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The Spaceplane Equation

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Abstract

Revenue earning space missions for deployment of space solar power satellites are economically competitive only when launch costs are in the region of $100-200 / kg in orbit. Nearly one half the capital cost of a solar power station is attributed to space transportation. To meet the objective of safe, affordable space transportation, an abundance of reusable space vehicle design concepts and programs emerged world-over, from 1985 to the present day. All these have been either abandoned or sub-optimally supported. Their designs emerged by the application of the ideal rocket equation derived in 1903 by Konstantin Tsiolkovsky. It had served as the scientific foundation for the design of multi-stage space rockets and ballistic missiles throughout the 20th century. The rocket equation emerged from a simple systems concept: the expendable space rocket that carries all the oxygen needed from earth for propulsion into space with low fuel efficiency rocket engines.

New fully reusable space vehicle concepts use the earth's atmosphere to enhance fuel efficiency and reduce/avoid carrying oxygen on board from earth to orbit. Their shape and aerodynamics call for adopting aircraft design practices. The classical rocket equation is unable to provide a satisfying and adequate theoretical framework to guide design of more complex systems concepts. A modification of the Tsiolkovsky rocket equation is developed here that introduces a "mass ratio multiplier factor," which enables a better understanding of spaceplanes. The Spaceplane Equation shows how mass ratios obtainable only by 2-and 3-stage rockets can be realized in a single stage. Novel spaceplane design and technology domains emerge from parametric mapping using the Spaceplane Equation that could synergize more effective design and development of space transportation systems for space solar power missions.

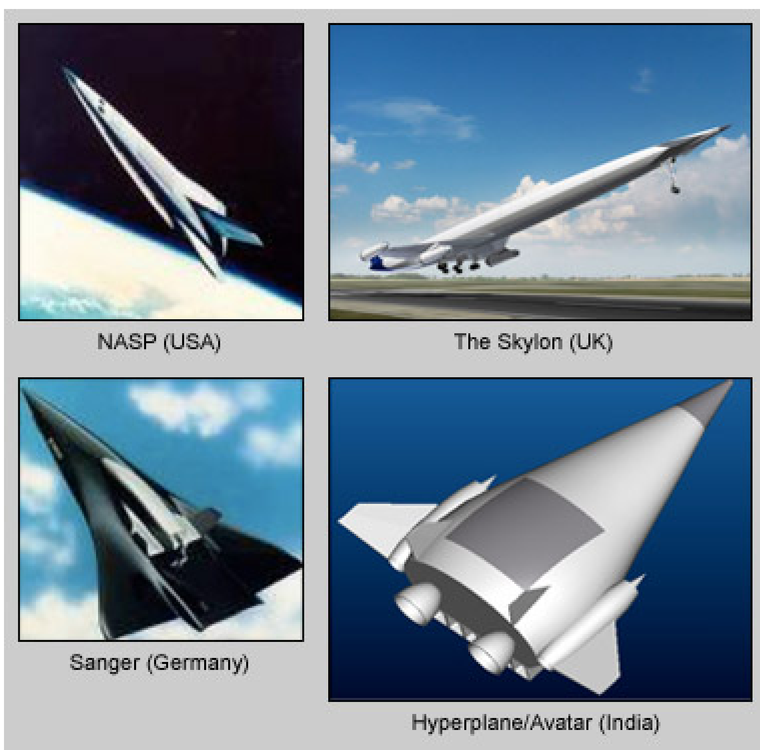
Introduction

Throughout the 20th century, the advancement of multi-stage space rocket and ballistic missile systems and technologies were founded on the Tsiolkovsky equation[1], the presumed "Ideal Rocket Equation" that relates the increase in velocity of a rocket vehicle to the effective exhaust velocity and the initial and end masses of a rocket when all propellant is consumed. The equation is named after Konstantin Tsiolkovsky who independently derived it and published it in 1903.

Expendable Space Rockets & Missiles of the 20th Century

Towards the end of the 20th century, in undertaking such revenue earning missions as those involving installation and management of solar power satellites, the cost of access to space emerged as an important economic factor in space transportation. The Space Shuttle was a movement in that direction. However, the expectations of low cost access to space remain unfulfilled even today as the Shuttle is to be phased out in 2010. From 1985, design concepts for fully reusable launch vehicles were advanced from several space faring countries.

Spaceplane Design Concepts and Programs: 1985 to the Present



The earlier ones included the US (NASP spaceplane), the UK (Hotol: now Skylon), German (Sanger), India (Hyperplane/Avatar), Japan (Unnamed

Airbreathing SSTO) and France (Hermes). All these design concepts gained visibility after the tragic loss of Space Shuttle Challenger in 1986.

An SSTO (Japan)

X-33 (USA)

Black Horse (USA)

Orbital Spaceplane (USA)

In the mid-1990s, a different class of rocket-engine powered aircraft-like space vehicles, sometimes called Rocketplanes, emerged. These included the X-33, Black Horse, Orbital Spaceplane, Pathfinder, Spacecruiser, Roton, Astroliner Space Access SA-1, Kistler K1 and Argus Maglifter. For a variety of reasons, none of them has been found suitable for extended design and development.

At the dawn of the 21st century, with the advent of global warming and rapid depletion of fossil fuel reserves, the demand for abundant and non-polluting energy has intensified work on revenue earning missions such as space based solar power satellites, calling for cost reductions by 100 times and more. Glaser[2] (1970-73) estimated that the capital cost of a space solar power station would be about \$150 per kg; and 53% of this capital cost or about \$80 per kg was attributed to space transportation.

The cost of access to space for a variety of space application missions was reported by Ashford[3] (1987), who estimated (Table 1) the cost for a space solar power mission to be $100-200 / kg. Estimates made in India in 1993-1996[4-7] were in reasonable agreement with that of Ashford. In 2000, Mankins[8] testified before the US Congress the cost would approximate $200 per kg.

The Ideal Rocket Equation

The ideal rocket equation for flight from earth to orbit in vacuum is:

$$Vi = g.\ Is.Ln\ R \ \ldots\ldots\ (1)$$

(For Symbol Explanation see Reference 1)

For obtaining an ideal orbital velocity of 7800m/s, using a rocket engine with I_s= 450s, it can be seen from (1) that R=5.85. However, the mass ratio of a single stage rocket is at best 2.5 to 3.5, which is not adequate to deliver the requisite orbital velocity.

Velocity Losses during Ascent from Earth to Orbit

In practice, velocity losses (ΔV_L) are incurred during flight from earth to orbit, due to atmospheric drag (ΔV_D) and gravity (ΔV_G). The orbital velocity requirement to be delivered by the rocket is thus:

$$Vo = Vi + (\Delta Vl)$$

Defining a Velocity Loss Factor as:

$$k = (\Delta Vl)/ Vi$$

we get $Vo= (1+k)^{-1}.g.\ Is.Ln\ R$ ...... (2)

In practical cases, the velocity loss may range from 20-30% of the ideal velocity, that is $k_L = 0.2$ to 0.3. Hence, the actual velocity required to be delivered by the rocket would be 9,360 to 10,140m/s as compared to 7,800m/s in drag and gravity free environment. Then from (1), the actual mass ratio required would be 8.33 to 9.94 as compared to the ideal case of R= 5.85

Multi-Stage Rocket Equation

The mass ratio of a single stage rocket vehicle is at best 2.5 to 3.5. To increase the mass ratio of a rocket vehicle, several single stages are stacked vertically. The mass ratio of a multi-stage rocket vehicle is then:

$$R = R1 \times R2 \times R3....Rn = \pi(Ri) \quad ....(3)$$

The Rocket Equation for a multi-stage

rocket can be written as:

$$Vo = (1+k)^{-1}.g.\ Isr\ Ln\ \{\varepsilon.\ R\} \quad ......(4)$$

Where $\varepsilon = \pi\ (Ri\ )\ /\ R$

The term ε is thus a Mass Ratio Multiplier Factor, where the mass ratio of a single stage is amplified using multi-stage rockets. A three-stage rocket, each stage with r=2.5, the "Mass Ratio Multiplier" $\varepsilon = (2.5)^3/5.85 = 2.67$.

Utility of the Rocket Equation for Airbreathing Ascent to Earth Orbit

One of the reasons for an overabundance of non-implementable spaceplane and RLV design concepts is that the Tsiolkovsky rocket equation continues to be the only conceptual framework for space vehicle design, whereas there have been revolutionary changes in aerospace technologies and materials towards the end of the 20th Century.

In a space vehicle that makes best use of the earth's atmosphere, the rocket equation in its original multi-stage rocket form is no longer usable because a rocket vehicle ascends to orbit propelled by a rocket engine whose performance is relatively unaffected by atmospheric flight conditions. In a spaceplane, however, the space vehicle needs to fly for sustained periods in an endo-atmospheric first phase, during which the airbreathing engines are 6-7 times more efficient than rocket engines needed for second exo-atmospheric phase of flight and its performance is highly sensitive to the endo-atmospheric flight regime.

Thereafter, as the vehicle ascends from the end of endo-atmospheric flight through the exo-atmosphere to earth orbit, the propellant mass is continuously decreasing. In addition, in a rocket propelled vehicle, the propellant (oxidizer+fuel) is continuously consumed as the vehicle ascends to orbit. In a spaceplane, however, oxidizer need not be carried on board the vehicle at take-off. Oxidiser could be added in different ways (by external means with an aerial tanker or internally by new aerocryogenic technologies) and at different flight regimes up to the end of endo-atmospheric phase of orbital ascent.

Clearly, the ideal rocket equation was not conceived for such complex vehicles, whose mass increases during part of its flight to orbit; whose fuel efficiency is widely different in different phases of flight to orbit; and whose velocity losses (especially drag losses due to sustained flight in the atmosphere) are far greater than that of a rocket vehicle and most sensitive to the flight regime. Hence, there is need for a Spaceplane Equation that will better serve evolving 21st century design concepts of space vehicles based on advanced technologies.

Single Stage to Orbit (SSTO) Spaceplane

The fuel efficiency of rocket engines reached an upper limit decades ago. Further, the mass ratio of the reusable space vehicle that has to ascend from earth to orbit in a single stage (to enhance reliability, safety and reduce costs) needs to be as high as that of a multi-stage rocket.

Enhanced fuel efficiency is feasible by making best use of the oxygen in the earth's atmosphere. This is conceivable by using airbreathing engines, as in an aircraft, that use atmospheric air for propulsion. It has been shown[9] that for an aircraft to ascend directly from earth to orbit in a single stage, at least 56% of its mass at take-off has to be hydrogen fuel. The mass of a rocket at take-off is constituted by about 65% liquid oxygen and 21% liquid hydrogen; the remaining 14% being empty structure and useful payload. By eliminating the 65% oxygen at take-off, the rocket vehicle, shaped like an aircraft, would hold 21/35 (i.e., 60%) hydrogen mass at take-off, more than the minimum requirement for direct ascent from earth to orbit. The atmosphere contains 23% oxygen that can be collected, liquefied, fractionated and stored on-board in flight. To accomplish this goal requires new aerocryogenic systems and technologies.

The mass ratio of such a single space vehicle is thus no longer limited because of the need to carry liquid oxygen (accounting for over 65% of mass) at take-off. Airbreathing engines with the highest specific impulse and thrust-to-weight ratio along with compact, light weight liquid oxygen collection systems that enable liquid oxygen be collected in flight (as late, as high and as fast as technically feasible) have emerged as advanced spaceplane technologies that could enable flight within the endo-atmosphere. Thereafter, at the limits of such airbreathing systems performance, the spaceplane continues its flight to orbit with conventional, high performance rocket engines using liquid oxygen collected and stored on board during ascent to orbit. But can the rocket equation be used to study and analyze these new concepts rigorously?

The Spaceplane Equation

The basic difference between the rocket and the spaceplane is that in the latter case, a significant part of the orbital velocity can be added more efficiently and effectively within the atmosphere. The spaceplane may be deemed to "take-off" at a higher altitude and higher speed (V_A). Hence for the spaceplane, the orbital velocity is gained in two distinct phases, the endo-atmospheric (airbreathing) phase (V_A-0), and the exo-atmospheric (rocket) phase (V_o - V_A).

$$Vi = (Va - 0) + (Vo - Va) = \Delta Va + \Delta Vr \ldots\ldots\ (5)$$

$$\text{where } \Delta Va = g.\ Isa.Ln\ (Ma / Mo)\ ;$$

$$\text{and } \Delta Vr = g.\ Isr.Ln\ (Ma / Me$$

Hence, for the spaceplane,

$$Vi = g.\ Isa.Ln\ (Ma / Mo) + g.\ Isr.Ln\ (Ma / Me)$$

$$= g.\ Isr\ \{(Isa / Isr)\ Ln\ (Ma / Mo) + Ln\ (Ma / Me)\}$$

$$\text{Let } \beta = Isa / Isr;\ Ra = Ma / Mo;\ Rr = Ma / M_E$$

$$\text{Then, } Vi = g.\ Isr\ \{\beta.\ Ln\ (Ma / Mo) + Ln\ (Ma / Me)\}$$

$$= g.\ Isr\ \{Ln\ (Ra)^\wedge \beta + Ln\ (Rr)\}$$

$$\text{Let } \zeta = (Ra)^\wedge \beta$$

$$\text{Then, } Vi = g.\ Isr\ \{Ln\ \zeta + Ln\ (Rr)\}$$

$$= g.\ Isr\ Ln\ \{\zeta.\ Rr\}$$

Taking velocity losses factor $k_L = (\Delta V_L)/ V_i$ into account is written as the Spaceplane Equation:

$$Vo = (1+k)^{-1}.g.\ Isr\ Ln\ \{\zeta.\ Rr\} \ldots\ldots\ (6)$$

$$\text{Where } \zeta = (Ra)^\wedge\beta,\ \beta = Isa / Isr,$$

$$Ra = Ma / Mo;\ Rr = Ma / M_E$$

Equation 6 is thus the spaceplane equivalent of the original rocket equation.

Comparing the Rocket and Spaceplane Equations

The two equations are:

Rocket Equation:

$$V_o = (1+k)^{-1}.g.\ I_{sr}\ Ln\ \{.\varepsilon.\ R_r\}\ldots\ldots \text{ from (4)}$$

Spaceplane Equation:

$$V_o = (1+k)^{-1}.g.\ I_{sr}\ Ln\ \{\ \zeta.\ R_r\}\ \ldots\text{from (6)}$$

It can be seen that these two equations have assumed similar forms with the introduction of the Mass Ratio Multiplier Factor. For the expendable conventional rocket, the Rocket Mass Ratio Multiplier Factor: $\varepsilon = (\pi R_i) / R$. This is introduced to account for the technology of Multistage Rockets of the 20th century.

For the reusable spaceplane, the Spaceplane Mass Ratio Multiplier Factor: $\zeta = (R_A)^{\beta}$. Here, the Mass Ratio Multiplier Factor is introduced to account for new airbreathing and oxygen collection technologies, where $\beta = I_{SA} / I_{sR}$ the specific impulse ratio; while $R_A = M_A / M_0$, the Mass Addition Ratio; and together they take into the technology of Two-Phase spaceplanes (with endo-atmospheric and exo-atmospheric flight phases) of the 21st century.

Numerical Explorations of the Spaceplane Equations

Airbreathing space planes in the last part of the 20th century emerged with the following alternate (and widely different complexity of) concepts to avoid carrying liquid oxygen as an inert mass onboard from take-off when flight to orbit could more effectively make use of the earth's atmosphere. These were:

a. In-flight re-fuelling at subsonic speeds and altitudes 15-20 kms e.g. the Black Horse Spaceplane[10] that uses a rocket engine with hydrogen peroxide/aviation kerosene as propellants. It takes-off with just sufficient oxidizer only till Aerial Tanker rendezvous. High density of oxidizer and very high oxidizer-to-fuel mixture ratio enabled compact spaceplane design, but the penalty is use of low efficiency propulsion systems.
b. Air-breathing rocket engines up to hypersonic speeds (Mach 5) and 28 kms altitude followed by pure lox/hydrogen rocket engine: the Skylon Spaceplane[11]. Take-off with 25% hydrogen fraction, and 55% liquid oxygen fraction on board. Fuel efficiency is enhanced several fold, but flight in atmosphere is restricted to Mach 5.
c. Airbreathing hydrogen fueled turbojet/ turboramjet engines with high fuel efficiency up to Mach 8 and 30 kms altitude, followed by pure lox/hydrogen

rocket engine: the Hyperplane[12, 13] and Avatar Spaceplane[14, 15]. Take-off is with 58% hydrogen fraction and zero liquid oxygen on board; and entire lox mass is added in high-speed flight. Its earlier (1988/1996) heavy-lift version was the Hyperplane.

Some Typical Results of Numerical Explorations of Spaceplane Performance Using the Spaceplane Equation

Although these different spaceplane concepts with internal mass addition (virtual in the case where lox is directly consumed by the engines and hence not stored on board, or real where the lox is stored on board) and external mass addition (using aerial tanker) are widely different conceptual approaches, they can more be coherently analyzed using the Spaceplane Equation. It is seen that:

a. Starting a new spaceplane development program based on the smallest designable vehicle makes both sound technical and economic sense. The effects of scaling and size on spaceplane payload performance are readily determined and compared using the Spaceplane Equation (with Excel spreadsheets) as a pre-cursor to numerical simulation of designs (A typical result in Figure 1). This reduces the time and cost involved in numerical simulation and yields valuable direction for simulation studies.
b. Sensitivity studies for system performance domain mapping (e.g. effect of rocket engine lox/hydrogen mixture ratio, and engine specific impulse on total velocity losses in spaceplane flight) are readily enabled by the simple Spaceplane Equation and Excel spreadsheets, thus enabling the spaceplane concept designer explore technologies for the most viable airbreathing/rocket engine combinations that maximize orbital payload delivery capabilities. This would enable the designer to proceed further in conceptual design to minimize the cost of access to space minimizing expensive use multivariable trajectory optimization studies (A typical Result is shown in Figure 2).
c. It can be seen that the Mass Ratio Multiplier Factor of a spaceplane with an airbreathing engine phase can be 2 to 4 times that of a multi-stage rocket for the same take-off weight (Table 2).
d. It can be shown that the use of airbreathing up to Mach 8 with lox collection and storage can yield mass ratios 3 to 6 times higher than a vehicle with only airbreathing engines to Mach 5 (without lox collection and storage), depending on the size of the vehicle (Table 3).
e. System performance comparison is enabled with a widely different range of spaceplane system design concepts for mass addition in flight (Table 4). For example, the effect of aerial refueling with high density, high mixture ratio oxidizer (hydrogen peroxide) from KC-135 tanker as compared to Skylon with airbreathing rocket engines and Avatar with new technologies for internally generated liquid oxygen may be discerned. The aerial refueling technique cannot be scaled-up to higher payloads, as reported[10] and hence is useful only for small satellite launching, and not for space solar power missions.

Conclusions

The Spaceplane Equation discussed herein enables the conventional Rocket Equation of 1903 be adapted for use with more complex systems and technologies of reusable spaceplanes of the 21st century. The use of this equation is illustrated employing performance data of different classes of spaceplanes as reported in the open literature. The equation will be helpful in discerning trend-lines and in obtaining immediate insights as to the limits, limitations and promises of alternate spaceplane system design concepts. It will also serve as a basic tool for rapid exploration of alternate spaceplane design concepts. The Spaceplane Equation thus paves the way to evolving a realistic technology vision for safe, affordable access to space for revenue earning missions.

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NOTES

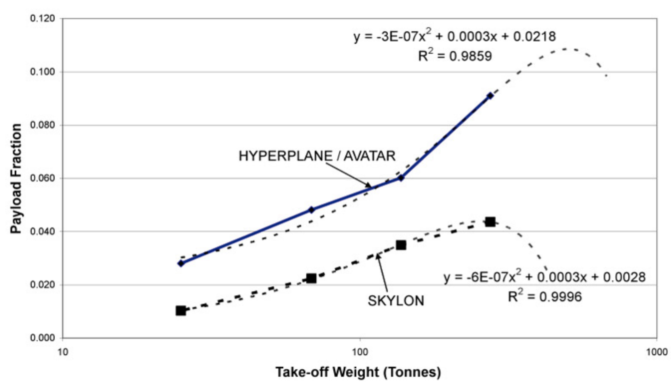

Figure 1. The Effects of Scaling and Size on Spaceplane Payload Performance.
(click image for larger view)

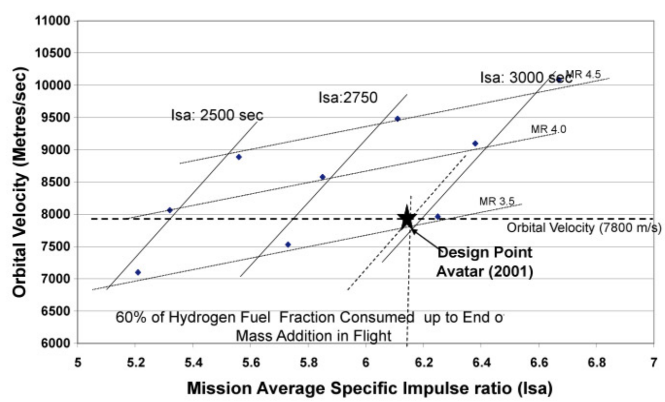

Figure 2. Typical Result of a Sensitivity Study Using the Spaceplane Equation Airbreathing/Rocket Engine Combinations to Maximize Orbital Velocity and
Payload Delivery Capability. (click image for larger view)

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   Symbols used in the Equation discussion are:

   - g: acceleration due to gravity, m/s.
   - I_s: Vehicle mission average specific impulse, (s)
   - I_{sA}: Vehicle mission average specific impulse air breathing engines, (s)
   - I_{sR}: Vehicle mission average specific impulse rocket engines, (s)
   - k_L: Velocity loss factor
   - M_o: Mass of space vehicle at take-off (t)
   - M_A: Mass of space vehicle at end of air breathing phase (t)
   - M_E: Mass of space vehicle in orbit (t)
   - R: Vehicle mass ratio (overall) = M_o / M_E
   - R_R: Vehicle mass ratio (Start Rocket Phase) = M_A /M_E
   - Π (R_I) =R_1 x R_2 x R_3…Rn: Mass ratio of multi-stage rocket with 'n' stages i=1, 2, 3…n
   - R_A: Vehicle mass ratio (End Airbreathing Phase) = M_A /M_0
   - β = I_{Sa} / I_{sR}: ratio of endo/exo atmospheric mission average specific impulse
   - ε = π (R_i) / R: Mass ratio multiplier factor (Multi-Stage Rocket)
   - $\zeta = (R_A)^\beta$: Mass ratio multiplier factor (Single-Stage-to-Orbit Spaceplane)
   - V_I: Vehicle ideal orbital velocity (m/s)
   - V_A: Vehicle Velocity at end of the endo-(Airbreathing) phase (m/s)
   - V_0: Vehicle delivered orbital velocity (m/s)
   - ΔV_A =V_A-0 =Vehicle velocity increment from take-off to end air breathing phase (m/s)
   - ΔV_R =V_0-V_A=Vehicle velocity increment end-air breathing phase to end rocket phase (m/s)
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