Optimization of the Concrete Composition Mix at the Design Stage

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Abstract

The problem of the composition optimization of concrete mixes seems to be quite urgent as errors at the composition design stage can lead to problems of concrete at the stage of exploitation such as delamination, cracking etc. Reasonable selection of concrete mix components guarantees the required strength of concrete and reinforced concrete structures in the future. This paper investigates the influence of the concrete mix composition on the strength of concrete. Firstly, typical risks that can occur on the composition design stage have been identified through the experts' interviews. Secondly, this risks were associated with indicators and characteristics that can be tested experimentally. Running of several mathematical models has allowed to outline concrete mix parameters of highest importance and formulate an empirical equation for the dependence of the strength of the concrete mixture on the values of the coarse aggregate quality factor, the fine aggregate fraction and the consumption of the Portland cement has been proposed. As a result, a methodology for controlling the quality of concrete at the stage of the composition design has been formulated.

Keywords: Concrete Mix; Coarse Aggregate Quality Factor; Fine Aggregate Fraction; Portland Cement.

1. Introduction

Design of concrete mixes composition remains one of the most difficult but obligatory stage in the production of concrete now. At this stage the durability and reliability of designs are provided. The development of science on concrete, computerization of technological and technical-and-economic calculations allow to develop and improve the calculation methods of design of compositions of concrete. Estimated compositions of concrete mixes are determined by the known empirical formulas [24]. The calculation methods of design of composition of concrete mixes are expedient, especially in the need of expeditious justification of requirement of resources and the efficiency of source materials, the decrease in labour input of the laboratory selections of structures.

The problem is that on the one hand the calculation methods of design are necessary for the production of concrete mixes, and on the other hand – the known design techniques often do not provide the accuracy of calculations and demand in each case of experimental specification before their production use.

The specified problem is caused by the lack of development and improvement of design techniques on the basis of mathematic-and-statistical methods and information technologies. The universal design methods and optimization of composition of concrete mixes are necessary. Expeditious change of the production of concrete mixes demands introduction at the enterprises of exact and universal techniques of design with the use of information technologies. The quality of the received material depends on a technique of design of a concrete mix.

Therefore, the development and improvement of calculation methods of forecasting of composition of concrete mixes is relevant, has scientific novelty and the practical importance. Optimization of composition of concrete mixes does not demand considerable capital investments, but provides the improvement of products quality and the decrease in the material inputs due to the reduction of cement consumption.

The model of design of concrete mixes composition is the mathematical model of forecasting of the key properties of concrete. It is obvious that objective and universal techniques should be based on some multiple-factor models and provide the accuracy of forecasting of composition of concrete mixes. However, the increase in the quantity of the influencing factors in models of forecasting leads to unreasonable complication of calculations, the possibility of emergence of errors and methodical mistakes, decrease in accuracy of the end results of calculations. The research and improvement of techniques of forecasting of composition of concrete mixes should consider:

- Risk-oriented approach, when choosing parameters of optimization of composition of mix;
- The rational choice of parameters of concrete mix on the basis of expert methods;
- Features of technology of receiving designs.

It is obvious that design errors in the composition of the concrete mix lead to quality decrease of the finished concrete mix as well as reinforced concrete structures. Consequently, the problem of optimization of the concrete composition mix at the design stage is of high interest. Justified selection of concrete mix cam guarantee the required strength of concrete and further reinforced concrete structures.

The purpose of the work is to study and optimize the composition of the concrete mix at the design stage. To achieve this goal, the following tasks have been completed:

- An analysis of the design model for the composition of the concrete mixture;
- An investigation of the causes of inconsistencies and assessment of risks arising from the design of the composition of the concrete mixture;
- Experimental studies of the strength of the ready-mixed concrete «BST B22.5 P2 F200 W8» when exposed to various factors;
- Optimization of the composition of the concrete mixture «BST B22.5 P2 F200 W8»;
- Development of a methodology for quality control of the concrete mix design process.

The practical significance lies in the development of a methodology for optimizing the design process for the composition of a concrete mixture, which allows managing project risks proactively before the building project starts. A common process of concrete mixture composition include a binder, water, aggregates and special additives, which are mixed together to a homogeneous state [1]. The production process of reinforced concrete structures is impossible without the stage of designing the composition of the concrete mixture [2].

The process of concrete mixes production contains the following operations [3]:

- Dosing of bulk materials;
- Mixing of bulk materials with the addition of water;
- Unloading and packaging of the finished mixture.

Analysis of the production technology of prefabricated reinforced concrete structures shows that in Great Britain, France, the Netherlands and other countries commonly use self-compacting concrete mixtures, which do not delaminate after being laid in a mold and occupies a design position under the influence of gravity without the use of vibratory installations [4-10]. A self-compacting concrete mix was developed at the University of Tokyo [11] and involves the use of chemical additives based on copolymers of acrylates and carboxylates with given grain size characteristics.

The Dutch company Schokindustrie uses a self-compacting concrete mixture based on blast-furnace slag cement «CEM III 52.5» [12], which significantly reduces the risk of structural failures during their hardening. The German company Schenk produces reinforced concrete structures of high functionality from self-compacting concrete mixture based on sulfate-resistant cements of two grades: «CEM I 42.5R-HS» and «CEM III / B 42.5 NW / HS», which reduces the risk of delamination of the concrete mixture [13]. Reinforced concrete structures of high functionality have a compressive strength of up to 150 MPa with a maximum aggregate size of 0.5 mm.

The Research University of Kassel [14] hold continuing development of reinforced concrete structures of high functionality with a compressive strength of over 150 MPa and a bending strength of about 50 MPa. This is achieved by adding different types of metal and plastic fiber reinforcement [15]. The main disadvantage of self-compacting concrete mixtures is the need for ac-curate dosing of the initial components, which is ensured by the experienced team of professionals or high-precision equipment [16-18]. Calculation of concrete strength on compression is the cornerstone of any technique of the concrete mix design. Accuracy of formulas of calculation of durability of concrete influences an optimum expense of components of concrete mix and compliance of the actual values of properties of concrete to design values.

The analysis of some known formulas, used during the calculation of durability of concrete has allowed to reveal various parameters influencing concrete durability. Several authors and researches suggest to use various parameters and empirical coefficients influencing concrete durability in design formulas: the coefficient depending on activity of cement, absolute volumes of cement and water; the volume of an air holes in concrete unit volume, the empirical coefficient considering influence of fillers on concrete durability; the activity of cement, value of the cement and water relation, density of a cement stone, empirical coefficients considering influence of fillers; consumption of cement and water, sizes of tested samples and others.

It should be noted that the majority of design formulas has common fault – they are focused on calculation of durability of concrete at the age of 28 days. Unfortunately it is not possible to calculate concrete durability at any age by the known formulas. Besides, complication of formulas and introduction of various corrections does not dismiss increase to accuracy because to consider all factors influencing durability of concrete it is almost impossible.

Thus, the following factors affect the strength of structures:

- The quality of the feedstock;
- Technological equipment;
- Qualification and expertise of production personnel;
- Technology of the production reinforced concrete structures.

Inconsistencies arising in the design of the composition of the mixture surely affect the strength of the finished reinforced concrete and structures. Coarse and fine aggregates are considered as important parts for the concrete mix composition [19, 20]. In ready-made reinforced concrete structures, the aggregate takes about 70–80% of the total volume, what significantly reduces the consumption of the «PC 400» hydraulic binder in the manufacture of «BST B22.5 P2 F200 W8» concrete mix. The coarse aggregate creates a rigid frame that absorbs stresses, which leads to a decrease in concrete shrinkage by approximately 10 times. Lack of coarse aggregate in mixture composition can lead to significant concrete deformations as it is shown on Figure 1.

Figure 1. Deformation of concrete due to lack of coarse aggregate

An important parameter of the quality of the concrete mix is the quality coefficient of the coarse aggregate. Table 1 shows the quantitative values of these coefficients.

Table 1. Coefficients of the quality of coarse aggregate

Characteristics of concrete aggregates	A1 (Cement/Water ratio < 2.5)	A2 (Cement/Water ratio > 2.5)
High quality	0.65	0.43
Medium quality	0.60	0.40
Low quality	0.55	0.37

Inconsistencies arising in the process of designing the composition of the concrete mixture, for example, a decrease in the quality factor of a large aggregate, lead to a de-crease in the strength of reinforced concrete structures. Another important parameter is the granulometric composition (sand fraction) of the fine aggregate, which affects the mobility, stiffness, workability, and formability of the concrete mixture.

One of the key indicators that determine the strength characteristics of finished reinforced concrete structures is the compressive strength of samples made from cement mortar (activity of the Portland cement). Errors leading to an underestimation of the grade (activity) of the Portland cement significantly increase the likelihood of failure of the finished reinforced concrete. An unreasonable overestimation of the grade of Port-land cement leads to its rise in price, which will ultimately affect the cost of reinforced concrete structures.

Another important parameter is the ratio of the mass of water to the mass of the Portland cement required to prepare the mixture (water-cement ratio). An overestimation of the water-cement ratio increases the delamination of the concrete mixture (Figure 2).

Figure 2. Delamination of concrete mix

Using a link diagram on Figure 3, a logical relationship has been established be-tween the design operations of the composition and inconsistencies in the finished concrete mixture.
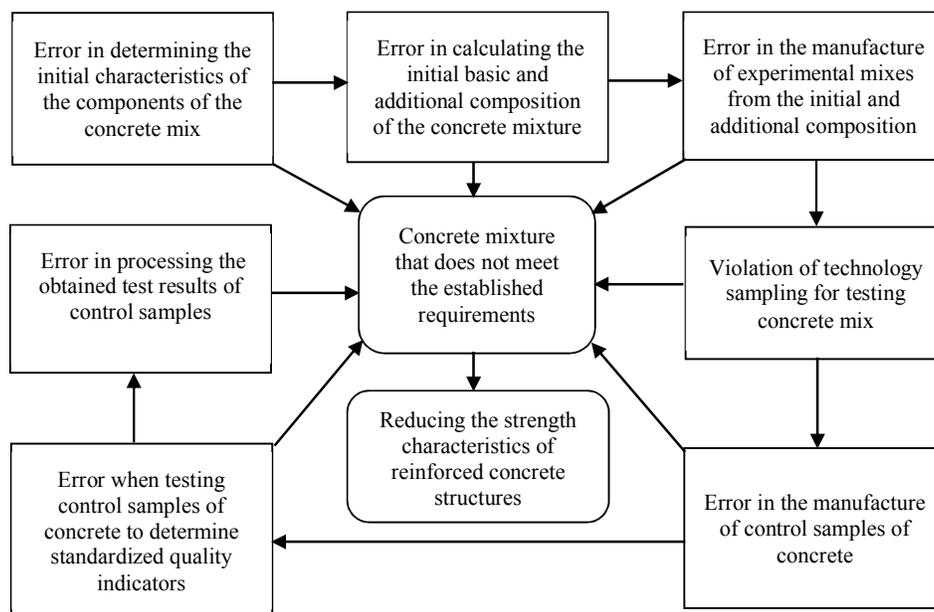

Figure 3. Relation diagram "Errors in the design of the composition of the concrete mix"

Before identifying risks, the following features of reinforced concrete structures production should be considered:

- It consists of a large number of operations (from the composition design of the concrete mixture to the stripping of ready-made reinforced concrete structures);
- Low automation of production therefore potential influence of the human factor;
- High variability of causes and consequences of failures.

2. Materials and Methods

2.1. Description of the Research Object

The object of the study is a heavy concrete mixture «BST B22.5 P2 F200 W8», the design and preparation of which is carried out in accordance with the standard requirements [21, 25, 26]. The concrete mixture is widely used for the construction of monolithic bearing walls, for the construction of strip, pile foundations, for the manufacture of stair flights, columns, reinforced concrete floor slabs, etc.

The strength of reinforced concrete structures made with «BST B22.5 P2 F200 W8» is required to be at least 28.9 MPa [19]. The quality indicators of the concrete mixture «BST B22.5 P2 F200 W8» and ready concrete made out of it are presented in Tables 2 and 3 respectively.

Table 2. Quality indicators of «BST B22.5 P2 F200 W8»

Quality indicator	Standard values		Permissible deviation of the indicator value	
Average density, kg/m^3	2 000 – 2 500		0.43	
Cone draft, cm	5 – 9		0.40	
Allowable delamination of concrete mix, %	Water separation: 0.4	Concrete segregation: 3	Water separation: +0.3	Concrete segregation: +0.5
Porosity, %	3 – 4		±1	

Table 3. Quality indicators of concrete B22.5 (made out of «BST B22.5 P2 F200 W8»)

Average density, kg/cm^2	Frost resistance, cycles	Water resistance, MPa
294.6	200	0.8

The main components of the concrete mixture «BST B22.5 P2 F200 W8» are hydraulic binder (the Portland cement «PC400»), water (free from alkalis and acids), crushed granite and clean sand (respectively - coarse and fine aggregate). The discrepancies arising at design of composition of mix influence durability of ready reinforced concrete and designs. For justification of the choice of the factors affecting quality of composition of concrete mix the formula is analysed widely applied (because of its simplicity) and recommended by normative documents:

$$R_{28} = A \cdot R_c \cdot \left(\frac{c}{w} \pm 0{,}5\right) \tag{1}$$

where R_{28} – durability of concrete aged 28, MPa; A – Coefficient of coarse aggregate quality; R_c – Activity of Portland cement, MPa; $\frac{c}{w}$ – Cement and water relation.

The analysis of Equation 1 shows that the discrepancies (mistake) arising in the designing process of composition of concrete mix including purposeful decrease in coefficient of quality of a coarse aggregate, lead to the decrease in strength indicators of reinforced concrete structures. Sand purified from clay and organic impurity is used as a fine aggregate at preparation of concrete mixes of heavy concrete. It is necessary to calculate optimum particle size distribution of a fine aggregate, namely fraction of sand during the designing process of structure. The particle size distribution (fraction of sand) substantially influences such indicators of quality of concrete mix as mobility, rigidity, placeability, molding capacity.

If the expense of the fine aggregate in concrete mix exceeds optimum value, and a consumption of granite crushed stone and the Portland cement mistakenly or is purposefully underestimated, then concrete durability considerably decreases. It is connected with the fact that the enveloping of fine particles requires a large number of a hydraulic binder. If they are not completely covered with Portland cement, then are badly linked among themselves and to a coarse aggregate. It leads to the fact that in designs there are weak points and under dynamic loads they collapse.

The activity of Portland cement is one of the key indicators defining strength characteristics of ready reinforced concrete structures. The activity (brand) represents durability on compression of the samples made of cement mortar. The mistakes leading to understating of the brand (activity) of Portland cement considerably increase the probability of refusal of ready reinforced concrete. Unreasonable overestimate of the grade of Portland cement leads to its rise in price that finally will affect the cost of reinforced concrete structures.

Water and cement are the basic components of concrete mix. We use the term water and cement ratio as the relation of mass of water to the mass of the Portland cement necessary for mix preparation. Wrong or purposeful overestimate of the water and cement ratio raises a delamination of the concrete mix that finally leads to sharp deterioration in strength qualities of ready reinforced concrete structures. The practice proves that the concrete strength index on compression of class B22.5 at the increase in mass of water decreases to class B15 concrete strength index. The excess water which has not reacted hydration in the course of solidification of concrete evaporates and is the reason of emergence of cracks and emptiness.

Water is a fluid part of concrete mix according to its properties, having small density therefore at non-compliance with proportions it filters to the very top of a design. The coarse aggregate by gravity falls down. As a result of the disproportion of mass of water and Portland cement loss of connectivity during the work with concrete mix begins. On the basis of the analysis given above for the experimental study the following the parameters of the concrete mixture have been selected: the quality coefficient of the coarse aggregate; water-cement ratio of the concrete mixture; consumption of coarse and fine aggregate, kg/m^3; the Portland cement consumption, kg/m^3; activity of the Portland cement «PC400», MPa; fine aggregate fraction, mm. The design process of the research object is shown in Figure 4.
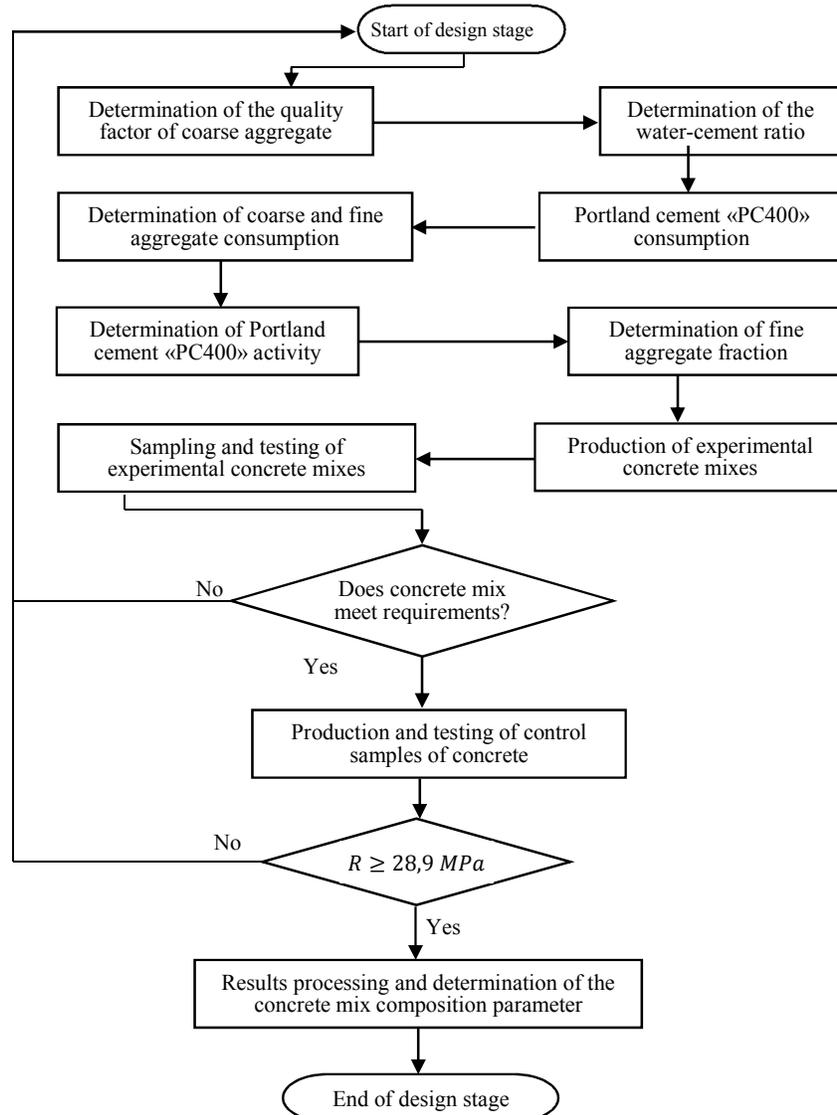

Figure 4. Design stages of concrete mix composition

2.2. Methods of Research and Optimization of Concrete Mix Composition

Expert and analytical methods have been used to detect, assess risks and optimize the heavy concrete mix composition. The expert method helped identifying potential failures at the design stage of the concrete mix composition, which can affect the quality of the concrete mixture and the strength characteristics of the finished reinforced concrete (Table 4).

Table 4. Potential risks at the design stage of the mix composition

Steps of concrete mix composition design	Risk	Reason for risk	Risk consequence
Determination of the quality factor of coarse aggregate	Decrease of quality factor	Human factor	Reduce of the concrete structures strength
Determination of the water-cement ratio	Increase of the water-cement ratio	Human factor	Tightening of the concrete mix delamination standards
The Portland cement «PC400» consumption for preparation of 1 m³ of final concrete mix	Overconsumption of the Portland cement	Human factor	Unjustified overestimation of the concrete tempering strength
Determination of coarse and fine aggregate consumption	Reduce of aggregate consumption	Human factor	Overconsumption of the Portland cement and a decrease of the concrete structures strength
Determination of the Portland cement «PC400» activity	Decreased of the Portland cement activity	Breakage of a hydraulic press	Understating the grade of the Portland cement and decrease of the concrete structures strength
Determination of fine aggregate fraction	Reduce of fine aggregate fractions	Human factor	Overconsumption of the Portland cement for the preparation of 1 m³ of concrete mixture
Production of experimental concrete mixes	Violation in manufacturing technology	Human factor	Non-compliance of concrete mix with requirements
Sampling and testing of experimental concrete mixes	Violation in test technology	Human factor	Increase of the concrete mix porosity
Production and testing of control samples of concrete	Malfunction of a hydraulic press	Breakage of the indicating device of the hydraulic press	Incorrect calculation of concrete compressive strength
Results processing and determination of the concrete mix composition parameter	Mischoice of statistical methods for results processing and optimization methods	Human factor	The composition does not provide a concrete of the required quality

Methods of mathematical statistics and design of experiments are used as analytical methods of a research for problem solving of optimization of composition of concrete mix of the BST B22.5 P2 heavy concrete of F200 W8. Application of such methodology consists in creation and the subsequent analysis of mathematical model for identification of optimum quantitative values of a subject of inquiry. The key indicator of quality of concrete in this research criterion of optimization is the durability of ready concrete of class B22.5, at compression. The realization of complete factorial experiment and creation of model of optimization of composition of concrete mix at the design stage will allow:

1) To reduce considerably the number of the laboratory tests which are carried out over controlled samples of concrete which size is 10×10×10 cm;

2) To use logical processes when carrying out the research of concrete mix of heavy concrete;

3) Apply the mathematical apparatus which formalizes actions of the experimenter;

4) To vary all the studied factors at the same time;

5) To use all factorial space properly;

6) To model the optimum composition of concrete mix taking into account the set conditions and resources.

Composition optimization of the heavy concrete «BST B22.5 P2 F200 W8» includes planning and analysis of the factorial experiment results, building a mathematical model and determining the optimal parameters of the research object. In this study, the optimization criterion was chosen to be the compressive strength of ready-mixed concrete of class «B22.5».

The choice of parameters and criteria of optimization of composition of concrete mix. By results of the carried-out expert analysis (Table 4) the major factors resulting in discrepancy of concrete mix to requirements of standard documentation, an unreasonable over expenditure of components that finally can lead to decrease in strength characteristics of ready reinforced concrete structures have been revealed. For the choice of the most significant factors having an impact on criterion of optimization (durability of ready concrete) in Table 5 the priority numbers of risks (PNR) of the studied factors are calculated.

Table 5. The studied factors with the indication of priority number of risk (PNR)

Stage of the process of the design of concrete mix composition of heavy concrete	Studied factor (parameter)	Units of measure	Priority Number of Risk (PNR)
1	2	3	4
Determination of the coefficient of quality of a coarse aggregate	Quality coefficient	–	147
Definition of fractions of a fine aggregate	Fraction of a fine aggregate	mm	135
Definition of the water and cement ratio of concrete mix	Water and cement ratio	–	108
Definition of a consumption of Portland cement for the preparation of 1 m^3 of concrete mix	PC consumption	kg per m^3	80

The factors given in Table 5 directly influence the indicators of concrete mix, strength characteristics of ready reinforced concrete and are the initial point of quality control. The wrong quantitative definition of factors will cause discrepancies at further design stages. This Table 5 has not included the following design stages:

- Production of pilot batches of concrete mix, PNR=300;
- Sampling and tests of concrete mix, PNR=288;
- Statistical processing of the received results, PNR=84.

The experience of the previous experiments has shown that the high priority number of risk of the factors stated above is caused by the fact that fabrication stages of pilot batches, sampling and test of concrete mix include a set of factors and have the high parameter of weight of consequences of discrepancies (10 and 8) respectively. Therefore it is not possible to set the range of variation of such factors. Realization of mathematical design of experiments requires performance of the following conditions:

1) The studied factors should be operated;

2) Factors should be unambiguous;

3) Lack of correlation between factors.

The first two conditions shown to the considered factors are feasible. For check of performance of the third condition correlation analysis is carried out. As a result of correlation analysis of the studied factors it is established that linear coefficients of correlation were statistically significant for the following combinations (interrelations) of factors:

– "Coefficient of quality of a coarse aggregate – a water cement ratio";

– "A water cement ratio – PC400 Portland cement consumption".

Therefore, to consider interrelation of the specified factors at mathematical design of experiments it is not allowed. Table 6 shows the controlled factors and the range of their variation during the experiment. The choice of factors is defined by the lack of a linear relationship between them, which will accurately solve the problem of optimizing the composition of the concrete mixture.

Table 6. Controlled factors and ranges of their variation

Controlled factor	Units	Legend	Factor variation range
Quality factor of coarse aggregate	–	X_1	0.51–0.65
Fine aggregate fraction	mm	X_2	2.0–2.5
Portland cement «PC400» consumption	Kg/m^3	X_3	300–400

2.3. Preliminary Results of Concrete Compressive Strength Tests

For determination of a mathematical model of a concrete mixture «BST B22.5 P4 F200 W8» with requirement to achieve the concrete strength of at least 28.9 MPa, a B-plan of the second order (Box's plan B_k) is used [22]. The planning matrix and the results of experimental studies are presented in Table 7. To reduce the influence of random errors, a uniform duplication of experiments has been carried out. Uniform duplication improves the accuracy of the experiment and greatly facilitates the statistical processing of experimental data.

Table 7. The planning matrix and the results of concrete compressive strength experimental studies

Test #	Factor value			Strength of concrete «B22.5» cubes, MPa								Dispersion, MPa²
	x_1	x_2	x_3	y_{j1}	y_{j2}	y_{j3}	y_{j4}	y_{j5}	y_{j6}	$\bar{y}_j$		s^2_j
1	-1	-1	-1	29.92	30.75	31.64	32.00	29.12	31.35	30.80		1.21
2	+1	-1	-1	28.94	31.00	31.98	29.25	32.15	29.95	30.55		1.89
3	-1	+1	-1	31.20	28.95	32.89	31.23	30.00	33.12	31.23		2.61
4	+1	+1	-1	35.50	31.09	33.00	33.45	34.00	32.08	33.19		2.35
5	-1	-1	+1	32.30	30.89	30.02	33.30	32.22	34.22	32.16		2.36
6	+1	-1	+1	30.25	32.10	33.50	31.57	34.40	31.10	32.15		2.39
7	-1	+1	+1	32.67	30.13	31.05	33.84	31.14	33.67	32.08		2.34
8	+1	+1	+1	36.42	33.00	32.22	33.38	35.58	32.00	33.77		3.32
9	-1	0	0	30.16	33.38	32.03	31.70	30.64	31.17	31.51		1.30
10	+1	0	0	32.45	35.78	34.00	33.97	36.40	33.24	34.31		2.27
11	0	-1	0	31.71	32.45	30.21	33.51	34.93	35.00	32.97		3.55
12	0	+1	0	36.69	32.85	32.50	34.12	32.20	31.95	33.39		3.20
13	0	0	-1	31.13	28.91	28.93	34.10	32.81	32.18	31.34		4.45
14	0	0	+1	35.45	31.71	32.00	36.01	31.95	31.90	33.17		3.97

2.4. Checking the Normality of the Distribution of the Output Values

To check the distribution normality, a random sample of a series of experiments have been carried out for samples-cubes with a size of 100×100×100 mm, made of a heavy concrete mix «BST B22.5 P4 F200 W8». Table 8 presents the results of this sample with n=60.

Table 8. Sampling results of experiments series on concrete «B22.5» cubes

Number of experiments series	Strength of concrete «B22.5» cubes, MPa									
	n_1	n_2	n_3	n_4	n_5	n_6	n_7	n_8	n_9	n_{10}
1	29.12	30.00	30.02	28.91	30.21	29.25	28.93	30.64	29.92	30.89
2	31.14	31.05	31.35	31.20	31.00	31.57	31.13	31.70	31.23	31.17
3	31.98	31.95	31.90	32.08	32.00	32.00	32.10	31.95	32.00	32.03
4	32.30	32.50	32.67	32.81	32.89	32.45	32.45	33.00	32.85	33.00
5	33.97	33.51	34.00	33.50	34.00	34.10	33.67	33.84	34.22	34.11
6	34.40	35.00	35.45	35.78	34.93	36.42	35.50	36.01	35.58	36.40

To check the normality of the experimental values, a statistical series have been prepared and analyzed (Table 9).

Table 9. Statistical series

Interval #	Interval boundary, MPa		Middle of interval y^*_i, MPa	Number of experiments in interval m_i	$y^*_i \cdot m_i$, MPa	$(y^*_i - \bar{y})^2$, MPa²	$m_i \cdot (y^*_i - \bar{y})^2$, MPa²
	y_{ilow}	y_{iup}					
1	28.91	29.99	29.45	5	147.25	9.30	46.50
2	29.99	31.07	30.53	7	213.71	3.88	27.16
3	31.07	32.15	31.61	18	568.98	0.79	14.22
4	32.15	33.23	32.69	10	326.90	0.04	0.40
5	33.23	34.31	33.77	10	337.70	1.61	16.10
6	34.31	35.39	34.85	3	104.55	5.53	16.59
7	35.39	36.47	35.93	7	251.51	11.76	82.32
				$n = 60$	$\sum_{i=1}^{k} = 1950$		$\sum_{i=1}^{k} = 203.29$

The results of hypothesis testing about the normality of the experimental distribution of the studied values according to the Pearson criterion x^2_{qf} are given in Table 10, which are calculated according to the commonly used statistical formulas [22]: k is the number of intervals into which the experimental data are divided; p_i is theoretical probabilities of data getting into the i-th interval; $p_i n$ is theoretical frequencies of hitting the experimental data in the i-th interval; $\varphi_0(Z)$ is distribution function for the normal law.

Table 10. Results of calculations by Pearson's test

Interval #	Interval lower boundary y_{ilow}, MPa	Interval upper boundary y_{iup}, MPa	Empirical frequency m_i	z_1	z_2	$\Phi_0(z_1)$	$\Phi_0(z_2)$	p_i	$p_i n$	$(m_i - p_i n)^2$	$(m_i - p_i n)^2 / p_i n$
1	28.91	29.99	5	-1.93	-1.35	0.0268	0.08851	0.06171	4	1	0.25
2	29.99	31.07	7	-1.35	-0.77	0.08851	0.22065	0.13214	8	1	0.12
3	31.07	32.15	18	-0.77	-0.19	0.22065	0.42465	0.204	13	25	1.92
4	32.15	33.23	10	-0.19	0.39	0.42465	0.65173	0.22708	13	9	0.69
5	33.23	34.31	10	0.39	0.97	0.65173	0.83398	0.18225	11	1	0.09
6	34.31	35.39	3	0.97	1.55	0.83398	0.93943	0.10545	6	9	1.50
7	35.39	36.47	7	1.55	2.13	0.93943	0.98341	0.04398	3	16	4.33
											$\sum_{i=1}^{k} = 8.9$

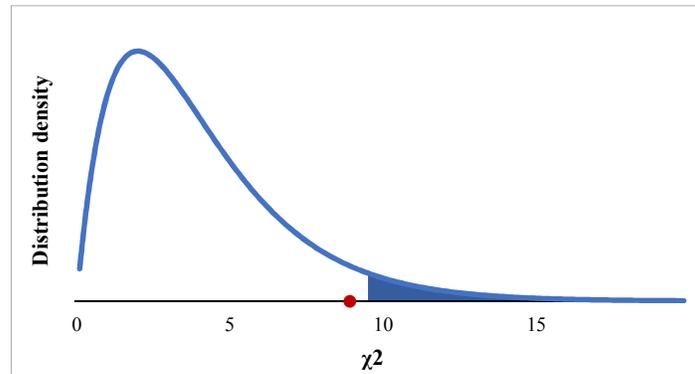The distribution graph of random variables is given on Figure 5.

Figure 5. Distribution graph of random variables

The tabular value of the Pearson test is $x^2_{qf} = 9{,}49$ [22]. Figure 5 shows that the value of the criterion $x^2_{calculated} = 8.9$ does not go beyond the specified limits ($x^2_{qf} = 9{,}49$.). The condition $x x^2_{calculated} \leq x^2_{qf}$ is met, therefore, the hypothesis of the normal distribution of the sample is accepted.

3. Results

It is necessary for modeling of a designing process of composition of concrete mix to provide that the chosen class of function had some "smoothness", i.e. "small" changes of values of factors (arguments) should lead to "small" changes of values of function (response). The durability of ready reinforced concrete structures acts as the response. The nonlinear equation of regression which statistical analysis is made by method of "nonlinear regression" was investigated for creation of model of optimization.

Now two main methods for the set of the nonlinear equations of regression are known, including linearization and approximation. Further creation of mathematical model has been based on approximation of the used dependences by polynoms (polynomials) of the second degree. The previous experience of design of experiments has shown that approximation of dependences polynomials of a higher order leads to complication of model of optimization and increase in labour input at insignificant increase in accuracy of processing of experimental data. The process of designing the composition of the concrete mix can be represented as a functional dependence $Y_i = f(x)$, the output parameter of which will be the optimization criterion - the strength of the structure (Y_i). Then the model of the concrete strength change from the factors under study (see Table 6) can be represented by the following regression equation:

$$\hat{y} = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 + b_{12} x_1 x_2 + b_{23} x_2 x_3 + b_{13} x_1 x_3 + b_{11} x_1^2 + b_{22} x_2^2 + b_{33} x_3^2 \tag{2}$$

The analysis of the mathematical model (1) has been carried out according to the regression coefficients with the normalized (coded) values of the factors (Table 6). To find the coefficients of the regression Equation 2, the system of Equations 3 has been solved:

$$\begin{cases} b_0 = T_1(Oy) - T_2 \sum_{i=1}^{k}(iiy) = T_1 \sum_{j=1}^{N} \bar{y}_j - T_2 \sum_{j=1}^{N}[x_1^2 \bar{y}_j + x_2^2 \bar{y}_j + x_3^2 \bar{y}_j]; \\ b_1 = T_3(iy) = T_3 \sum_{j=1}^{N} x_1 \bar{y}_j ; \\ b_2 = T_3(iy) = T_3 \sum_{j=1}^{N} x_2 \bar{y}_j ; \\ b_3 = T_3(iy) = T_3 \sum_{j=1}^{N} x_3 \bar{y}_j ; \\ b_{12} = T_6(iuy) = T_6 \sum_{j=1}^{N} x_1 x_2 \bar{y}_j ; \\ b_{23} = T_6(iuy) = T_6 \sum_{j=1}^{N} x_2 x_3 \bar{y}_j ; \\ b_{13} = T_6(iuy) = T_6 \sum_{j=1}^{N} x_1 x_3 \bar{y}_j ; \\ b_{11} = T_4(iiy) + T_5 \sum_{i=1}^{k}(iiy) - T_2(Oy) = T_4 \sum_{j=1}^{N} x_1^2 \bar{y}_j + T_5 \sum_{j=1}^{N}[x_1^2 \bar{y}_j + x_2^2 \bar{y}_j + x_3^2 \bar{y}_j] - T_2 \sum_{j=1}^{N} \bar{y}_j; \\ b_{22} = T_4(iiy) + T_5 \sum_{i=1}^{k}(iiy) - T_2(Oy) = T_4 \sum_{j=1}^{N} x_2^2 \bar{y}_j + T_5 \sum_{j=1}^{N}[x_1^2 \bar{y}_j + x_2^2 \bar{y}_j + x_3^2 \bar{y}_j] - T_2 \sum_{j=1}^{N} \bar{y}_j; \\ b_{33} = T_4(iiy) + T_5 \sum_{i=1}^{k}(iiy) - T_2(Oy) = T_4 \sum_{j=1}^{N} x_3^2 \bar{y}_j + T_5 \sum_{j=1}^{N}[x_1^2 \bar{y}_j + x_2^2 \bar{y}_j + x_3^2 \bar{y}_j] - T_2 \sum_{j=1}^{N} \bar{y}_j, \end{cases} \tag{3}$$

where $T_1, T_2, \dots, T_5$ – coefficients of different types of experimental design, the value of which is given in Table 11.

Analyzing the Equation 2 it is possible to draw a conclusion that application of the plan of the second order allows to describe dependence of output size – concrete durability at compression from each studied factor or their combinations in the form of the parabola equation.

According to the known formula have expressed natural X_i values of entrance factors the coded x_i values and have received the system of the Equation 4.

$$\begin{cases} x_1 = \frac{X_1 - 0,58}{0,07}; \\ x_2 = \frac{X_2 - 2,25}{0,25}; \\ x_3 = \frac{X_3 - 350}{50}. \end{cases} \tag{4}$$

Table 11. Values of $T_1, T_2, \dots, T_6$ coefficients

Ti	B-plan (*k*=3, *N*=14) (Box's plans)
T1	0,40624
T2	0,15624
T3	0,1
T4	0,5
T5	−0,09375
T6	0,125

By substitution of the natural values with the factors from the Table 6, the following values of the coefficients of the regression Equation 2 have been calculated:

$$\begin{cases} b_0 = 0,40624 \cdot 452,62 - 0,15624 \cdot 964,48 = 33,18; \\ b_1 = 0,1 \cdot 6,19 = 0,62; \\ b_2 = 0,1 \cdot 5,03 = 0,50; \\ b_3 = 0,1 \cdot 6,22 = 0,62; \\ b_{12} = 0,125 \cdot 3,91 = 0,49; \\ b_{23} = 0,125 \cdot (-1,53) = -0,19; \\ b_{13} = 0,125 \cdot (-0,03) = 0,004; \\ b_{11} = 0,5 \cdot 321,75 - 0,09375 \cdot 964,48 - 0,15624 \cdot 452,62 = -0,26; \\ b_{22} = 0,5 \cdot 322,29 - 0,09375 \cdot 964,48 - 0,15624 \cdot 452,62 = 0,008; \\ b_{33} = 0,5 \cdot 320,44 - 0,09375 \cdot 964,48 - 0,15624 \cdot 452,62 = -0,92. \end{cases} \tag{5}$$

It should be noted that for the polynom of the second degree which is written down in the normalized (coded) designations, extent of influence of the considered factors or their combinations on function of a response – durability of concrete is defined by the size of the found coefficients of b_i of the equation of regression (model). For further

evaluation of the significance of the regression coefficients the variances of the reproducibility of the experiments have been calculated

$$G_{calculated} = \frac{s_{max}^2}{s_1^2 + s_2^2 + s_3^2 + \cdots + s_{14}^2} = \frac{4.45}{37.21} = 0.12 \tag{6}$$

Considering that the number of samples N=14, the value $G_{table} = 0,23$. The condition $G_{calculated} < G_{table}$ is met; therefore, the hypothesis of homogeneity of variances is accepted. To assess the significance of the coefficients of the regression Equation 3, the reproducibility variance $s^2\{y\}$ has been calculated as the arithmetic mean of the variances of the experiments:

$$s^2\{y\} = \frac{s_1^2 + s_2^2 + s_3^2 + \cdots + s_{14}^2}{N} = \frac{37.21}{14} = 2.66 \, MPa \tag{7}$$

In this study, the coefficients b_{23}, b_{13}, b_{11}, b_{22} turned out to be insignificant; therefore, the corresponding variables $b_{23}x_2x_3$, $b_{13}x_1x_3$, $b_{11}x_1^2$, $b_{22}x_2^2$ are to be excluded from the regression Equation 2 from updated regression Equation 8:

$$\hat{y} = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_{12}x_1x_2 + b_{33}x_3^2 \tag{8}$$

Evaluation of the equation according to the Fisher criterion confirms the adequacy of Equation 8, which is further used to optimize the composition of the concrete mixture.

4. Discussions of Concrete Mix Composition Optimization

The mathematical model (6) in normalized (coded) notation is described using Equation 9:

$$\hat{y} = 33,18 + 0,62x_1 + 0,50x_2 + 0,62x_3 + 0,49x_1x_2 - 0,92x_3^2 \tag{9}$$

For the graphic interpretation of the results of the experiment, the canonical transformation of Equation 9 has been performed [23]. Necessary conditions for the analysis of mathematical model are presented in Table 12.

Table 12. Conditions for the analysis of mathematical model [22]

№	Condition	Description
1	2	3
1	$b_{ii} > 0$	The polynom of the second degree (the parabola equation) describes concave function which branches are directed upwards
2	$b_{ii} < 0$	The polynom of the second degree (the parabola equation) describes convex function which branches are directed downwards
3	$\|b_i\| > 2\|b_{ii}\|$	The top of function is out of the range of variation of the studied x_i factor – the mathematical model describes monotonic function
4	$b_i > 0$	Function monotonously increases on the explored site
5	$b_i < 0$	Function monotonously decreases on the explored site
6	$\|b_i\| < 2\|b_{ii}\|$	Function has an extremum in the range of variation of the studied factor x_i: if $b_{ii} < 0$ – maximum; if $b_{ii} > 0$ – minimum

In an assay value of the Equation 9 the following results are received. Three families of graphs of the dependence of the response function $\hat{y}$ on each factor (x_1, x_2, x_3) have been constructed (Table 13).

Table 13. Results of the canonical transformation of Equation 9

$\hat{y} = f(x_i)$	Coded values of factors x_i	Regression equation	Dependence of concrete strength on factors x_i	
$\hat{y} = f(x_1)$	$x_3 = 0;$ $x_2 = -1; 0; 1$	$\hat{y} = 33,18 + 0,62x_1 + 0,50x_2 + 0,49x_1x_2$	$\begin{cases} \hat{y} = 32,68 + 0,13x_1; \\ \hat{y} = 33,18 + 0,62x_1; \\ \hat{y} = 33,68 + 1,13x_1. \end{cases}$	(9.1)
$\hat{y} = f(x_2)$	$x_1 = 0;$ $x_3 = -1; 0; 1$	$\hat{y} = 33,18 + 0,50x_2 + 0,62x_3 - 0,92x_3^2$	$\begin{cases} \hat{y} = 31,64 + 0,50x_2; \\ \hat{y} = 33,18 + 0,50x_2; \\ \hat{y} = 32,88 + 0,50x_2. \end{cases}$	(9.2)
$\hat{y} = f(x_3)$	$x_2 = 0;$ $x_1 = -1; 0; 1$	$\hat{y} = 33,18 + 0,62x_1 + 0,62x_3 - 0,92x_3^2$	$\begin{cases} \hat{y} = 32,56 + 0,62x_3 - 0,92x_3^2; \\ \hat{y} = 33,18 + 0,62x_3 - 0,92x_3^2; \\ \hat{y} = 33,80 + 0,62x_3 - 0,92x_3^2. \end{cases}$	(9.3)

Analyzing the systems of Equations 9.1 to 9.3, the following conclusions can be drawn. The first family of functions (9.1) in Table 13. The graphic dependence was considered $\hat{y}$ from x_1 at various values $x_2 = -1; 0; 1$ and the fixed level of the factor $x_3 = 0$. Taking into account these conditions the equation of regression is described by the means of Equation 10. $\hat{y} = f(x_1); x_3 = 0; x_2 = -1; 0; 1$.

$$\hat{y} = 33.18 + 0.62x_1 + 0.50x_2 + 0.49x_1x_2 \tag{10}$$

At various values of the factor $x_2 = -1; 0; 1$ the dependence of the concrete durability on coefficient of quality of a coarse aggregate is linear and is described by the system of the Equation 9.1 in Table 13. Analyzing the received system of the Equation 9.1, we can draw the following conclusion. With an increase in the value of the quality factor of the coarse aggregate corresponding to the coded value of the factor x_1, the concrete strength increases. The optimal value of the factor is $x_{1optimal} = 1$, since in all equations of system (9.1) the coefficient $b_1 > 0$.

The second family of functions. The graphic dependence was considered $\hat{y}$ from x_2 at various values $x_3 = -1; 0; 1$ and the fixed level of the factor $x_1 = 0$. Taking into account these conditions the equation of regression is described by the Equation 11.

$$\hat{y} = f(x_2); x_1 = 0; x_3 = -1; 0; 1$$
$$\hat{y} = 33,18 + 0,50x_2 + 0,62x_3 - 0,92x_3^2 \tag{11}$$

At various values of the factor $x_3 = -1; 0; 1$ the dependence of the concrete durability on the average diameter of particles of a fine aggregate is linear and is described by the system of the Equation 9.2 in Table 13. Analyzing the received system of the Equation 9.2, we can draw the following conclusion. With an increase in the fraction of fine aggregate corresponding to the coded value of the factor x_2, the strength of the concrete will increase. The optimal value of the factor is $x_{2\ optimal} = 1$, since in all models in Equation 9.2 the coefficient $b_2 > 0$.

The third family of Equation 9.3 in Table 13. The graphic dependence was considered $\hat{y}$ from x_2 at various values $x_1 = -1; 0; 1$ and the fixed level of a factor $x_2 = 0$. Taking into account these conditions the equation of regression is described by means of Equation 12.

$$\hat{y} = f(x_3); x_2 = 0; x_1 = -1; 0; 1$$
$$\hat{y} = 33,18 + 0,62x_1 + 0,62x_3 - 0,92x_3^2 \tag{12}$$

At various values of the factor $x_1 = -1; 0; 1$ the dependence of durability of concrete on a consumption of PC400 Portland cement is square and is described by the system of the Equation 9.3 in Table 13. The analysis of this system of the Equation 9.3 has shown change of the free member upwards. Therefore, it is possible to draw the following conclusion. With an increase in the consumption of the Portland cement in the heavy concrete mix «BST B22.5 P2 F200 W8», corresponding to the coded value of the factor x_3, the strength of the finished concrete will increase. The optimal value of the factor x_3 is determined using the following Equation 13:

$$x_{optimal\ 3} = \frac{-b_3}{2 \cdot b_{33}} = \frac{-0.62}{2 \cdot (-0.92)} = 0.34. \tag{13}$$

To find the optimal strength of concrete of «B22.5» class, the optimal values of the quality factor of the coarse aggregate, the average particle diameter of the fine aggregate (fine aggregate fraction), and the consumption of the Portland cement in the concrete mixture of heavy concrete were added into Equation 9. As a result, the optimal concrete strength was $y_{optimal} = 34.89$ MPa. The optimal values of the studied factors have been checked in the program for statistical analysis Statistica. Sections of the surfaces of optimal values are presented in Figure 6.
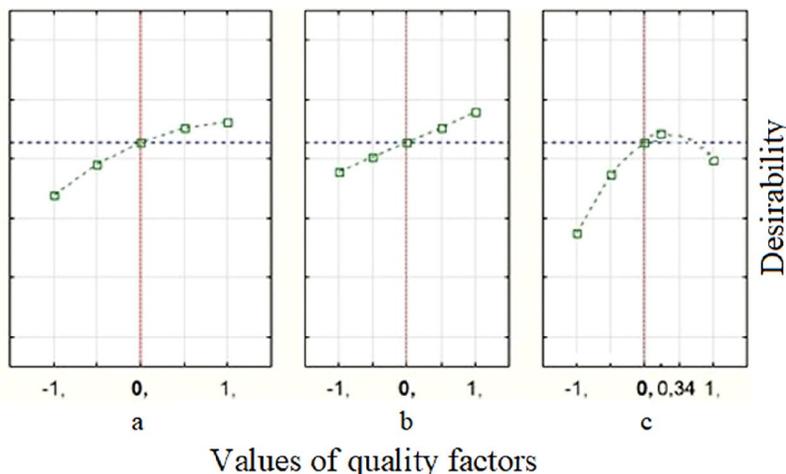

Figure 6. Surfaces of optimal values of the factors under study: a) quality coefficient of coarse aggregate, b) average particle diameter of fine aggregate (fine aggregate fraction), c) consumption of the Portland cement

The final stage of checking the optimal values of the quality factors is to construct surfaces and desirability contours, which are shown in Figures 7 to 9. Table 14 presents the standard estimates on the desirability scale.

Table 14. Standard estimates on the desirability scale

Desirability level	Estimates
Very well	0.80 d < 1.00
Good	0.63 d < 0.80
Satisfactory	0.37 d < 0.63
Bad	0.20 d < 0.37
Very bad	0.00 d < 0.20
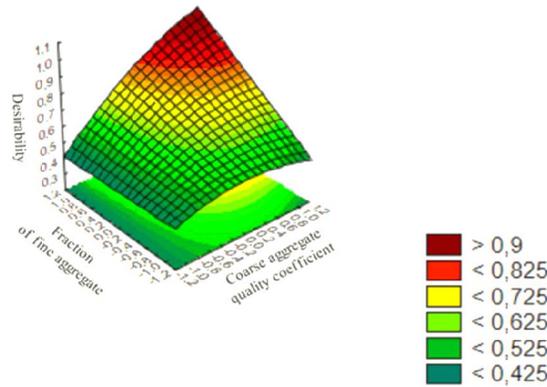
Figure 7. The surface and the desirability contour of the two-factor dependency: "Fraction of fine aggregate – Quality coefficient of coarse aggregate"
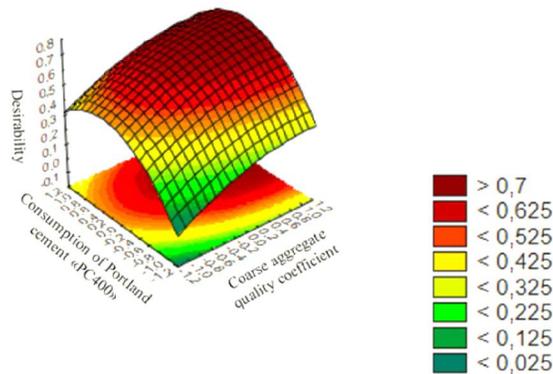

Figure 8. The surface and the desirability contour of the two-factor dependency: "Consumption of the Portland cement «PC400» – Quality coefficient of coarse aggregate"
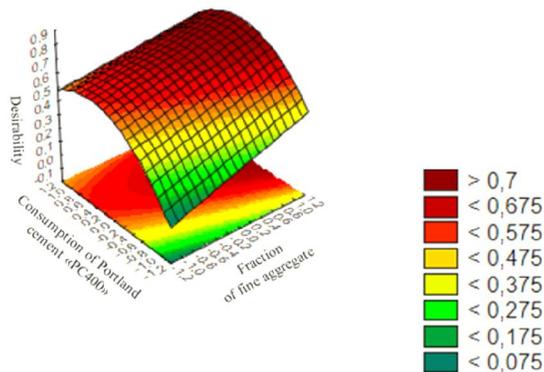

Figure 9. The surface and the desirability contour of the two-factor dependency: "Consumption of the Portland cement «PC400» – Fine aggregate fraction"

Analysis of Figures 7 to 9 showed that the assessment on the desirability values vary from 0.7 to 1. This allows concluding that all the studied factors of the concrete mix are selected correctly and can provide the optimal strength of the finished reinforced concrete. Thus, the optimal values of the quality factors for the preparation of 1 m^3 of «BST B22.5 P2 F200 W8» are as follows:

1. The coarse aggregate quality factor - 0.65;

2. The average particle diameter of fine aggregate (aggregate fraction) - 2.5 mm;

3. The consumption of Portland cement «PC400» - 367 kg/m^3.

5. Conclusions

As a result of conducted research, several conclusions can be outlined. The reasons of errors in the process of concrete mix composition design have been investigated as well as the importance of monitoring of the following factors:

● The coarse aggregate quality factor;

● The water-cement ratio of concrete mix;

● The consumption of coarse and fine aggregate, kg/m3;

● The Portland cement consumption, kg/m3;

● The activity of Portland cement, MPa;

● The fine aggregate fraction, mm.

Potential risks of the design stage of concrete mix composition, the consequences of which are mentioned below, have been identified:

● Tightening of the standards for the delamination of concrete mix;

● Non-compliance of the concrete mixture with regulatory requirements;

● Increase of the concrete mix porosity;

● Composition does not provide concrete mix of required quality.

During the implementation of the full factorial experiment, a second-order regression model has been defined:

$\hat{y} = 33.18 + 0.62x_1 + 0.50x_2 + 0.62x_3 + 0.49x_1x_2 - 0.92x_3^2$.

The following coefficients (X$_{1-3}$) of the regression equation establish the dependence of the output parameter (concrete strength on the input factors): X$_1$ – quality coefficient of coarse aggregate, X$_2$ – fine aggregate fraction, X$_3$ – consumption of the Portland cement «PC400».

Using mathematical planning of the experiment and the Statistica software environment, the output parameter of the regression equation has been optimized, and the optimal values of the factors under study have been calculated for the preparation of 1 m^3 of concrete «BLS B22.5 P2 F200 W8»:

● The coarse aggregate quality factor - 0.65;

● The average particle diameter of fine aggregate (aggregate fraction) - 2.5 mm;

● The consumption of Portland cement PC400 - 367 kg/m3.

The optimization results allowed developing a procedure for quality control of the concrete mix design process, which includes the following stages:

1. The first preparatory stage includes evaluates the selection of the characteristics of the initial components and monitor of the concrete mix composition calculations;

2. The second main stage includes control over the manufacture and testing of experimental concrete mixes and samples of heavy concrete;

3. The final third stage of quality control includes statistical processing and optimization of the results, such as the quality coefficient of the coarse aggregate, the average particle diameter of the fine aggregate (aggregate fraction), and the consumption of the Portland cement.

6. Declarations

6.1. Author Contributions

L.L. responsible for development of the research design what includes the overall methodology for error identification and risk management in the design of the composition of the concrete mix; N.O. the experiment itself – she has formed the questionnaire, organized the expert interviews, and processed its results. Moreover, she has provided the first draft of the article; T.S. was responsible for experiment design and organization; L.K. has processed all the research results with required statistical methods, calculated models and equations, provided graphics for research illustration and finalized article text. All authors have read and agreed to the published version of the manuscript.

6.2. Data Availability Statement

The data presented in this study are available in article.

6.4. Conflicts of Interest

The authors declare no conflict of interest.

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