motive will be investigated to elucidate the interactions that influence abstraction.

An activity occurs in response to a need (see Figure 4.2). A need excites activity, but it does not organize it (Kaptelinin, 2005; van Aalsvoort, 2004). A motive is how a need is shaped by context, consequently organizing activity. For example, water purification may be excited by a community's need for clean water, but how the water purification is carried out (how it is organized) depends on the specifics of the context—for example, the nature of the contaminant in the water and the local infrastructure. The motive may be a need for water free of lead, which then guides how the activity is carried out. Activities consist of discrete steps or actions—for example, collecting the water, transporting it to the purification plant, carrying out the purification itself. Each of these actions are driven by their own discrete goals, such as "bring water to plant" is the goal that drives the action of "transportation." Therefore, an activity consists of a sequence of goal-directed actions, and is excited and organized by an overall motive.

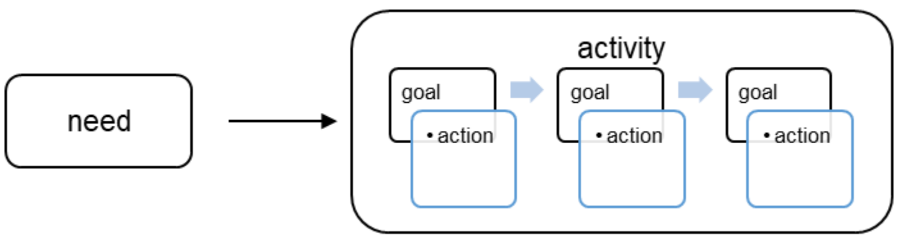


Figure 4.2. Simplified generalized structure of the need structure of a first-generation activity.

Here, abstraction is also considered to be an activity (Hershkowitz et al., 2001). During problem solving, a need for abstraction may arise from the student running into some barrier that prevents them from solving the problem using a procedural approach. For example, they may encounter two disparate concepts that they cannot reconcile, which causes them to feel stuck, exciting a *need* to try something different. Depending on how they approach overcoming this barrier, the need is transformed into a *motive*—the student may try to overcome their "stuckness" by connecting the two concepts or may reframe the problem through a different type of problem space. This is done through a series of epistemic actions that individually work towards this motive, and each of these actions is directed by a goal. The epistemic actions that make up the abstraction activity are captured by the Epistemic Actions of Abstraction framework (see Figure 4.3). If the activity that occurs in response to the need involves epistemic actions oriented toward that need, and that either change the horizontal or vertical abstractness of the problem space, then the abstraction activity is realized. If the activity that occurs does not involve successful horizontal or vertical abstraction, then the abstraction activity was impeded.

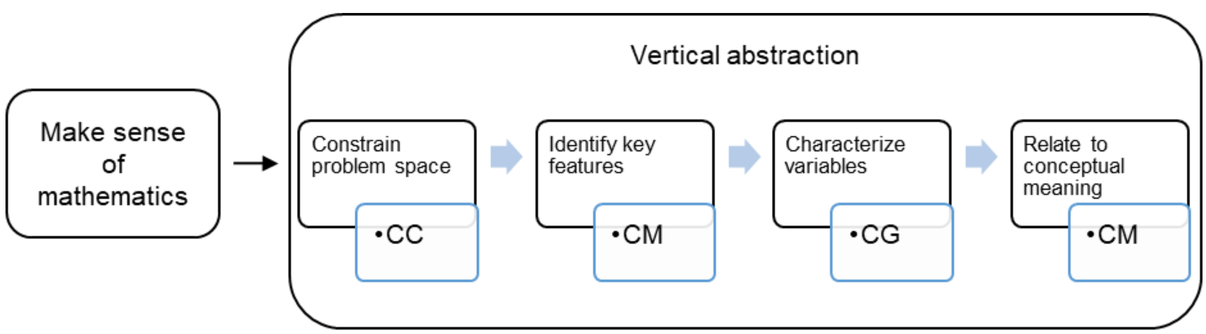


Figure 4.3. Example structure of an abstraction activity. Each of the acronyms represents an epistemic action: CC (conceptual concretizing), CM (conceptual manipulation), CG (conceptual generalizing), and CM (conceptual manipulation. This example was taken from Chao's kinetics interview.

Operationalizing the Abstraction Activity

Although *need* and *motive* are linked, motive can be difficult to observe, because it relies on how the agent is thinking through what they are doing and what is done deliberately. For this study, I focus on the *needs* that excite abstraction. Needs were easier to identify because a need excites activity, so they are generally the initiators of an activity and recognized by the problem solver themselves. Second, a motive is very dependent on context, so to identify something that may be slightly more generalizable I look at need.

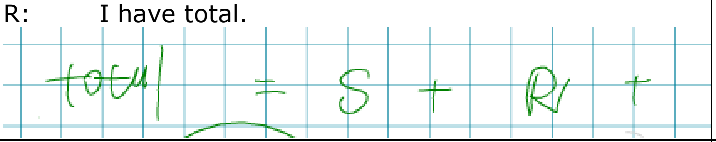
To operationalize an activity, I had to understand how it was initiated (through a need) and how it ended (by being realized or not). Thus, an abstraction activity was characterized by being initiated through an explicit or implicit need that could be fulfilled through abstraction, and which concluded either by abstraction occurring (the need being fulfilled) or by the agent switching focus to another task (the need staying unfulfilled).

Activities were identified following the structure of an activity in first generation activity theory. To that end, three features were sought out: how was the activity initiated (what seems to be the need for abstraction?), how was it carried out (what task did the participant seem to be focused on?), and how was it resolved (was the need met?). First, the entire transcript was reread closely, paying attention to how participants' problem solving developed through the actions coding. To support interpreting participants' problem solving, two types of notes were taken. First, interpretative descriptions of what the participant was doing were written in the notes column. How that line furthered the participant's problem solving was described in the goals column. Descriptions for goals and notes were used to support interpretation of the activity by helping to identify when a participant was focused on some subtask during problem solving. Identification of activities was supported by previous coding that focused on identifying the phases of problem solving. As coding developed and the idea of an activity crystallized, the goals and notes columns were utilized less.

Below, an example of an abstraction activity is presented with these analytic memos (see Table 4.1). This table is provided as an exemplar to show how the idea of goal provides insight into the role of the action in both the activity and in the problem solving at large. The table represents one coherent activity.

This participant (Vivian) began by trying to interpret the variables and equations given in the problem. She was particularly focused on trying to figure out what kind of mathematical constructs the variables in the problem are, and reasoned that there were two possibilities: the variables *S*, *I*, and *R* could be "numbers" or they could be a "ratio." This was coded as a need: understand the assumptions of the problem, and quickly results in a symbolic restructuring move as she tries to redefine the problem space. Her reasoning is supported by an interviewer hint that there is a constant population (lines 60-62).

Transcript Line	Code	Goal
48. I: Okay, then we can move on to the second one. All right, so here's the second question. So what are you thinking about?		
49. R: So *SIR* is the ratio of, it's the number.		
50. I: So these *S*, *I*, and *R*, you mean?		
51. R: Hmm hmm.		
52. I: They're the numbers.		
53. R: They're the numbers.		
54. I: Does it make a difference for you whether the ratios are the numbers? I'm just wondering why you asked.		
55. R: Yeah. Yeah, I'm just thinking, if the ratio, then we also need to know the total number, I guess.	SR	Defining/changing the problem constraints by identifying the types of mathematical constructs

56. I: Do you think knowing the total number is important?		
57. R: I guess yes, because the ratio can be changed over the time. But I guess yeah, here is counting the ratio, the slope.		
58. I: So there is an embedded assumption in the question that the population is constant,	Int. SC	Providing a hint to constrain the problem space.
59. R: Okay		
60. I: but I don't know if that helps. So that the total of *S*, *I*, and *R*	Int. SC	Providing hint to constrain the problem space
61. R: --added together		
62. I: will always be one constant number.	Int. SC	See above
63. R: I have total. 		Accepting the constraint and writing it out as an equation
[[long pause around ~1 minute after writing this relation]]		
64. I: Can you tell me what you're thinking about?		
65. R: I'm thinking about how to figure out the ratio of *a* and *r*. And if I'm thinking, I was thinking if I add those numbers together, they would be equal zero. But if I add—	SM	Mentally substituting the rates for the differential expressions to solve for the ratio; realizes that the result is unexpected
66. I: Okay. Can you tell me a little bit more about that? Oh. Sorry, you just made me realize something about this problem by saying that that I hadn't thought about. So can you tell me a little bit more about that, if you add these up, they equal zero?		
67. R: So when I was thinking the questions, and here is when the time goes bys and we get negative number here, which means the number going a bit decrease, and I mean— Yeah, actually I was assuming the total number is equal *S* plus *R* plus *I*, and then I was thinking to add those things together to see what happens, and I realized that when I add all those things together, they will be equal to zero.	SM	Solving the equation to see they equal zero.
68. I: Okay. That they'll equal zero on one side, but what about on the other side?		
69. R: The other side would be the total number divided by *t*. That's my assumption still. *t*'s [?] things.	SM	Following logic to solve for the other side
70. I: Why did you think to do that? Or did you just notice that if you add these together, they equal zero? I guess I'm wondering if you looked at the problem and you realized that they equalled zero, or if you wondered first, what would happen if I add them, and you realized that they'd equal zero?		

71. R:	So when I look at that, I was thinking if that's my— So here, they was acting in a ratio to decide whether the disease spread at all, die out.	SM	Making sense of the ratio relationship
	So I was thinking that then we can look at what happened for the total peoples there, if we're assuming that that's our total number.	CM	Explaining how the ratio relationship gives useful information for the physical scenario
	So if we add all those things together— Yeah, so that's why I think I'm going to [put] those things together.	SM	Explaining choice to add the values together

Table 4.1. Example of an abstraction activity in Vivian's kinetics interview. Acronyms stand for the epistemic action— those starting with "S" indicate a symbolic action, and those starting with "C" indicate a conceptual action. Those ending with "C", "M", and "R" represent concretizing, manipulating, and restructuring respectively. Epistemic actions made by the interviewer are indicated with "Int." (e.g., "Int. SC" indicates that the interviewer is symbolically concretizing, which may be picked up by the problem solver).

After her abstraction was initiated by a need (identify the type of mathematical construct "*SIR*" is), she and the interview go back and forth reasoning through her thoughts. Each of her epistemic actions work toward the goal of identifying the construct. For example, she wants to test whether the values given are ratios. To figure this out, she decides she needs to find the total population, so she can figure out the ratio for *S, I,* and *R* (lines 57-65). She initially views the differential equations as equivalent to the values *S*, *I*, and *R*, and tests this by adding the three differential equations together in her head (65-67) to find the total. Originally, she believes this will give her the total value; however, instead they give her zero, which violates her expectation. In line 71, she begins to shift her understanding of what *SIR* is ("I *was* thinking"). This abstraction (symbolic restructuring to symbolic manipulation to conceptual manipulation) gives her a deeper understanding of *SIR* (meeting her need), which she carries forward in her sense-making. The goals column helped make this focus on figuring out the math visible throughout her problem solving, because it explicitly laid out what each step was accomplished both in the moment and in the context of the broader problem solving.

Methods.

The interviews were designed as teaching interviews, as teaching interviews have previously been used in studies investigating abstraction.

Positionality and Reflexivity in Interviews

In this section, I will discuss my positionality and describe why first-person and third-person are both used to describe the role of the interviewer. My epistemological stance on interviews is that they are interactional experiences co-created between the interviewer and the participant (Potter & Hepburn, 2005; Tanggaard, 2009). The interview is an interactive artifact that is analyzed as the unit of analysis for this study. In this analysis in particular, I play a dual role of insider and outsider. As an insider, I was an active member of the interview, and thus I co-produced the interactional event that is being analyzed. When conducting the interviews, I brought a theoretical lens about what abstraction is and what I hoped would emerge from the interviews that shaped how I encouraged participants' meaning making, how I responded to their questions, and how I framed my own questions. In return, participants brought their own framing of what it means to participate in an interview and how they viewed me, potentially as an unknown authority, which shaped how they responded to my probes and how they decided what was appropriate to discuss (Russ et al., 2012). Our interactions, from verbal and nonverbal communication to power dynamics, shaped the production of the interview artifacts. This is common in qualitative interviews and is important to acknowledge.

Particularly relevant for this chapter, my interactions as the interviewer are a key aspect of the analysis at hand. However, I am also the interpreter of the data, and in doing so I bring an outsider perspective that is informed by literature and a removal in space and time. In particular, when writing about my interactions with participants as the interviewer, I draw on my memos and memories of the interactions to inform interpretations of why I said or did things the way I did (insider-status), while analyzing my role as the interviewer as an external person who made situated choices and as a dialogic piece of an artifact of data (outsider-status). To that end, and to honor this dual status, I will use the third person when talking about myself as the interviewer (Kapon, 2016). The interviewer-subject will be referred to as "Interviewer" and actions and thoughts from her perspective, as informed by research memos, will be described in the third person (e.g., "She believed the participants were ready to end the interview"). "Interviewer" is distinct from the first-person "I", which is the perspective I as the interpreter of data will take.

Data Collection and Participants

Data collection for this analysis is described extensively in Chapter 2. Twenty-six total interviews were conducted (see Table 4.2). Although the majority of transcripts were used in this analysis, a subset were excluded. Specifically, this study did not deeply examine the activity structures in the paired entropy interviews. This was done because the entropy task was not well-designed to elicit abstraction and tended to result in algorithmic problem solving. Because collaborative problem solving dynamics can be very complex, it was more fruitful to more deeply examine a subset of the data (the kinetics interviews), which contained richer problem solving and more theoretically relevant data.

	# Individual Interviews	# Pair Interviews
Entropy Interview	9	4
Kinetics Interview	8	5

Table 4.2. Number of interviews. Pair entropy interviews were excluded from analysis.

Data Analysis

The approach to data analysis occurred in multiple steps. First, interviews were coded with the epistemic actions of abstraction to identify the actions that took place during problem solving. These interviews were coded in NVivo, a qualitative data analysis software (see Chapters 2 and 3 for more details). Then, the actions coding was used to support identifying abstraction activities (see the section *Operationalizing an Activity* above).

After the abstraction activities were identified, they were inductively coded for sustainers and constrainers of abstraction, guided by looking at the three salient types of interactions identified in the research design: participant-task (P-T) interactions, participant-interviewer (P-I) interactions, and participant-participant (P-P) interactions. These interactions were attended to in order to identify which interactions were important for sustaining or constraining abstraction, and what about that interaction sustained or constrained abstraction in that moment. Themes, guided by our literature review on problem solving, were emerged to characterize the core of these interactions.

To analyze these themes, a holistic coding approach was taken to maintain the situated nature of abstraction. First, for each activity, a narrative summary was written of (1) how the abstraction activity was carried out, and (2) the salient aspects of interaction that influenced it. This was done to understand how the abstraction activity was situated within the context of that problem solving. Based on whether or not the abstraction activity was realized, the interactions were coded as either sustainers (facilitated the activity) or constrainers (interrupted the activity). The summary was used to help identify what aspect of the interaction sustained or constrained abstraction. These categories were identified through the lens of the literature review laid out above, e.g., when reading P-T interactions, I did so through the lens of epistemological framing and the use of conceptual versus mathematical resources. When reading P-I interactions, I attended to the role of the interviewer in scaffolding reasoning, and to the role of the interviewer as an authority who influenced the problem solver's framing. At the same time, these were not set, deductive categories, and the analysis was open enough to allow for the emergence of unexpected findings.

After coding for activities, all codes were compiled into an Excel document that had columns for the type of interaction (P-I, P-T) theme (e.g., framing, interviewer scaffolding), the need, whether the interaction was a constraint or sustainer, the pattern of actions, and narrative summary, as well as identifier tags (e.g., line numbers and transcript file name). This was a living document, and additional columns were added that helped refine what aspects of the different constructs (e.g., interaction and need) were important to attend to, such as function [of the need], who introduces need, and motive.

In particular, these additional categories informed the analysis of needs. The “needs” that were inputted into the Excel document were the highly situated and contextually dependent situations participants encountered (e.g., make sense of mathematics). These different columns helped make sense of the role the needs played during problem solving. “Motive” described to what end the need served the problem solving. Together, the “needs” and “motive” column were used to identify patterns in the role the need played during problem solving. Patterns were identified across the dataset, which led to the development of the categories in “function” (see Table 4.3 for an excerpt).

Need	**Function**	**Who introduces need**	**Motive**
figure out conceptual meanings of *a* and *r*	task-directed	Participant	directly solving the "guiding question"
figure out conceptual meanings of *a* and *r*	situational-insufficient	Interviewer	interviewer-driven
understand assumptions of problem	task-directed	Participant	trying to figure out how to move forward by figuring out whether *SIR* are ratio or numbers
unexpected result from prior manipulation led to need to make sense of math/physical scenario	situational-emergent	Participant	shifted problem solving from trying to calculate ratio to thinking about physical scenario

Table 4.3. Example of coding from Excel document. Each row represents the coding for a different abstraction activity. The examples above are all from a single participant (Vivian).

A note on data saturation: two caveats should be noted about the data analysis. First, this is an exploratory study designed to examine the role of interaction dynamics on problem solving. As such, the goal was primarily to identify which interaction dynamics influenced abstraction in problem solving, rather than to deeply characterize them. Second, this study is rooted in activity theory, which conceptualizes an activity as contextually situated. To that end, my goal was not to identify trends that are generalizable, but rather to identify rich examples that further a theoretical understanding of abstraction. That is, I sought theoretical saturation, which is met "when the complete range of constructs that make up the theory is fully represented by the data" (Starks & Brown Trinidad, 2007, p. 1375). Here, theoretical saturation was considered to be met when there were emergent themes for each of the constructs we identified as parts of the activity system that can potentially affect abstraction (task, peer interaction, and interviewer interaction).

Findings

Research Question 1.

A need, as characterized in activity theory, is something that excites activity. Thus, I looked for needs that excited abstraction. Because activity theory is the theoretical framework, I assume a priori that abstraction occurs in order to fulfil something that is otherwise unmet. By investigating needs, I am trying to understand what sparks abstraction, and to infer what seems to be going unmet that is satisfied through abstraction. This is important, because the literature on physical chemistry suggests that abstraction is a valued process through which students and instructors can build conceptual knowledge and make mathematical sense, and that one way professors strive to do so is through problem solving. However, *how* to elicit abstraction is not well understood. To foster abstraction processes, it is important to understand what sparks abstraction to occur in the first place.

Looking across all of the needs that arose in the data, three types of needs emerged, guided by the questions, "What function does abstraction play? Why is there a need for abstraction?" The question of "function" provided a lens to think about the how abstraction moved problem solving progress forward and to infer what abstraction fulfilled at that moment.

Overall, the findings suggest that abstraction served a larger goal of moving forward in solving the problem. A need for abstraction arose because abstraction seemed to be perceived by the participants in the interaction as a way to move forward in solving the problem based on the barriers they faced in that moment. That is, abstraction seems to have been a tool for problem solving rather than a goal in and of itself. The findings for RQ1 could be categorized in three ways based on how the abstraction activity served the larger goal of problem solving, and what seemed to spark the need.

Task-directed needs.

The first type of need related directly to features of the problem. These *task-directed needs* arose when the participant interpreted the problem as one that demanded abstraction, generally as part of their initial problem space. Task-directed needs were cued directly by features of the task. For example, the kinetics problem tasked students with understanding complex equations within a biological context and asked them to figure out the conceptual meaning of two variables, a and r. For several participants, this led to the task-directed need "make sense of mathematics," because the problem text explicitly directed them to do so (a task-directed need for horizontal abstraction).

Below is an example of a task-directed need from the entropy interview. The participant, Alex, recognized that to solve the problem, he needed to understand the connection between the different aspects of the problem task he currently framed as unconnected:

204. Alex:	Oh so it's saying that if each rung of the, each rung of this, this whole thing were random, what would be the residual entropy. Right? It's hard to think about. Um. So it's already kind of random. Not really. Ah. Ah. It's hard to think about
205. Interviewer:	Yeah, talk
206. Alex:	I can't relate, I don't know I can't relate it. I don't know how I would relate these two ideas.

Alex identified a need to relate the ideas of entropy and DNA together, but realized that he did not have a framework to do so ("I don't know how"). This is a task-directed need for abstraction, because he framed the task as requiring him to do this in order to move forward, however he does not have a procedural way to do it, so it requires abstracting.

Situational-insufficient needs.

The second type of need arose when the participants' current problem solving approach was no longer sufficient. These *situational-insufficient needs* occurred when the participant's previous problem solving approach was no longer useful to move forward in problem solving and they became stuck. A situational-insufficient need emerged when an initial problem solving approach that did not involve abstraction, e.g., a procedural approach, did not yield expected results. Needs that were coded as situational-insufficient included "can't find a useful equation for entropy" and "manipulation strategy was insufficient." A typical discursive marker of a situational-insufficient need was that it was explicitly vocalized (e.g., "I'm not sure what to do").

In this dataset, the interviewer typically co-constructed the situational-insufficient need. That is, participants would identify that they felt they could no longer move forward in solving the problem but were not sure what to do, and the interviewer would suggest abstracting as a way to overcome this barrier. The following excerpt from Joshua's problem solving is a typical example of how the interviewer would scaffold abstraction as a way of moving forward in the problem. Prior to this moment, Joshua had been trying to engage in horizontal abstraction to define the conceptual meanings of the different variables. However, he eventually got to a point where he felt stuck:

111. Joshua:	I mean, as far as this goes, I guess, it would I guess be the amount of people susceptible minus the amount that have recovered makes sense. I just don't know exactly what the variables mean.
112. Interviewer:	But this might have something to do with transmission, you said, and this might have something to do with people who are healthy.
113. Joshua:	Yeah.
114. Interviewer:	Okay. Let's take a step back from the math, maybe, for a second. So it seems like you know a lot about biology. And I know nothing about biology. So can you just tell me a little bit about how disease is spread? What you know about how disease is spread.

Joshua hit a barrier in his problem solving when he was trying to figure out the conceptual meaning of the different variables. As a means to move forward with the problem solving, the interviewer introduces a need to vertically abstract, by suggesting that he recontextualize the problem task in a larger biological framework (ll. 114). He later reengaged with the horizontal abstraction more productively, because he could draw on the mediational tools introduced to the problem space through his vertical conceptual abstraction.

Although situational-insufficient needs generally led to a discussion that was scaffolded by the interviewer, participants could also independently move forward with abstracting after recognizing the need. For example, when she began solving the entropy problem, Hoa wanted an equation she could use to solve for ΔS; however, none of the equations on the equation sheet provided by the interviewer matched how she interpreted the problem:

61. Hoa:	Well most of these has temperature, and there is no temperature in here. And others have volume, and they don't have this here. I'm not really sure how to go about it. I just wonder, if you're just multiplying this to get the order of the system.
62. Interviewer:	Can you tell me a little bit more about that?
63. Hoa:	So I'm guessing that like how orderly a system is, as in like how many possibilities there is. So if they're asking for possibility there's four different kinds of these binucleotides, so I would just multiply it. So like this is saying that the average DNA molecules has 5 times 8 to the 10 binucleotides. But these four of these, so let's say this is A, this is B. I'm not sure if I'm doing this right

She identifies a barrier to solving the problem using her current procedure ("I'm not really sure how to go about it"), and immediately shifts away from a manipulating, procedural strategy to approach the problem to restructuring it in terms of thinking about how ordered the system is and in terms of probabilities.

Situational-emergent needs.

The third type of need for abstraction came about when an opportunity for abstraction emerged as a result of the participant's problem solving. These *situational-emergent needs* sparked abstraction as something that could occur to deal with the unexpected, or because some piece of information came up that sparked a new idea, which created an opportunity to abstract that the participants could capitalize on. These were the least common type of need found.

For example, consider the excerpt below from Avni and Hoa's interview. Immediately prior to this moment, they had been trying to figure out the conceptual meanings of *a* and *r*, but were not able to reach consensus. After several minutes of back and forth, they both revealed that they were stuck and had a brief discussion with the interviewer. The interviewer interpreted that they were ready to stop problem solving and, following her protocol, summarized what they had done so far during their work, and moved to ask them if they had any final thoughts:

261. Avni:	I cannot think about it any [?: further].
262. Interviewer:	Okay. So I want to sort of look at what you've done so far, see if it makes sense. Okay, so you started by trying to figure out what you could do with these equations. Okay, so you started by trying to manipulate these equations to get the ratio.
263. Avni:	Yeah, but I really couldn't like get anything. How does the *a* and *r* mean, so I'm not sure what I'll get with this.
264. Interviewer:	Okay. And then we talked about how, the way rate and disease spreading has to do with the specific way that the disease affects people biologically. Right? Okay. And then we talked a little bit about, that these three equations represent something about how the population can be affected by the disease. Okay. And that this *dRdt* has something with the rate of people dying or otherwise being removed from the population. So it has something to do with— *So I would say that it maybe has something to do with infected people turning into removed people*. Okay. ((interviewer is moving to end the interview))
265. Avni:	*This is telling us about the rate law*. I'm not sure if the disease is like spreading. That's—
266. Hoa:	((mumbled)) *Like how fast it's spreading*?
267. Avni:	Yeah.
268. Hoa:	*So maybe [I could] make the correlation*.

Although what sparked this discussion was the fact that they got stuck during their problem solving, this need was coded as *situational-emergent* rather than *situational-insufficient*. When the interviewer summarizes what they had been working on, she highlights several ideas that had not been as foregrounded in their immediately preceding discussion: "the *rate* of people dying," "it maybe has something to do with infected people turning into removed people" (ll. 262-264). The idea of "rate" had been a feature of Avni and Hoa's negotiation of meanings immediately prior to this. However, they had disagreed about what "rate" meant—Avni related it to the idea of the growing rate of bacteria, and Hoa related it to the rate at which the disease was spreading. In her summary, the interviewer implicitly affirmed Hoa's definition. Hoa had also previously been grappling with the idea of how the disease spread through the population, however, they had not explicitly discussed how this related to the way the rates related to each other. In her summary, the interviewer inadvertently added her own interpretation ("it maybe has something to do with infected people turning into removed people") (ll. 264). These two pieces of information supported Hoa's realization: "So maybe [I could] make the correlation" (ll. 268) This idea of "correlation" became the basis for an extended abstraction activity, in which the pair uses hypothetical values for each rate to compare them and how they affect the change in the population over time.

This was identified as a situational-emergent need because this discussion led Hoa to have a revelation about the data that supported her abstraction. Situational-insufficient needs led to abstraction as a way to shift away from an unproductive problem solving approach. For example, the participant may have been using a symbolic manipulation approach that was no longer productive, and the interviewer suggested that they shift to thinking about the problem and the equations conceptually instead (a shift to horizontal abstraction). Situational-insufficient needs generally represented a distinctly new phase during problem solving, even when they built on the participant's previous reasoning. For example, in Joshua's problem solving above, he had previously been thinking about the variables conceptually. The interviewer's introduced need continued along the lines of reasoning conceptually, but changed the approach to reasoning within a different framework of biological knowledge (vertical abstraction). These new strategies were deliberately introduced to approach the problem. Situational-emergent needs, however, were initiated by recognizing a previously unrealized opportunity for abstraction. They built off of the previous line of reasoning, but represented a shift due to the connection that they recognized.

When a need for abstraction does not arise

Up until this point, I have described three types of needs that may spark abstraction during problem solving. However, there were also a handful of cases where a need for abstraction seemed as if it would arise, but was subverted by participants shifting their problem solving approach. For example, during their pair kinetics interview, Jamila and Akeyo were discussing what the meaning of *a* and *r* are. Please note that the acronyms represent the epistemic actions coding for each line (see Table 4.4), and are included because this example will be returned to in the discussion of RQ2:

106. Jamila:	*dR* over *dt*.
107. Akeyo:	Is equals to aI. aI. So what are they asking? Find the conditions on the ratio a/r that decide whether the disease will spread or die out? What do you think *a* and *r* mean?

108. Interviewer: The second's more of a guiding question that might be helpful to think about, but the first is the—

109. Jamila: The main question. [[8]] These sort of remind me of, what you call?

110. Akeyo: It reminds me of a [?].

111. Jamila: (SR) Oh, I didn't take (?: plant bio)— I'm thinking of like fundamental equations, like the way we're given, and then you can like input one to the other. It's sort of like kind of like that, and solve it.

112. Akeyo: So we know—

113. Jamila: (SM) Let's begin with the second one guides the first one. What do you think *ar* means? Well maybe get *a* and *r*. And then we can divide *a* over *r*, because how will you get *a* over *r* if you don't get *a*, separate *a* and separate *r*?

114. Akeyo: Okay, so [[8]] what's *a*?

115. Jamila: ((laughs)) What do you think *a* is? First, I'm thinking they gave us these mathematical questions so we can solve it, [[2]] right?

116. Akeyo: If you think about this as like *R* would be removed class, *S* would be susceptibles, and *I* would be infected, right?

117. Jamila/Akeyo: (CC) ((together)) So what is *a*?

118. Akeyo: (CC) If *I* is infected, what's *a*? [[12]]

119. Jamila: (CC) *a* may be the rate?

120. Akeyo: (CC) No.

121. Jamila: (CC) At which they occur?

122. Akeyo: (CC) Can't be. That wouldn't make sense, right?

123. Jamila: (CC) If this were rate, that I understand— Well it can be because like *dS* over *dt* would be the, you know, susceptible over time.

124. Akeyo: (CC) Yeah, but infected over time.

125. Jamila: (CC) So this should be the rate. Hmm? Maybe?

126. Akeyo: (SM) We have to figure out what *a* is. This one.

127. Jamila: (SM) Since this is 1 and that is 1, if *dR* over *dt* is equals to *aL* [sic], can we put this in here?

128. Akeyo: (SM) So just plug in? Okay. It's up to you.

At first, it seems like a task-directed need for abstraction will arise, because they express a need to understand what the variables *a* and *r* mean (a need for horizontal abstraction) (ll. 106-125) However, this needs to understand the conceptual meanings shifts when they agree on taking a manipulation approach. Rather than trying to understand the conceptual meaning, they shift to "figuring out what *a* is" through solving for *a* (ll. 126-128). This marks the beginning of an extended period of symbolic manipulation called "magic math," in which they use an approach they seem to have adapted from their unit on Maxwell relations that involves taking derivatives to relate different variables. The features of their interaction that seem to influence this decision not to abstract will be discussed further under the results for RQ2. This example shows that even when a need for abstraction seems to arise, if that need is not sustained by the problem solvers themselves, abstraction will not occur.

Code	Description	Acronym
Conceptual Concretizing	Putting a conceptual constraint on the problem space	CC
Conceptual Manipulation	Advancing toward a solution by thinking through meanings and connections between conceptual aspects in a procedural way	CM
Conceptual Restructuring	reimagining the meaning of the mathematics, which transformed the relationship between the math and the variables it stood for (focuses on the changing the meaning and conceptual underpinnings)	CR
Conceptual Generalizing	Connecting ideas and meanings to develop a new concept/idea within the problem space	CG
Symbolic Concretizing	Putting a mathematical constraint on the problem space	SC
Symbolic Manipulation	Advancing toward a solution by thinking by working with math or variables in a procedural way	SM
Symbolic Restructuring	transforming mathematical relationships to represent something that they were familiar with from another context, applying those constraints and meanings (focuses on changing the representation and mathematical relationships)	SR
Symbolic Generalizing	Connecting ideas and meanings to produce new mathematical relationships	SG

Table 4.4. Description of each of the epistemic actions codes and the acronyms used for them through the transcript examples.

Research Question 2.

To answer Research Question 2, I attended to features of interaction that influenced whether an abstraction activity, once initiated by a need, was fulfilled or interrupted. To preface this discussion, the reader is reminded that this study is rooted in activity theory, and that abstraction is conceptualized as an activity. This has several implications for how the results are presented. Vitally, learning is socially mediated and socially constructed. How a student encounters a problem and how they make sense during the problem solving process, and in particular how they abstract, is determined by their personal histories, the personal histories of the others involved with the interaction, and the socially constructed mediational tools they bring to solve the problem. To that end, our interpretation of the data to answer

RQ2 was not hypothesis driven, but rather it was inductive and specifically attended to abstraction processes that were dynamic and shifted by interaction.

Briefly, I found that analyzing the data by interactions revealed that it was difficult to parse the influences of these interactions from each other.

Framing (Participant-Task and Participant-Interviewer)

As discussed in the introduction, framing is the answer to the question, "What is going on here?" In multiple occurrences, framing was found to be an interaction that influenced students' abstraction during problem solving. Participants' framing often determines what type of knowledge and procedures are appropriate to use to solve the problem, so framing seemed to play a consistent role in whether or not the student would engage in horizontal abstraction. First, I will discuss the role framing played in constraining interaction, and then discuss how shifts in framing could sustain abstraction.

A recurring frame that constrained participants' horizontal abstraction was the idea of what is considered appropriate for a p-chem problem. The problem tasks for both of the interviews had a biology context and, based on what the participants had studied in their physical chemistry class at the point of the interview, were dissimilar to the types of problems they would have solved in their course. This meant they had not necessarily learned procedures that could be directly applied to solving the problem during the interview. Furthermore, the problems had been selected because reasoning about them biologically could support leading to a solution state. However, several students framed the problem as one that had to be solved in a "p-chem" way; that is, using an approach that they had learned in their course. For example, Chao encounters a situational-insufficient need for abstraction, because the manipulation approach he had been using to solve for *a* and *r* had not led to a solution he could easily make sense of. The interviewer tried to support him to horizontally abstract:

70. Interviewer:	Okay. And so the question's asking, when will the disease spread become an epidemic, or when will it die out? So what do you think that means?
71. Chao:	If, from I got the information from the question, it should be when it will finish the *R*. It should be that [?].
72. Interviewer:	But can *R* also be people who are—
73. Chao:	It's has the disease, recorded, died, or immune.
74. Interviewer:	So when the reaction goes to completion, so to say, when everyone's an *R*.
75. Chao:	Hmm hmm. So when the reaction completion, it completes, this will be died. This disease will be died.
76. Interviewer:	Okay. What about when it becomes an epidemic?
77. Chao:	Mm. What? What? At the beginning? No, no, asking those kind of questions is— I mean, it's not a p-chem question. No. [laughs] What does that mean? No. It's more biology, like last time also. I have no idea about this question, actually. Because usually if I got the questions, I should be imagine what's that, is, connects with the class, how it's [?], but this one, too many things I don't understand. I can make me know some, like the symbols mean, like the *SI* something means, but still.

Here, Chao is framing the problem in the interview as something that has to be solved using a familiar approach ("should…connect with the class") (ll. 77). Because of this belief about how the problem was supposed to be solved, he resists following the interviewer's cue to horizontally abstract, because this violated his expectation. To Chao, thinking about the biological aspects of the problem is a *biology* question, not a physical chemistry one, and thus is not appropriate to use while solving a physical chemistry problem.

Framings and the notion of what could be used to solve the problem could also shift over the course of the problem solving. For example, when Jamila first encountered the entropy problem, she connected the idea of entropy change to the ideas of disorder and Gibbs free energy. This led her to experience a situational-insufficient need for abstraction, as she realized that this "math way" may not be the right approach to solve the problem. She begins to draw on her biological knowledge to puzzle out how the idea of entropy as disorder and DNA are related:

65. Jamila:	I'm thinking like how to approach it, cause I understand the first part. I know that it's a binucleotide. I've been given a number, and I've been told that it has four nucleotides. That's good. What is the residual entropy associated with this typical DNA molecule? Unless I have all the other information, I don't think I would able to be like calculate like math way, to maybe explain it.
66. Interviewer:	Okay. Yeah, so why don't you try to explain like this, how you're thinking about this problem to me.
67. Jamila:	(CR) Okay. I know entropy is the law of disorder, like in the system, and disorder in a DNA molecule, I, like thinking in a biological way, isn't DNA pretty ordered? It's base paired. Like if there's like something missing, you have like the polymers just come and fix everything, and it's coiled, and it's coiled again, and again. Is that sort of the disorder they're asking? But then if you coil them, is in that order you're putting it together?
68. Interviewer:	That's a great question. How do you make sense of that?
69. Jamila:	Cause I'm thinking, when you put things into order, you bring it together, right?
70. Interviewer:	Hmm hmm.
71. Jamila:	(CM) And this thing, it's like rungs on a DNA ladder, so maybe it's opened up, and it's not coiled.
72. Interviewer:	All right. Can you show me what you mean?
73. Jamila:	(CM) Like you know how DNA is initially, like you have your base pairs, and then it goes into the helix, and then it goes into

like the coil, and then it goes into like the supercondensed coil. So it's like as it increases, I would think the order increases.

Here, Jamila draws on several types of knowledge, which allow her to fulfill the situational-insufficient need. The interviewer supports this shift by asking her to explain her reasoning (ll. 66). She uses her biological understanding of different stages of DNA packaging (e.g., ladder, coiled, supercoiled) to conceptually restructure the notion of entropy by relating these stages to the idea of disorder (ll. 67). Although prior to this moment, she had framed the problem as something that had to be solved in a math way, aligning with her previous experience calculating ΔS using the Gibbs equation, she shifted to trying to make sense of the idea of disorder and how disorder applies to DNA.

Next, she and the interviewer go back and forth for several minutes as Jamila talks through how disorder can be related to the different confirmations of DNA. Eventually, Jamila asks what exactly the idea of residual entropy refers to, and the interviewer clarifies that it refers to the idea that there are two energetically equivalent states (a conceptual concretizing move, as she is constraining the definition of entropy). Jamila then tries to relate this new idea with her previous understanding:

93. Jamila: (CM) So these are two different states. this is when you're like adding another, like you're growing your other strand and you're like replicating, and then you start to initially coil and then have like more proteins help you coil, and then you're like super coiled. So maybe like the two different states, how I would think about it, is maybe like a different base bearing, but then DNA is very specific, and A has to bind with a T and a B has to bind with a C. Is it maybe that maybe GC or CG in your sort of like explanation, maybe that's where the residual entropy? I don't know. It's an interesting question.

[…]

97. Jamila: (CG) I don't know. This is the only like explanation I can think of right now, maybe because you can get like any order of orientation, maybe this. Maybe the residual entropy is. [[draws the different conformations of base pair pairings]]

Jamila conceptually manipulated using the new constraint to try to reason through how the idea of two states may relate to the reasoning she had done so far about entropy (ll. 93). Eventually she connected the idea of residual entropy as relating to different states to the orientation of the base pairs ("Is it maybe that maybe GC or CG?"), which she consolidated in a conceptual generalizing move (ll. 97). At this point, Jamila had fruitfully engaged in conceptual vertical abstraction, in which she performed multiple restructurings of the relationship between entropy and DNA to arrive at the canonically correct idea, that the residual entropy is related to the orientation of the base pairs. This conceptual vertical abstraction may have been sustained by the interviewer in two ways: (1) the interviewer continually affirmed her meaning making by asking her to elaborate, and (2) when there was a concept Jamila did not understand, the interviewer constrained its meaning by providing a definition, which gave Jamila something to manipulate and allowed her to move forward.

The interviewer then requested Jamila to horizontally abstract and relate this meaning making to the mathematics:

138. Interviewer: If you were asked to come up with a numerical answer, how would you approach that?

139. Jamila: (SM) Hmm. I'd first have to like write down the formula, *G* equals delta *H* minus *T* delta *S* [[writes out the equation for Gibbs free energy]]. I'm looking for a delta *S*. So it would be this on the other side, [?] minus delta *H* divided by *T* [[solves for ΔS]]. I don't have the temperature. I don't have delta *G*. I don't have delta *H*. So I would definitely like be stuck at this point, until maybe somebody threw a hint. I don't know. Cause

there's no other way, so I would think there's no, there's no way for me to get it other than explain the concept.

There is a sharp shift here in how Jamila is approaching the problem. When she is asked to provide a mathematical or numerical solution, she shifts away from her thinking of entropy as disorder (a probabilistic interpretation) to using the Gibbs free energy formula (a thermodynamic interpretation, as well as her initial problem space). Despite the shift from conceptual to symbolic reasoning, this was not horizontal abstraction, because Jamila began attending to a new object (the Gibbs free energy equation), rather than mathematizing a statistical formulation of entropy she had previously articulated.

Insights from two other participants' interviews may shed further light into the role framing may play constraining in abstraction. Similar to Jamila, after experiencing a situational-insufficient need for abstraction, Hoa engaged in vertical conceptual abstraction to make sense of the relationship between DNA and entropy, and she generated a model through conceptual and symbolic restructuring to represent this relationship:

61. Hoa:	(CC) Well most of these has temperature, and there is no temperature in here. And others have volume, and they don't have this here. I'm not really sure how to go about it.
62. Hoa:	(SC) I just wonder, if you're just multiplying this to get the order of the system.
63. Interviewer:	Can you tell me a little bit more about that?
64. Hoa:	(CR) So I'm guessing that like how orderly a system is, as in like how many possibilities there is. So if they're asking for possibility, there's four different kinds of these binucleotides, so I would just multiply it. So like this is saying that the average DNA molecules has 5 times 8 to the 10 binucleotides.
65. Hoa:	(SR) But these four of these, so let's say this is A, this is B. I'm not sure if I'm doing this right

66. Interviewer: I'm following so far.

67. Hoa: (SR) So there's four of these, A, B, C, D. So if each rung—

68. Hoa: (SM) So there's four kinds, and there's 5 times 8 rungs. Yeah, I'm [?] just multiplying this, and then I would get the number of possibilities. If I divide by four, I'd have the answer.

Hoa cued in on the idea of "possibility" and conceptually restructured the problem to be about probability, and then horizontally abstracted to a symbolic restructuring move (ll. 64-64). In this symbolic restructuring move, she generated a representation that brought together all of the features of the problem she had been trying to reconcile: each type of possibility (A, B, C, D), which is a proxy for the four types of base pairs; the length of the DNA strand (5×10^8); and how the ideas of "rung" and "possibility" relate (see Figure 4.4). However, when she symbolically manipulated and generated an equation that showed how these two numbers (4 and 5×10^8) relate, she ran into the limits of this representation and how she defines "1 rung = 4 possibilities" (line. 68). This led to a new situational-insufficient need for abstraction, because she could not use her current line of reasoning to move forward in solving the problem.

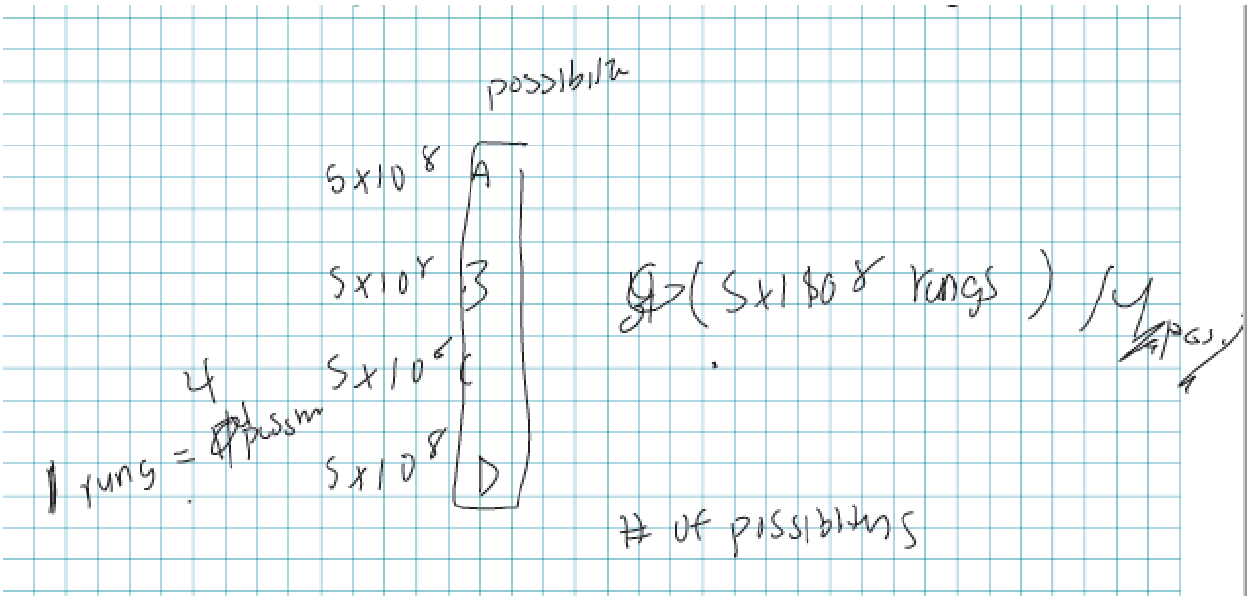


Figure 4.4. Hoa's written work during the entropy problem. The left side of the figure shows the initial representation Hoa generated that shows how all of the ideas fit together. The equation on the right side shows how she related rungs to possibilities. The equation she wrote reads $(5 \times 10^8 \;\; rungs)/\, 4\, pos[sibilities]$.

To fulfill this need, Hoa and the interviewer go back and forth defining and clarifying terms, but she gets stuck which leads to this interaction:

111. Interviewer:	Okay. Let's take a step back from the numbers. Can you sort of, like conceptually, what do you think this question is about?
112. Hoa:	It's, I don't know what they mean by how I would get entropy, because all the equations you gave me don't really match this. Yeah.

The interviewer made a bid for vertical abstraction by asking the participant to step away from the specifics of the problem and think about what the problem might mean in a bigger picture. However, Hoa resisted this bid by referring to the equation sheet, and that she was unsure how what she and the interviewer are discussing relates to the equation sheet. In particular, Hoa mentioned they are "all the equations *you* gave me." The equation sheet was tended to be a mediational artifact provided to participants to support their calculations so they would not have to rely on memorized procedures. However, Hoa and several other participants framed the equation sheet as something the interviewer *expected* them to use, and that their task as problem solvers was to identify the correct equation. That is, the equation sheet constrained abstraction by shifting the problem to be about identifying the right tool for symbolic manipulation. This was particularly true for the entropy interviews, as the kinetics task did not neatly map onto any equation.

The relationship between framing and a reluctance to horizontally abstract is well captured by the following instance from Imani's kinetics problem solving. Her approach was heavily mathematical; she engaged in extended symbolic manipulation, in particular trying to integrate the three rate laws to solve for a and r in terms of R, S, and I. The resulting equations did not align with what she expected, because they implied neither a nor r relied on the population of the infectives. The interviewer asked her to elaborate on this thought:

135. Interviewer:	Okay. So let's go back to what the question is asking us. So it's asking us to find a over r to figure out if the disease will spread or die out. How does that relate to those equations, that question of spreading or dying out?
136. Imani:	[CM] So I think it takes into account the change in population of people who are susceptible versus those that are removed. And in this case, we're not looking at the infected, which doesn't make sense, because we should be, and that's a correlation of the other thing too. But it takes, it primarily, this equation is taking into account the population, changing population, the susceptibles versus removed class.
137. Interviewer:	Does that makes sense to you?
138. Imani:	[CM] I mean, if you're trying to find out whether it will spread or die out, I assume that you also need to take account of the population that is infective, because the removed class are not going to spread it anymore. And then the susceptible versus

infectives are those that you really have to take into consideration. And that makes more sense to me.

139. Interviewer: Okay, cool. So if that's what we need to be considering, what's going on with the math?

140. Imani: I don't know. Math doesn't lie. It's factual, so I think it's something that I'm doing wrong, definitely.

Even though her conceptual reasoning and her mathematical problem solving do not align, Imani assumed that the mathematics must be correct. She seemed not only to be framing the problem as one that she must use mathematics to solve, but she valued the results of her mathematical manipulations over conceptual reasoning. Rather than rethinking whether her approach was correct, she automatically defers to the mathematical result. The fact Imani does not horizontally abstract here, despite the fact that she is faced with an opportunity to do so, is explicitly related to her framing that math is the proper way to solve a physical chemistry problem.

Interviewer Interventions (Participant-Interviewer)

A second key interaction that influenced participants' abstraction processes was how the interviewer intervened to support or constrain participants' meaning making. Above, I discussed how the interviewer could influence participants' problem solving by indirectly encouraging them to shift their framing or utilize other resources, and in turn how this framing influenced participants' abstraction. In this section, I will explore how the interviewer's own epistemic actions more directly influenced participants' abstraction through conceptual restructuring to scaffold connections as a basis for abstraction, as well as how conceptual concretizing moves could both constrain and sustain student abstraction.

First, the interviewer could sustain participant abstraction through scaffolding. As part of the teaching interview design, the interviewer asked questions intended to support participants' abstraction. These questions tended to be asked when the participants got stuck; that is, when they encountered a situational-insufficient need for abstraction. For example, consider this excerpt from Joshua's kinetics interview:

114. Interviewer: Okay. Let's take a step back from the math, maybe, for a second. So it seems like you know a lot about biology. And I know nothing about biology. So can you just tell me a little bit about how disease is spread? What you know about how disease is spread.

115. Joshua: (CM) It would be either like bacterial or viral, and it's spread, it's changed. It's different I guess depending on the disease, if it's airborne or contact through like hand to hand contact or contact through like something else, or droplet precautions from like sneezing or coughing in somebody's vicinity, and that depends on the organism that is doing the infection. So some can survive a lot longer outside the body, so they can like, it's easier for them to spread. But at the same time, diseases that may be able to transmit a lot better are not as infective.

116. Interviewer: Really? Oh, I guess cause everyone would die.

117. Joshua: (CM) Well, diseases that can like— how do I explain this. Something can like be able to transmit very easily, but cannot infect people upon like getting to them as easily, I guess, like its ability to actually infect and like colonize or whatever isn't as good.

118. Interviewer: Oh, so the ability to infect and the ability to transmit are not the same?

119. Joshua: Yes, they're different like factors.

120. Interviewer: Oh, I did not know that.

121. Joshua: (CM) Like something like, diseases like, diseases that are like blood borne, like it's not as likely that you'll get something, like something blood borne from another person very easily, but like if you do, it's, the chance of you getting it is very high, you know, compared to like walking by somebody who just

sneezed with the kind of cold, it's less likely that you'll get it just, unless you like maintain contact.

122. Interviewer:	(CR) Okay. So if we think about that in terms of these three classes of people, how do those two models work together? So not necessarily thinking about the math, but just thinking about infected people, people who can catch it and people who can't catch it. How does that work with this model of disease spread that you were just telling me about?
123. Joshua:	(CR) Well susceptible people have a higher chance of getting the disease, so it would require like less transmission to get to them, I guess, than an average person, it would require more interactions with people who are infected, so I guess that's why it's infected people times the— Well I guess, you know what? The derivative doesn't necessarily mean like that, like S doesn't have to equal— DS over DT doesn't have to equal like S. So it's like the number of susceptible people times the number of infected people. So this ratio actually could be the number of people, number, the increasing number of people becoming, or reducing the number of people becoming less susceptible, because, like what am I saying? So susceptible people times infected people times a variable that's negative. So it's reducing the susceptibili-, the number of people being susceptible over time is decreasing, because of the negative number, I think, just because they're becoming infected as the number of people that are infected increase. There are like the higher chance that they can be— I guess, the transmission can happen. I don't know.

In this example, the interviewer performs two important scaffolding moves after Joshua experiences a situational-insufficient need for abstraction (prior to this moment). The interviewer initially explicitly shifts the framing of the problem by proposing that Joshua use biology knowledge and positioning him as an expert (ll. 114). This move invites Joshua to identify relevant information through a series of conceptual manipulation moves, bringing that information explicitly into the problem space (ll. 115-121). She then makes an explicit bid for conceptual restructuring by asking Joshua to bring this reasoning back to the problem (ll. 122). This supports Joshua's vertical abstraction through a conceptual restructuring move in line 123, where he uses his prior reasoning about transmission to rethink the rate law model. Before this moment, he had been thinking about the rate law expressions ($\frac{dS}{dt}, \frac{dI}{dt}, and \frac{dR}{dt}$) as conceptually equivalent to the magnitude of the populations (e.g., that $\frac{dR}{dt} = R$). Being explicitly asked to consider *how* diseases spread biologically may have supported Joshua's shift to thinking about change and spread over time.

The interviewer supported Joshua's vertical abstraction through her own conceptual restructuring move. Another interviewer move that could support participant abstraction was *conceptual concretizing*. For example, let us revisit Jamila's problem solving. Above, I discussed that she incorporated the interviewer's conceptual concretizing move to support her meaning making. Below are the interviewer utterances that led up to her conceptual generalizing move that marked the conclusion of her conceptual vertical abstraction:

86. Interviewer:	(Int. CC) Yeah. I think that's a good way of thinking about it. So residual entropy is, so you know, at *T* equals zero, entropy is zero. But if there are multiple states that are equally energetically favorable, that, even at *T* equals zero, there might be some entropy associated with that system. So think about it this way. Here's how I think about it. If I had the diatomic molecule, AB.
87. Jamila:	Okay.
88. Interviewer:	(Int. CC) It can be AB or BA.
89. Jamila:	Yes.
90. Interviewer:	(Int. CC) Since neither of those are more favorable, there's two possibilities, and so there's still some entropy, because there's possibility for disorder still. So that's how I understand residual entropy.
91. Jamila:	So now bringing it back to this, well these are two different states. I wouldn't think of them as like—

92. Interviewer:	Yeah, talk me through that.
93. Jamila:	(CM) So these are two different states. this is when you're like adding another, like you're growing your other strand and you're like replicating, and then you start to initially coil and then have like more proteins help you coil, and then you're like super coiled. So maybe like the two different states, how I would think about it, is maybe like a different base pairing, but then DNA is very specific, and A has to bind with a T and a B has to bind with a C. Is it maybe that maybe GC or CG in your sort of like explanation, maybe that's where the residual entropy? I don't know. It's an interesting question.
94. Interviewer:	It is. I like it.
95. Jamila:	Yeah, it's made me think.
96. Interviewer:	Okay.
97. Jamila:	(CG) I don't know. This is the only like explanation I can think of right now, maybe because you can get like any order of orientation, maybe this. Maybe the residual entropy is. [[draws the different conformations of base pair pairings]]

Conceptual concretizing by the interviewer serves two purposes here. First, it reduces the abstractness of the problem, because she defines a term that Jamila did not know. Rather than Jamila trying to guess at the meaning of residual entropy, the interviewer constrains the problem space by providing her the definition (ll. 86-90). Second, this definition becomes something that Jamila can further manipulate and use when she abstracts (ll. 91-93). Concretizing provides an anchor point to support Jamila's reasoning, which is evidenced in her conceptual generalization action, where she incorporates the interviewer's definition of residual entropy (ll. 97).

However, anchor points introduced through interviewer conceptual concretizing can also constrain abstraction. During one of the kinetics pair interviews, the interviewer unintentionally introduced a constraint that was internalized by one member of the pair. Avni and Hoa experienced a situational-insufficient need for abstraction when their initial approach (solving for *a* and *r*) did not yield a result that they could make sense of. The interviewer was guiding them through horizontal abstraction as they tried to figure out what the conceptual meanings of *a* and *r* might be:

229. Avni:	(CM) Maybe we can make that, we can try to take out the bacteria that is causing the disease and like working in the lab, and like see their growing rate. So then we can look at it like people's genes, like whether they have the immunity or not. What do you think? Yeah, I cannot think about any other way. What's this *a* and *I*? I still didn't get it. Did you get anything?
230. Hoa:	I didn't either. So *S* is the one who can catch the disease.
231. Avni:	Can you write down over here?
232. Hoa:	(CC) So can catch the disease. So this is *S*, right? Why does it have *aI*? Why doesn't it have *a*? Does *a* have to do with like—
233. Avni:	(CC) Who has the disease, or who—
234. Hoa:	That's what I'm thinking. So S can catch it. I can, I is what?
235. Avni:	They have it, or they can transmit it. And *R*, they already had it and they're like dead or—
236. Hoa:	So they're dead. ((laughs))
237. Interviewer:	(CC) They can also be immune. ((laughs))
238. Avni:	(CC) Oh yeah. So they already have it, or like they're infected. I still didn't get *R* and *I*.

To reason about what *a* and *r* could be, Avni and Hoa first discussed the meanings of the three population variables (*S*, *I*, and *R*) provided by the text (ll. 229-234). While they were reviewing the definitions, both Avni and Hoa agreed that the removed class (*R*) represented the populations that were dead (ll. 235-236). The interviewer reminded them that the removed class could also represent people with immunity (as they are still removed from being able to spread the disease throughout the population) (ll. 237). Avni seemed to internalize this idea that the removed class is immune, and used it as an anchor point while the pair continued to reason:

243. Hoa:	(CC) Cause what if *r* is spread? Because *S* can still catch, right?
244. Avni:	Hmm hmm.
245. Hoa:	And it could be infected.
246. Avni:	Yeah.
247. Hoa:	(CM) But it's not dead. That's why it doesn't have the capital *R*.
248. Hoa:	(CG) So maybe *r* equals spread and *a* equals die out. Because if this is the rate of infected—
249. Avni:	(CC) But we cannot tell. If they're infected, they will like die or— Because they might have the immunity too.
250. Hoa:	Right.
251. Interviewer:	(CR) But they do catch it, right? The remove class had the disease at some point.
252. Avni:	(CC) So maybe they have the ability to catch.
253. Hoa:	(CM) ((sounds confident, speaking faster)) Cause if you look at this rate, right, so the infective can spread, because it, they are infected, and they can spread people, right?
254. Avni:	(CC) Yeah, [for] I. But look at this one. If they have the immunity or they are dead—
255. Hoa:	(CC) This one is they're dead. So *a* could mean die out, and then *I* is the infected.
	(CM) So the number of people that die out, die out times the number of people infected, equals the rate of people removed. That makes sense, right? No?
256. Avni:	Like who are, who have the immunity? How do you mean by removed?
257. Hoa:	(CM) I'm looking at this, right? So this is the rate of people that died, right?
258. Avni:	(CC) I don't think they are necessarily dead.

259. Hoa: ((tone changes, sounds unsure whereas before she was very confident)) Oh, okay. Who have either had the disease— Hmm. All right, never mind. Okay, I'm lost now.

Redefining *R* to include immune people (and not just deceased people) seems to shift Avni's perception of the characteristics of the removed class (ll.249). Hoa is vertically abstracting to figure out the conceptual meanings of the parameters *a* and *r*, and specifically to relate them to the nature of the populations they control (e.g., if *S* can catch the disease, then *r* must be related to spread, because it controls the rate of change of *S*) (ll. 245-250). However, she is reasoning using the idea that *R* means dead, which contradicts Avni's conception that *R* must include immunity, and that immune people have a different relationship to disease than dead people. The interviewer again affirms this meaning, when she interjects that "But they do catch it, right? The remove class had the disease at some point" (ll. 251). Her intention was to restructure the problem space as one that included time, and that individuals in the *R* class were at one point individuals in the *S* class. However, this seemed to confirm Avni's hunch that immune people could get reinfected (ll. 252: "So maybe they have the ability to catch."). Avni used this understanding as an anchor point to continually push back against Hoa's reasoning (ll. 255: "Who have the immunity? …I don't think they're necessarily dead"). Because the two do not resolve this contradiction, they become stuck again, and their abstraction is constrained.

Although the anchor point was the primary source of the constraint, the interviewer's conceptual restructuring move demonstrates a second way in which interviewer discourse moves could constrain abstraction. The source of this contradiction was that the interviewer's understanding of the objects (the rate laws and the population variables) differed from Avni's understanding of the object. This contradiction could lead to the interviewer providing a hint or inadvertently pushing the participant in a direction that derailed their current abstraction. For example, consider the following excerpt from Chao's kinetics interview:

47. Interviewer:	Why do you think this way?
48. Chao:	(SM) No, because I, first way is I thought you should, we should be find out *da*, *a* divided by *r*, so we should be from here. So when we come to this way divided by a, it should change something else. So now I should be, think, *dI*, *I* equals minus *dI*. So *R* equals *a* under *I*. *I* is constant.
49. Chao:	(CR) No, no. It should be. *I* should be constant. Because it's a who. Who has the disease and can catch it. *dR R* divided by *t*. I equals what? *I* equals I don't know.
50. Interviewer:	So why do you think *I* should be constant?
51. Chao:	(CR) Yeah, because *dR/dt*, like if this one likes the change, is the change. So it means when *R* change, how, the equation is, *R* equals some *t*, and then when *t* is changed, how *R* change for that? When *t* change a little bit, how *R* change. And as here, if we will keep going for the *a* divided by *r*, how it looks like? It should be, like *I* should be, I thought if *I* would be constant, the reaction will be, maybe can keep going to so if *I* is not constant,
52. Chao:	(SM) this reaction will be, if we want to calculate *a* divided by *r*, it should be a little bit [complexed].
53. Chao:	(CM) That I thought, and also it's that, who? Who has the disease and can transmit. It now means, mm, yeah, it means just some things can catch it. Right? I don't know. It's made me confused.
54. Interviewer:	So you keep referring to a reaction. What is the reaction?
55. Chao:	For this one?
56. Interviewer:	Hmm hmm. Or what is the reaction that you're talking about?
57. Chao:	Which part? Which reaction [did] I talk about?
58. Interviewer:	You were talking about this *dR/dt*, and then you were talking about a reaction.

59. Chao:	(CM) This one is the further reaction [relates] the *t* with *R*. Yes. Calculate *t* with *R*. So we're reaching a little bit of *t*, how the *R* change.
60. Interviewer:	Okay. And what is t?
61. Chao:	*t* should be time. Usually should be time, but I don't know. It's not this [?] was a *t*. Usually should be time.
62. Interviewer:	Well it says that they're rate laws, so I think *t* is also time here.

Chao vertically abstracts to reshape problem space to include idea of "*I*" as a "constant" or "who," and horizontally manipulates to test how this idea works (ll. 49-53). However, this line of reasoning gets interrupted because of an interviewer question that derails the line of reasoning—asking about the "reaction" (ll. 54). The interviewer picked up on an idea that was salient to her for two reasons: (1) one way the problem can be solved is by restructuring the relationships between the variables to be modelled as a chemical reaction, and (2) because other participants had mentioned it in recent interviews and that was still fresh in the interviewer's mind. However, the idea of "reaction" did not seem as salient to Chao (ll. 57), and this may have shifted him away from engaging with his prior ideas further. It is possible that he had been using the word "reaction" as a stand-in for something like "equation" or as a general word to refer to what was happening in the problem, rather than specifically meaning "chemical reaction" as the interviewer interpreted.

Peer Interaction (Participant-Participant, Participant-Task)

The final major interaction that influenced abstraction was the interaction between members of a pair during group problem solving. In pair interviews, a salient influence on both horizontal and vertical abstraction was the way in which participants shared attention. I will illustrate the influence of peer interaction on abstraction through three examples, to illustrate how both shared attention and the use of diverse resources sustained abstraction.

Avni and Hoa spent several minutes arguing about the meanings of different variables and could not come to a consensus (see discussion above). After several minutes of back and forth with the interviewer, Hoa initiated a new abstraction activity after she experienced a situational-emergent need by realizing that she could "make a correlation" between the rate laws and time:

273. Hoa:	(SC) Yeah, like hypothetically, we could put like *aI* equals like a number, right? So let's just say it's like 3. So that's on the *RdT*, right? This is for the people that are removed. So all the people that are infected, right? So *rSI* minus 3, hypothetically let's do this at like 5, minus 3. So the rate equals 2. So the rate that the people that are infected, and this is the people that can catch it, right, the rate of *dS*. So let's hypothetically, it's negative 5. So this is what the rate that we're looking at.
274. Hoa:	(CG) So we could compare it to determine whether like the disease will spread or die out, right?
275. Anvi:	So you mean like we can just like estimate any number?
276. Hoa:	Yeah, we can estimate, right?
278. Avni:	Yeah.
279. Hoa:	Because like they're all correlated, kind of. So we could say that—Yeah, now I'm stuck. Okay, um.
280. Avni:	(SM) I know how you mean, like let's say you have this number, then you can plug like this number over here. If we have one of these number, then you can see how many people have—

Hoa initiates the abstraction activity through a symbolic concretizing move, where she assigns arbitrary values to each of the rates (ll. 273). This action was enormously productive for their collaborative problem solving, because it gave them a shared object, they could both work with. Hoa leads the problem solving, but gets stuck (ll. 279). However, unlike in the previous section where their differences constrained their abstraction, because they could not come to an agreement about the variables, here their differences sustain abstraction. Hoa leads the abstraction by shaping the problem space, and Avni affirms her reasoning through a manipulation move. This pattern of collaboration persists for the rest of their problem solving, where Hoa leads the abstraction, and when she gets stuck, Avni advances their ideas, which supports Hoa to further abstract (ll. 280).

Compare this to Keith and Seth, who could not come to an agreement on a problem space. Their initial approach to solving the problem was guided by Keith, who encountered a task-directed need for abstraction. Their initial approach was a symbolic manipulation approach (solve the equations for *a* and *r*, then divide), and then to gain insight into the relationships the variables have with "spread" and "die out" based on the results. That is, the goal of the manipulation was toward an end of horizontal abstraction (lines 98-109):

98. Keith:	(SM) Oh, I see. Okay. Well then— Let me write. *a* over *r* equals *I* over *r, dS, dI*. [[18]]
99. Seth:	(SR) I keep on trying to go back to like combining these. [[5]] *rSI* minus *dR/dt*. Like cause I'm assuming you can like derive just one equation because like pretty much you can plug this in here [Keith: yeah] and put this guy in here.
100. Keith:	Yeah. That might have been helpful. Well yeah, so what if we solve this for—
101. Seth:	Probably.
102. Keith:	For, for what?
103. Seth:	(SR) Well you wouldn't have to solve that for anything. It's just like an identity, so you can just like plug it in to the other equation.

104. Keith:	(SM) Mm. [[7]] So *a/r* equals the amount of infected over little *r*, which is changing with respect, the susceptibles with respect to the infected *I*. I don't know how to make sense of all this. I feel like this makes sense [[taps paper]], some sort of sense to me.
105. Seth:	(CC) Yeah, I mean the *r* just looks like a just a general coefficient for like the susceptibles and the infectives, right? So it's like a constant that you multiply by, for each one. So why is it negative in the susceptibles with regard to time? That means because it's the negative, um. I don't know.
106. Keith:	I don't know. I think I'm as far as I can get with this for now.
107. Seth:	Yeah, same. I definitely need just a lot of time to sit down with this.

Similar to Avni and Hoa, there was a clear leader in the problem solving (Keith). Both Keith and Seth came up with potentially productive ways to approach the problem. Seth started to vertically abstract to try to make sense of the relationships between the rate equations (ll. 99). However, he and Keith focused on different objects within the problem space: Keith was focused on figuring out his manipulation method (ll. 100 & ll. 104), while Seth was trying to figure out the nature of the relationships between the objects (ll. 99, 103, & 105). Because of this lack of consensus and lack of joint attention, Keith did not advance Seth's abstraction, and they eventually became stuck because they could not agree on a way to move forward.

The final pair that gives insight into the role of interaction in abstraction is Jamila and Akeyo, whose joint problem solving is presented in the "When a need for abstraction does not arise" section above. Jamila and Akeyo worked very collaboratively, and shared attention to a single object, similar to Avni and Hoa. However, they did not abstract, and in fact seemed to take an "escape hatch" (Sohr, et al., 2018) and shifted from possible horizontal abstraction to symbolic manipulation. There are several reasons for this. First, they seem to frame the problem as one that needs to be solved mathematically ("First, I'm thinking they gave us these mathematical questions so we can solve it"). The presence of the equations support the view that this problem is one that they are supposed to solve, not reason about conceptually. Second, Akeyo and Jamila fully agree with their approach, and do not challenge each other to think differently about the problem. During the interview, they expressed that they often worked together on problem sets and that they studied together for physical chemistry. Consequently, during the interview they often had similar approaches to problem, and they worked together very collaboratively, often building on and finishing each other's thoughts. However, perhaps because they did not bring any different resources to the table, there was no need for them to abstract. Instead, the manipulation approach was suitable because it was one to which they could agree.

Discussion

In this chapter, I present the findings from an exploratory study of the facets of interaction that may affect abstraction during thermodynamics and kinetics problem solving. I investigated two features of the abstraction activity: what initiates abstraction (RQ1), and what influences whether abstraction occurs once it has been initiated (RQ2). I found that there were three types of needs: task-directed, situational-insufficient, and situational-emergent, and several features of interaction that influenced abstraction, such as framing, discursive anchor points, and peer interaction.

Task-directed needs emerged when the participant perceived abstracting to be something the problem explicitly required. There were a few different ways this could manifest. Both the entropy and the kinetics tasks were selected with the goal that they would be able to elicit abstraction. In particular, they were selected with the initial idea that abstraction involved recognizing similarities and connecting seemingly unconnected ideas through a new structure (Hershkowitz et al., 2001; Mitchelmore & White, 2007; Noss & Hoyles, 1996). During the kinetics problem, task directed needs were often associated with making sense of the mathematics (a task-directed need for horizontal abstraction), perhaps because there was an explicit bid in the problem text for (1) students to figure out the conceptual meanings of *a* and *r*, and (2) students to figure out how *a* and *r* related to the biological ideas of disease spread and die out. That is, the task itself provided an opportunity to horizontally abstract as a way to make sense of how to reach the solution state. During the entropy problem, task-directed needs seemed to be associated with figuring out a way to relate the seemingly unconnected ideas of entropy and DNA—that is, to figure out a larger conceptual structure that included both concepts (a need for vertical abstraction).

Situational-emergent needs arose when participants recognized an opportunity to abstract that emerged while they were solving the problem. These needs were uncommon in the dataset, but they tended to occur when either an expectation was violated (e.g., Vivian's problem solving shown in "Operationalizing an abstraction activity") or when a connection became clear, for example because another person picked up on it (e.g., Avni and Hoa's). These suggest that it is possible for students to independently abstract while solving a problem, particularly if they are reflective about their problem solving. However, the uncommon occurrence of situational-emergent needs suggests that other factors, such as the interactional factors identified in RQ2, influence whether or not these needs may arise.

There were also instances in which students abstracted, but the direct need they were responding to was unclear. For example, during the kinetics problem solving, Philip performed a conceptual and restructuring move to apply a "Maxwell's relation" approach to the rate laws. In this approach, he took the partial derivative of *a* and *r* to solve for a ratio of "*da/dr*." The "Maxwell relations" approach was related to a set of equations they had learned earlier in the semester, in which partial and second derivatives could be used to relate different natural variables with thermodynamic potentials. However, he had reified this idea into a generalizable problem solving approach that could "correlate uncorrelated things." The use of this approach may have also been sustained by the presence of derivatives he was unsure how to interpret. That is, he cued on surface features (the presence of derivatives) to identify a procedure from his physical chemistry class that met his need to identify a coherent conceptual structure (Chi et al., 1981).

The kinetics pair interview featuring Akeyo and Jamila also provides insight into how the role of framing (RQ2) influences whether a task-directed need is taken up to initiate abstraction. During their pair problem solving, it seemed as if they would engage in horizontal abstraction because they identified a need to figure out the conceptual meaning of the variable *a*. However, the notion of "figure out *a*" quickly shifted to "solve for *a*", because they framed the problem as one that had to be solved mathematically. In the findings for RQ2, evidence other interviews suggests that part of the challenge to horizontal abstraction was a belief that physical chemistry problems require mathematical problem solving, and that whatever result is obtained through mathematical manipulation must be correct, even if it contradicted a conceptual interpretation. The majority of problem solvers initially approached both the kinetics and entropy problems mathematically, often using approaches that were inappropriate for solving the problem at hand (e.g., algebra, integration, or in Philip's case a "Maxwell relations" approach) because they did not necessarily recognize the underlying mathematical structure (e.g., a set of differential equations). Simultaneously, many of these participants also showed an enormous capacity to engage in conceptual reasoning and to identify and leverage useful external knowledge, such as what they know from biology, to make sense of the task through vertical abstraction. Sometimes, this conceptual vertical abstraction, which seemed stable and salient to the participants, contradicted their results from the mathematical approach. When this happened, the conceptual reasoning was generally abandoned in favor of the original arithmetic approach (see Jamila's entropy problem solving), rather than trying to bridge between the conceptual reasoning and its mathematical counterpart. That is, students often leaned on familiar equations and methods (a symbolic manipulation approach) rather than mathematizing their reasoning (a horizontal abstraction approach). This disconnect aligns with the findings from Becker and Towns (2012), who also found that students were highly capable of rich conceptual understanding and mathematical reasoning, but struggled to bridge these two. Similar to Rodriguez and collaborators (2020), our findings suggest that the barrier to this bridge, what we refer to as horizontal abstraction, may be related to students framing.

The role of the zone of proximal development

Situational-insufficient needs tended to arise after students attempted a procedural approach and became stuck when it did not yield expected answers. In most cases, the need was introduced by the interviewer through scaffolding questions as a way to overcome this “stuckness.” Triangulating these findings with the findings on the role of the interviewer in supporting abstraction, I suggest that abstraction may occur within students’ zone of proximal development (see Figure 4.5).

According to Vygotsky, the zone of proximal development (ZPD) is "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem-solving under adult guidance, or in collaboration with more capable peers" (Vygotsky, 1978, p. 89). Wertsch (1984) uses the lens of intersubjectivity to reexamine this definition as the ways in which a learner and a more knowledgeable other reconcile different situation definitions to expand their understanding of an object. This definition works well with the understanding of problem space that underlies the idea of abstraction. That is, the zone of proximal development is the interaction between two subjects as they negotiate shared meanings (see Figure 4.5).

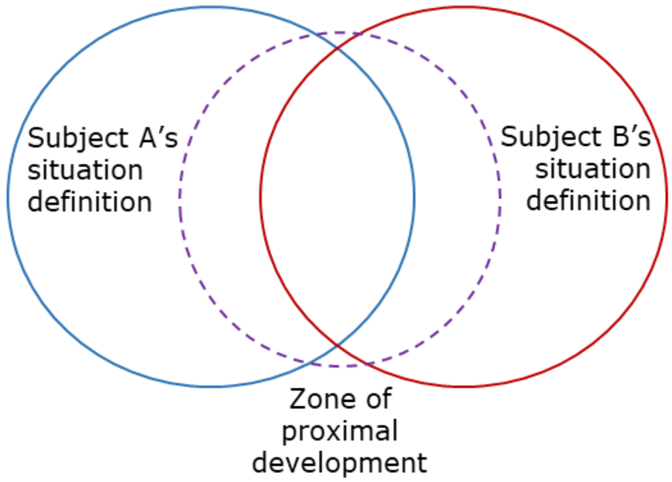


Figure 4.5. Diagram depicting the ZPD. The blue circle represents Subject A's (e.g., the problem solver) situation definition; the red circle represents Subject B's (e.g., the other problem solver or the interviewer) situation definition, and the dashed purple line represents the ZPD, where both subjects can expand their situation definitions.

During these interviews, situational-insufficient needs were met by the interviewer scaffolding abstraction. These scaffolding questions were in response to what seemed to be the source of the participant's challenge moving forward. For example, if the participant was stuck on mathematical sense making, the interviewer may have asked that they step back from the math. This negotiation occurred then as such: the interviewer would guide the participant toward an area of sense making that they had not previously considered, because that sense making match the interviewer's conception of the problem and of the problem situation. She viewed the problem as one that could be solved through conceptual sense making (vertical abstraction), and the connection of conceptual sense making with the mathematical formalisms (horizontal abstraction). These scaffolding questions pushed the participant to consider the problem space and the objects within the problem space as having relationships they may have otherwise neglected.

Abstraction that was achieved through interviewer scaffolding, then, was the result of this intersubjective negotiation. Consider both Joshua's kinetics problem solving and Jamila's entropy problem solving. Joshua and Jamila both brought an important resource to the table that the interviewer did not have: a deep familiarity with biology. Both Joshua and Jamila engaged in conceptual vertical abstraction by drawing on their formal biological knowledge about the mechanisms of disease spread and the structure of DNA, a type of reasoning the interviewer herself did not have the prior knowledge to achieve. However, her scaffolding questions foregrounded relationships that they themselves had not previously been attending to. Together, interviewer and participant(s) co-constructed and negotiated a problem space that facilitated abstraction—that is, they worked toward an intersubjectively negotiated object (the feature of the ZPD according to Wertsch).

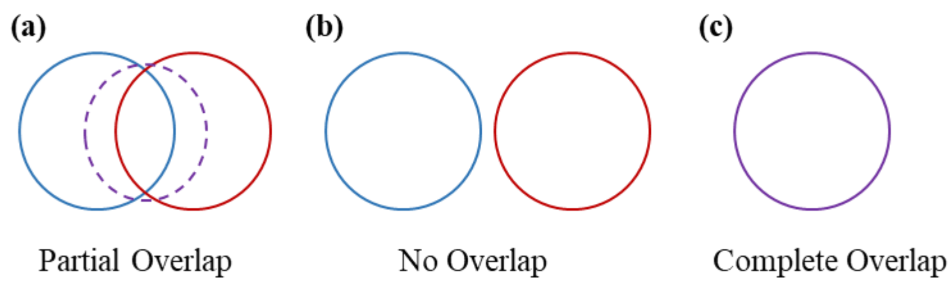


Figure 4.6. Diagram depicting the role of the ZPD and the overlap of participants' problem spaces. **(a)** Represents partial overlap (e.g., Avni and Hoa), **(b)** represents no overlap (e.g., Seth and Keith), and **(c)** represents complete overlap (e.g., Akeyo and Jamila).

The pair interviews provide further support for the role of the ZPD. The three pair interviews that are highlighted show differing extents of intersubjective negotiation. Avni and Hoa bring different perspectives to the problem solving, and consequently abstract in the space that they co-construct (their ZPD). Seth and Keith do not engage in collective meaning making; their problem solving does not involve intersubjective negotiation, and consequently they do not abstract. Finally, Akeyo and Jamila have complete overlap of their problem spaces (see Figure 4.6). There is no negotiation, because there are no disagreements. Because of this overlap, they also do not work within the ZPD, and they do not abstract. This suggests that participants bringing diverse perspectives can be fruitful for supporting abstraction, because it allows them to work within the ZPD and apply their knowledge in novel ways (e.g., abstract). This aligns with previous findings in mathematics education, which found that abstraction could be supported by one of the peers serving as a more knowledgeable other if the second peer supported their problem solving and shared attention (Dreyfus et al., 2001; Schwartz, 1995).

Conclusions

In this chapter, I use the activity structure of abstraction to hypothesize about reasons why abstraction may be initiated during problem solving (RQ1) and how it may be supported by dialogic and dynamic interactions (RQ2). I show that there were three types of needs that could be characterized that initiate abstraction: task-directed needs, which occur because the participant views the problem as one that requires abstraction; situational-insufficient needs, which occur because a procedural approach ceases to be productive for the problem solving; and situational-emergent needs, which arise when the participant recognizes an opportunity to abstract.

Guided by activity theory, I also analyzed the interactions between three different components of the activity: participant-task, participant-participant, and participant-interviewer. Using these interactions, I identify several themes that influence whether or not participants abstract. First, in the theme of framing, I found that a common frame that constrained student problem solving was the idea that the problems had to be solved in a "physical chemistry way," that is, using known and familiar algorithms and procedures that they had acquired during their physical chemistry course. I also found that this framing could be shifted to support abstraction, particularly when the interviewer affirmed this shift and supported participants to view non-mathematical resources, such as biology concepts, as relevant and appropriate to use in the problem at hand.

Under the theme of interviewer interactions, I found that the interviewer could support abstraction in two ways: first, by scaffolding the participant to consider other relationships that supported abstraction, e.g., by making a bid for restructuring; and second, by reducing the abstractness of the problem through a concretizing move and generating a conceptual anchor point. These anchor points, however, could also constrain participants' abstraction, depending on how they were internalized and interpreted by the participant. Finally, I briefly discuss the role of peer interaction, and suggest that for abstraction to occur during pair problem solving, the participants need to jointly attend to the objects within the problem space.

These findings suggest that it is not some inherent cognitive capacity that determines whether abstraction occurs. Rather, I suggest that abstraction is dynamic and situated, and can be influenced by a variety of interpersonal and dialogic interactions, such as how a participant interprets a task and the task situation, and how they interact with peers and individuals such as more knowledgeable others.

Limitations

There are several limitations to this study. First, this study examined problem solving for only two problems (an entropy problem and a kinetics problem), both of which were specifically selected for their potential to elicit abstraction. To understand these findings and to generalize beyond these problem types, it may be important to examine a wider range of problems. Second, because abstraction is so contextually rooted, it can be challenging to make generalizable claims about what influences abstraction. There are also undoubtedly factors that influence abstraction that are not captured in this analysis, either because we did not observe them or because we did not observe them in enough detail to fully explain them. For example, our analysis and study design attended to the interaction between different elements of the activity system. However, what we observed could not always fully explain our results, such as why some participants cued on specific features whereas others did not, or why the same scaffolding question pushed some participants to abstract and not others. To understand this, a more in-depth study and analysis may need to be conducted.

Implications for Practice

These findings, particularly the identification of task-directed needs, suggest that it may be possible to design for abstraction. From these findings and from my observations during interviews, I would make three suggestions to design for abstraction. First, the problem ought to be somewhat unfamiliar, but allow for the student to leverage prior knowledge in ways that allow them to engage in genuine sense making. The goal is that there is not an easily identifiable procedural or algorithmic solution, but rather, that the student has to grapple with making sense of the problem and identify the conceptual underpinnings and meanings of the mathematics. This may point to the utility of a task design for abstraction as a formative assessment (one given at the beginning of the unit to foster student learning) rather than a summative assessment (one given at the end to assess student learning).

Second, students can benefit when there is an explicit bid for abstraction in the problem text, either through a request for students to make conceptual sense of something or for students to identify a structure that encompasses the relationships between the objects. This not only sets up a task-directed need, it also may subvert the framing that physical chemistry problems need to be solved in a "physical chemistry way," that is, using a procedural mathematical approach. If there is not an explicit bid for abstraction in the problem text, it may be worthwhile to explicitly acknowledge that the problem may require students to think outside of their toolbox of memorized formulae and procedures.

Third, I suggested that abstraction may occur in students' zone of proximal development; that is, in the region of their knowledge in which they are best supported by a more knowledgeable other, such as an instructor or peer, in thinking through and solving a problem. This has two implications relevant to task design. First, opportunities to abstract may be opportunities to learn and to expand their understanding. Second, it may be challenging to abstract without the use of additional resources, such as a more knowledgeable other, or access to resources such as the textbook or the internet. This again points to the potential utility of abstraction tasks for formative assessment, e.g., on a problem set, rather than on a high-stakes assessment such as an exam.

CHAPTER 5

THE ROLE OF ABSTRACTION IN PHYSICAL CHEMISTRY INSTRUCTION: A MULTIPLE CASE STUDY

Introduction

Motivation

In chemistry, the relationship between instruction and students' abstraction processes has been noted but not fully explored. For example, a study of organic chemistry students' mechanistic reasoning found that students proposed reaction mechanisms by mimicking what their professors modeled in class; however, because their prior knowledge differed from that of their professors, they often applied abstract concepts and methods inappropriately (Caspari et al., 2018). Blackie (2014) proposed using Legitimation Code Theory, which characterizes concepts based on two axes, semantic density (complexity) and semantic gravity (abstractness) to support chemistry teaching. In her theory paper, she laid out several examples of how separating concepts on these two dimensions could be used to support students' navigation of concepts that are simultaneously abstract and complex, such as the idea of a Grignard reaction. Santos and Mortimer (2019) also used Legitimation Code Theory to develop taxonomies that can be used to characterize the complexity and abstractness of chemistry ideas. These studies suggest that abstraction may be a salient factor to consider in teaching, and that it may be productive to be intentional about modeling abstraction in chemistry teaching. However, abstraction was either not a focal point of the study (Caspari et al., 2018), or the studies lack empirical data to support their hypotheses (Blackie, 2014; Santos & Mortimer, 2019).

Chapters 3 and 4 of this dissertation focused on examining how students abstract, and defined abstraction as a process of movement that gives meaning to or extracts meaning from an object. However, as motivated in the introduction section, there exists a disconnect between professors' pedagogical approaches and students' learning outcomes as assessed through problem solving. To understand this disconnect, and the role abstraction may play in bridging it, it is not sufficient to examine student problem-solving approaches alone. This chapter presents a multiple case study of two instructors at different institutions in order to explore how physical chemistry professors model abstraction in different contexts.

Abstraction in Teaching

Two important factors that have been found to influence how abstraction is fostered in the classroom are social context (Dreyfus et al., 2001; Hershkowitz et al., 2007; Schwartz, 1995; Tabach et al., 2017) and pedagogical approach (Schmittau, 2011; White &

Mitchelmore, 2010). First, classroom social interactions influence abstraction. In small group work, Schwartz (1995) and Dreyfus, Hershkowitz and Schwarz (2001) found that peer interaction changed and often increased the abstractness of the representations that participants co-constructed compared to those participants constructed individually. In whole class discussion, students supported each other's abstraction by contributing to and building off of each other's contributions as they worked to co-construct collective knowledge (Hershkowitz et al., 2007; Tabach et al., 2017).

Second, pedagogical choices may also influence abstraction. This has been explored primarily in mathematics education, where abstraction is associated with developing mathematical knowledge. For example, teaching elementary mathematics from an abstract, "algebra-first" stance can improve students' abilities to tackle abstract problems (Schmittau, 2011). There are also several instructional models that explicitly aim to foster abstraction. White and Mitchelmore (2010) developed a four-part instructional model for teaching empirical mathematics concepts, such as angles or decimals. The four parts guide students through exploring multiple contexts to generalize about individual contexts (Familiarity), recognize similarities and differences (Similarity), draw out general principles (Reification), and use the concept (Application). The different phases of this model feed back into each other; for example, application can help students recognize similarities and differences. This instructional approach frames abstraction as grounded in some real or tangible experience. Similarly, Moreno, Ozogul, and Reisslein (2011) found that students were most successful in developing abstract concepts in elementary algebra classes when they are given both abstract and concrete representations to work with.

The role of abstraction in physical chemistry instruction

In undergraduate physical chemistry, students face several challenges related to abstraction. Proficiency in abstract mathematics such as calculus is often necessary to understand thermodynamics concepts (Derrick & Derrick, 2002; K. E. Hahn & Polik, 2004). However, mathematical proficiency does not guarantee conceptual understanding in physical chemistry (Becker et al., 2015), as even well-performing students often use algorithmic approaches that do not necessarily reflect a deep understanding of a problem's conceptual underpinning (Becker & Towns, 2012). Sevian and colleagues (2015) identified thermodynamics as a potential "abstraction threshold" course in undergraduate chemistry curricula; that is, thermodynamics is a course in which the cognitive demand of the course may exceed that which the students are capable of providing.

Instructional scaffolding may help bridge some of these challenges related to abstraction. For example, a study on a physical chemistry laboratory course found that students were better able to make connections between mathematics and chemistry concepts when instructors helped scaffold their understanding (Hernández et al., 2014). Similarly, deliberate scaffolding by the instructor in a Process Oriented Guided Inquiry Learning (POGIL) course (Becker et al., 2015) as well as scaffolding using molecular dynamics simulations (Schwedler & Kaldewey, 2020) facilitated students making conceptual connections across representational levels (submicroscopic, microscopic, macroscopic).

Despite the usefulness of instructional scaffolding, especially that which is does in reform curricula such as POGIL, much of physical chemistry instruction is traditional, e.g., lecture-based. Research has shown that faculty teaching physical chemistry believe the goal of teaching physical chemistry is to help students develop conceptual understanding, proficiency with models, and development of skills such as scientific communication that are important for working as a chemist (Mack & Towns, 2016). However, a recent review of literature on physical chemistry teaching and learning found that while there is quite a bit of literature published about student difficulties and student success in physical chemistry, there is much less work published on how instruction affects student problem solving (Bain et al., 2014). Thus, it is important to better understand instructors' current and dominant pedagogical practices and examine how they affect students' abstraction.

Modeling practices in instruction

One pedagogical technique that instructors may employ is the practice of modeling, or demonstrating a practice for students to emulate, e.g., how to solve a problem (Caspari et al., 2018; Leonard et al., 1996). In teacher education, modeling refers to the ways in which instructors demonstrate how to develop and perform certain skills in the classroom by practicing them themselves (Moore & Bell, 2019). In teacher education, Lunenberg and colleagues (2007) identified 4 types of modeling: implicit modeling, or demonstrating a practice without drawing attention to it; explicit modeling, or giving meta-commentary on pedagogical choices during teaching; explicit modelling and with reflection, or facilitating student teachers to connect the modelled practices to their own practice; and explicit modelling with reflection and connection theory, or connecting practice to literature. Similarly, I hypothesize that abstraction is a practice that can be modelled implicitly and explicitly. This study seeks to explore that hypothesis and expand the work that has been done on physical chemistry instructional practices.

Research Questions

This brief literature review suggests that students face challenges in developing conceptual knowledge through abstraction. It also suggests that in physical chemistry, instructors can play a key role in supporting and scaffolding student conceptual understanding. Borrowing the concept of implicit and explicit modeling from teacher education, I seek to study how professors model abstraction during instruction. To that end, I pose two research questions:

RQ1. How do professors of physical chemistry model the process of abstraction in teaching thermodynamics and kinetics (a) implicitly or (b) explicitly?

RQ2. What role does abstraction play in teaching thermodynamics and kinetics?

Methods

Multiple Case Study Methodology

This study was designed using a multiple case study methodology (Yin, 2018). Case studies have been demonstrated to be very rich for building theory (Eisenhardt, 1989), and have been used in previous studies of chemistry instructors (Sandi-Urena & Gatlin, 2013). A case study "focuses on understanding the dynamics present within single settings" (Eisenhardt, 1989, p. 534). Here, each case encompassed one professor teaching at a different university over the course of a semester. Multiple case studies follow replication methodologies. That is, cases are selected as either literal replicates (similar results expected between the cases) or theoretical replicates (selecting cases with deliberate differences informed by theory) to provide insight into a phenomenon of interest (Yin, 2018). This study followed a literal replication design, because there was no theoretical basis on which to expect a priori differences between instructors of physical chemistry in how they model abstraction.

Site Selection.

In Fall 2019, data were collected from two Physical Chemistry 1 classes at different institutions (one private and one public institution) in the Northeastern United States. At the private institution ("Private College"), the physical chemistry course included thermodynamics and kinetics. Private College is a small, private liberal arts college that only services undergraduate students. At the public institution ("Public University"), the first semester of physical chemistry only covered thermodynamics. Public University is a public land-grant research university with a Carnegie classification of R1—high research activity.

To select the sites, I identified all of the colleges in the Greater Boston Area (defined as either within a 2 hour drive of Boston, or within the state of Massachusetts) that offered Physical Chemistry 1: Thermodynamics and Kinetics ("PC1") during the Fall 2019 semester. These institutions were divided into 3 tiers based on their driving distance from Boston, where Tier 1 institutions were the closest to Boston and Tier 3 institutions were the furthest. Potential sites were contacted in order of tier (Tier 1 institutions first, followed by Tiers 2 and 3). Institutional gatekeepers (the department chair and/or department assistant and the institutional review board (IRB) officer) were emailed to assess whether they would allow an external researcher to collect data on-site. If they approved, the course instructor was contacted to gauge interest in participating in the study.

Five Tier 1, five Tier 2, and eight Tier 3 institutions were identified (18 institutions total). Of these institutions, two departments denied permission, IRB requirements at four institutions were too stringent to pursue participation, and two faculty members were not interested in participating in the study. Of the remaining sites, there was either no response to emails or follow-up emails, or conversations were discontinued once instructors at the two final sites agreed to participate.

The two sites that agreed to participate primarily did so out of interest in learning more about their students (instructors were incentivized with a de-identified summary of the research results on their students after the data were analyzed). There were two other factors that seemed to influence their participation. First, officials at both sites expressed that they valued and/or had were familiar with chemistry education research. Second, informal trust and a shared network seemed to be a key role in getting these sites to participate in the research study. At both sites, I had a shared acquaintance with either the department head or with the instructor. Although these acquaintances did not play a formal role in introducing me to the institutional gatekeepers, they came up in conversation when negotiating access to the site, and I believe this shared network helped establish trustworthiness.

IRB protocols differed at the two data collection sites. For Public University, UMass Boston's IRB oversight was accepted, and there was no local IRB overview. For Private College, UMass Boston's IRB protocol, as well as site-specific documents, had to be submitted for review, and the department chair served as a faculty sponsor for the proposal. Additionally, minor site-specific changes had to be implemented to the data collection protocol (e.g., students for the problem-solving aspect of the study could only be recruited while the professor was out of the room to avoid perceived pressure from the instructor). To maintain consistency in data collection, these changes were implemented at both sites and did not substantially change the planned data collection procedure.

Data Collection.

Three strands of data were collected at each site: video-recordings of lectures, stimulated recall interviews with professor participants, and semi-structured problem solving interviews with student participants enrolled in the class.

Strand 1: Video-recording of lectures. Selected lectures that focused on kinetics and the second and third laws of thermodynamics were recorded. Five lectures on thermodynamics were video recorded at Public University, and four lectures on thermodynamics and four lectures on kinetics were recorded at Private College (see Figure 5.1). Both courses met three times per week (MWF) for 50 minutes. Recording dates were negotiated with the instructors. When there were conflicting dates, a colleague not affiliated with the research study video-recorded one of the sites (Private College).

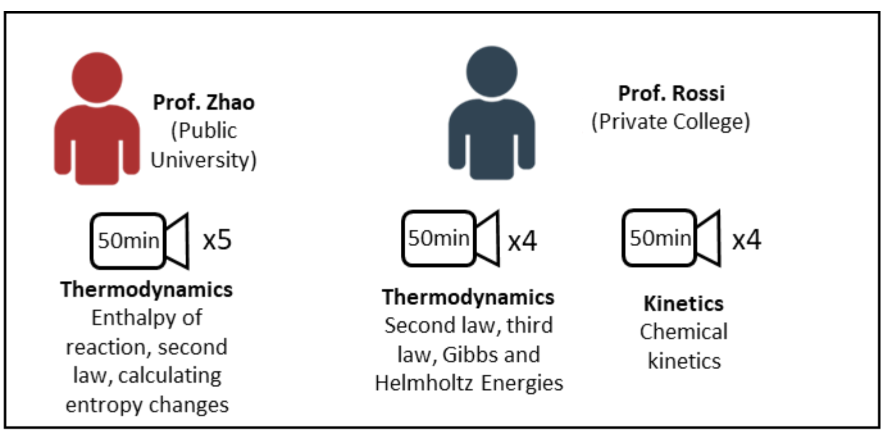


Figure 5.1. Topics covered in recorded lecture videos at both sites.

Lectures were video recorded using a Swivl robot and an iPad mini. A Swivl is a robot system designed for classroom video recording that uses IR to motion to follow an individual wearing a tracking device (*Swivl Tools: Robot + Teams*, 2021). The tracker also includes a local audio recorder. The Swivl software merges the video and audio tracks into a single high quality output. The Swivl can integrate multiple audio recorders; however, only one audio recorder can be designated as the tracker. During recording, I held the tracker to manually control the Swivl's movement, in order to safeguard against the possibility that the connection between the Swivl and the tracker might be broken if the instructor faced toward the whiteboard. The instructor wore one of the other audio recorders.

The recording system was set up in the back of the classroom on a tripod, such that the iPad was able to capture both the entire board and as few student faces as possible. It was also set up to capture the instructor's body language and gesticulations. During recordings, I took informal field notes to refer back to later, such as noting down timestamps of salient moments, or copying what was written on the board to ensure that I had a copy.

Strand 2: Problem Solving Interviews. Think-aloud problem-solving interviews were conducted with a small sample of students enrolled in each course (n=3 for Private College, n=4 for Public University). All participants volunteered to take part in the study, and student participants were given a $25 Amazon gift card as a token of appreciation. A surface analysis of these interviews revealed that although these data were rich, the connection between student problem-solving approaches and instruction was more complicated than initially hypothesized. Thus, analysis of these data was not included for this dissertation. For the sake of documentation, more information about how these data were collected is included in Appendix 4.

Strand 3: Video-stimulated recall interviews. After each unit, one-hour semi-structured video-stimulated recall interviews (VSRI) were conducted with the instructors (Calderhead, 1981). A VSRI is a type of semi-structured interview that is especially useful for probing how participants make decisions (DeKorver & Towns, 2016). As opposed to think-aloud interviews, in which the interviewer interviews a participant while they are actively engaged in a task, a VSRI is a retrospective interview that takes place after the participant completes the task in a natural setting. During the interview, the interviewer plays selected video clips to jog the participants' memory, and then conducts a semi-structured interview based on the clip. Because selecting the clips is an analytical process (the clips are deliberately selected by the researcher to probe a phenomenon of interest), the clips and how they were selected are described in more detail in the data analysis section below.

The interview protocol for each clip included a set of three questions:

1. Can you tell me what you were thinking about during this part of the lecture?
2. I noticed _____ . What was your goal in walking through ____ this way?
3. What do you think students took away from this? What was your goal for what they would take away?

The goal of the first question was to elicit the instructor's thinking and decision-making around the clip of interest. The second question was meant to draw the instructor's attention toward the aspect of the instructional moment that I viewed as relevant to the construct of abstraction, and to solicit the instructor's thoughts around this, in particular to probe whether they made these decisions intentionally (RQ1) and what the purposes of these decisions were (RQ2). The third question was geared toward understanding how the instructor viewed a potential disconnect between their intentions for student learning and what students actually learned (evidenced by the instructor's observations as well as insights gleaned from the problem-solving interviews).

Analytic Framework.

The analytic framework for this study was the Epistemic Actions Framework. The Epistemic Actions framework draws on first-generation activity theory to operationalize abstraction as a series of actions that involve movement between objects that are more concrete (e.g., an equation) to those that are more abstract (e.g., a framework that explains how that equation fits into a broader conceptual landscape). The abstractness of an object is defined relative to the context in which the activity is taking place. For example, if a student is solving a limiting reagent problem, they are expected to use the ratios of reactants to products and the amounts of each reactant provided to figure out which will be used up first. The ideas of ratio and chemical reaction are concrete in this situation, because both concepts need to be used to solve the problem. However, to solve a limiting reagent problem, one does not necessarily need to understand why and how those reactants lead to that product, so reasoning regarding bonding would be more abstract. Now consider a student trying to figure out whether the same reaction is spontaneous. To figure out whether the reaction is spontaneous, they may have to think about the relative change in enthalpy and entropy and figure out whether the reaction is entropically driven, energetically driven, or neither. To do so, they may think about whether bonds are made or broken and which side of the reaction has more particles. Here, thinking about bonding may be more concrete because it is part of the procedure to address the problem task, while thinking about the relative amounts of substance may be more abstract, because it is not directly relevant to the problem at hand.

The Epistemic Actions framework described in Chapter 3 identifies eight mental, goal-mediated actions that can be used to operationalize this movement between objects that are more and less concrete within a context. *Concretizing* involves placing a constraint (making something more concrete), *manipulating* is a procedural transformation (does not change abstractness), *restructuring* is the mediated transformation of an object through prior knowledge (increases abstractness), and *generalizing* is the construction of an object through connecting ideas and structures (increasing abstractness). Depending on the type of object, these actions can be classified as conceptual (having to do with conceptual knowledge) or symbolic (having to do with mathematical or representational knowledge). Together, these actions are used to define two types of movement: *horizontal* abstraction (movement between conceptual and symbolic objects or frameworks), and *vertical* abstraction (movement between more or less abstract objects) (see Figure 3.3).

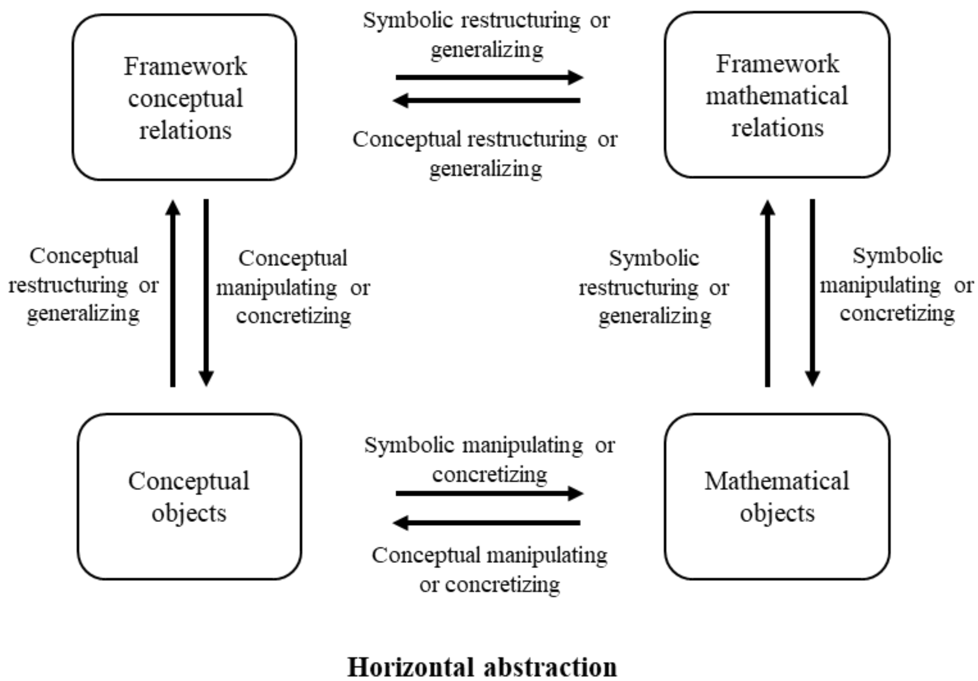


Figure 3.3. Depiction of horizontal and vertical abstraction. The Epistemic Actions of Abstraction framework.

This Epistemic Actions framework provided an initial deductive codebook that could be applied to analyze both streams of data (see Appendix 3). Specifically, it could be applied to identify instructors' epistemic actions while they teach, which allows us to identify instances of abstraction (RQ1), and to the VSRIs to analyze instructor intentions for abstraction (RQ2). However, the Epistemic Actions framework was developed from student problem solving interviews, and so it had to be adapted to be used in these two contexts (see Table 5.1).

	Initial Codebook (from problem solving interviews)	Codebook for Classroom Videos & VSRs
Definition of "abstractness"	Defined relative to the problem space; something is more abstract when it is not directly cued by the problem space	Defined relative to what instructor assumes students know; something is more abstract when it is a new concept or one the instructor does not think students have mastered
Dimension of codes	Two dimensions: conceptual (based on conceptual knowledge) and symbolic (mathematical or representation knowledge)	Three dimensions: conceptual, symbolic, and blended (use of mathematics and symbols as conceptual tools)
Total number of codes	8 (Conceptual and symbolic forms of concretizing, manipulating, restructuring, generalizing)	*Classroom videos*: 12 (Conceptual, symbolic, and blended forms of concretizing, manipulating, restructuring, generalizing) *VSRs*: 14 (all of the above, and two general codes, "vertical abstraction" and "horizontal abstraction")

Table 5.1. Adaptation of coding to classroom videos. Overview of changes made to the Epistemic Actions coding to adapt it for use in analyzing classroom instruction and the video stimulated recall interviews (VSRIs).

Applying the epistemic actions framework to a new context.

A methodological goal of this study was to investigate how the Epistemic Actions framework could be applied to a novel context. The Epistemic Actions framework was developed from teaching interviews in which students were solving novel problems. That means that the Framework was developed in a context in which the subjects were actively abstracting to make sense of and to construct novel connections in-the-moment toward a broader goal (successfully solving a problem). In this study, the professor is already a content expert who has consolidated knowledge structures and does not have a need to abstract for their own sakes during teaching. That is, if the professor abstracts, the purpose of this abstraction is not to actively make sense for himself, but perhaps to demonstrate how to make these connections for the students. This difference resulted in three changes in how the Epistemic Actions framework was applied (see Table 5.1 for an overview).

To apply the Framework to this context, we thus had to shift how we determined whether something was considered abstract. For student problem solving interviews, an object was considered more abstract if it was not part of the initial problem space. During coding, this was determined by asking the question, “Does this idea and/or connection already exist within the problem space?” During the first attempts to apply the Framework to the lecture transcripts, we framed “what already exists within the problem space” as “what has already been discussed during the lecture.” However, initial analysis revealed that this did not seem to align with how the instructors were teaching. Instead, we had to shift this question to be, “Does the professor take for granted that students will have internalized this idea or connection?” If the professor seemed to be treating an idea as “taken for granted,” their use of it was coded as manipulating (procedural, more concrete); if it seemed to be something they were actively reinforcing or [re]constructing, it was coded as restructuring or generalizing (more abstract).

A second change was the addition of a new category of codes. In the student problem solving interviews in Chapters 3 and 4, there tended to be a clear distinction between symbolic and conceptual codes. However, this line was blurrier during the professor interviews. This led to a new dimension of codes: blended actions, which used mathematical representations as a tool for conceptual thinking (Kuo et al., 2013).

The final adaptation arose when applying the Framework to analyzing the stimulated recall interviews instead of instruction or problem solving, i.e., when applying it retrospectively. The unit of analysis in the Framework is one epistemic action, a mental action that is oriented toward a goal. These actions are most easily identifiable when the subject is actively acting toward a goal or an end; however, the VSRIs are reflections on previous actions. During these reflections, the instructors often referred to intentions that were related to vertical and horizontal abstraction more generally, for example referring to the importance of students understanding the meaning of mathematics (horizontal abstraction) or seeing the big picture (vertical abstraction). This led to the addition of two codes: horizontal abstraction and vertical abstraction.

Data Analysis.

Data Analysis for Instructional Moments.

Transcripts of the classroom videos alongside the video recordings were analyzed using the epistemic actions as a deductive codebook (see Appendix 3). For this study, only a subset of the classroom videos were analyzed (the nine 3-6 minute long clips that were used in the stimulated recall interviews). These data were selected because there was the additional data source of the VSRIs could provide insight into the instructors' decision-making process during these moments.

Clip Selection. To select the clips for the VSRIs, I reviewed the lecture videos and selected three 2-5 minute clips during which the instructor seemed to be modeling abstraction based on my understanding of abstraction at the time of the interviews. For example, one clip was chosen because the instructor seemed to be modeling abstraction explicitly by intentionally demonstrating the logic with which the students can make connections between concepts and mathematics in order to generalize from data to develop concepts and theories (e.g., Clip B1). Other clips were chosen because they seemed to be modeling abstraction implicitly, for example by generalizing directly from an example problem, but not talking through the logic flow to the generalization (e.g., Clip B2). Tables 5.2 and 5.3 contain brief summaries of the class content covered in each clip; the wording of question 2 from the interview protocol, which probes participants about the pedagogical instantiation I saw as relevant to abstraction; and a brief summary of the justification for choosing the clip by relating the pedagogical instantiation vocalized in Q2 to the conceptual framework.

Clip	Class Material	Question 2	Connection to Abstraction
Clip B1	Introduction of the concept of entropy and derivation of the second law of thermodynamics	You started by introducing entropy as a state function and then focused on entropy as a function with absolute values.[…] the next thing you did was walk through the mathematical derivation for ΔS=q/t from the Carnot cycle.	Vertical abstraction (conceptual generalization of definition of entropy)
Clip B2	Calculating entropy change for a process	You talked through reasoning and justifying designing reversible processes for doing the calculations	Horizontal abstraction (modeling how to mathematize in different situations)
Clip B3	Defining the second law of thermodynamics	You introduced the definition of the 2nd Law, that entropy change is greater than zero, before going through the proof	Horizontal abstraction (connecting mathematics and definitions)

Table 5.2. Description of VSRI Interview Clips for Prof. Zhao (Public University)

	Class material	Question 2	Justification for choosing clip
Clip A1	Reinforcing a statistical mechanical definition of entropy by drawing a grid to represent microstates	You drew out explicitly a model to show how the entropy changes through gas compression[…] How does this related to what they have been learning about entropy and Gibb's energy?	Multi-modal representation to support vertical abstraction
Clip A2	Determining the spontaneity of a process using the Gibbs equation	When you're working through Gibbs, you work through a specific example.[…]When you walked through figuring out ΔH and ΔS signs, you work through the reasoning	Concretizing to support concept development
Clip A3	Deriving the Maxwell relation for Gibbs energy at constant temperature and pressure	When you were deriving dG, you walked through the meaning of each variable[…] and at the end of the clip you point out the equation is not just an equation, but relationships between variables.	Conceptual generalizing to manipulation
Clip A4	Deriving the integrated rate law for first order kinetics	What was your goal in deriving the integrated rate law and connecting that to these two plots?	Horizontal abstraction (connecting different types of representations)
Clip A5	Showing how to calculate the rate for a two-step reaction	Part of what you seem to be doing here was relating the idea of a mechanism to the idea of a complex reaction being multi-step	Modeling a problem solving process
Clip A6	Walking through Michaelis-Menten kinetics	You were very explicit in talking about when things were being created or destroyed and the role that plays in writing the rate law.	Horizontal abstraction (connecting mathematics with their conceptual underpinnings)

Table 5.3. Description of VSR Interview Clips for Prof. Rossi (Private College). Clips A1-A3 were used in the first round interview (Thermodynamics) and clips A4-A6 were used in the second round interview (Kinetics)

Epistemic Actions Coding. To analyze each clip, the clip as well as several minutes of instruction preceding it were re-watched in their entirety. This was done to understand the instructional context of the clip, in particularly what immediately preceded it. After watching the clips, they were coded using the epistemic actions framework. Analytic memoing supported coding decisions (Birks et al., 2008). In particular, analytic memos were used (1) to contextualize the instructional moment in the broader context of the class, and (2) to flesh out how the codes were adapted. These memos were used to guide decisions around what the instructor seemed to be taking for granted ("concrete") and what was considered abstract.

Table 5.4 illustrates an example of this coding supported by analytic memos. On the left side of the table are quotes from the professor's lecture. The two quotes in the table occurred subsequently during the same instructional moment. Coding was conducted in NVivo and supported with analytic memoing in a word document. To see the actions patterns for all clips, see Appendix 5.

So the example I gave you guys last time was after I went through all of the examples, there was a question saying, well how do you calculate entropy change for a system, for process and that's kind of slightly deviated from equilibrium [also] irreversible. Um, because we have to use q(rev) divide by T to calculate entropy change for system. What if we have something like this? Okay. So we had this strategy. […] What we did was we designed a reversible process . Okay. So the process we design was really, you think about this is a starting point. You want to get to the final point, okay. And then you'll say, well, I'm going to go through another path.	**Memo:** He starts by reminding the class that they have to use qrev/T to calculate entropy change for a system, but points out that the process they are working on (freezing of liquid water below the freezing point) is an irreversible process, and that means you can't use q/T to calculate it directly. This means you have to use the strategy of designing a reversible process.
	Code: Conceptually restructuring (we need to think about it like this, this way we have previously learned, based on the constraint of the system at hand) followed by symbolic restructuring (the way he represent that new way of thinking about the process is like this, designing this reversible process)
Go from here to here, make sure this is a reversible step, and then go from here to here as zero C. we know that this is going to be a reversible step. Okay? And then from here to here, we can make that into reversible step. We know, again, this is irreversible	**Memo:** He then applies this restructuring to the specific problem at hand—you first do this and make sure it's reversible, then do this and make sure it's reversible, and do this and make sure it's reversible.
	Code: He alternates between symbolically and conceptually manipulating, using the ideas he has introduced about how to design a reversible process through the restructuring moves and applying them to the specific example at hand.

Table 5.4. Transcript excerpt with example of analytic memoing.

Implicit and Explicit Coding. To answer RQ1, after identifying abstraction processes through coding for epistemic actions, instructional clips where abstraction occurred were coded holistically to characterize whether or not abstraction was being implicitly or explicitly modeled. Definitions of implicit and explicit modeling were adapted from teacher education (Lunenberg et al., 2007; Moore & Bell, 2019). Because each clip represented roughly one abstraction activity (e.g., the professor was modeling abstraction toward a single end), the unit of analysis for this coding was one clip, and each clip was coded as either implicit or explicit. There were three exceptions: clips B2, A1, and A4. During these clips, there seemed to be two separate abstraction activities with different motives and which seemed to be modeled differently by the instructor.

A clip was coded as *implicit* if the instructor modeled abstraction without drawing attention to that fact. That is, a clip was coded as implicit if the instructor modeled abstraction, but did not justify, explain, or acknowledge that they were doing so. A clip was coded as *explicit* if the instructor referred to the abstraction they were doing and its purpose. Table 5.5 shows two examples of this coding. In the implicit coding example, the instructor models horizontal abstraction at the level of manipulation, to reinforce the meanings of the variables in the equation. However, he neither draws attention to the fact that he is doing so for this reason, nor discusses that alternating between thinking about the objects (e.g., variables) conceptually and mathematically was a deliberate choice. Consequently, this moment of instruction was coded as implicit modeling. In the explicit coding example, Prof. Rossi conceptually concretizes and draws out microstates as a grid. He deliberately discusses this choice, and refers to its utility as a tool that students can adopt when thinking through a process. Because of this reflection, this moment was coded as explicit modeling.

Code	Definition	Example
Implicit	Instructor models abstraction without drawing attention to it. There is no reflection on what they are doing or its purpose	**A3 (modeling horizontal abstraction):** "Or derivative or *d*? Right? And can figure out what it is using an integral though, right? So I'd write it as *dG*, it's some small, tiny change in the Gibbs energy due to a change in our system. And so if we take kind of the derivative of *G*, we have to do it for all this. So we have, um, *dH*, um, what I doing here? Okay. *dH* minus *TdS* minus *SdT*, yeah. Right."
Explicit	Instructor models abstraction and draws attention to the abstraction process and/or reflects on the purpose of what they are modeling	**A1 (modeling conceptual concretizing for vertical abstraction): "**So like I said, I'll draw out kind of a simplified grid. So say, and you can always do this when you're trying to think through a process. So say we have 16 places the molecules can be and we have four molecules, right?"

Table 5.5. Examples of the implicit and explicit coding. The definitions for the codes are adapted from Moore and Bell (2019).

Analysis of Stimulated Recall Interviews.

RQ2 revolves around the role abstraction plays in physical chemistry instruction. To address this question, the VSRIs were also coded using the Epistemic Actions codebook. This coding took place after coding all of the instructional clips to reduce the extent to which the instructor's insight into their own instruction influenced coding decisions.

To code the interviews, I attended to what the professor's intention was for that moment of instruction. The unit of analysis ranged from one to several lines of discourse in which the instructor explicitly referred to an epistemic action they performed during instruction. For example, if the instructor talked about connecting an equation back to previous classes, this action may have been coded as "conceptual restructuring." Because the professor did not always describe an intention that could be mapped to a specific epistemic action, two additional codes were added: horizontal abstraction and vertical abstraction.

Reliability.

Two procedures were followed to ensure reliability in the coding. First, several meetings with outside researchers were held to support adapting the codebook for use with instructor videos and to develop a procedure for triangulating findings from the stimulated recall interviews with those from the classroom videos (Dalgety et al., 2003). Second, three of the clips (33% of the data) were given to an external researcher to code. Coding was discussed until we reached 100% agreement.

Findings

Research Question 1.

To answer the question of how instructors implicitly and explicitly model abstraction while teaching kinetics and thermodynamics, 9 instructional clips were analyzed. Each instructional clip contained both horizontal and vertical abstraction. In both instructors' datasets, conceptual epistemic actions tended to be more frequent than symbolic actions, except for in the case of manipulating. There were also examples of blended actions for each of the four types of actions (see Table 5.6).

	Conceptual		**Symbolic**		**Blended**	
	Zhao	*Rossi*	*Zhao*	*Rossi*	*Zhao*	*Rossi*
Generalizing	5	4	1	2	1	3
Restructuring	5	7	3	2	0	2
Manipulating	8	16	8	22	1	9
Concretizing	2	7	1	4	0	1

Table 5.6. Frequency of epistemic actions across both instructors. The counts for Prof. Zhao are drawn from 3 clips, and the counts for Prof. Rossi are drawn from 6 clips.

Examples of both implicit and explicit modeling of abstraction were found in the data. All of the clips except for one included examples of implicit modeling. One clip (B1) was exclusively coded as explicit modeling, and three clips (B2, A1, and A4) were double coded because they contained two distinct abstraction processes that were modeled differently. Below, I illustrate each of these findings with an example from the data.

Implicit Modeling.

As mentioned above, implicit modeling was more common than explicit modeling. This means that when the clip was coded using the Epistemic Actions framework, some form of abstraction was identified. However, when the professor was teaching, he did not elaborate on or explain how he was doing these abstraction actions or why.

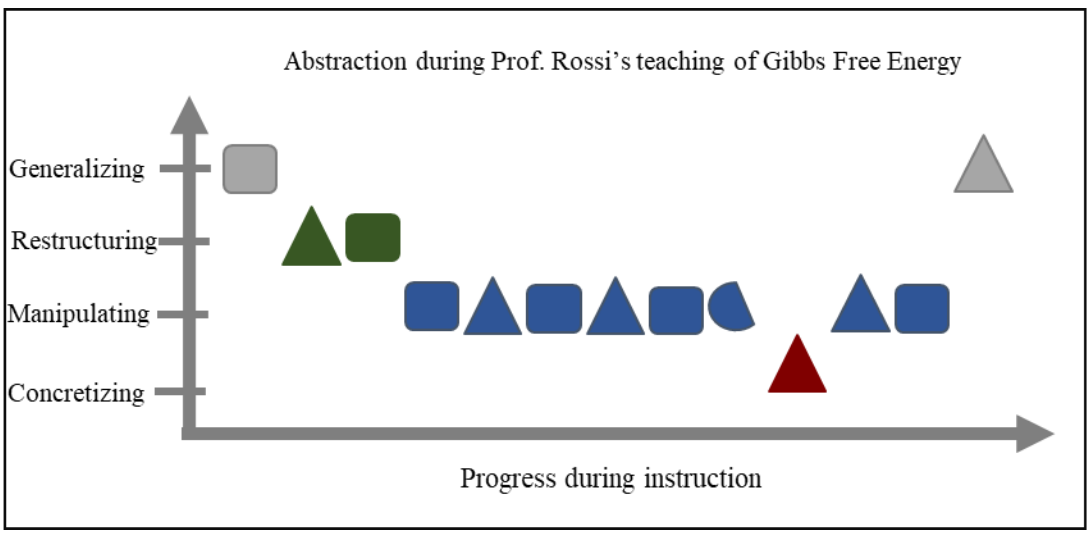


Figure 5.2. Prof. Rossi's epistemic actions during Clip A3. Triangles represent conceptual actions, squares represent symbolic actions, and half moons represent blended actions.

For example, consider this excerpt from Clip A3 (see Figure 5.2). In the lectures preceding this clip, Prof. Rossi taught the students a molecular interpretation for Gibbs free energy that focused on qualitatively interpreting the relative change in entropy and enthalpy by comparing inter- and intra-molecular interactions in the initial and final state. During the lecture for Clip A3, his goal was to develop a physical interpretation of Gibbs energy, and in particular to relate it to the maximum non-expansion work that can be done by a system under constant pressure and temperature. To derive this relationship, he starts with the Gibbs free energy equation ($G = H - TS$):

Prof. Rossi:	(CR) And so we're going to start again with kind of our basic definition where our fundamental definition of what Gibbs energy is relationship between entropy and energy. In this case, Gibbs is an or energy change at constant pressure. So it's enthalpy (SR) and we're going to do some, um, infinitesimal, infinitesimal change. And how would we indicate mathematically and an infinitesimal change in the Gibbs energy?
Student:	[response]
Prof. Rossi:	(SM) Or derivative or d? Right? And can figure out what it is using an integral though, right? So I'd write it as dG, (CM) it's some small, tiny change in the Gibbs energy due to a change in our system. (SM) And so if we take kind of the derivative of G, we have to do it for all this. So we have, um, dH, um, what I doing here? Okay. dH minus TdS minus SdT, yeah. Right. (CM) Because we have the product of TS. So first observed of second, second times derivative of first, and then we can use our relationship between internal energy and um, enthalpy (SM) because H is equal to U plus PV. And so we can plug in for that. So our dG becomes dU plus PdV plus VdP minus TdS, minus SdT. [to himself] I get that all right. Yes. Okay. (CM/SM) And so again, H is equal to U plus PV. (CC) So we're going to do this process for, again, constant temperature and pressure. (CM) And that simplifies this a little bit, right? Because anything that depends upon a change in pressure and a change in temperature, um, goes away.

> (SM) So if we're at constant T and P, the dP term here goes away and the dT term here goes away. Okay? So we'll write this out with what we have and we'll talk about exactly what this means. dG is equal to dU plus PdV minus TdS.
>
> (CG) So what this equation is saying is essentially any change in Gibbs energy for pressure or for a process at constant temperature and pressure can be, um, determined by looking at how the internal energy changes, the volume changes at that pressure, and how the entropy changes at that temperature when we do the change to the system. So that's all that equation is saying. There's a lot of letters. Today's going to be basically an entire class of letters. Um, but keep in mind, it's not just an equation to be an equation, it's the equation that's telling us how different variables are affecting the free energy of our system when they change.

As Prof. Rossi walks through the derivation, he performs several abstracting actions. He starts the derivation by horizontally abstracting at the level of restructuring, which couches the example at hand in what has already been taught about Gibbs (conceptual restructuring), which elicits a particular mathematical form (symbolic restructuring). As he walks through the derivation, he horizontally abstracts at the level of manipulation—he performs a symbolic manipulation, conceptually manipulates to interpret it or raise a question about a physical condition, and then translates that question back into mathematics. At the end, he makes a jump in abstractness from symbolic manipulation (the final step of his horizontal abstraction) to conceptual generalization in order to interpret the results of the manipulation as a generalizable explanation of what the equation means.

During this clip, Prof. Rossi modeled two kinds of abstraction: vertical abstraction (from restructuring to manipulation to generalization) and horizontal abstraction (conceptual to symbolic restructuring, and conceptual to symbolic manipulation). Both of these are modeled implicitly. He did provide some meta commentary on what they are doing ("And let's see how exactly that relates back to our equation and how we're looking at our, um, Gibbs energy”). However, he neither narrates his strategy as he moves through the actions nor explains why he is making these abstraction moves. In particular, he does not explicitly narrate how he is connecting the concepts at hand to what they had previously learned or connecting what they are doing to a more general strategy.

It is particularly important to point out the role of horizontal abstraction in this clip, because it represents a larger finding. Horizontal manipulation was one of the most common forms of abstraction, occurring in 8 out of 9 clips, including all clips from Prof. Rossi’s course. When horizontal manipulation occurred, it was always implicit. That is, when the instructors used horizontal manipulation to either reinforce the meaning of an equation, or to mathematize a physical phenomenon, it was always done without meta commentary on what they were doing or why.

Explicit Modeling.

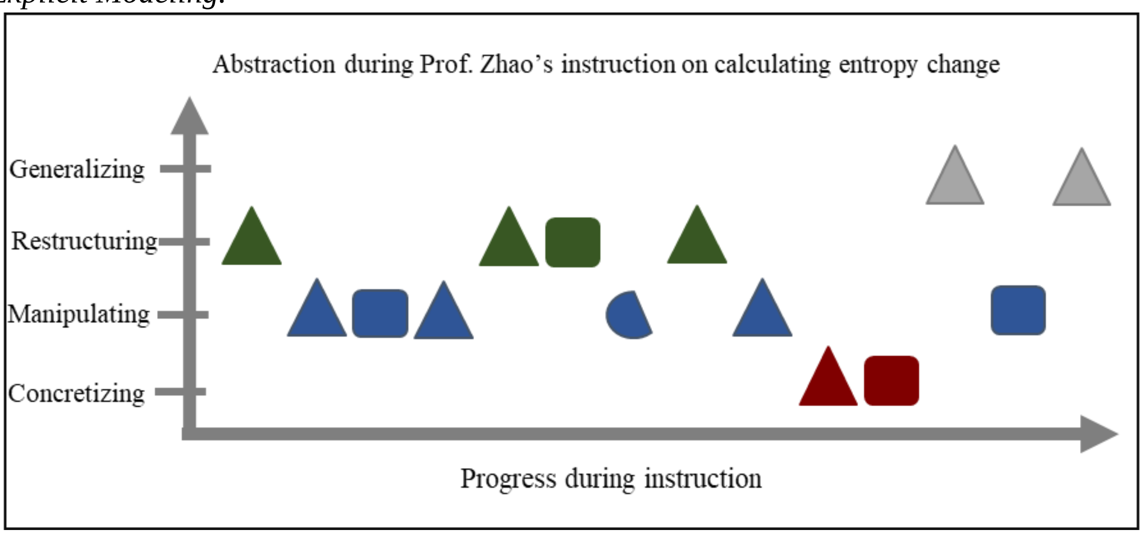


Figure 5.3. Prof. Zhao’s epistemic actions during Clip B2. Triangles represent conceptual actions, squares represent symbolic actions, and half moons represent blended actions.

While he was teaching about how to do entropy calculations, Prof. Zhao explicitly modeled an abstraction process (clip B2). In previous classes, Prof. Zhao taught the class how to calculate the entropy change of several physical processes, including a state change and the isothermal expansion of a gas. To prepare the class for thinking about how to calculate the change in entropy of a chemical reaction, he gave them a slightly more complicated example: the superfreezing of water at -10° Celsius. He began the example by referring back to these earlier calculations through a series of explicit restructuring moves (see Figure 5.3):

Prof. Zhao: (CR) So the example I gave you guys last time was after I went through all of the examples, there was a question saying, well how do you calculate entropy change for a system, for process and that's kind of slightly deviated from equilibrium [also] irreversible. Um, because we have to use q(rev) divide by T to calculate entropy change for system. What if we have something like this? Okay. So we had this strategy,

(CM) we know that this is not going to be reversible. Okay. So I'm getting write down this is certainly irreversible. Okay.

(SM) So you really cannot use that to calculate entropy change for the system.

(CM) Okay. Um, we know there is a, there might be a *q* for or something, but that *q* is not reversible so you can not use it.

(CR/SR) What we did was we designed a reversible process.

Prof. Zhao: (SR) Okay. So the process we design was really, you think about this is a starting point. You want to get to the final point, okay. And then you'll say, well, I'm going to go through another path.

(SM) Go from here to here, [[draws arrow]]

(CM) make sure this is a reversible step,

(SM) and then go from here to here at zero C. [[draws arrow]]

(CM) we know that this is going to be a reversible step.

Prof. Zhao: (SM) Okay? And then from here to here, we can make that into reversible step. [[draws arrow]]

> (CM) We know, again, this is irreversible. Let me just write down here.
>
> Okay. So that's a strategy we took. Um, let me just erase this one to make it simple. Our strategy was, uh, the question mark was calculate, uh, entropy change for the irreversible step. Okay. But in order to apply this equation in calculating entropy change for a system, we bypass the difficulty, we say well, let's design, a multiple step process, okay? And make them reversible of the individual step.
>
> (CR) And then we can calculate based on the fact of entropy is a state function. So entropy change from here to here will be the same as entropy change you go through another path. Okay? So that's our strategy. And again, I want to highlight this strategy because later in the semester when we talk about Gibbs energy change, you'll find out this is a very, very useful strategy, okay

In this clip, Prof. Zhao introduced a strategy for calculating entropy changes for the class to build on to learn how to calculate entropy changes of a chemical reaction (see Figure 5.4 for a picture of the work he presented on the board). When he first introduced the process, he explicitly conceptually restructured to situate the process within a strategy the class had previous used to calculate entropy change of a process: “So the example I gave you guys last time was after I went through all of the examples. […] Um, because we have to use q(rev) divide by T to calculate entropy change for system. What if we have something like this? Okay. *So we had this strategy*.”

After talking through the assumptions that underlie and facilitate the process (e.g., that the process will be “certainly irreversible”), he elaborated on the restructuring move by stating that they were designing a reversible process. He previewed the box structure he drew through a symbolic restructuring move: “So the process we design was really, you think about this is a starting point. You want to get to the final point, okay. And then you'll say, well, I'm going to go through another path."

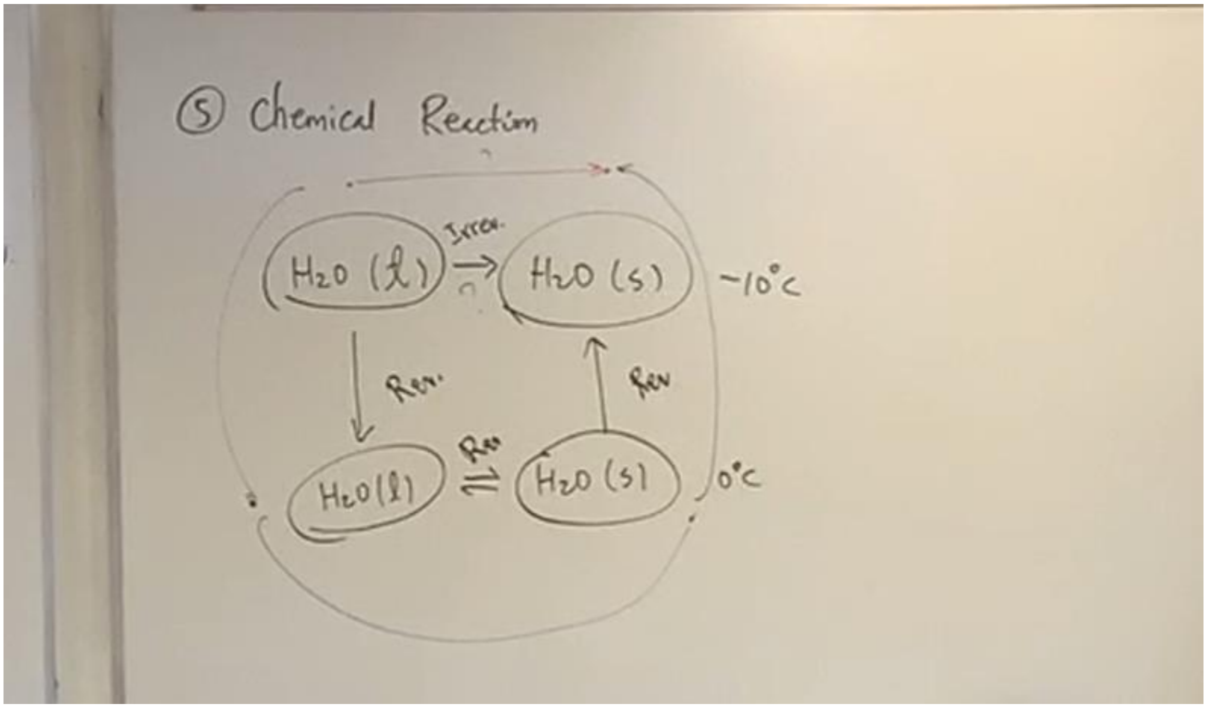


Figure 5.4. Screenshot from classroom video. Depicts the process Prof. Zhao developed for calculating the entropy change of the superfreezing of water in Clip B2.

In his third conceptual restructuring move, Prof. Zhao deepened the connection between the process he drew out and entropy's status as a state function that enables this type of calculation. He modeled each of these restructuring moves explicitly. For each restructuring action, he reinforced that he was drawing on prior knowledge to develop a process that they can use to solve the problem. He explicitly referred to other examples that were modeled in similar ways to draw parallels between them, and he made the choice to set up the calculation as a box (see Figure 5.4), a structure that he had used for all similar calculations, and referred to it as a generalizable strategy. During his interview, he explained that he also uses this procedure when modeling calculations for other state functions, such as Gibbs free energy.

Explicit and Implicit Modeling Together

Although Prof. Zhao's restructuring moves were explicit, a second abstraction process during Clip B2 was coded as implicit. Above, he used the idea of state functions to justify his restructuring move ("And then we can calculate based on the fact of entropy is a state function"). However, at the end of the clip, he shifted how he used the idea of state function from being the foundation of restructuring to the object of conceptual generalizing:

> Prof. Zhao: (CG) Uh, but the whole point is because entropy change is a state function.
>
> So in order to figure out something going from here to here, okay, there's a question mark.
>
> (SM) We actually designed something like, I go from here to here, okay. And then go from here to here and then go from here back here.
>
> (CG) Okay. So it's just two different path, okay. Because entropy is a state function, okay? So entropy change from here to here will be the same as here.

Prior to this, he had been using the fact that entropy is a state function to justify the design of the process. However, in these actions, he shifts from using this to justify restructuring to it being a connection he was emphasizing and reinforcing. In the symbolic manipulation move, he referred back to the process they had designed as support for the conceptual generalization move in the last line ("Because entropy is a state function, okay?"). This shift in goal and this abstraction process was implicitly modeled. Unlike with the restructuring moves, he did not provide commentary on how he was doing the generalizing or how it fit into the bigger structure of what they were learning. Rather, the conceptual generalization seemed to be an almost taken-for-granted leap—he moved directly from the "evidence" (the symbolic manipulation move) to the point he was reinforcing (the conceptual generalization).

Research Question 2.

To answer RQ2, I triangulated the analyses of the classroom clips and the stimulated recall interviews to find what role abstraction plays in physical chemistry instruction. Three themes emerged related to the purposes of the instructional moments analyzed.

Theme 1: Abstraction as a way to develop a mathematical tool that is grounded in conceptual understanding.

The first major role that abstraction seemed to play in the clips was to develop a mathematical tool while reinforcing the meaning of the mathematics. Four clips fell into this category. Each of these clips were characterized by an action that reshaped the problem space at the outset of the clip. Two of the four clips began with restructuring moves (B2 and A4), and the other two began with concretizing moves (A1 and A5). Three of the four clips included symbolic vertical abstraction, and all four clips included horizontal abstraction at the level of manipulation. During the interviews, the instructors referred to the purpose of this part of instruction as being related to developing a mathematical tool or process (see Table 5.7).

Clip	Explanatory Quote
Prof. Zhao: B2	So the reason I highlight this box again, answer is one I want to highlight this is a state function because going you're from here to here is the same, that's two different paths. Number two was again, it's more for problem solving. I actually on Wednesday, so good timing, on Wednesday I'm going to build this connection for them. […] If they don't design the box, they will say, well what kind of equations are you using? Why should I use the combination of these two equations? Why? They just don't know why. I mean you, they have a, if I gave them the equation, they just don't know why. But if I bring this up, pretty much these are off of exam number two. They know much better. The temperature difference, pressure difference. They're all related to something we can like describe in the boxes. So that's really a reason.
Prof. Rossi: A1	*[from classroom video transcript]* Right? So you're changing, but you're changing the available places the molecules can exist. So like I said, I'll draw out kind of a simplified grid. So say, and you can always do this when you're trying to think through a process.
Prof. Rossi: A4	So ultimately when I'm, you know, the, the initial goal is to show them, you know, ways or basically how to get a time dependence. Right? Um, so how do you start with your rate law and then end up with a time dependence. So taking through that derivation, um, but then also showing what the usefulness of that integrated rate law is, right? Because we talk about showing them graphically what the concentrations look like, especially to be able to compare, show them how the *k* [rate constant] affects things. Um, but then, you know, we talked about the second one in showing how it's useful in, um, now, and also I'm getting ahead of myself here, but it's, um, I don't know if you have, but I'm showing them the usefulness because if you have concentration and time data, you can actually get information like the rate constant of the reaction, um, which is the second, the second plot.
Prof. Rossi: A5	So this whole thing leads up to the steady state approximation. Right? So it's kind of introducing that, um, before this I had just talked about, um, elementary versus complex step. So it's kind of working with the difference between the two. You have a complex reaction and then you look at the individual steps. Um, also it's just practice in writing out kind of these little rate laws, um, to get an overall rate law. So starting with basic steps and then writing out individual rate laws to get the rate law for the overall complex reaction and um, and yeah, and getting them to think about, so I do the, the reactant and the product first because each of them are only affected by one reaction, but then do the intermediate last because that one is affected by both the first and the second. So it was created and destroyed. Um, and then, yeah, so it's kind of combining of all of those things, um, in one step kind of distinguishing between different types of multistep, two step reactions, um, which again leads to those plots of, um, concentration versus time for everything. And then, um, just writing out rate laws for co- for elementary steps and that sort of stuff.

Table 5.7. Excerpts from interview quotes for each clip coded as Theme 1.

Although they were all oriented around developing a mathematical tool or process, each clip had a slightly different purpose. In Clip B2, Prof. Zhao explicitly modeled a problem solving process he hoped the students would adopt and use, in part because the process itself reinforced entropy's conceptual structure. In Clip A1, Prof. Rossi uses a symbolic concretizing move (representing the concept of microstates as molecules in a grid) to clarify a misconception he believed the students had about the nature of microstates. However, he also explicitly referred to this concretizing (drawing the grid) as an epistemic tool students can use when they are trying to think through a process. In Clip A4, Prof. Rossi walked through how to derive the integrated rate law from the differential equation. In his interview, he revealed that he intended for students to be able to replicate this process (similar to B2). Clip A5 had slightly different intentions. Again, Prof. Rossi discussed that part of his purpose was to demonstrate a process for the students to imitate ("just writing out rate laws […] for elementary steps and that sort of stuff"). However, the primary purpose of A5 seemed to have been demonstrating where a useful mathematical tool for multi-step reaction kinetics, in particular enzyme kinetics (the steady state approximation), comes from. That is, while the role of abstraction in the other clips was to model *how to use* processes and tools, abstraction in A5 primarily served to *develop* a tool that students could later use.

The second major distinguishing features of these clips was that the intention was not simply mathematical or processual in nature. Rather, the instructors emphasized the importance of connecting the mathematics to conceptual understanding. In the interviews for A1 and B2, the instructors explicitly said that these clips had the dual purposes of developing a tool and addressing conceptual understanding. Clips A4 and A5, which were more focused on the mathematics, still had the secondary purpose of emphasizing the meaning of the mathematics through horizontal abstraction. For example, when deriving the steady state approximation in A5, Prof. Rossi continually reinforced the physical meaning of the differential rate law. During his interview, he framed this as a deliberate choice:

Prof. Rossi: But I also, um, if you notice, I also a lot of times say it out, not just as like *da/dt* right? But I tried to say, you know, how do we write out how the concentration is changing with time? So I try to really, and I do that with all my, like anytime I do derivatives or integrals. […] I actually talk about what derivatives are like, why, why do we take the derivative of something? We're not just doing it for fun, right? Sometimes we kind of are just doing it to get it to look like something else. But that's, but there's a, the actual, you can think of an actual like description. You're looking at how it changes when you change this and you change this. So I really try to not just let sit with these like equations or terms in their heads and try to say it differently at different times or explain it a little more.

This reinforcement of the variables' physical meaning, which he did during the lecture through horizontal abstraction, suggested that it was important to him not just that the students can do the calculation, but that they also understood what the calculation means.

Theme 2: Abstraction as a way to reinforce and foster the development of conceptual knowledge, particularly using mathematics.

The second major role that abstraction played during instruction was to reinforce and foster conceptual understanding. Five clips fell into this category (see Table 5.8), including two clips that also fell into the category of developing a tool (B2 and A1). Each of these clips were characterized by conceptual vertical abstraction and at least one instance of horizontal abstraction. Three of the five clips had horizontal abstraction at the level of restructuring, and four had horizontal abstraction at the level of manipulation. Four of the five clips had at least one instance of conceptual generalizing, and two had blended conceptual and symbolic generalizing. During the interviews, the instructor often emphasized the importance of understanding the "bigger picture."

Clip	Explanatory Quote
Prof. Zhao: B1	I was trying to relate how, how we calculate this three quantities [[ΔH, ΔS, ΔG]] and these two are identical [[$\Delta_r H$ and $\Delta_f H$]] and then you use the this right and this one you use, uh, the molar enthalpy of formation, molar Gibb's energy of formation. But this one [[$\Delta_r S$ and S°]] is different. This one is we know the absolute value so we don't bother to define this one. […] Um, so those are the two reasons. One reason I wanted to show, highlight the difference of the three calculations. And again, as I said, probably I think last week I talk about the three differences, the three equations, the differences, similarities. Um, and then the second purpose is really to speak to my original flow of the course. Always bring them the big picture. Like why we do this, why would do that?
Prof. Zhao: B2	Um, and I was wanting to bring up throughout the whole semester, we have beautiful state functions based on this, like only depends on the two states, doesn't matter what kind of path you go. […]So we use the box to solve it. So the reason I highlight this box again, answer is one I want to highlight this is a state function because going you're from here to here is the same, that's two different paths.
Prof. Rossi: A1	Yeah, because I talk about how I, I don't like the deck of cards analogy. Um, because there are, there are a lot of different […] where they say if you shuffle a deck of cards, you've increased the entropy because you go from order to disordered. And I talked to them about how that's not true because you're just taking it from one state to another. Like the, because in that case ordered and disordered is arbitrary, right? You can find any state as ordered. It's just one state that we decided that if they're in order by suit and so somehow he got in his mind that because, that somehow a deck of cards only has one microstate so it took a couple of different times to get through to him that no, it doesn't have just one micro state, we're in one particular micro state with a given order. Right?
Prof. Rossi: A2	I really just keep trying more and more to get them to think about what the physical picture is before they start trying to put the math on it because they get so hung up on the math. When I ask for, you know, the sign of Delta G sometimes it just doesn't connect that, oh, is it going to happen or not under these conditions. […] I say we're going to bring in all the different stuff as you've learned so far. So don't forget you don't forget your gen chem. Don't forget your orgo. Um, because you've learned all of that stuff. You know what Delta G is. Because they do learn about spontaneity and Delta G like the, this equation, they get in orgo, right? Because they talk about free energy, changes in reactions, but they don't know where it comes from. So they apply it, but they don't see it. So now we're just kind of trying to fill in the gaps. Like why do we talk about free energy when we talk about reactions, right? Why don't we talk about these different things? Why does it depend on energy? Why does it depend on enthalpy or, um, entropy.
Prof. Rossi: A3	So the whole point of that part of the lecture is to give them kind of an interpretation of what a Gibbs energy is. Right? So, yay, this thing is spontaneous and the negative and the Gibbs energy is negative 64 kilojoules per mole. What does that energy actually mean? […] Or not even understand what Gibbs is, but understand what this equation is. Cause we ended up with this big long equation and it's kind of like, well why, you know, I want, but yeah, so I'm relating everything back to what exactly, um, is affecting the energy, like how all these things tie together.

Table 5.8. Excerpts from interview quotes for each clip coded as Theme 2.

Similar to the first theme, the precise function of abstraction depended on the instructional context. In Clip B1, Prof. Zhao introduced the concept of entropy to the class for the first time. He did this by comparing it to another state function they were already familiar with, enthalpy, and drew a comparison between how ΔS and ΔH are calculated to build toward a conceptual generalizing move (that entropy has known absolute values):

Prof. Zhao: (CG) We know the absolute value of entropy. So think about what we have here.

(CR) Enthalpy, because we don't know the absolute value of enthalpy.

(CR/SR) So we have to design all of this direct method, indirect method.

(SR) We even come here, use the BE values, try to calculate the delta(r)H values.

(CM) That's really only because we don't know the absolute value of enthalpy, okay? But entropy is easy, right? Now.

(SG) I told you we, if we have a chemical reaction, okay? Entropy change is going to be the total entropy of products minus the total entropy for reactants.

[…]

(CG) I just wanted to highlight entropy value that we know this value, absolute value.

Here, Prof. Zhao generalized the property of entropy ("entropy has absolute values") through vertical and horizontal abstraction to compare entropy to the more familiar state function enthalpy. In Clip B2, Prof. Zhao similarly used mathematical properties to reinforce conceptual understanding. As discussed above, the primary pedagogical goal of B2 was to develop a mathematical process that students could emulate. However, in his interview, Prof. Zhao discussed that he designed this process to highlight that calculating the change in entropy and Gibbs free energy is path-independent, reinforcing the idea that they are state functions (a goal of conceptual understanding).

Prof. Rossi similarly used mathematics to build concepts. In Clips A2 and A3, he was trying to develop an understanding of Gibbs free energy. In Clip A2, he used hydrogen decomposition as an example of how to determine a reaction's spontaneity by reasoning through changes in enthalpy and entropy. He did this through horizontal manipulation (continually reinforcing the conceptual meaning of the mathematical manipulations) and vertical conceptual abstraction (restructuring to situate the discussion in what they already know about enthalpy and entropy). During his interview, he emphasized that the purpose of this was to shift students from thinking about the math in isolation and instead relying on a conceptual understanding of the physical phenomena. In Clip A3, Prof. Rossi used Maxwell relations to model how free energy change is affected by different natural variables, such as temperature. His goal was to develop a physical interpretation of Gibbs free energy. The fifth clip was less focused on using mathematics to develop a concept. In Clip A1, Prof. Rossi used conceptual vertical abstraction to try to shift students to a more canonical model of microstates.

Theme 3: Other roles of abstraction.

Two clips did not fit into either of the above categories (see Table 5.9). In Clip B3, Prof. Zhao introduced the statement of the second law *before* walking through how to derive it. The clip captured a brief part of the derivation. During the interview, when asked to explain this choice, he talked about how his goal was to focus on the application and use of the second law and to deemphasize the derivation. Choosing to introduce the law first (conceptual and symbolic generalizing) and then show where it came from (derivation through horizontal abstraction) foregrounded the use of the second law as a tool rather than as an outcome. However, he did not develop the tool through abstraction; rather, he introduced it first as a given and then justified it through horizontal abstraction. In Clip A6, Prof. Rossi walked through an example of enzyme kinetics to demonstrate how to use the steady state approximation. During this example, he horizontally abstracted to explain the meaning of the mathematics. Similar to Prof. Zhao, he explained that the purpose of this moment of instruction was to show how to *apply* the tool. The horizontal abstraction supported this goal by continually reinforcing the meaning of the mathematical manipulations he was carrying out.

Clip	Explanatory Quote
Prof. Zhao: B3	Um, this approach, again, I bring them the law up front just because I think I want to emphasize that says this law and then deemphasize the derivation. I always tell them derivation, I mean you read textbook is better than my lecture. Really. I mean you just read how it is done uh. Many other derivation, many other equations, I totally skipped the derivation and just don't even do the math at all because sometimes they would focus too much on the math. Like why we go from this step to the other step, rather than like focusing on the very important implication application of the second law. […]I want to bring [the second law] up front, just giving them a big picture and then deemphasize the derivation so they know the importance of this second law rather than, Oh is this going to be quiz, test, exam? Should I memorize this?
Prof. Rossi: A6	Yeah, it's just more practice and showing them how you can apply what, you know, we learned with the generic reaction to an actual one. […] It's a really good example of applying what they've learned to a real system.

Table 5.9. Excerpts from interview quotes for each of the clips coded as Theme 3.

Triangulating results from RQ1 and RQ2.

Triangulating the results for research questions 1 and 2 provide two interesting insights about how instructors may utilize abstraction. Four of the five clips that fell into “developing conceptual knowledge” included abstraction processes that were implicitly modeled. In contrast, most (three of the four) clips that fell into the category “develop mathematical tool or process” had explicitly modeled abstraction processes (see Table 5.10). That is, when the two instructors demonstrated procedures to develop tools and processes that could be used for mathematical problem solving, they tended to be more explicit in how these tools were derived and how they fit into the broader context of the material already covered in class. However, when the instructors abstracted to develop conceptual knowledge, they tended to more implicitly model how they drew connections that led to generalizing new pieces of information. This trend also held for the two clips that fell into both categories (B2 and A1). In these, the vertical abstraction that led to conceptual generalizing was implicitly modeled, while the vertical abstraction that included restructuring and led to the development of a tool was explicitly modeled.

	Theme 1: Develop Mathematical Tool	**Theme 2: Develop Conceptual Knowledge**	**Theme 3: Other**
Explicit modeling	B2, A1, A4	B1	None
Implicit modeling	A5	B2, A1, A2, A3	B3, A6

Table 5.10. Triangulating results from RQ1 and RQ2. Table shows how abstraction was modeled in the clips under theme 1 and theme 2.

There were three exceptions to this trend: B1, when Prof. Zhao first introduced the concept of entropy; and A4 and A5, when Prof. Rossi developed tools for use in chemical kinetics. As discussed above, in Clip B1, Prof. Zhao seemed to strive toward explicitly modeling how to generalize the concept of entropy by comparing it to another known state function, enthalpy, and drawing comparisons between how they are calculated to generalize a property of entropy, that entropy has an absolute value (see Figure 5.5). In Clip A4, there were two abstracting processes. One led to developing the tool, characterized by being initiated with a symbolic restructuring move; and one involved horizontally manipulating to connect the math and the concepts. In A5, Prof. Rossi derived the steady state approximation for Michaelis-Menten kinetics. Although he was deriving a mathematical tool, he did not explicitly model how he was doing so.

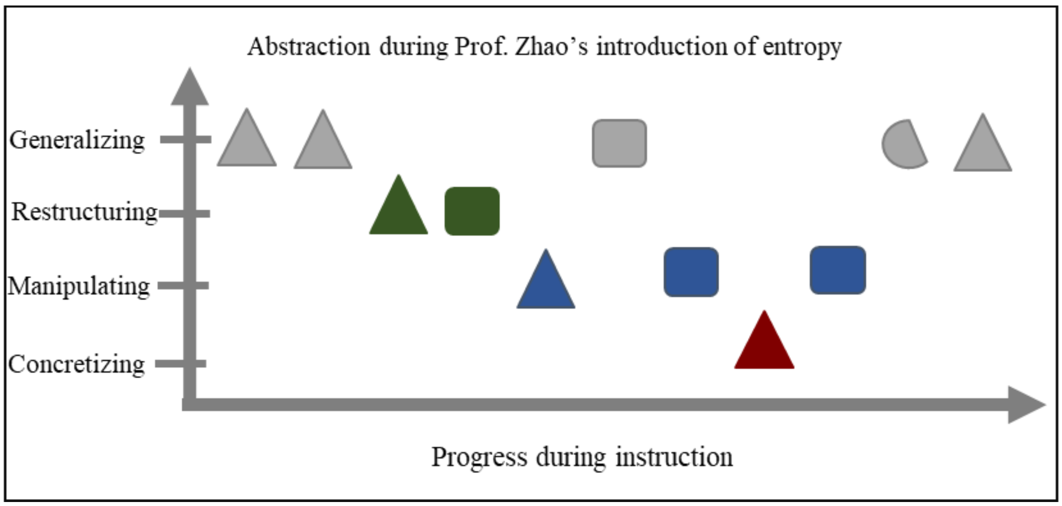


Figure 5.5. Prof. Zhao's epistemic actions during Clip B1. Triangles represent conceptual actions, squares represent symbolic actions, and half moons represent blended actions.

The interviews give insight into why these clips were the exception. During his interview, Prof. Zhao continually referred back to the "flow of the course" and the "big picture." He discussed how he designed his course to emphasize the connections between different concepts in thermodynamics, and that over the years he developed an approach that involved deemphasizing derivation of math and equations and foregrounded the big picture and the conceptual structure of thermodynamics. This was demonstrated in B2, where he modeled a specific problem solving process that he had the students utilize whenever they were solving problems that involved the change in a thermodynamic potential, to reinforce that they were all path-independent functions. In Clip B1, this seemed to manifest as striving toward explicitly modeling conceptual generalization. He explicitly refers back to a known state function, enthalpy, and discusses similarities and differences to extract an important quality of entropy. For Prof. Zhao, it was important to highlight the interconnectedness of the different concepts.

In contrast, Prof. Rossi implicitly models abstraction when the trend was to explicitly model. Similar to Prof. Zhao, Prof. Rossi thought it was important to emphasize the "big picture," and especially the connection between mathematics and the physical phenomena they represented. This emerged during A4, when he used horizontal manipulation to emphasize the interpretation of the mathematics as he moved forward toward generating the relationship between concentration and time. However, during clip A5, when he modeled how to develop the steady state reaction, he was less explicit about where this tool came from and how it fit into the larger framework of what they know. During his interview, Prof. Rossi talked about how he saw the steady-state approximation as a straightforward application of what the students already knew:

> Prof. Rossi: They already know how to write rate laws based off of reactions and so what it's kind of building on is kind of the general form of the rate law. […] So this is kind of the first time I think that we really talk about, um, well I talk about it a little bit. When I talk, when I mentioned the difference between complex and so this is the first time actually doing something with a multistep reaction and it's a two step one. So it's kind of straight forward. You just have products, intermediate, reactants products, intermediates involved. […] Because they've, before we get to it here, they've seen it in, you know, gen

> chem and they do a little bit of it in, I don't know if they do it in analytical and some in biochem and so they, they like, they're very familiar with the kinetics.

This excerpt from his interview gives two important insights: first, he saw deriving the steady state approximation as something that was an extension of what the class had already learned. Second, he expected the students to already have strong kinetics content knowledge, because they had extensively encountered similar material in other courses. It is possible that he implicitly modeled abstraction because he took for granted that students would be able to make the connections that he modeled during these examples, because they already had a strong base in the content knowledge.

Discussion

In this chapter, I applied the epistemic actions framework to analyze how two instructors of physical chemistry use abstraction while teaching thermodynamics and kinetics concepts. To answer RQ1, examples of both implicit and explicit modeling of abstraction were presented. Implicit modeling was more common across both instructors; however, there were several clips in which the instructors seemed to be modeling two distinct abstraction processes, and in these cases, they tended to be modeled differently (one implicit, one explicit). When instructors explicitly modeled abstraction, they referred to the purpose of the abstraction, as well as abstraction as a more general strategy students could utilize.

To answer RQ2, the interviews were analyzed in conjunction with the transcripts to examine the purpose of the abstraction. Two common themes were found around the role of abstraction: that abstraction could be used to develop a mathematical tool or strategy grounded in conceptual understanding, and that abstraction could be used to build conceptual knowledge that was often tied to mathematics. There were also two other clips that suggest that modeling abstraction may play other roles, such as modeling how to apply knowledge to a novel case. Triangulating the results from these two research questions revealed that abstraction geared toward developing tools tended to be explicitly modeled, while abstraction geared toward developing conceptual knowledge tended to be implicitly modeled. There were a handful of exceptions that seemed to be related to the individual instructors' beliefs and orientations toward teaching.

Connecting abstraction to the purpose of teaching physical chemistry

Situating this work in previous literature may also give insight into the role of modeling abstraction as an epistemic tool to teach physical chemistry. A national survey of physical chemistry instructors in the US found that instructors identified the connection between mathematics and concepts as one of the major struggles students face (Fox & Roehrig, 2015). Both of the emergent themes for RQ2 highlighted the importance of connecting mathematical and conceptual knowledge, suggesting that modeling abstraction is one way instructors may try to connect mathematics and concepts, e.g., by grounding mathematical tools in conceptual understanding.

However, these findings may also give insight into why this connection is challenging for students. Our findings suggest that instructors tend to implicitly model the connection between mathematics and concepts; for example, horizontal manipulation was a common and important strategy for instructors to reinforce the meaning of mathematics. However, the expectation that students would recognize and extract meaning from this horizontal abstraction was unspoken. For example, although Prof. Rossi emphasized in his interview that he horizontally manipulated to make the connection between math and concepts, he did not make this goal clear to his students during the lecture. Furthermore, horizontal abstraction was most common at the level of manipulation, i.e., for procedural transformations; however, it was much less common for restructuring or generalizing. For example, it was uncommon to explicitly discuss how physical differences between two systems (conceptual restructuring) may explain or justify the mathematical differences between them (symbolic restructuring). This may lead to students internalizing connections between surface features rather than the underlying mathematical and conceptual structures (Chi, Feltovich, & Glaser, 1981)

This leads into a second trend: conceptual vertical abstraction tended to be more common in this dataset than symbolic vertical abstraction. Vertical abstraction is a process by which connections between concepts or representations can be made and consolidated to support developing a new concept or equation. In general, vertical abstraction tended to be conceptual or diagonal (e.g., from a lower-level symbolic action, such as manipulating, to a higher-level conceptual action, such as generalizing). This means that while the instructors used mathematics to ground conceptual understanding, they did not necessarily develop the structure of mathematical relationships that can explain where the form of an equation comes from. For example, in Clip A3, Prof. Rossi used a symbolic restructuring move to derive the Maxwell relation for Gibbs by restating the Gibbs free energy equation through the differential form. However, he did not discuss *why* it was important to apply an infinitesimal change, or how that related to the physical phenomenon they were representing. Rather, it was a means to an end. At the end of the clip, he explains the meaning of the equation:

> (CG) So what this equation is saying is essentially any change in Gibbs energy for pressure or for a process at constant temperature and pressure can be, um, determined by looking at how the internal energy changes, the volume changes at that pressure, and how the entropy changes at that temperature when we do the change to the system. So that's all that equation is saying. There's a lot of letters. Today's going to be basically an entire class of letters. Um, but keep in mind, it's not just an equation to be an equation, *it's the equation that's telling us how different variables are affecting the free energy of our system when they change*.

The first part of his explanation gave a surface level explanation of the meaning of the equation based on the natural variables. At the end, he connected it to a more general meaning ("that's telling us how different variables are affecting the free energy of our system"). However, because he did not symbolically vertically abstract, he did not explicitly connect the form of the differential to this general meaning. Instead, it seemed like he expected students to make that connection themselves. This may lead to challenges in applying these equations to novel situations, because the meaning of the mathematical *form* and its connection to the physical phenomenon was not emphasized (Becker & Towns, 2012; Sherin, 2001).

Finally, a 2016 survey of physical chemistry instructors' beliefs about the purpose of teaching physical chemistry found that conceptual understanding was one of the core goals of physical chemistry instruction (Mack & Towns, 2016). However, our findings suggest that developing conceptual knowledge (drawing connections between familiar and emergent concepts) tends to be implicitly modeled. Clip B1 provides some insight into why this may be challenging. In Clip B1, Prof. Zhao strived to model conceptual generalization explicitly; this involved frequently switching back and forth between thinking about the constraints on the meaning of different thermodynamic potentials and how these constraints could be used to extract a property of entropy. His actions pattern (see Figure 5.5) shows that this involved frequent movement between levels of abstractness. Two challenges have to be overcome for this approach to be successful for student learning: first, modeling conceptual generalizing requires the instructor to attend to what kinds of connections will be most salient for the students. This may require the instructor to have a strong pedagogical content knowledge, as well as insight into how their own understanding of concepts differs from those of students (Chi et al., 1981). Second, as demonstrated by the map, this type of reasoning involves frequent shifts in levels of abstractness to make connections between different concepts, and may be difficult for students to follow if it is not done clearly.

Conclusions

This chapter reports the findings from a multiple case study of two physical chemistry instructors. These findings provide insight into how instructors model abstraction in teaching thermodynamics and kinetics (RQ1), and the role this modeling plays in instruction (RQ2). In the introduction, I motivated a gap between instructors and students, and suggested that a deeper understanding of how abstraction occurs in instruction may help bridge that gap. Here, I showed how the epistemic actions framework can help make abstraction in instruction visible. Juxtaposing the findings from this study and from the studies focusing on student problem solving will be discussed in more detail in Chapter 6.

Limitations

There are several limitations to this study and analysis. First, the findings are based on a subset of the data, the clips used in the VSRIs. This poses two limitations. First, the clips were selected on my understanding of abstraction at the time of data collection—that is, they were theoretically sampled based on an emergent understanding of abstraction to support furthering that understanding. Because they were theoretically sampled, they are not necessarily representative of typical practices in teaching thermodynamics and kinetics. Second, the clips were selected because they were salient to the researcher, not necessarily because they were salient to the instructor. Third, there may be additional purposes and roles that abstraction plays beyond the scope of what was reported in this study. This is suggested by the presence of "Other themes," which show that abstraction can be used as a tool for purposes beyond the two reported here.

Implications for Practice

The findings from this study make a previously invisible process (abstraction) visible in instruction. The Epistemic Actions framework provides a tool for instructors to think through how they make connections between new and familiar content and to be explicit in how they make decisions about how to present this content. Furthermore, based on the results of this study, I recommend several questions for instructors to reflect on:

What do I assume students know? What do I take for granted? As discussed in the methods, the concept of "abstract" had to be shifted from what seemed to be relevant and salient to the student, to what the instructor seemed to assume students already know. This presents a possible gap between instructors and students, if what students see as relevant to the topic at hand differs from what the *instructor* takes for granted that the students think about while learning the topic. Connections that seem obvious to the instructor are not always clear to students, because the instructor has already thought through and consolidated these connections into concepts. That is, what is abstract to the instructor may be different from what is abstract to the student; however, it is how the instructor understands the latter that informs their teaching practice. To help model abstraction, the instructor should reflect on what they assume is "abstract" to their students.

What do I want students to be able to do? How do I want to show that? As discussed, the majority of abstraction processes were not explicitly modeled, and those that were tended to be related to developing processes and mathematical tools. It is worthwhile to reflect on what types of processes are important to model for students. For example, if a goal of instruction is that students successfully apply their knowledge to a new case, it may be worthwhile to explicitly model how to think through constraints on a novel system and how these constraints require shifting between different conceptual structures (e.g., explicitly model restructuring). This could be done in problem solving through scaffolding a design for abstraction or during lecture through explicit modeling of abstraction.

CHAPTER 6

IMPLICATIONS AND CONCLUDING REMARKS

Summary of findings from work on abstraction in physical chemistry

Abstraction has been variously (and non-exhaustively) defined as a cognitive capacity learned by young children that undergirds the ability to categorize and develop concepts, the process of vertical mathematization, and the extraction of the "essential qualities" of an object (Hershkowitz et al., 2001; Piaget, 1964; Scheiner, 2016; Scheiner & Pinto, 2016). This dissertation research builds on several research traditions that frame abstraction as a dynamic and socially constructed process, to operationalize a definition of abstraction that can be used to study physical chemistry teaching and learning.

Development and Application of the Epistemic Actions framework

In Chapter 2, I report the development of the Epistemic Actions framework through a process of constant comparison. In this process, I iterated through multiple cycles of data analysis with different grain sizes for the unit of analysis and used chemistry and science education literature to interpret these findings. Each cycle led to a deepening understanding of what abstraction is. The final framework reported in Chapter 3 characterizes abstraction as a dynamic process of giving meaning to or extracting meaning from an object (e.g., an equation or concept) by shifting the context or form of the object. Two types of abstraction were identified (see Figure 3.3). The first was horizontal abstraction, which involves shifting between a conceptual and symbolic (e.g., mathematical) form of an object, for example through mathematizing. Some literature suggests that conceptual and mathematical knowledge are often siloed in physical chemistry learning (Becker & Towns, 2012; Rodriguez et al., 2020). Horizontal abstraction is a process by which these knowledge sources can be bridged, which enriches one's understanding of an object by evoking the connections and processes associated with the mathematical or conceptual form of the object. The second process was vertical abstraction, which involves developing epistemic structures by making connections to pieces of prior knowledge. There was also empirical evidence to support the existence of diagonal abstraction, which involves developing conceptual meaning straight from mathematical objects or vice versa. These three processes are identifiable through coding for epistemic actions of abstraction (conceptual and symbolic forms of concretizing, manipulating, restructuring, and generalizing).

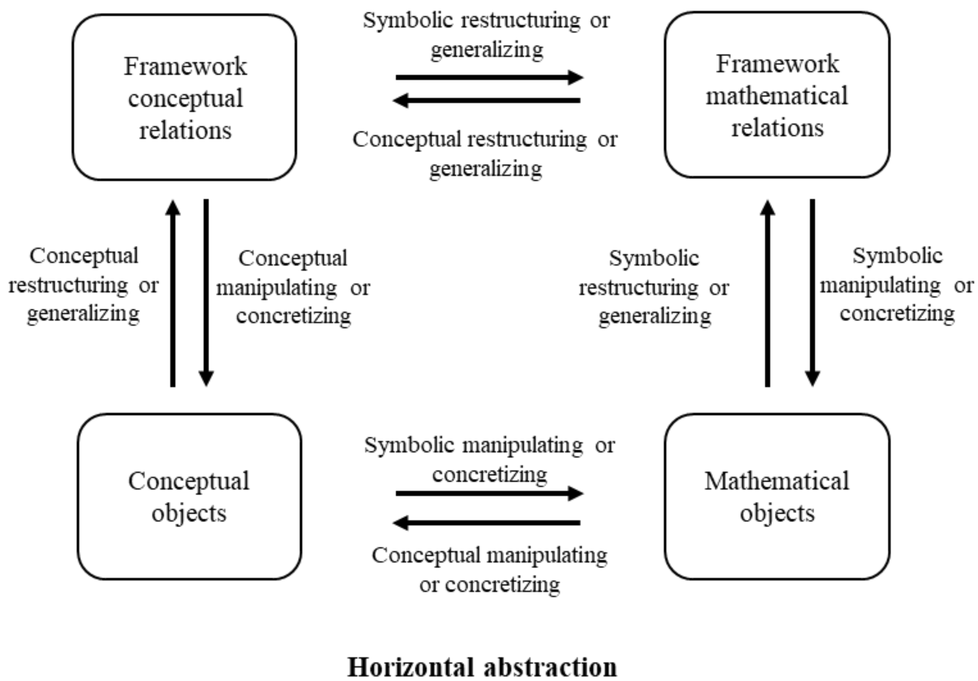


Figure 3.3. Depiction of vertical and horizontal abstraction.

In Chapter 4, I expand on the structure of abstraction in problem solving through an activity theoretical lens by defining the scope of the abstraction "activity" as a unit of analysis. Here, I lay out that abstraction (operationalized by a series of epistemic actions that are oriented around a coherent task) is initiated by a need. The activity is completed when abstraction is realized or when the problem solver shifts to another task. I identified three types of needs that initiate abstraction: *task-directed needs*, which are directly related to the problem solver's interpretation of the task; *situational-insufficient needs*, which arise during problem solving when a known procedure falls short; and *situational-emergent needs*, which arise because the problem solver recognizes an opportunity to abstract based on relationships revealed during problem solving.

Chapters 4 and 5 also demonstrate the utility of the Epistemic Actions of Abstraction framework to analyze two distinct forms of data: problem solving teaching interviews and classroom instruction. These chapters discuss practical considerations for applying the framework. For example, one of the foundational constructs underlying the framework is the idea of what makes something abstract. Through our activity theoretical lens, abstractness is contextual and related to the knower. In problem solving interviews, the role of abstraction is to support the problem solver in moving toward a solution state and making sense of an unfamiliar situation. The abstractness of an object (something on which a transformation can be applied) is consequently related to what the problem solver knows and how they interpret the situation at hand. That is, the abstractness of an object is determined in relation to the problem space, the set of resources and possible problem solving paths a problem solver identifies as relevant in the moment (Jonassen, 2010). When teaching, the instructor is not actively making sense of concepts for themselves. Rather, they are modeling how to develop tools and make connections to develop conceptual knowledge, solve problems, etc. Abstraction during instruction is a pedagogical tool, not a sense-making tool. Consequently, "abstractness" when studying instruction was considered to be relative to what the instructor takes for granted that the students know.

Findings on interactions that influence abstraction in problem solving

In the introduction to this thesis, abstraction was presented as a socially mediated process. Identifying actions operationalized the process of abstraction as one that is dynamic. Each epistemic action is guided by the problem space, but they also operate on and change the problem space. For example, when a student restructures, they draw on an idea from outside of the problem space (something more abstract) to make sense of the problem. Through the restructuring move, they change the problem space, changing the scope of what is considered abstract during that episode of problem solving.

To understand how abstraction is socially mediated, Chapter 4 reported the findings of an exploratory inductive analysis that attended to three types of dialogic interactions: those between the participant and the interview (P-I), the participant and the task (P-T), and the participant and their partner (P-P) (for pair interviews). Each of these types of interactions interacted with each other and could sustain or constrain abstraction. Three main themes emerged: framing (P-I, P-T), interviewer interventions (P-I), and peer interaction (P-P, P-T).

First, framing (how an individual interprets what a situation is) tended to be a salient influence on abstraction in problem solving when it constrained abstraction. Some participants framed the problem as one that had to be solved in a "p-chem" way, e.g., using procedures learned in class, which was generally synonymous with "plug and chug." Even when these approaches fell short, participants with this framing searched for a "p-chem" solution. This particularly constrained horizontal abstraction, as horizontal abstraction involves shifting to think about conceptual and symbolic forms of an object, and students with p-chem framing tended to prefer symbolic actions. This framing was likely strongly influenced by the context; participants knew they were being interviewed about physical chemistry, and I as the interviewer was perceived as an arbiter of physical chemistry knowledge (Russ et al., 2012).

However, that authority role could also be leveraged to shift the framing to support abstraction. The interviewer supported framing shifts by actively affirming participants who considered using other types of knowledge and by directly encouraging students to shift their framing through leading questions. Sometimes this was temporary—for example, participants who engaged in biological sense making sometimes returned to a p-chem frame when their attention was called to the part of the problem that cued that frame in the first place (such as the presence of an equation sheet).

Two other interviewer moves that influenced abstraction were (1) scaffolding abstraction by drawing participants' attention to salient connections or requesting that they change their approach, and (2) reducing the abstractness of the problem space by introducing an anchor point through a concretizing move. Again, whether these constrained or sustained abstraction were contextually situated, and depended on factors such as how the participant interpreted the interviewer's utterance. Interviewer interventions that were most supportive of abstraction built on connections that were salient to the participant, and were interpreted as encouraging genuine sense making. Interviewer interventions that constrained abstraction built on the interviewer's rather than the participant's interpretation of the problem, and/or were interpreted as pushing the participant toward thinking of the interview as an oral examination.

The final finding centered on the role of peer interaction. Previous studies suggest that abstraction in peer interaction is supported when one member of a group can serve as a more knowledgeable other who can support and sustain abstraction and help their peer expand their understanding of an object (Dreyfus et al., 2001; Schwartz, 1995). Other literature has suggested that it is important for both members of the pair to be at equal levels of competency so that they can problem solve together (Hatano & Inagaki, 1991). These findings fell somewhere in the middle. While a more competent peer was not necessary for group abstraction, having diverse knowledge resources was. There were two features of pair problem solving that promoted abstraction. First, there had to be some element of negotiation and reconciling different understandings of the problem situation, which aligns with our notion of the zone of proximal development (ZPD) (Wertsch, 1984). Second, the pair had to be actively collaborating and focused on the same object—they had to be negotiating *together*.

One of the key implications from Chapter 4 was the role of the ZPD in abstraction (see Figure 4.5). The ZPD refers to how two knowers reconcile their situation definitions to expand their understanding of an object, and has particularly been used to understand how a more knowledgeable other supports a learner in expanding their understanding of an object. Abstraction in our findings seemed to occur primarily within the ZPD. For example, when participants experienced a situational-insufficient need for abstraction, part of the role of the interviewer was to scaffold abstraction that occurred within the range of what students were capable of, but what they did not recognize alone. These findings point to the importance of instructors supporting actively supporting abstraction by attending to the social dimension of their teaching.

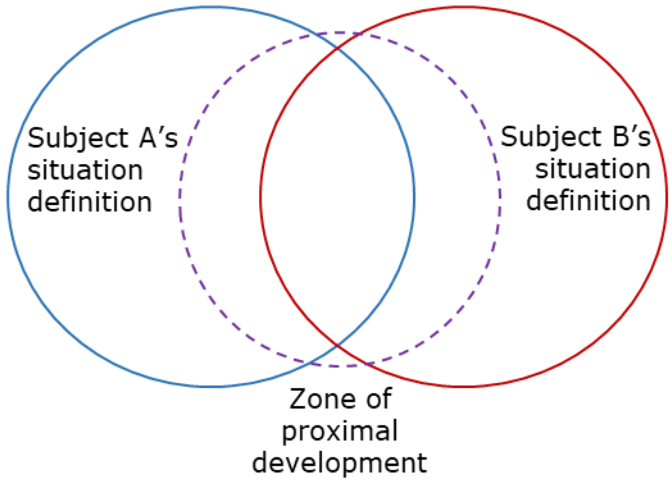


Figure 4.5. Diagram depicting the ZPD.

Findings on the role of abstraction in physical chemistry instruction

Chapter 5 focused on analyzing abstraction in instruction as a way of starting to characterize the conditions under which domain-specific abstraction is fostered. The analysis focused on two things: how instructors modeled abstraction, and what the purpose of the abstraction was. Two roles of abstraction were identified: abstraction to develop a mathematical tool grounded in conceptual understanding, and abstraction to reinforce and foster the development of conceptual knowledge, particularly using mathematics. There seemed to be a correlation between whether instructors implicitly or explicitly modeled abstraction and the purpose of the abstraction. For example, abstraction for developing a mathematical tool or procedure was typically explicitly modeled, whereas abstraction for developing conceptual knowledge was typically implicitly modeled. Furthermore, there were also specific patterns about types of abstraction that were generally more common. For example, the most common form of horizontal abstraction occurred at the level of manipulation, e.g., connections were made between mathematics and concepts primarily in a procedural way. The most common form of vertical abstraction was conceptual, e.g., connections between objects to create a framework of relationships that could be consolidated and used in further abstraction was primarily done between conceptual ideas and not with mathematical representations.

Looking across the findings

Triangulating the results from Chapters 3 and 5 reveals some more general patterns that may be worth investigating. Chapter 3 reported that conceptual actions were much more frequent in our dataset than symbolic actions, and I speculated that this could be related to the problems' embedded biological context. However, Chapter 5 reported that conceptual actions, and in particular conceptual vertical abstraction tended to be more commonly utilized than symbolic vertical abstraction, and that horizontal abstraction tended to happen on the level of manipulation. These commonalities existed despite the fact that study participants in Chapters 3 and 4 were located at a different institution from the professor participants. This suggests that this finding, that conceptual actions are more frequent than symbolic ones, is more than spurious and may in fact reflect some broader trend in how physical chemistry is taught. If meaningful horizontal and symbolic vertical abstraction is a goal of physical chemistry instruction, e.g., understanding the conceptual underpinnings of mathematical constructs and formulae and subsequently applying these to novel and unfamiliar situations, it may be worthwhile to attend to symbolic vertical abstraction in a more meaningful way. This may involve being more explicit about how the form of an equation meaningfully represents an underlying physical phenomenon (Becker & Towns, 2012; Sherin, 2001) to avoid students cuing on surface features to make sense of math and other representations (Chi et al., 1981).

Suggestions for Further Research

As previously noted, the power of the Epistemic Actions of Abstraction framework is that it can make abstraction visible. Based on the findings in this dissertation, there are several threads of research that may be interesting to pursue.

Further Research on Physical Chemistry Teaching and Learning

The findings in this work provide insight not just into the process of abstraction, but also into how students make sense of concepts and solve problems in thermodynamics and kinetics. The first natural extension of this work would be to use the Epistemic Actions Framework to look at the third major domain of undergraduate physical chemistry: quantum mechanics. As a discipline, quantum mechanics is highly abstract. The teaching of thermodynamics and kinetics is often removed from the observable, but the core laws and principles are rooted in empirical data and measurable phenomena. The teaching of quantum mechanics, on the other hand, is often rooted in philosophy and first principal derivations, as the physical phenomena they represent are generally not observable in the lab (Garritz, 2013; Scerri, 2004). Heat of reaction can easily be calculated through an experiment; a wavefunction cannot. Understanding how students abstract in quantum mechanics may help support developing best pedagogical practices.

Second, the findings in Chapter 5 are based on a multi-case study examining only two instructors. In the introduction to Chapter 5, I motivated that process of abstraction is important in physical chemistry instruction, in part by identifying gaps between instructors and students that may be bridged through abstraction, such as a disconnect between how professors intend to provide opportunities to develop conceptual knowledge and the design of these opportunities (Fox & Roehrig, 2015), and by naming challenges identified by both faculty and students that are related to abstraction (Becker & Towns, 2012; Sözbilir, 2004). However, aside from Sözbilir's (2004) finding that the abstractness of concepts is a salient difficulty in learning physical chemistry and Sevian and collaborators' (2015) hypothesis that thermodynamics is an "abstraction threshold" course, the concept of "abstraction" or "abstract" is rarely named or explored explicitly in the physical chemistry education literature. The constructs identified here, including the two themes related to the role of abstraction in teaching and the epistemic actions themselves, could be used as a basis to develop a wider study that better generalizes how instructors in physical chemistry use abstraction while teaching. For example, a quantitatively-minded researcher could develop a survey instrument that probes instructors' pedagogical practices related to abstraction. This could explore whether the two themes I found (abstraction as a process to develop mathematical tools and as a process to develop conceptual knowledge) are used by instructors beyond those interviewed in this study. These findings also speak to the importance of more deeply investigating physical chemistry instructors' pedagogical content knowledge and developing research-driven best practices for teaching physical chemistry.

Third, the Epistemic Actions framework provides a tool to more deeply probe this disconnect between instructors and students. One original goal of the study presented in Chapter 5 was to examine how instructors' epistemic actions during class influenced how their students solved problems in an interview setting. The student data were not analyzed for this dissertation, because a preliminary analysis revealed that the relationship between instructors' modeling and students' actions was more complicated than hypothesized. For example, I realized during this preliminary analysis that it would have been more fruitful to examine students' understanding of concepts and mathematical tools that were presented during class rather than their problem solving procedures. The Epistemic Actions framework can be used to understand the alignment between instructors' modeling practices of abstraction and students' abstraction in naturalistic or interview settings, to understand the role of instruction in fostering abstraction.

Applicability of the Epistemic Actions Framework beyond Thermodynamics and Kinetics

I hypothesize that the Epistemic Actions framework may be generalizable to other domains. When developing the framework, I drew on different sources to develop the horizontal and vertical dimensions. First, the actions in the vertical dimension (concretizing, manipulating, restructuring, generalizing) were primarily derived from mathematics education literature and systemic-functional linguistics (SFL). For example, *restructuring* bears similarity to the *building-with* action proposed in the Abstraction in Context framework (Hershkowitz et al., 2001). B*uilding-with* uses "available structural knowledge to build with it a viable solution to the problem at hand" (p. 215), while restructuring involves the mediated transformation of relationships between objects in the problem at hand to reimagine the structure of those relationships. *Concretizing* was drawn in part from the SFL idea of semantic gravity and that a piece of knowledge has heavier "gravity" when it is more closely related to observable reality, i.e., when it is more concrete (Maton, 2013; Santos & Mortimer, 2019). To develop the horizontal dimension (conceptual and symbolic), I drew primarily on data and physics and physical chemistry education literature. For example, a common framework in this literature is blended processing, which suggests that students solving problems in mathematically-dense science disciplines may draw on conceptual resources, mathematical resources, and/or resources that are a blend of these two, e.g., an equation imbued with conceptual meaning (Kuo et al., 2013; Rodriguez et al., 2020). This may suggest that the two dimensions of the Framework are distinct, and in particular, I hypothesize that the vertical dimension may be domain-general, whereas the horizontal dimension is domain-specific.

I suggest that it may be utile to apply the Framework to other disciplines to investigate abstraction processes. Above, I discussed the applicability to other physical chemistry courses, such as quantum mechanics. This framework could also be applied to other chemistry courses, such as inorganic chemistry or analytical chemistry. In particular, it may be interesting to examine laboratory learning. The Framework currently has two dimensions, based on the type of knowledge that arose as most salient to physical chemistry learning (conceptual and mathematical/symbolic). In laboratory learning, there is a third dimension: psychomotor skills, which describes how students make sense of and use physical tools (Bretz et al., 2013; Hofstein, 2004). Research has shown that there is a disconnect between lab tasks and conceptual understanding, in part because students' goals are to finish the lab as quickly as possible, and that this is true for both introductory and upper-level students (DeKorver & Towns, 2015, 2016). The Framework may help make these barriers and possible abstraction processes during laboratory learning visible, by examining how students contextualize their laboratory tasks in their lecture content (e.g., restructuring) or how they form new understandings from doing laboratory tasks (e.g., generalizing). Adding this third dimension (psychomotor) may help identify this barrier, as it would support researchers in identifying where the disconnect lies (e.g., between concrete tasks and concepts, between concrete tasks and mathematizations, etc.).

I also suggest that it may even be extended beyond STEM disciplines. Consider a hypothetical example of a student reading Kafka's Metamorphosis. They may do so at the same level of abstractness as the text by retelling it as the story of a man who turns into a bug, and through comparing scenes may *generalize* a theme of isolation, and construct the knowledge that stories can be allegories for broader political issues. Although some actions in our framework are named in a domain-specific way (e.g., manipulating is a common term in arithmetic and algebra), our definition of abstractness as relative to a problem space or text can be generalized across domains as a means to explore student sense-making. Similar to how Avni and Hoa navigated multiple levels of abstractness to make sense of the kinetics problem in Chapter 2, in this example it is clear that a student may have to work through understanding what happened in the story (*manipulating*) and how this fits into a genre (*restructuring*) to generalize themes. Furthermore, how abstract something is considered to be emerges from the relationship between individual, task, and the individual's perception of the task. For the student, the idea of text-as-allegories emerges as an abstraction they take away from the experience of reading Kafka; for the teacher, this is an idea taken for granted as an aspect that can belong to a work of literature, a piece that they can manipulate.

Implications for Practice

This work also has several implications for teaching thermodynamics and kinetics. As discussed at length, the Framework helps make abstraction visible. This leads to two major implications for instructor: designing for abstraction and reflecting on abstraction.

Designing for Abstraction

The findings in this thesis suggest that instructors can design for abstraction. This can be done in two ways: by designing tasks that elicit abstraction, and by designing situations that support abstraction. Several features may be incorporated in task design to elicit abstraction. First, the task may contain an explicit bid for vertical or horizontal abstraction by requiring students to connect pieces of information that do not seem related at first (vertical abstraction), or to draw on different types of knowledge (horizontal abstraction). Second, the task ought not to be solvable using memorized or familiar procedures, and rather require application of knowledge in a novel way. Third, the instructor should make clear that application of knowledge and abstraction are appropriate ways of solving the problem. As noted in Chapter 4, a salient barrier to abstraction is not whether the students are *capable* of abstraction, but whether they view abstraction as *allowed*. When designing the situation, it may be prudent to support abstraction through group work or whole class discussion, to support students' expansion of knowledge through the zone of proximal development. This suggests that tasks designed for abstraction may be more appropriate as formative rather than summative tasks.

Reflecting on Abstraction

The epistemic actions provide a framework for instructors to reflect on their own practice. For example, when introducing or teaching a new concept or procedure, an instructor may want to be intentional in modeling where the concept or procedure came from and how the structure and form relate with the meaning. The epistemic actions provide a way for instructors to be deliberate in how they draw on already familiar concepts, make connections, and to be explicit with what they take for granted in students' understanding.

Based on the utility of the Framework for characterizing talk during instruction, it may be fruitful to use it as the basis for developing a professional development or pedagogical model to support instructors to intentionally model abstraction while teaching. White and Mitchelmore (2010) developed a 4-phase model called "Teaching for Abstraction" that is designed to promote teaching abstract concepts in elementary mathematics. The four phases of this model are Familiarity, Similarity, Reification, and Application. This model can be adapted to be compatible with the epistemic actions. For example, *similarity* involves recognizing similarities and differences that arise when an object is situated in different contexts. In our framework, this may be observed through the epistemic action of *restructuring*, as an instructor changes the context to see how the meaning and form of an object changes. The conceptual compatibility of our framework to other models demonstrate the flexibility of the Epistemic Actions Framework to be used in a wide range of studies.

Concluding Thoughts

The aim of this dissertation was to characterize abstraction in order to explore how abstraction processes occur in student problem solving and instruction. It had the secondary aim of extending theory around abstraction to characterize abstraction as a dynamic, socially mediated process. I demonstrated that not only does this definition and operationalization work well to characterize abstraction processes in a variety of contexts, it can also provide powerful insights into the socially mediated nature of problem solving and abstraction. I show that not only can abstraction be shifted by in-the-moment interactions, abstraction as a process is dynamic and a process of learning. When students abstract during problem solving, they test understandings, make connections, and apply knowledge. They shift what a problem means to them and push boundaries on how they use their knowledge. It is important to move away from thinking of abstraction as a static capacity: something students can do or something they cannot. Instead, let us reimagine abstraction as an activity: something that draws on tools to generate dynamic sense making, that is sensitive to context but may generate new understandings that transfer beyond the situation at hand. Through this dissertation, I hope I have made the idea of "abstraction" and the "abstract" more concrete, to give readers a tool they can consolidate and apply to generate new understandings and advance education in undergraduate physical chemistry and beyond.

APPENDIX

1. Sample Solutions for Student Problems
2. Equation sheet provided during entropy interviews
3. Actions Coding Codebook
4. Description of Problem-Solving Interview Protocol and Student Tasks in Professor Study
5. Actions Patterns for all Professor Clips

1. SAMPLE SOLUTIONS FOR STUDENT PROBLEMS

One possible solution for kinetics problem:

$$\frac{dS}{dt} = rate\ of\ change\ for\ susceptibles\ population$$
$$\frac{dI}{dt} = rate\ of\ change\ for\ infectives\ population$$
$$\frac{dR}{dt} = rate\ of\ change\ for\ removed\ population$$

If disease is epidemic, the number of infectives is increasing. If it is dying out, the number of infectives is decreasing. Therefore:

Epidemic: $0 < rSI - aI$ (rate is positive)

$$aI < rSI$$
$$\frac{a}{r} < S$$

Dying out: $0 > rSI - aI$ (rate is negative)

$$aI > rSI$$
$$\frac{a}{r} > S$$

One possible solution for entropy problem:

1 rung = 1 of 4 possibilities

Total length of DNA ladder = $5x10^8$ binucleotides

Total number of possible combinations = 4^{5x10^8}

Total number of combinations = total microstates = W

$$S = k \ln W$$

$$S = k \ln 4^{5x10^8}$$

$$S = k \ln 4^{5x10^8}$$

$$S = 5x10^8\, k \ln 4$$

2. EQUATION SHEET PROVIDED DURING ENTROPY INTERVIEWS

The following is the equation sheet that was provided during Round 1 (Entropy) of the teaching interviews. The equation sheet was provided to students upon request, and was adapted from the course textbook.

Property	**Equation**	**Comment**
Thermodynamic Entropy	$\mathrm{d}S = \mathrm{d}q_{rev}/T$	Definition
Entropy change of surroundings	$\Delta S_{sur} = q_{sur}/T_{sur}$	
Boltzmann formula	$S = k \ln W$	Definition
Carnot efficiency	$\eta = 1 - T_c/T_h$	Reversible processes
Thermodynamic temperature	$T = (1 - \eta)T_h$	
Clausius inequality	$\mathrm{d}S \geq \mathrm{d}q/T$	
Entropy of isothermal expansion	$\Delta S = nRln(\frac{V_f}{V_i})$	Perfect gas
Entropy of transition	$\Delta_{trs}S = \Delta_{trs}H/T_{trs}$	At the transition temp
Variation of the entropy with temperature	$S(T_f) = S(T_i) + C \ln(\frac{T_f}{T_i})$	

3. ACTIONS CODING CODEBOOK

The following table and examples of coded student responses were created during the coding process for the student problem solving interviews. The codebook served as a way to define codes and ensure consistency, particularly when conducting inter-rater reliability.

Code	Description	Examples
Conceptual concretizing	Putting a conceptual constraint on the problem space	**Interviewer:** Where do you notice a on these equations? **3979:** So it's only for the infective and the removed class. So it's people that can actually be infected. Oh, wait, susceptible, sorry, they catch the disease. So it's people who have the disease and/or have recovered, died, immune, or isolated. So a equals have disease. Okay.
Symbolic concretizing	Putting a mathematical constraint on the problem space	**8848:** So since we just said that r, the constant, is depending on the number of people were infective. So [[10s]] if you plug in like random numbers. So if you said that S is 5 and I is 10, that's 50, negative 2. Negative 100. It can't be negative 100 people. [[7s]] So let's say in a population we have 300 people.
Conceptual manipulation	Advancing toward a solution by thinking through meanings and connections between conceptual aspects in a procedural way	**2039:** Um, well it's a combination of the other two. It's the susceptible times the infected times r minus the people who are recovered, dead, or immune. It's the rate of people becoming— It's the rate of the susceptible patients minus the rate of the recovered or etc. patients leaving the rest of the people to be infected over time, I guess, the amount of people infected kind of thing. Yeah.

Symbolic manipulation	Advancing toward a solution by thinking by working with math or variables in a procedural way	**1765:** I thought about first that, if we want to calculate a divided by r, it should be something related with a and r, right? So here has an r, here has an a. I tried to just substitute to this equation, because this, you can, is an r and a. But what I substituted is only can [?] an issue with R and an I, like here, I, it's only R, S, and I. It's not a and r. So, but if I try to substitute again, it will be back. It will be back to here, and not meaning anything. Meaningless. So maybe some some some things I didn't find out from these questions. But.

Conceptual restructuring	reimagining the meaning of the mathematics, which transformed the relationship between the math and the variables it stood for (focuses on the changing the *meaning* and conceptual underpinnings)	**2039:** Well susceptible people have a higher chance of getting the disease, so it would require like less transmission to get to them, I guess, than an average person, it would require more interactions with people who are infected, so I guess that's why it's infected people times the— Well I guess, you know what? The derivative doesn't necessarily mean like that, like S doesn't have to equal— dS over dt doesn't have to equal like S. So it's like the number of susceptible people times the number of infected people. So this ratio actually could be the number of people, number, the increasing number of people becoming, or reducing the number of people becoming less susceptible, because, like what am I saying? So susceptible people times infected people times a variable that's negative. So it's reducing the susceptibili-, the number of people being susceptible over time is decreasing, because of the negative number, I think, just because they're becoming infected as the number of people that are infected increase. There are like the higher chance that they can be— I guess, the transmission can happen. I don't know.

Symbolic restructuring	transforming mathematical relationships to represent something that they were familiar with from another context, applying those constraints and meanings (focuses on changing the *representation* and mathematical relationships)	**3979:** So I decided to differentiate it because it made me think of like the Maxwell equations for p-chem. It made me think of how you have formulas that you can, you can make them, you can change them after you differentiate them, because then you can pretty much combine like terms that aren't pretty much defined as like correlating. You can kind of like, like with Maxwell equations, it's like, I can't think of one off the top of my head, but it's one that equals another, and then if you're, if you find one of them inside an equation, you then know if you have to get it in specific terms of say N or G or pressure or temperature, you then know you can plug that in to be able to figure it out to find out the answer, like the, yeah, the fundamental equations.
Conceptual generalizing	Connecting ideas and meanings to develop a new concept/idea within the problem space	**1765:** a and r, mmm, by the time [you released]. It maybe means— Ah, and also. Hey maybe it's only the constant, how the, how new things can, how stable for these things catch disease, because it's relatable if it's remove and catch, right? And the R should be the [?] of disease, and this one also how catch it, and, I don't know. R is too complex. **Interviewer:** I actually don't know what a and r are, so that's— **1765:** Okay, so I thought— **Interviewer:** It's definitely what your thoughts are. **1765:** I don't know. **Interviewer:** But you think that r has something to do with how people catch the disease?

		1765: No no, I said a. a is how stable, how stable the things catch disease, catch some things, because if it's relatable, it's catch, and is remove, right? This means, where maybe a is less, it's easy to remove. When a is high, it's hard to remove.
Symbolic generalization	Connecting ideas and meanings to produce new mathematical relationships	**1732:** I think that makes more sense, because the disease will spread less, and more people, I mean less people will be affected by it, and maybe like people over here, they already had the immunity when the disease is like spreading. They are making their immunity in their body. **Interviewer:** Okay. That could definitely be what's happening. **7018:** So like it's telling us what condition. So I think it really depends on rSI, I guess, because if you look at this— [[10]] Because if rSI is greater than aI, which is right here, that means that the rate is larger. What does it mean when the rate is larger? **1732:** That it will spread more. **7018:** Spread more? Do you think that's right? Do you think that's a good statement? ((directed toward her partner)) And then rSI is less. This is 2, and 1. This is 1 right here. Do you get what I'm saying or no? So this will die out. What do you think? I guess this is the condition. What do you think? **1732:** So in order to be, in order to increase this one— **7018:** Yeah, so this one would be like 3, and then this would like 1, so rSI is greater.

4. DESCRIPTION OF PROBLEM-SOLVING INTERVIEW PROTOCOL AND TASK WITH STUDENTS

During the data collection for the professor study reported in Chapter 5, interviews were also conducted with a small number of student volunteers. These interviews were semi-structured think aloud interviews. Participants were asked to talk through their process while they worked on solving the problem. In contrast to the teaching interviews from the first phase of this dissertation work, the interviewer tried to play a relatively passive role in the interview process, primarily asking questions to expand on or explain their reasoning, or encouraging participants to talk when they had been quiet for an extended period of time.

During the problem-solving interviews, participants were asked to solve two problems related to the material at hand: a well-structured problem that was similar to the types of problems they solve in their physical chemistry course, and one problem with multiple possible solutions that was selected to elicit abstraction. For each site, only one interview was conducted for each student. Although two rounds of interviews were planned for Private College, only one round of interviews was conducted due to difficulties in recruiting student participants in time for the end of the thermodynamics unit.

The thermodynamics interviews at Public University were video recorded; however, the video recordings did not yield much additional useful information, and participants at Private College seemed reluctant to participate if they were video recorded, so the kinetics interviews at Private College were only audio recorded. All participants solved the problems using a LiveScribe pen, which creates a playable video of what an individual wrote, synced with what they were saying as they wrote.

Thermodynamics task (Public University):

Well-structured problem. This was adapted from the course textbook (Chang & Thoman, Jr., 2014) and based off of the types of problems the professor taught in class:

> An ideal sample of neon gas (Ne) at 1.0 atm is heated from 15°C to 55°C and simultaneously expanded from 3.2 to 5.6 L. Calculate the entropy change for the process. The C_p (Ne) = 20.79 J/(mol*K), C_v(Ne) = 12.47 J/(mol*K)

Ill-structured / open-ended problem. This was adapted based on the theoretical entropy measurement device described in *The second law* (Atkins, 1984).

> You've just gotten a job as a chemical engineer. Your boss wants to know the entropy change of a chunk of metal when it is being heated, and has tasked you with designing a device to do so. Imagine what aspects this entropy meter might need to include, and propose a design.

Kinetics task (Private College):

Well-structured problem. This was adapted from the course textbook (Chang & Thoman, Jr., 2014) and based off of the types of problems the professor taught in class.

The following kinetic data were obtained for the production of HI from hydrogen and iodine gas at 400°C, $H_2 + I_2 \rightarrow 2HI$. Write the rate law for the reaction and calculate the rate constant k.

$[H_2]$ (M)	$[I_2]$ (M)	Rate (M/s)
0.15	0.20	7.26×10^{-4}
0.30	0.20	1.45×10^{-3}
0.15	0.37	1.34×10^{-3}

Ill-structured / open-ended problem. This was taken from an equivalent undergraduate physical chemistry textbook (Atkins & de Paula, 2014).

Many biological and biochemical processes are catalyzed by the presence of the product (this process is called autocatalysis). In the SIR model of the spread and decline of infectious diseases the population is divided into three classes: the 'susceptibles,' S, who can catch the disease; the 'infectives,' I, who have the disease and can transmit it; and the 'removed class,' R, who have either had the disease and recovered, are dead, are immune, or are isolated. The model mechanism for this process implies the following rate laws:

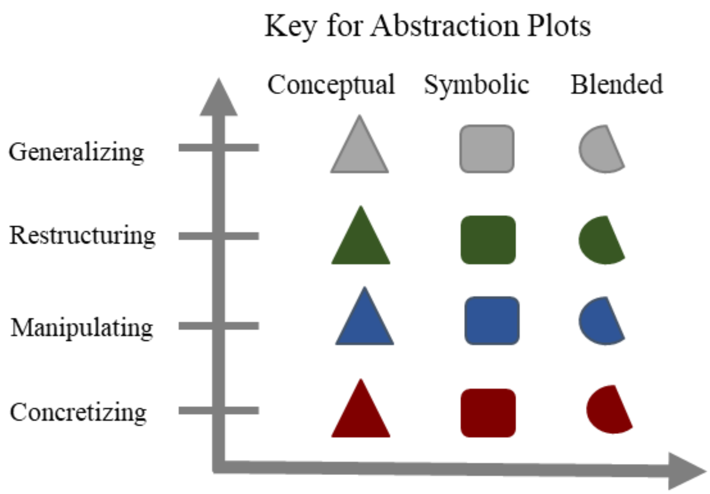
$$\frac{dS}{dt} = -rSI$$

$$\frac{dI}{dt} = rSI - aI$$

$$\frac{dR}{dt} = aI$$

Find the conditions on the ratio a/r that decide whether the disease will spread (an epidemic) or die out.

5. ACTIONS PATTERNS FOR ALL PROFESSOR CLIPS



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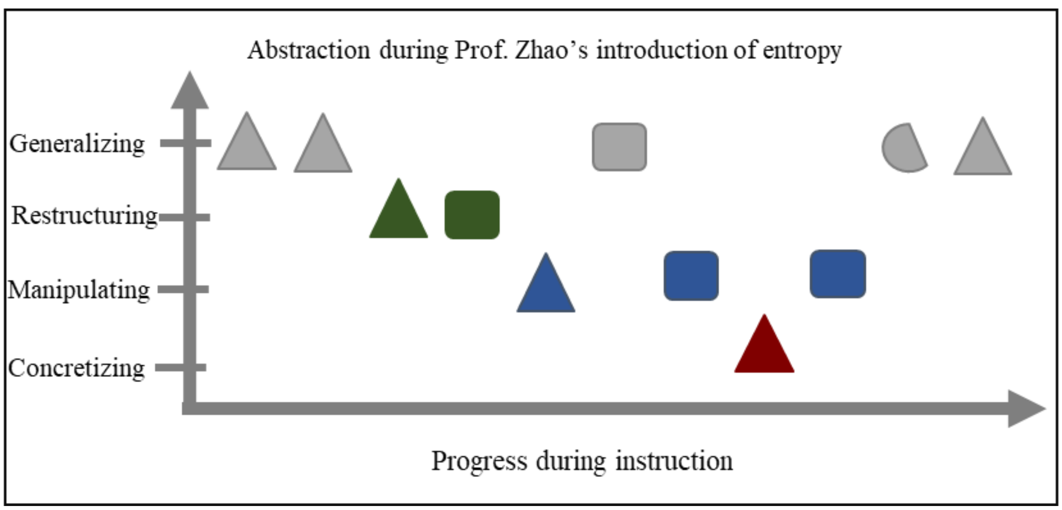

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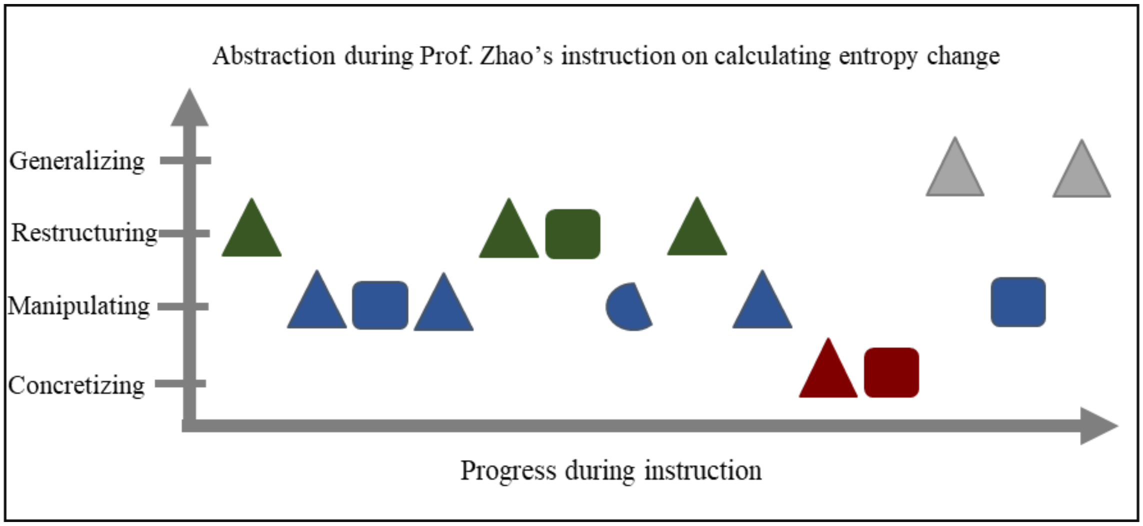

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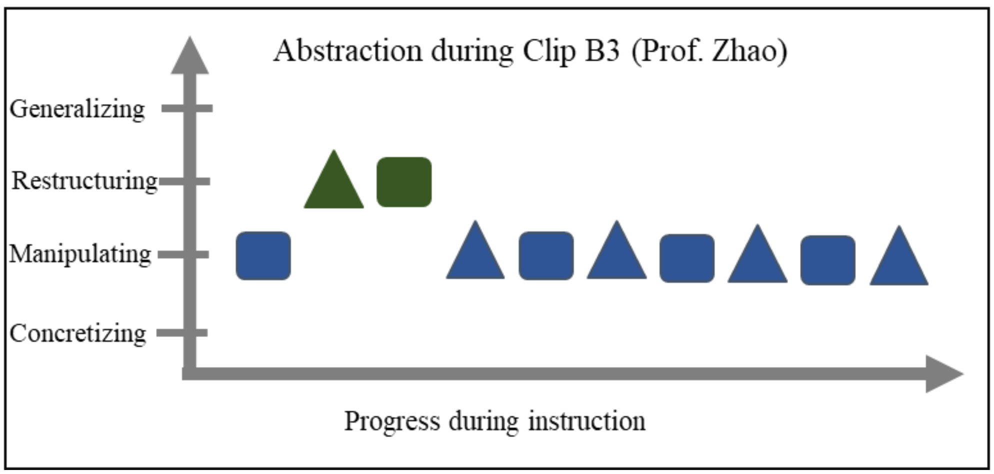

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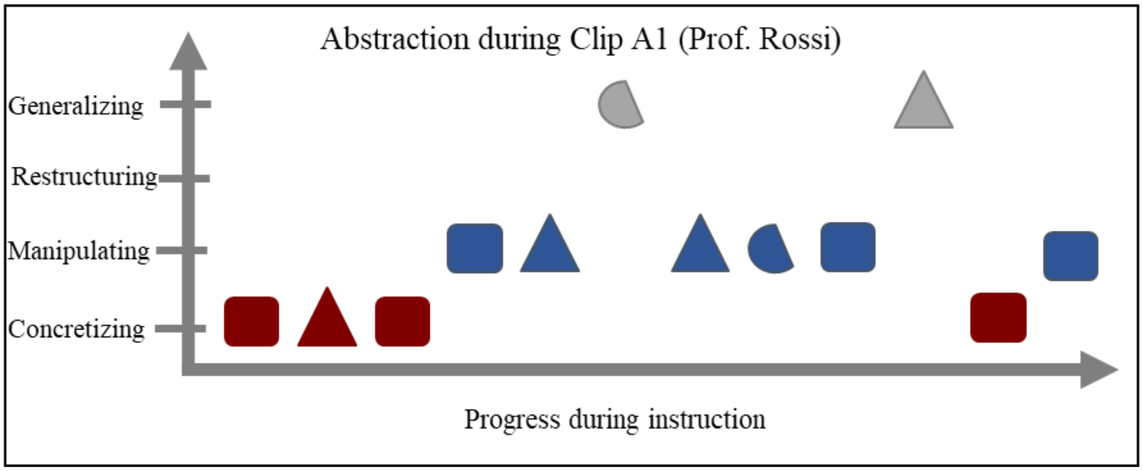
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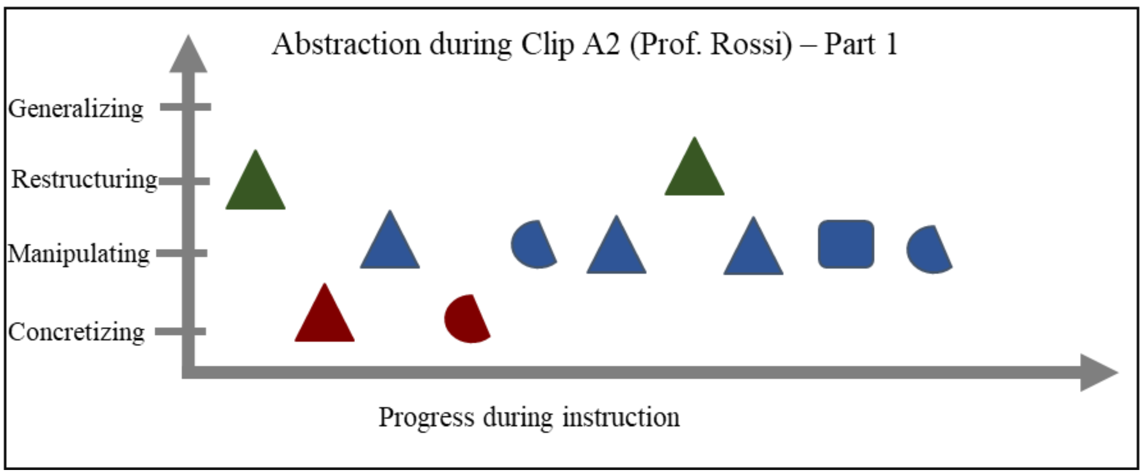
Generalizing
Restructuring
Manipulating
Concretizing
Progress during instruction

A2: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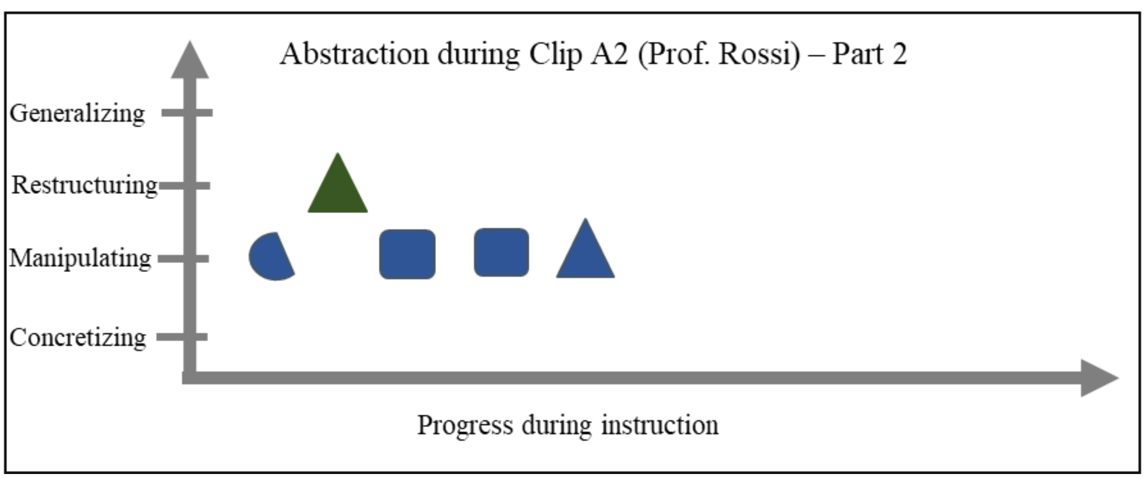
Generalizing
Restructuring
Manipulating
Concretizing
Progress during instruction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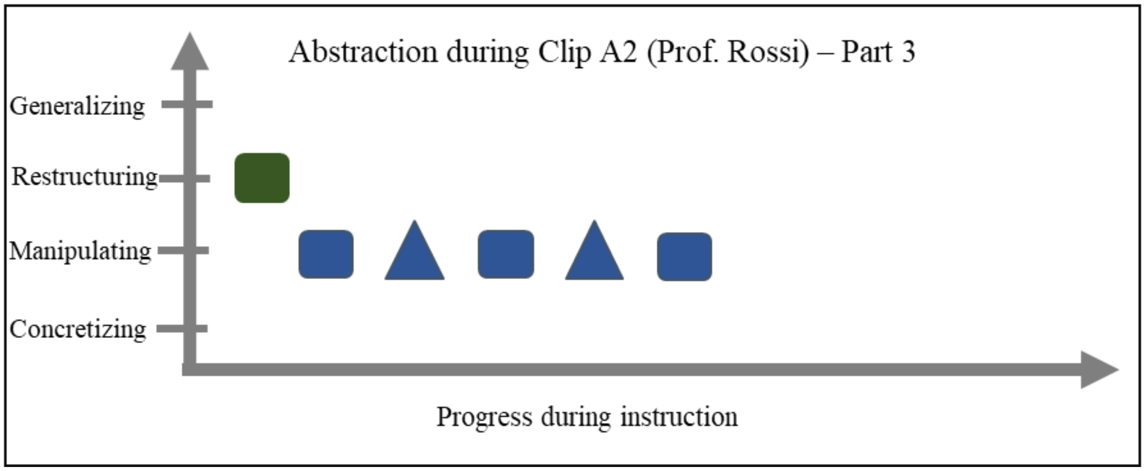
Restructuring
Manipulating
Concretizing
Progress during instruction



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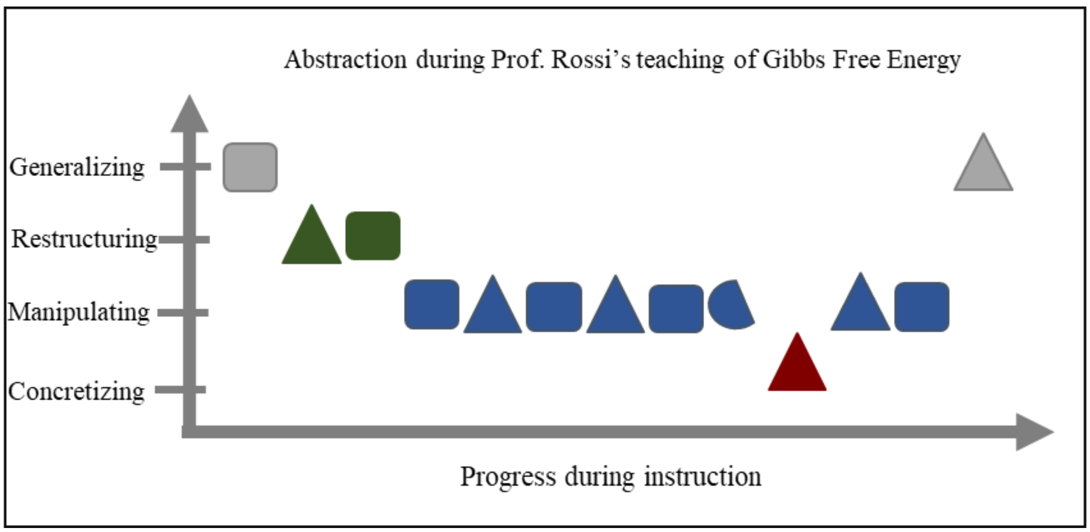

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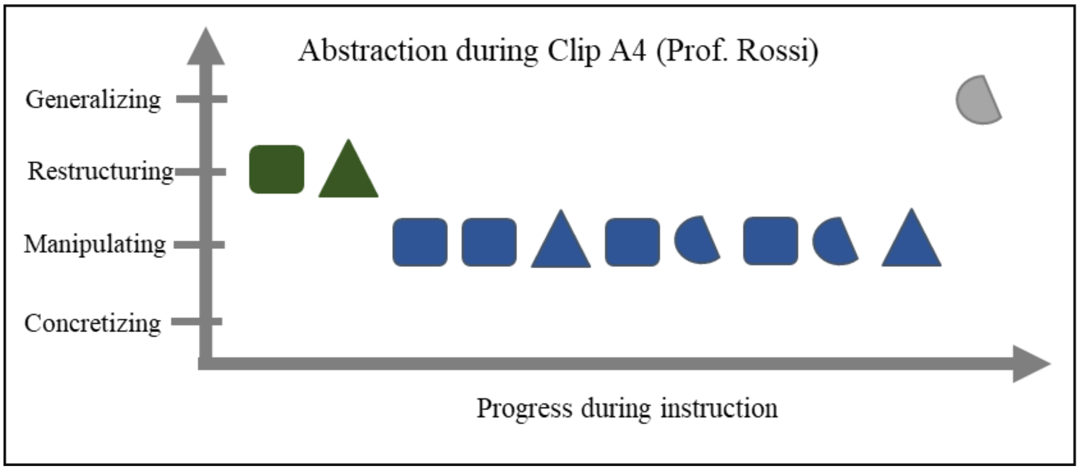

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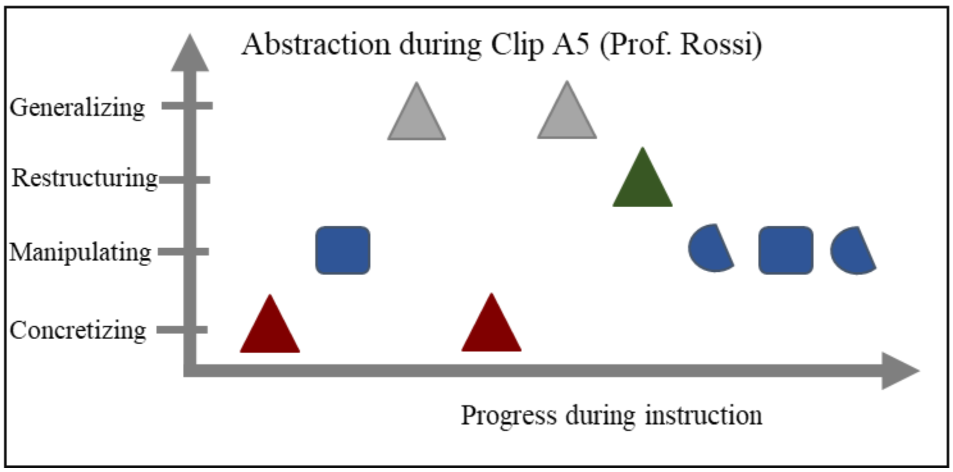

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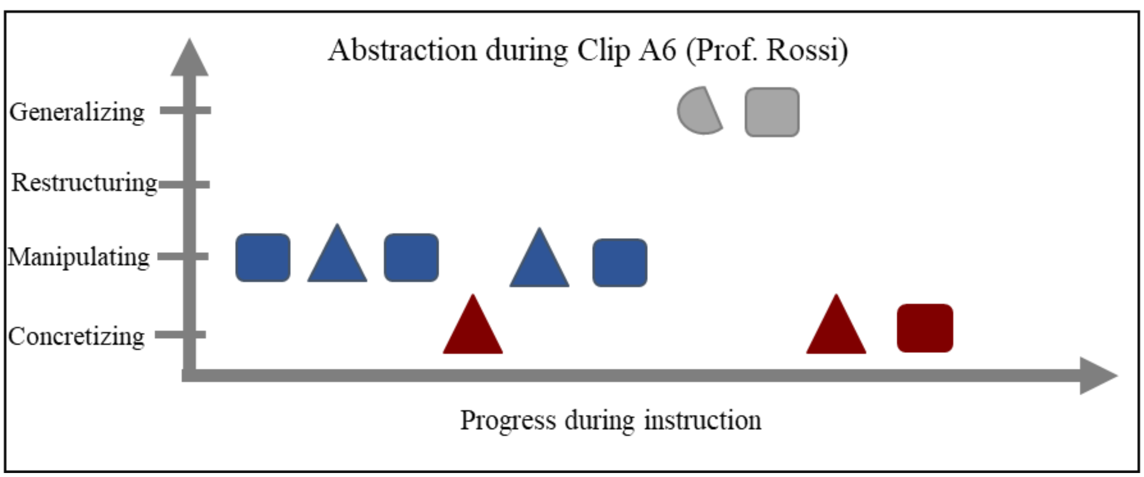

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