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FACULTY WORKING
PAPER NO. 1064

A Multi-Output Analysis of the Iron and Steel Industry

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August 1984

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ABSTRACT

A multi-output analysis of the Australian iron and steel industry is carried out. The two outputs considered are production for domestic use and production for exportation. The translog functional form is chosen to compare the results of various specifications: joint cost function, multi-product function and profit function. Conclusions about complementarity or substitutability remain the same. Finally the joint cost function is used to test for separability between inputs, and outputs, separability between inputs, and Hicks neutral technical progress. Separability between inputs and outputs and non-jointness are not rejected at the 1% level of significance which could suggest that exports may not represent a distinct output.

Most manufacturing models of production or cost are based on the assumption that the manufacturing sector produces one single homogeneous output. But such an assumption imposes arbitrary restrictions on the structure of production. Moreover in the past, the transformation functions available were also very restrictive. Multi-output cost functions corresponding to general production structure have now been developed and used in a number of empirical studies. With such functions, a priori restrictions need not be imposed on the elasticities of substitution. Not only can all kinds of substitution be appraised but moreover separability between input and output as well as non-jointness can be tested.

Multiproduct models have been applied in mainly two situations. First, very aggregated production sector models of the economy were developed. The pioneering work of Christensen, Jorgenson and Lau (1973) testing a production possibility frontier and a price possibility frontier deserves particular mention. More recently studies by Burgess (1974), and by Kohli (1978), both summarized in Woodland (1982), and another study by Denny and Pinto (1978), are variations of a model that considers various combinations of consumption, investment and/or exports as outputs and capital, labor, material and/or imports as inputs.

Second, specific industrial sectors of the economy have also been modeled. Griffin (1977, 1978) studied the oil industry and estimated a price possibility frontier with so-called pseudo-data. Brown, Caves and Christensen (1979), Caves, Christensen and Tretheway (1980), Caves, Christensen and Swanson (1980, 1981) concentrated their efforts on modeling U.S. railways. This is obviously an ideal case for multi-output experiments.

It would seem that multi-output models are tested only either on a very aggregated level or with very disaggregated data. Indeed it would be very difficult to estimate a multiproduct function at the level of an industrial sector because of the existence of interindustry transactions: such data are usually not available on a year to year basis. It would also be difficult to aggregate the various final products of the manufacturing sector into a few categories.

In this study we experiment on an hybrid between the two approaches tested in the literature, i.e., an international multi-output model for a specific industry, the Australian iron and steel industry. The two outputs are exports and production for domestic use. Although the composition of these two outputs is similar, they are considered as two distinct outputs of the iron and steel industry on the grounds that the products sold domestically are intermediate goods for some other Australian industry while the exports represent a final good from the point of view of the Australian economy.

Does such an argument actually justify the assumption that domestic consumption and exports are two distinct outputs of the production process itself? Indeed the same goods might enter either the export or the domestic consumption category. Kohli (1978) acknowledges the conceptual difficulty involved and argues that the two aggregates are different because they come through different channels of distribution. The other studies treating consumption and exports as two different outputs make no attempt at justification.

In the case of the iron and steel industry, the empirical evidence gives rise to two distinct price indices corresponding to production for domestic use and to exports respectively. If they represent the same good the cause for the difference in price must be investigated. For instance the price difference may be due to international price discrimination. The assumption of perfect competition in the output market would thus be untenable and we could not estimate revenue share equations as they are based on the assumption of equality between marginal cost and price: the full joint cost function along with the system of cost share equations would have to be estimated. Unfortunately the existing data set precludes such an attempt.

For the purpose of this study, it is obviously more fruitful to argue that perfect competition exists in the output market and to find another form of justification. The two different price indices for exports and for production for domestic use may arise because the composition of the two outputs is different: different weighting in the aggregation results in two different price indices. This assumption is still not entirely satisfactory because domestic consumption and exports cannot form two separate aggregates if the same goods are included in each aggregate. In such cases the marginal rate of transformation between two goods in one aggregate will not be independent of the level of the goods in the other aggregate.

An alternative approach is to argue that the goods included in the export bundle may have the same denomination as the goods included in the production for domestic use bundle but they are somehow different: they are especially produced for the export market, for instance they are usually of better quality, they may have to meet certain foreign requirements (right hand drive versus left hand drive cars, metric versus non-metric specifications, etc.). In sum they exhibit different attributes (see Lancaster, 1966) and thus exports and production for domestic use form two different aggregates.

As noted in an earlier paper by Turnovsky and Donnelly (1984) the assumption of perfect competition in the input market is subject to questioning because of the concentrated structure of the iron and steel industry within Australia. On the other hand, if the Australian iron and steel industry is cast in an international context, in which it competes with the rest of the world, the assumption of perfect competition in the output market may not be too unrealistic. Finally, imports are not treated separately as a specific input because they represent a negligible component of the materials used; indeed Australia possesses ample resources of iron ore and of coking coal and various other domestic sources of energy for the production of iron and steel. Imports are thus directly included with the material or the energy inputs for the purpose of this study.

The structure of the paper is as follows: The theory underlying the multi-output model is developed in a first section. The econometric estimation procedure and various related problems is discussed in a second section. The models and the relevant data basis are then surveyed and the econometric estimates as well as the various regularity tests of the underlying economic theory are also presented. Finally the findings concerning various forms of separability and of non-jointness, the possible biases of technical change, the substitution possibilities between inputs and the marginal rate of transformation between the two outputs are discussed.

I. THE ECONOMIC MODEL

One basic issue raised in this study concerns the substitutions possibilities: will the introduction of two outputs in the iron and steel model have a substantial impact on the elasticities already calculated in a single output iron and steel model of Turnovsky and Donnelly (T&D) (1984). To extend the original model, we assume the existence of a transformation function $T(y;x) = 0$ where y is a vector of outputs and x is a vector of inputs. If the transformation function is strictly convex in inputs and if it is defined and continuous for all non-negative values of x and y, there exists a unique joint cost function $C = C(y;p)$ dual to the transformation function (McFadden, 1972, 1978) where p is a vector of input prices and C is total cost, $C = p'x$. The cost function is non-decreasing, linear homogeneous and concave in the input price vector p.

In order to determine the substitution possibilities implied by the technology and to test for separability, a flexible functional form will be specified for the joint cost function. To make the results comparable with our previous study (T&D) the transcendental logarithmic functional form (Christensen, Jorgenson and Lau, 1971) interpreted here as a second-order approximation to a more general joint cost function is chosen:

$$\text{Ln } C = \alpha_0 + \sum_i^m \alpha_i \ell n y_i + \sum_j^n \beta_j \ell n p_j$$
$$+ 1/2 \sum_i^m \sum_k^m \gamma_{ik} \ell n y_i \ell n y_k + 1/2 \sum_j^n \sum_h^n \delta_{jh} \ell n p_j \ell n p_h$$
$$+ \sum_i^m \sum_j^n \theta_{ij} \ell n y_i \ell n p_j \qquad \begin{matrix} i,k = 1,\ldots,m \\ j,h = 1,\ldots,n \end{matrix} \qquad (1)$$

A number of restrictions must be imposed on the parameters of the joint cost function: symmetry of the Hessian of the function requires $\gamma_{ik} = \gamma_{ki}$ and $\delta_{jh} = \delta_{hj}$.

As the cost function must be homogeneous of degree plus one in factor prices, the following restrictions (given symmetry) must further be imposed:

$$\sum_j^n \beta_j = 1, \; \sum_j^n \delta_{jk} = 0 \text{ and } \sum_j^n \theta_{ij} = 0.$$

Finally (given all the above restrictions) linear homogeneity in output prices implies

$$\sum_i^n \alpha_i = 1, \; \sum_i^m \gamma_{ik} = 0 \text{ and } \sum_i^m \theta_{ij} \; 0.$$

Assuming perfect competition in input markets, the partial derivatives of the cost function with respect to the input price yields the cost minimizing input levels (Shephard, 1953), i.e., a set of input demand equations that lend themselves readily to econometric estimation can be derived. By differentiating the joint cost function with respect to the output quantities, the marginal cost of the outputs are obtained and if the assumption of perfect competition in the output markets can also be sustained these marginal costs equal output prices and a set of output supply equations can be similarly derived. With the translog specification these equations take the form of revenue share (R_i) and of cost share (S_j) equations

$$R_i = \frac{y_i z_i}{C} = \frac{\partial \ln C}{\partial \ln y_i} = \alpha_i + \Sigma \gamma_{ik} \ln y_k + \Sigma \theta_{ij} \ln p_j \quad (2)$$

$$S_j = \frac{x_j p_j}{C} = \frac{\partial \ln C}{\partial \ln p_j} = \beta_j + \Sigma \delta_{jh} \ln p_h + \Sigma \theta_{ij} \ln y_i \cdot \quad (3)$$

where z_i are output prices and x_j are input

In order to investigate whether technical change is Hicks neutral or not, a time trend t may be added to each share equation.[1] To retain homotheticity in the model, two further restrictions on the coefficients of the time trend, τ_i, in the revenue share and ω_j, in the cost share equations, have to be imposed

$$\sum_i^m \tau_i = 0 \quad \text{and} \quad \sum_j^n \omega_j = 0.$$

Finally the Allen-Uzawa elasticities of substitution, σ, can be readily estimated from the following formulas (Berndt and Christensen, 1973)

$$\sigma_{ii} = (\delta_{ii} + S_i^2 - S_i)/S_i^2 \quad \text{and} \quad \sigma_{ij} = (\delta_{ij} + S_i S_j)/S_i S_j$$

Another purpose of this study is to test for separability between inputs and outputs, separability between inputs and for non-jointness. Separability between inputs and outputs is usually assumed in empirical works involving production functions. If a vector of output y and a vector of input x are related by a production structure, T(y;x) = 0,

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[1]The biases are assumed to occur at a constant rate over time (see Binswanger, 1976).

imposing additive separability between inputs and outputs implies the existence of aggregator functions H(y) and F(x) such that H(y) = F(x), hence the original structure becomes T(y;x) = H(y) - F(x). In terms of the marginal rate of transformation (MRT) the interpretation is as follows: the MRT between outputs depends only on the composition of the output mix and is independent of the factor intensities or of the factor prices (if we assume perfect competition). As a result separability between outputs and inputs is not a very desirable restriction to impose on a production structure. With a flexible functional form it is possible to test the validity of such an assumption on the data. Separability between inputs and outputs for a production function which is linear homogeneous in outputs is warranted only if all the coefficients of the cross products between inputs and outputs are equal to zero, i.e., all $\theta_{ij} = 0$.

Separability between inputs is also investigated. Weak separability implies that the marginal rate of substitution of the two inputs be independent of the composition of the other inputs. This means that the elasticities of substitution between one of the separable inputs and each of the other inputs respectively must be equal to that of the other separable input. This can be translated into specific restrictions on the coefficients of the function. Instead of testing for overall weak separability (see Berndt and Christensen, 1973), we investigate the case of weak separability at the point of approximation (Denny and Fuss, 1977). This approach is chosen because of the existence of a fundamental difficulty with such testing when the translog functional form is used (Blackorby, Primont and Russell, 1977). In sum, inputs 1 and 2 are approximatively and weakly separable from input 3 and 4 if $\beta_1\delta_{23} = \beta_2\delta_{13}$ and $\beta_1\delta_{24} = \beta_2\delta_{14}$. As weak separability is only tested at the point of approximation, the results obtained will not be necessarily comparable to those of the single output cost function of T&D.

Finally non-jointness has been described as a situation where no economy of jointness nor any diseconomy of jointness will arise from the multi-output production process. Hall (1973) shows that this is equivalent to stating that total cost is the sum of the individual cost of production of each output. If for each output i we have $C_i = c_i(y_i;p)$ where y_i is the individual output and p is a vector of input prices, the total cost function is such that $C = C(y;p) = \Sigma\ c_i(y_i;p)$. Alternatively, non-jointness implies that the marginal cost of each output is independent of the level of the other outputs.

Hall proves that a technology which is homogeneous cannot be simultaneously separable in input and output and non-joint. If this was the case, the isoquants corresponding to each output would be identical, i.e., only one kind of output would be present. Non-jointness can be analyzed in the following manner: Denny and Pinto (1978) show that it is equivalent to a set of restrictions on the parameters of the translog: $\gamma_{ik} = -\alpha_i\alpha_k$ $i,k = 1,\ldots,m$.

II. CRITIQUE OF THE MODELS AND THE DATA

The models estimated includes exports and production for domestic uses, as outputs and capital services, labor services, energy, and materials, as inputs. The basic function estimated is the joint cost function: output quantities and input prices are assumed to be exogenous. This is probably the more realistic approach for the iron and steel industry. Indeed one can argue that the firm is a price taker in the input market and that the production of steel depends on the economic conditions. Thus if the various industries using steel as material plan some desirable output to meet the demand, they will submit specific orders to the iron and steel industry. The joint cost function is also adopted in all the four papers modeling the railways and in the aggregate studies by Burgess and by Denny and Pinto mentioned above. The joint cost function can be interpreted as the dual of a multi-product function which can be estimated by assuming output prices and input quantities to be exogenous. From a purely economic point of view these two approaches are not the same as the fundamental causality assumptions are different. But if perfect competition exists in both markets, the results should be equivalent from an estimation point of view. However even if such conditions prevailed, these two approaches would not yield the same estimates when the functional specification is a translog. Indeed the translog is not self-dual except at the point of approximation (see Burgess (1975)). Since the direction of the causality is not necessarily obvious, an alternative model treating output prices and input quantities as the exogenous variables will also be experimented with. (This model will be referred to as the multi-product function.) The concepts of duality in production can be invoked again and a profit function or a price possibility frontier could be estimated following Christensen, Jorgenson and Lau (1973) or Griffin (1977, 1978). With such a function, constant return to scale and perfect competition imply zero profit in the long run. Given the fact that the iron and steel industry in Australia has experienced a very rapid development from a young industry to a mature one in the past few decades, the time span used in this study should be capable of describing long run relations between the variables considered and no attempt was made to estimate a restricted profit function as in Kohli (1978). Finally, the joint cost function is tested for various forms of separability between inputs in order to find out whether any pair of inputs could be aggregated; a three-input function is consequently estimated. Most of the data series have originally been constructed for a previous Australian manufacturing model (Turnovsky, Folie and Ulph, 1982) and for a single output iron and steel model (Turnovsky and Donnelly, 1984) and are described in Turnovsky (1984). For the purpose of this study, the existing series were updated. This study spans the years 1959-1960 to 1979-1980. Prices and quantities time series had to be constructed for the two outputs, goods produced for domestic use and exports. Very detailed exports data are published yearly in Overseas Trade. Quantity and value series for nine categories[1] of exports were constructed and were used to estimate price and quantity Divisia indices for exports. The value series for production was obtained as the residual between total value of output and value of export. A Divisia price index for iron and steel basic products for domestic use was constructed with the help of data published yearly in

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[1]Scrap, coke, pig iron, ingots-blooms-slabs, bars-rods-angles-tees, plates-sheets, hoops, railways-tramways material, pipes-tubed including fittings. Source: Australian Bureau of Statistics, formerly Commonwealth Bureau of Census and Statistics: Overseas Trade, Australia.

the Australian Mineral Industry yearbooks.[1] Finally a quantity index was calculated with the value data and the price index.

III. ECONOMETRIC MODEL

Given the econometric models discussed in section I and II, let us assume that the actual shares diverge from the predicted shares due to the stochastic nature of the behavior of the firms. Independently and identically distributed random disturbances, u_i and v_j, are added to the revenue share and value share equations under the condition that $\Sigma u_i = 0$ and $\Sigma v_j = 0$.

As the disturbance covariance matrix of the revenue shares equation and that of the value shares equations are respectively singular since the shares sum up to unity, one of each set can be omitted. Consequently, only one revenue share equation corresponding to exports and three cost share equations corresponding respectively to labor, energy and materials are retained.

The basic econometric model used to estimate the coefficients of the joint cost function takes the following form:

$$R_X = \alpha_X + \gamma_{XX}\ell n(Q_X/Q_D) + \theta_{XL}\ell n(P_L/P_K) + \theta_{XE}\ell n(P_E/P_K)$$

$$+ \theta_{XM}\ell n(P_M/P_K) + \tau_X t + u_X$$

$$S_L = \beta_L + \delta_{LL}\ell n(P_L/P_K) + \delta_{LE}\ell n(P_E/P_K) + \delta_{LM}\ell n(P_M/P_K)$$

$$+ \theta_{LX}\ell n(Q_X/Q_D) + \omega_L t + v_L$$

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[1] The following categories were available: structural mill and heavy mill, merchant bar, heavy rail, plate and strip and tinplate. Source: Bureau of Mineral Resources, Geology and Geophysics, formerly Department of Mineral and Energy: Australian Mineral Industry--Annual Review.

$$S_E = \beta_E + \delta_{LE}\ell n(P_L/P_K) + \delta_{EE}\ell n(P_E/P_K) + \delta_{EM}\ell n(P_M/P_K)$$

$$+ \theta_{EX}\ell n(Q_X/Q_D) + \omega_E t + v_E$$

$$S_M = \beta_M + \delta_{LM}\ell n(P_L/P_K) + \delta_{EM}\ell n(P_E/P_K) + \delta_{MM}\ell n(P_M/P_K)$$

$$+ \theta_{MX}\ell n(Q_X/Q_D) + \omega_M t + v_M$$

where Q are quantities, P are prices and D corresponds to output for domestic use, X to exports, K to capital services, L to labor services, E to energy, M to materials and t is time.

The resulting estimate will be invariant to the equation excluded if an iterative Zellner efficient estimation procedure is used.

Since the model is based on a time series at a rather disaggregated level, the possible existence of first order autocorrelation is also investigated. In the presence of autocorrelation, the residuals would be correlated in the following manner

$$u_{it} = A_I u_{it-1} + \varepsilon_{it} \quad \text{and} \quad v_{jt} = A_O v_{jt-1} + \varepsilon_{jt}$$

where A_I and A_O are the matrices of autoregressive coefficients for the inputs and for the outputs respectively.

As the system of equation is singular, Berndt and Savin (1975) have pointed out that the parameters of the autoregressive process are restricted, i.e., each column of A must sum up to a constant K. Consequently, if A is specified to be diagonal, all the non-zero elements of A must be equal and such approach will only use one coefficient ρ_O for each output and one coefficient ρ_I for each input. A more elaborate specification for A might be desirable, unfortunately the complexity of such approach precludes the use of anything but the simplest specification mentioned above which already makes the system non-linear in its coefficients.

IV. EMPIRICAL RESULTS

The estimates for the coefficients of the joint cost model and the main statistics concerning the system and the individual equations are presented in Table 1. The coefficients are mostly significant and the coefficients of determination for the individual equations are quite high. The coefficients are similar to those calculated in the single output iron and steel model reported in T&D. Similarly the KE coefficient is the only non significant cross-coefficient between inputs. One must note that some of the traditional single equation statistics, although usually reported along with the estimates, are not as meaningful in the case of a system of equations. As in most studies of this kind the single equation Durbin Watson deteriorates with the imposition of linear homogeneity and symmetry on the system. An effort was made to correct this problem using the Berndt and Savin approach but it was not entirely satisfactory.

The autoregressive coefficient for the input was the only significant one and the overall improvement for the system was mixed. As expected, the standard deviations become larger with the correction. However, the significance or non-significance of the estimates is not really affected on the whole. Moreover, the results of the capital equation are improved but those of the labor equation deteriorate. The respective single equation Durbin Watson before the correction suggests that the capital, labor and material equations exhibit positive autocorrelation while the labor equation exhibits negative autocorrelation. Consequently as the autoregressive coefficients are specified to be equal in each value share equation, the use of the positive ρ_I calculated above has an adverse impact on the labor equation. As a result, this attempt at correcting for autocorrelation was not pursued any further.

i. General Tests

The empirical results must be analyzed to see if the function fitted is well behaved in the sense that it does not violate any of the basic economic assumptions underlying the theory of production. First are the imposed restrictions of symmetry and of linear and homogeneity in the inputs accepted by the data set? Given the nature of the translog, these two sets of restrictions cannot be tested separately hence if we assume that symmetry is the maintained hypothesis, we test for linear and homogeneity given symmetry. This assumption is supported by the data in all the specifications with the exception of the multiproduct function where it is not overwhelmingly rejected. Then monotonicity is tested: the predicted values must always be non-negative. The test is straightforward and always yields satisfactory results. Finally concavity in input prices is investigated: the Hessian of the cost function must be negative semi-definite. As the translog is a very general function containing higher order terms, such requirement is not expected to be met everywhere. However it would be sufficient to have the concavity assumption accepted at each observation point, i.e., in the range of the data set. Unfortunately, as the number of variables in the system estimated increases, such requirement becomes harder to meet. Moreover Wales (1977) has pointed out that its complete fulfillment usually makes very little difference to the results. In this study, the concavity assumption for the joint cost function was accepted at all points by a preliminary data set and was only partially accepted by a much superior and refined data set; the latter data set also yielded better results from an econometric point of view, e.g., better coefficient of determination, higher log of likelihood function, etc. Furthermore the conclusions based on the calculated elasticities were practically identical in the two models. Although it is also possible to impose concavity on the data (see Lau (1978)), there did not seem to be any practical advantage in adding further restrictions on the model. In sum, for the joint cost function the results yielded by the best data set will be reported. On the other hand the multi-product function is well behaved at all points while the profit function and a three-input joint cost function exhibit only partial concavity.[1] The tests of the behavior of the function fitted are presented in Table 2.

ii. Various Separability Tests on the Joint Cost Function

As the translog can be considered as a generalized Cobb-Douglas, one can check whether the technology is Cobb-Douglas in inputs or in outputs by testing whether the second order coefficients corresponding

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[1] It seems that the function treating input quantities as exogenous is well behaved while the corresponding functions considering input prices as exogenous are not as successful. This inconsistency is indeed another symptom of the lack of self-duality of the translog.

respectively to all the inputs or to all the outputs are jointly significant. Both F tests were rejected (see Table 3). Two levels of separability tests were also performed: specific test concerning multi-output functions and various separability tests on the inputs themselves. The results are presented in Table 3. First are inputs and outputs separable? This is equivalent to inquiring whether there exists a separate cost function. A joint test of significance on all the cross-coefficients of input and output was thus performed. The hypothesis that all the θ_{ij} were equal to zero was rejected at the 5% level (but accepted at the 1% level). Denny and Pinto obtained the same results with their data on exports for Canada.

The test for non-jointness using Denny and Pinto's specification mentioned above had to be performed with a non-linear system of equations since the additional restrictions are non-linear in the parameters. Then a χ^2 test was performed on the log of likelihood of the non-joint and of the original system estimated. Non-jointness was accepted in the main model at the 5% and at the 1% level. It was also accepted by the Denny and Pinto data. These results are quite interesting for the following reason: if separability between input and output and nonjointness are both accepted, it means that the two outputs have the same isoquants, i.e., they are not really distinct outputs (cf. Hall 1973). Since our study as well as the Denny and Pinto study strives at interpreting domestically consumed goods and exports as two separable goods, it is noteworthy to point out that the data does not support unquestionably such an assumption: if we choose the 1% level of significance for our tests instead of the usual 5% level, exports and domestically consumed goods are shown to represent one single output by the production frontier.

Finally we tested the model for separability between inputs. Neither capital and labor, KL, (the value added specification) nor capital and energy, KE, (the capital utilization specification) form an aggregate. However the separability of energy and materials from capital and from labor is accepted and a more conventional three-input model including energy as material can thus be estimated. The resulting coefficients for EM are closer to the coefficient of M than to the coefficients of E; this is due to the fact that the value shares for M are more important than the value shares for E in the production process. The estimates for the three-input, KL(EM), joint cost function are presented in Table 1.

iii. Technical Progress

In the joint cost function, the coefficients of the technical progress variable, i.e., of the time trend, are all very small. With respect to their individual t-statistics, the only unquestionably significant coefficient is the energy coefficient; it suggests that technical progress is energy using. The same conclusion was reached in the manufacturing model of Turnovsky, Folie and Ulph (1982) and in the single output iron and steel model. No other conclusion concerning the effect of technical progress on the other inputs is warranted. As it is necessary to impose symmetry with the translog setup, the time trend cannot be removed from the equations which yield non-significant coefficients. Thus the only relevant test for significance is a joint test of the null hypothesis

$$\tau_i = 0 \qquad i = 1,..,m$$

and $$\omega_j = 0 \qquad j = 1,..,n$$

The test was rejected (see Table 2) at the 5% level and thus the time trend remained in the model.

iv. Elasticities of Substitution and Marginal Rate of Transformation

The elasticities of substitution at the mean of the sample for the various inputs are presented in Table 4. When the elasticities from the joint cost function are compared to those yielded by the cost function (T&D) we find that the own elasticities remained quite similar. On the other hand, the cross elasticities exhibit greater differences in their levels and one cross elasticity--between labor and energy--actually changes sign. (It is very small anyway.) Such an occurence is not surprising because the labor input is an aggregate of administrative and of production labor. When included separately, these two forms of labor exhibit respectively substitutability and complementarity with energy as shown in the T&D model. Finally all the elasticities between input and output are positive. When examining the elasticities yielded by the various specifications experimented upon, it is quite interesting to note that the estimates are rather stable in the sense that the signs are the same (with the exception of the LE case already mentioned). The magnitudes of most of the elasticities are also quite similar in the case of the two joint cost functions and of the profit function. On the other hand, the multiproduct function did exhibit greater disparities.

The marginal rate of transformation between the goods produced for exportation and the goods produced for domestic consumption at each observation point are calculated in the following manner. The marginal rate of transformation can be approximated as the ratio of the marginal costs. The translog revenue share equations yields estimates of the logarithmic marginal cost $\frac{\partial \ln C}{\partial \ln y}$, i.e., of the cost elasticity with respect to each output. So the marginal cost can be estimated as the cost elasticity multiplied by C/y (cf. Brown, Caves and Christensen (1979)). Consequently the marginal rate of transformation, MRT, between two outputs i and k at each observation point is given by the following equation

$$MRT = \frac{\partial \ln C}{\partial \ln y_i} \Big/ \frac{\partial \ln C}{\partial \ln y_k} \cdot \frac{y_k}{y_i}$$

The marginal rates of transformation calculated at each data point using the joint cost function are presented in Table 5. The estimates vary between .64 and .99

V. CONCLUSIONS

This study which is an extension of the original iron and steel single output translog cost function, searches for answers to a number of questions concerning the translog. First, it seems that the use of a multi-output cost function does not have any great impact on the elasticities of substitution already calculated in the single output model although the input-output cross coefficients are jointly significant.

A second observation concerns the validity of using exports as a separate output. The literature offers many examples of such an approach although it is questionable from a purely "productive process" point of view. Indeed the results are not conclusive on this question as non-jointness and separability are both accepted at the 1% level of significance, but not at the 5% level. This could imply that exports are not a distinct output and thus explain why the elasticities of substitution remain similar to those calculated in the single output model.

As the various assumptions about the existence of perfect competition and about the casting of specific variables as exogenous or endogenous can be questioned at length, a multi-product and a profit function were also estimated. The profit function yielded elasticities of substitution that were very similar to those of the cost function. This is not surprising considering that only one series[1] is different when the system is estimated. On the other hand, the elasticities calculated with the multi-product function are further apart from those calculated with the joint cost function. In this case all the independent variables are different and the lack of self-duality of the translog is more apparent. However, except for one elasticity involving labor,[2] none of the elasticities changes sign.

Finally, the three-input model also behaves in a very predictable manner; all the elasticities involving the energy-material aggregate becoming some form of average of the corresponding elasticities involving either energy or material, respectively.

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[1]The independent variable corresponding to output is a ratio of output prices instead of output quantities and the signs of the value shares is changed.

[2]It was mentioned previously that labor did not form a satisfactory aggregate anyway.

In conclusion, all these experiments point to a certain robustness of the translog which could be considered as a strength but perhaps also as a weakness if this is due to the stringent restrictions imposed on the various systems of equations. However, we must point out that the basic linear-homogeneity restrictions that have to be imposed on the translog in order to estimate the parameters from each system of equations are either accepted or very marginally rejected by the data. This means that the data imply perfect competition and thus support theoretical duality between the models. An extension of this study would be obviously to experiment with alternative functional forms and more specially with the approach preconised by Gallant (1981, 1982), unfortunately none of these functions are as convenient to handle as the translog.

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Table 1

Parameter Estimates and Main Statistics

	Joint Cost Function	Multi-Product Function	Profit Function[a]	Joint Cost Function Corrected For Autocorrelation		3-Input Joint Cost Function[b]
γ_{XX}	.0737	-.0604	-.0491	.0729	γ_{XX}	.0732
	(7.22)	(-1.63)	(-1.37)	(7.79)		(7.31)
δ_{KK}	.0572	.0353	-.0520	.0947	δ_{KK}	.0689
	(5.46)	(2.69)	(-4.56)	(12.53)		(5.35)
δ_{LL}	.0722	.0422	-.0746	.0874	δ_{LL}	.0559
	(5.02)	(1.54)	(-4.47)	(4.82)		(3.77)
δ_{EE}	.0948	.0132	-.0936	.1095	$\delta_{M'M'}$	.1714
	(10.89)	(.42)	(-10.56)	(8.76)		(7.71)
δ_{MM}	.1962	.0887	-.1984	.1887		
	(8.69)	(1.37)	(-7.90)	(7.44)		
δ_{KL}	.0235	.0089	-.0235	.0114	δ_{KL}	.0233
	(3.78)	(1.01)	(-3.70)	(1.56)		(3.25)
δ_{KE}	-.0032	.0106	-.0005	-.0266		
	(-.48)	(.70)	(-.08)	(-3.76)		
δ_{KM}	-.0775	-.0548	.0760	.0795	$\delta_{KM'}$	-.0922
	(-6.37)	(-2.13)	(5.41)	(na)		(-6.27)
δ_{LE}	-.0343	-.0205	.0349	-.0363		
	(-3.82)	(-.91)	(3.59)	(-2.83)		
δ_{LM}	-.0614	-.0306	.0632	-.0625	$\delta_{LM'}$	-.0793
	(-6.32)	(-1.46)	(5.80)	(-4.90)		(-5.34)
δ_{EM}	-.0572	-.0033	.0592	-.0467		
	(-5.41)	(-.09)	(5.42)	(-3.58)		
θ_{XK}	.0047	-.0117	.0148	.0035	θ_{XK}	.0050
	(1.06)	(-1.09)	(1.80)	(1.24)		(.99)
θ_{XL}	.0036	-.0157	.0072	.0021	θ_{XL}	.0046
	(1.60)	(-2.49)	(1.50)	(.75)		(2.00)
θ_{XE}	.0100	-.0122	.0126	.0068		
	(3.29)	(-.92)	(1.90)	(2.15)		
θ_{XM}	-.0183	.0396	-.0346	-.0125	$\theta_{XM'}$	-.0096
	(-2.66)	(1.57)	(-2.54)	(-1.95)		(-1.60)
α_D	.9078	.9170	.9170	.9139	α_D	.9081
	(138.61)	(76.53)	(78.68)	(135.3)		(142.11)
α_X	.0922	.0830	.0830	.0859	α_X	.0919
	(14.08)	(6.93)	(7.12)	(12.73)		(14.38)
β_K	.0664	.0467	-.0651	.0848	β_K	.0681
	(20.37)	(10.26)	(-19.87)	(38.79)		(18.08)
β_L	.1789	.1708	-.1793	.1786	β_L	.1797
	(105.8)	(57.91)	(103.44)	(83.45)		(99.15)
β_E	.1297	.1254	-.1288	.1289		
	(59.40)	(22.97)	(-52.51)	(53.47)		

	Joint Cost Function	Multi-Product Function	Profit Function	Joint Cost Function Corrected For Autocorrelation		3-Input Joint Cost Function[a]
	.6241	.6571	-.6268	.6076	$\beta_{M'}$	.7522
	(127.65)	(65.71)	(-121.85)	(128.91)		(168.89)
	-.0003	-.0051	-.0040	-.0009	τ_D	-.0005
	(-.38)	(-4.33)	(-3.26)	(-1.26)		(-.69)
	.0003	.0051	.0040	.0009	τ_X	.0005
	(.38)	(4.33)	(3.26)	(1.26)		(.69)
	-.0006	.0016	.0005	-.0008	ω_K	-.0006
	(-1.13)	(3.24)	(.86)	(-2.89)		(-1.09)
	-.0001	.0040	.0000	-.0002	ω_L	.0002
	(-.20)	(5.36)	(.15)	(-.60)		(.36)
	.0019	.0018	-.0022	.0009		
	(4.07)	(2.09)	(-4.82)	(2.66)		
	-.0017	-.0074	.0017	.0001	$\omega_{M'}$	.0009
	(-1.41)	(-6.53)	(1.95)	(.12)		(.46)
stem R^2	.99	.97	.99	.99		.99
uation $\bar{R}^2$	1 .88	.60	.60	.90	1	.89
	2 .88	.60	.60	.90	2	.89
	3 .85	.79	.87	.94	3	.91
	4 .98	.95	.98	.96	4	.98
	5 .95	.74	.93	.93	5	.95
	6 .97	.89	.96	.97		
uation rbin- tson	1 1.45	1.80	1.77	1.85	1	1.52
	2 1.45	1.80	1.77	1.85	2	1.52
	3 .82	1.43	1.05	1.54	3	.49
	4 2.45	1.99	2.41	3.02	4	2.55
	5 1.38	.98	1.40	1.31	5	.99
	6 1.23	1.02	1.35	2.03		
g of kelihood nction	324.05	283.48	307.82	312.62		233.03
	.1425	.0691				
	(.45)	(.26)				
	.5981	.3375				
	(3.82)	(1.48)				

ote for comparison with the other models that the signs of all the coefficients nvolving inputs should be reversed because negative value shares are used for he estimation.

' represents the energy-material aggregate.

Table 2

Results of the Regularity Tests

	Joint Cost Function	Multi-Product Function	Profit Function	3-Input Joint Cost Function		Critical Values at 5% level	Critical Values at 1% level
Positivity[a]	21	21	21	21			
Linear Homogeneity-Symmetry[b]	F 2.287 DF(6,60)	4.122 (6,60)	1.475 (6,60)	4.271 (3,48)	F(6,60) F(3,48)	2.45 2.80	3.12 4.22
Concavity or Convexity in Input Prices or Quantities[c]	8	21	10	6			

[a]Number of predicted value shares that are positive.

[b]This is a test for linear homogeneity given symmetry or for symmetry given linear homogeneity.

[c]Number of observations passing the test.

Table 3

Results of Separability Tests
Joint Cost Function

			Critical Values at 5% level	1% level
Cobb-Douglas				
in Inputs	$F(6,66)$	25.56	2.24	3.09
in Outputs	$F(1,66)$	52.06	3.99	7.04
Separability	$F(3,66)$	4.00	2.75	4.10
Input-Output	$\chi^2(3)$	9.26	7.81	11.34
Non-Jointness	$\chi^2(1)$	.86	3.84	6.63
Input Separability				
(KL)	$\chi^2(2)$	15.60	5.99	9.21
(KE)	$\chi^2(2)$	18.48	5.99	9.21
E(KLM)	$\chi^2(2)$	47.88	5.99	9.21
(EM)	$\chi^2(2)$	6.70	5.99	9.21
Technical Progress-Joint Test				
$\tau_j=0$ $j=1,\ldots,m$ $\omega_i=0$ $i=1,\ldots,n$	$\chi^2(4)$	15.56	9.48	13.28

Table 4

Elasticities of Substitution at the Mean

	Joint Cost Function	Multi Product Function	Profit Function	Single Output Cost Function		3 Input Joint Cost Function[a]
σ_{DK}	.93	1.17	.79		σ_{DK}	.93
σ_{DL}	.98	1.09	.96		σ_{DL}	.97
σ_{DE}	.93	1.09	.91		$\sigma_{DM'}$	1.02
σ_{DM}	1.04	.92	1.07			
σ_{XK}	1.39	.02	2.24		σ_{XK}	1.42
σ_{XL}	1.12	.49	1.23		σ_{XL}	1.15
σ_{XE}	1.44	.47	1.55		$\sigma_{XM'}$	.91
σ_{XM}	.77	1.50	.56			
σ_{KK}	-2.73	-5.93	-3.49	-2.73	σ_{KK}	-1.02
σ_{LL}	-2.11	-2.78	-2.06	-2.23	σ_{LL}	-2.48
σ_{EE}	-1.53	-4.79	-1.58	-1.97	$\sigma_{M'M'}$	-.07
σ_{MM}	-.17	-.53	-.16	-.24		
σ_{KL}	2.34	1.51	2.34	1.39	σ_{KL}	2.33
σ_{KE}	.76	1.81	1.04	1.01	$\sigma_{KM'}$	-.58
σ_{KM}	-.71	-.21	-.68	-.11		
σ_{LE}	-.02	.39	-.04	.03	$\sigma_{LM'}$	.47
σ_{LM}	.47	.74	.45	.48		
σ_{EM}	.34	.96	.32	.29		

[a] M' represents the energy--material aggreate.

Table 5

Marginal Rate of Transformation
Joint Cost Function

1959-60	.8279
1960-61	.8256
1961-62	.8607
1962-63	.9639
1963-64	.9430
1964-65	.9374
1965-66	.9545
1966-67	.7913
1967-68	.8729
1968-69	.8277
1969-70	.8432
1970-71	.9900
1971-72	.9105
1972-73	.8053
1973-74	.8435
1974-75	.6746
1975-76	.6453
1976-77	.6384
1977-78	.6366
1978-79	.6449
1979-80	.7522