



Optimization of the Ultimate Bearing Capacity of Reinforced Soft Soils through the Concept of the Critical Length of Stone Columns

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Abstract

The objective of this paper is to develop an analytical equation based on the concept of the critical-length of columns in order to optimize the ultimate bearing-capacity of soft soils, supporting a strip footing and reinforced by a group of floating stone columns. Optimization procedure was performed on three-dimensional numerical models simulated on the Flac3D computer code, for various soft-soils with different undrained-cohesions ($C_u=15-35\text{kPa}$), reinforced by columns of varying lengths (L) and area replacement ratio ($A_s=10-40\%$), considering different footing widths B . Obtained results indicate that the optimal bearing-capacity ratio (Ultimate bearing-capacity of reinforced soil/unreinforced soil) is reached for the column critical-length ratio (L_c/B) and increase with increase of the later ratio, depending on A_s and C_u . Analysis of results also showed that the optimal values of the bearing-capacity ratio in the reinforced soils remain bounded between the lower and higher values (1.28-2.32), respectively for minimal and maximal values of the critical-length ratio (1.1) and (4.4). Based on these results, a useful analytical equation is proposed by the authors, for the expression of the critical-length; thus ensuring an optimal pre-dimensioning of the stone columns. The proposed equation was compared with the data available in the literature and showed good agreement.

Keywords: Stone Columns; Critical-length; Footing; Ultimate Bearing Capacity; Numerical Simulation; Soil Reinforcement.

1. Introduction

Stone column technique has been widely used to improve the mechanical behavior of difficult foundation sites throughout the world. The method has a particular application in soft cohesive soils and loose silty sands [1]. The general concept of the technique is to replace 10 to 40% of the soft-soil with cylinders that contains compacted granular material, which has more strength, better stiffness as well as higher permeability compared to the native soil. Stone column method has the advantage of increasing the bearing-capacity, reducing both reduces the total and the differential settlements. In addition, the positive effect of this technique has been revealed by accelerating soil consolidation and reducing the liquefaction potential.

Hanna et al. (2013) [2] and Hu (1995) [3], indicate that the ultimate bearing-capacity of the reinforced soil and the failure mode of stone columns under vertical loads depend on both the geometric and materiel parameters of the native soil as well as the stone column. Furthermore, some investigations have reported that the potential failure modes of a single stone column can be categorized as bulging, general shear failure and punching [4-6]. While for the group of stone columns and because of the interactions between them, the columns fail in a conical form under the foundation [7].

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Many analytical and numerical approaches have been developed for predicting the ultimate bearing-capacity of soft-soils reinforced by stone columns, in which the physical model was usually proposed uses five principal approaches, summarized as: 1) unit cell method, which consider only the stone column and the surrounding soil attributed to it [8]; 2) plane strain approach, in the later, the group of stone column is converted to gravel trenches, and used generally for embankments and long loads [9]; 3) Homogenization is the approach, where the columns and the surrounding soil are assumed as an equivalent homogeneous soil with equivalent properties [10, 11]; 4) Axial-symmetry is a technique, which convert the cylindrical stone column to gravel ring and is generally used for circular foundations [12, 13]; 5) The three-dimensional model, for this case the reinforced-soil is modeled as a real configuration and geometry [14].

In the same context, Bouassida et al. (2009) [15] introduced the limit analysis method for evaluation of the stone column bearing-capacity by using homogenizations technique. Etezad et al. (2015) [11] developed a theory based on Terzaghi's equation using limit-equilibrium method.

For numerical studies, finite element and finite difference method codes were used for predicting the bearing-capacity of stone columns in reinforced-soils for two and three-dimensional models [16]. Zhou et al. [17] adopted a numerical simulation to study the effects of surcharge on the mode of failure, and investigated the bearing-capacity factors of a strip footing supported by a group of floating stone column. Lee and Pande (1998) [18] have used the homogenization technique to predict the bearing-capacity of the soil-reinforcement system based on elastoplastic analysis. Besides, In order to investigate the effects and the performances of stone column reinforcement on the bearing-capacity of a raft foundation supported by a single and group of stone columns: a parametric study was conducted by Hanna et al. (2013) [2].

In addition to the previous approaches, many experimental investigations have been carried out to predict the mechanical behavior of soils reinforced by stone columns [19, 20]. A number of experimental tests have been conducted by Bora (2014) [21], in soft clay improved by a group of stone columns. Based on these results, an analytic relation was proposed to relate the bearing-capacity of reinforced and unreinforced soil. Hu (1995) [3] realized a large experimental study and investigated both mechanical behavior and failure mode of stone columns. Whereas; a general equation to calculate the ultimate bearing-capacity of single and group of floating stone columns has been derived, based on a large experimental performed by data Fattah et al. (2017) [22].

The previously mentioned methods showed that the geometric and material properties of the foundation, the soft-soil and the stone column are important factors that may affect the bearing-capacity of stone columns. Some studies have shown that the maximum bearing-capacity has been reached for a special length of the stone column which was called the critical/optimum length. Furthermore, for stone column length greater than the critical-length, the ultimate bearing-capacity does no improve [4]. Values of critical-length were reported in the literature by many authors. Najjar et al. (2010) and Black et al. (2007) [23-25] reported that the value of the critical-length could be reached for a ratio ($L/D=6$), where (L) and (D) are the length and the diameter of the column respectively. Hughes and Withers (1974) [4] Found that no increase in the bearing-capacity was noted up to ($L/D=4$), while Zhou et al. (2017) and Das (1989) [17, 26] used the ratio of column length to footing width, in order to define the critical-length; it was reported that this value is reached for ($L/B=2$) and ($L/B=3$), respectively.

It should be noted that in these studies, the values of the critical-length were determined without taking into account the effects of varying other parameters such as: Area replacement ratio (A_s), undrained-cohesion of the surrounding soil (C_u), and footing width (B). The analysis of the critical-length is frequently performed using the column length to diameter ratio (L/D), which is less representative because of the small influence of this ratio on the load-displacement behavior. For better presentation of the results it is recommended to replace (L/D) by the length to footing width ratio (L/B) as reported by Castro (2014) [27]. Therefore, the presented values could be considered as approximate values that differ from one study to another.

Based on the different methods mentioned above, it could be concluded that the existent analytical methods developed for the estimation of the bearing capacity of soft-soils reinforced by a group of stone columns do not take into account the type of stone column (i.e., floating or end bearing) as well as the effect of the length ratio of the column taken either over its diameter, or over the width foundation [8, 22].

Moreover, the prediction of the bearing capacity by the homogenization technique is more precise compared to the analytical models, Since for this method, the influence of the columns is distributed overall the reinforced zone, the length of the stone column could be greater than its critical length for certain cases, resulting in a none optimal design of stone columns. Besides, Experimental studies are more reliable and relatively precise approaches. However, for model that incorporates a large number of parameters, these studies become more expensive and their achievement takes much more time because of very large number of small scale tests to perform.

The increase in computer power has encouraged the use of the Finite Element and Finite Difference Method in the prediction of the bearing capacity of foundation placed on soil reinforced by a group of stone columns. The three-

dimensional numerical models is more appropriate and has the advantage to preserve the physical nature of the problem and. However, three dimensional models involve greater complexity in the model setup and therefore require good numerical knowledge to perform a reliable analysis. Nevertheless, these methods take a lot of time for large geometry models implicating a higher number of iterative tests.

The aim of this paper is to develop a simple equation to estimate the critical-length of column, for the determination of the optimal bearing-capacity of soft cohesive soils reinforced by a group of stone column, by taking into consideration the effects of the geometric and material properties. The proposed equation is built up using the data gathered from a series of numerical simulations, using a finite difference code Fast Lagrangian Analysis of Continua in 3 Dimensiones [28]. Effects of influencing parameters on the bearing-capacity ratio are also presented and analysed. The proposed equation was validated with the available experimental and numerical data and it showed a good agreement.

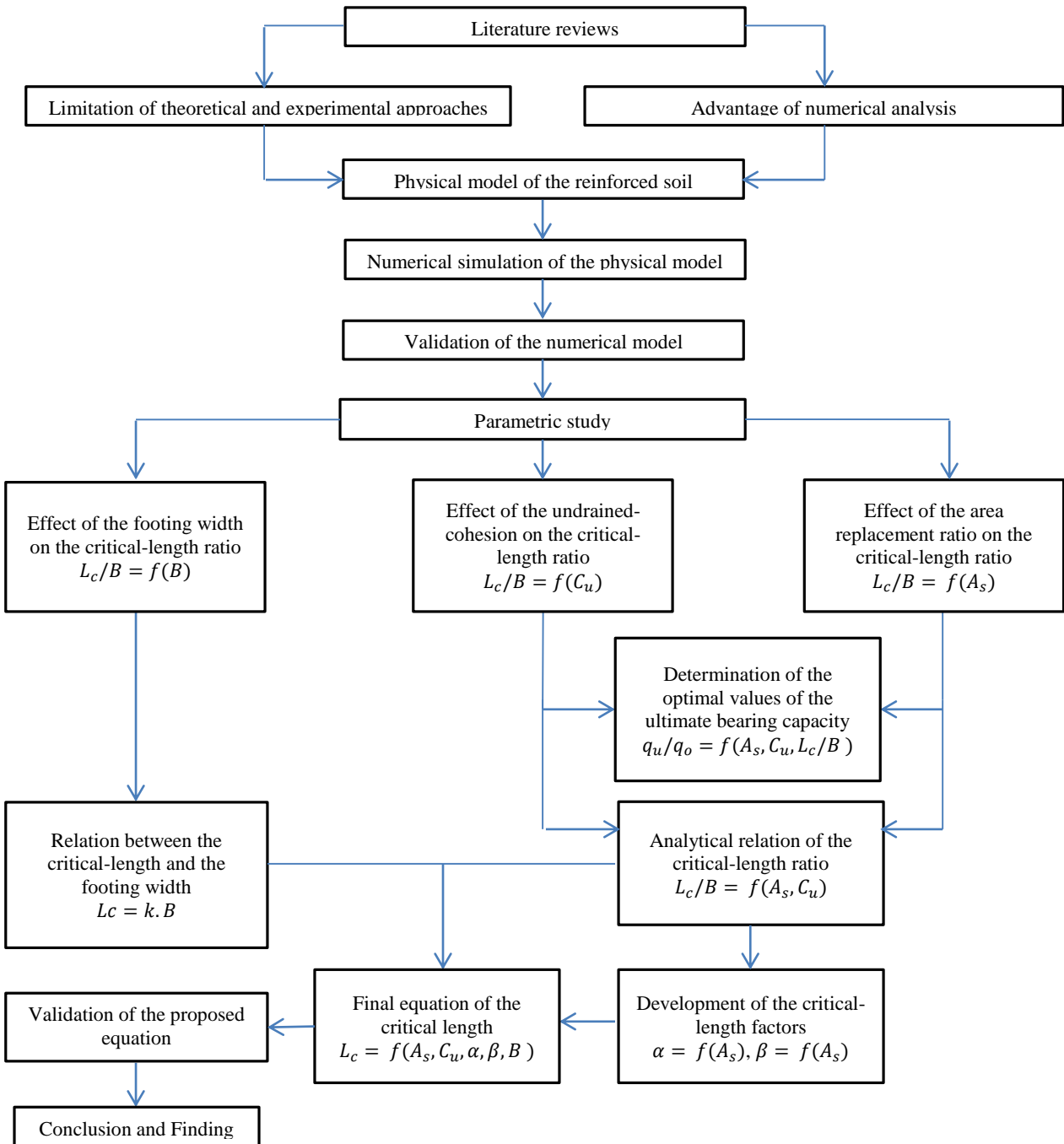


Figure 1. Flowchart of the research methodology

2. Numerical Modeling

2.1. Physical Model of Parameters

The physical model of study consists of a strip rigid footing of width (B) subjected to a vertical load (P), resting on a soft cohesive soil reinforced by a group of (N) floating stone columns of diameter (D), length (L) and spacing (S) as shown on Figure 2. The area replacement ratio of the model is then defined as the area of stone columns over the area of the footing [2], and expressed in Equation 1 as:

$$A_s = (N \cdot \pi \cdot D^2) / (4 \cdot B \cdot S) \quad (1)$$

The mechanical and physical properties of the materials used are denoted as: friction angle γ_i : unit weight, Ψ_i : angle of dilatancy, C_u : Cohesion, E_i : modulus of elasticity, ν_i : Poisson ratio; where the subscripts $i=c, s$ refers to the stone column and the soft-soil respectively. The stone columns are formed by a cohesionless material ($C_u=0$), while the soft-soil is considered frictionless ($\varphi_i=0$).

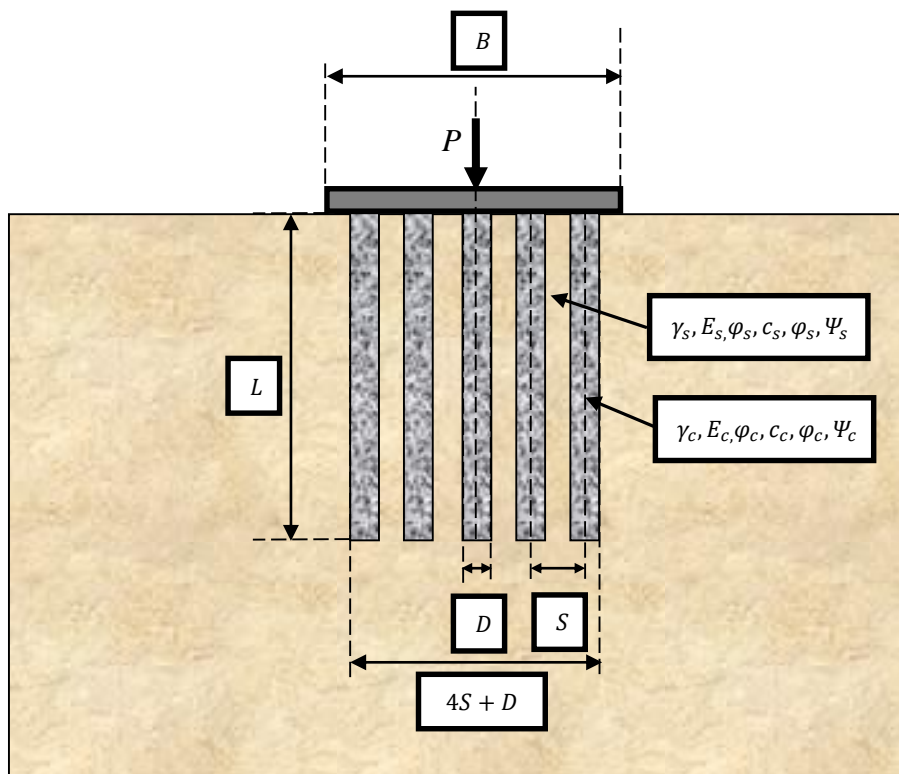


Figure 2. Geometrical and mechanical properties of the model

2.2. Numerical Procedure

The physical model presented in the figure above is numerically simulated, using a three-dimensional Finite Difference code [28] (Fast Lagrangian Analysis of Continua) in order to estimate the ultimate bearing-capacity of a strip footing resting on stone column reinforced-soil. Figure 3 shows the numerical model with the mesh discretization in addition to the boundaries, as well as the geometry and the fixation conditions. In this numerical approach, vertical loads are not directly applied on the footing, but they are generated through a uniform vertical prescribed displacement applied to the footing nodes. Numerical ultimate bearing-capacity is obtained from the load-displacement curves when the load reaches a steady value that indicates the limit load.

The present simulated model integrates the geometry of the footing, the stone columns and the soft-soil, as well as their physical and mechanical properties. It also includes the constitutive laws of the materials. It should be noted that the origin of the coordinate system is placed at the center of the foundation and because of the double symmetry conditions in the plan (x, y); only a square of the model is considered in the simulation procedure as illustrated in Figure 3.

The present model is appropriately discretized, using two types of meshes. Radial cylinder elements are used to discretize the foundation, stone columns as well as the part of the soft-soil located between the foundation interface and the lower boundary of the model. While the brick elements are used in the remaining part of the soft-soil which lies from the right side of the footing to the lateral boundary.

In order to ensure a good accuracy to the results, a series of preliminary tests were carried out on the geometric effects to set appropriate boundaries; where the influence of horizontal and vertical stresses are insignificant if not equal to zero. The tests show that both vertical transversal and longitudinal boundaries can be placed respectively at a distance of 30m (x direction), and of 0.7m (y direction) from the center of the footing. While the horizontal boundaries at the base of the model are at a depth of 40 m below the center (z direction). It could be noted that the displacement of the transverse side of the model is fixed in the horizontal direction $U_x=0$, while the base of the model is restrained in all directions $U_x=U_y=U_z=0$. To simulate the strip footing, the horizontal displacement of the longitudinal side is also fixed $U_y=0$.

The friction contact between the soil and the footing is simulated by connecting the soil to the footing through interface elements defined by Coulomb shear-strength criterion. Moreover, as the footing is assumed to be rigid, the soft-soil and the stone columns will have the same amount of displacement (settlement), then, no differential settlement will occur, hence interface element between the stone columns and soft-soil is not necessary [2, 29]. In addition, because of the high loading rate and the low permeability the soft-soil is assumed to be undrained [17].

Different models of materials behaviors are incorporated in the Computer Code Flac3D. For this analysis the appropriate behaviors for the elements that constitute the model are selected directly from the Code. The footing behavior was modeled by a linear elastic model, while the soft-soil and the backfilling material of stone column are assigned a linearly elastic-perfectly plastic material, obeying the Mohr-Coulomb criterion.

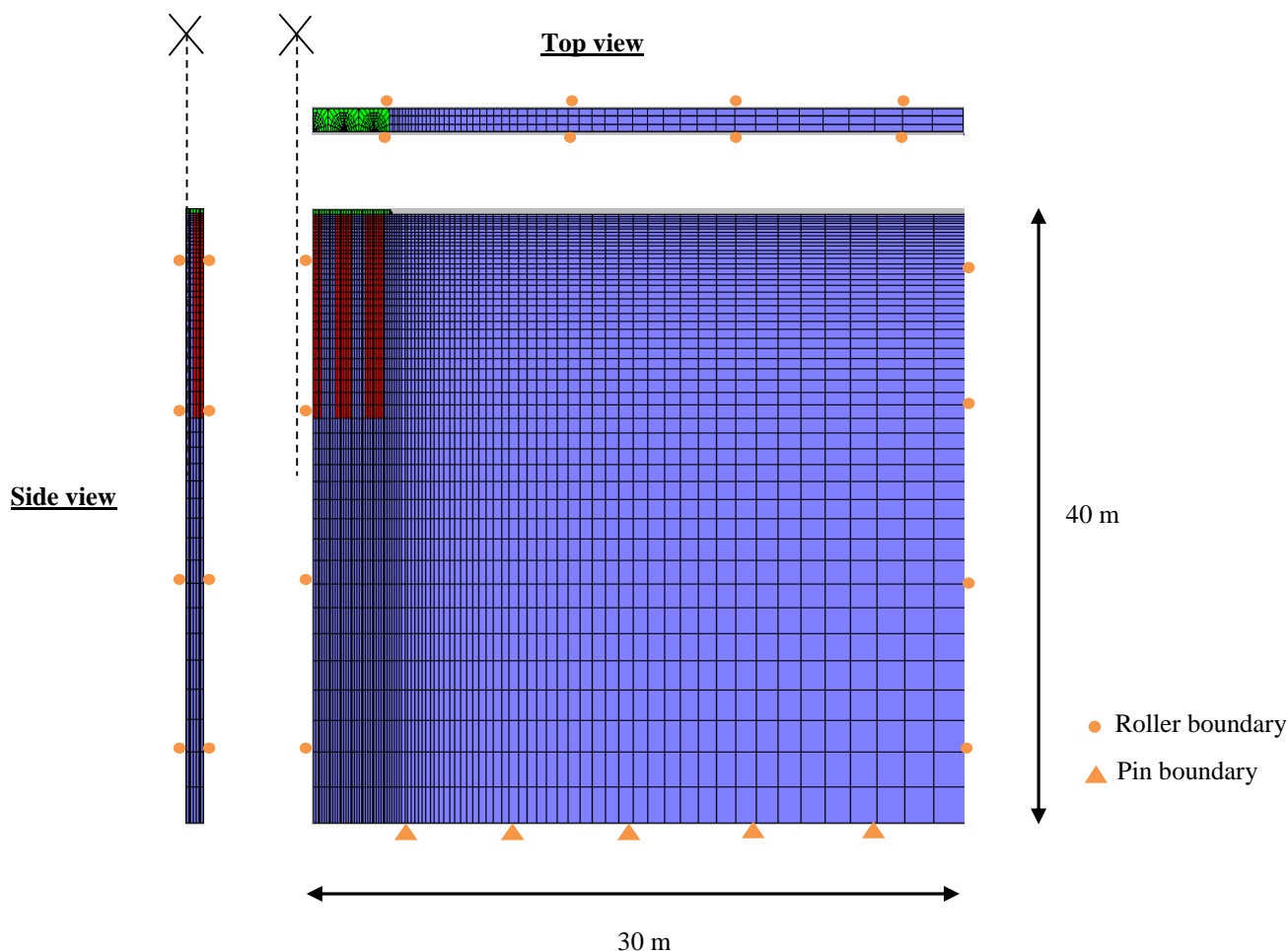


Figure 3. Finite-difference mesh and boundary conditions of the numerical model

3. Validation of the Numerical Model

Table 1 shows the values of the ultimate bearing-capacity (q_u), calculated using the present numerical model, and compared to those obtained from the experimental tests of Hu (1995) [3], from the finite difference method of Zhou et al. (2017) [17] And from the theoretical approach (slices method) studied by Khalifa et al. (2017) [30]. Seven cases of soil-reinforcement system are considered, with different material properties and are presented in Table 1. Comparison among the results clearly shows that the values of (q_u) obtained from the present study are in good agreement with those calculated from previous studies.

Table 1. Validation of the numerical model: comparison of results for different values of material properties

Case	c_u	ϕ_c	ϕ_s	γ_c	γ_s	A_s	Previous Study	Present study	Difference
	(kPa)	(°)	(°)	(kN/m ³)	(kN/m ³)	(%)	q_u (kPa)	q_u (kPa)	(%)
Hu 1995	10.5	30	0	15.47	13.1	30	85	76	10.5
	11.5	30	0	15.47	13.1	30	90	83	7.7
Zhou et al 2017	32	34	0	17.3	14	24	243	236.6	2.6
	20.5	34	0	20.3	9.9	40	171	164	4
Khalifa et al 2017	15	35	0	20	18	20	124	131	5
	15	35	0	20	18	10	102.6	100	2.6
	10	35	0	20	18	10	63	71.6	13.1

4. Parametric Study

It is recognized that the main purpose in the reinforcement of soft-soils by stone columns is to find the optimal geometric and physical parameters of the stone column, providing a good and efficient improvement of the soft-soil. This essentially depends on the maximum bearing-capacity that the reinforced-soil can mobilize. This bearing-capacity is influenced not only by the properties of the stone column but also by those of the soft-soil and the foundation supported by this soil [7].

The present parametric study aims to derive a useful and reliable analytic form for the optimal length of the column, also called critical-length (L_c) which can be used for efficient dimensioning of the stone column, considering the influence of the other parameters such as the area replacement ratio (A_s), the undrained-cohesion of the soft soil (C_u) and the footing width (B). This critical-length is selected from the analysis and interpretation of numerical results performed in this parametric study.

4.1. Data and Parameters of Study

The parameters used in this study are shown in Figure 1. For convenience of presentation, results are presented in a form using dimensionless parameters:

- (L/B): The column length ratio defined as a length L of column to the footing width B .
- (L_c/B): The optimal or the critical column length ratio.
- (q_u/q_o): The bearing-capacity ratio, defined as the ultimate bearing-capacity of reinforced-soil over the ultimate bearing-capacity of unreinforced homogenous soil ($q_o = C_u \cdot N_c$), with ($N_c = 2 + \pi$).

The values of material and geometry parameters used in the analysis are reported on Table 2; they are selected according to the typical ranges adopted in most recent studies [2, 17]. The column length ratio (L/B) and the area replacement ratio (A_s) are respectively selected in the practical ranges of (0.5–4.5) and (10–40%), keeping the same spacing ($S=1.4m$) among the columns. Besides, the efficiency of stone column technique is more beneficial for soils having an undrained-cohesion between 7 and 50kPa [31], according to that, for the present study the undrained-cohesion is taken in the range ($C_u = 15-35$ kPa), and the internal angle friction is taken 38° . Additionally, the module of elasticity of the soft clay and the stone column are taken ($E_s=200 \cdot C_u$, $E_c=45000$ kPa) as suggested by Ladd (1964) and Ambily et al. (2007) [32, 29], respectively. It should be noted that the influence of the footing width (B) on the critical-length ratio is analyzed using three numerical models with different number of columns ($N=3, 5$ and 7) beneath the footing, respectively for the cases of width ($B= 4.2, 7$ and $9.8m$). While for the rest of the study, only the model associated to ($N=5$ and $B=7m$) is considered.

Table 2. Values of material properties used in the parametric study

Material Parameters	Range of values
Soft-soil module of elasticity (kN/m ²)	3000-7000
Stone column module of elasticity (kN/m ²)	45000
Clay Poisson's ratio	0.3
Stone column Poisson's ratio	0.495
Stone column friction angle (°)	38
Stone column Angle of dilatancy	8
Area replacement ratio (%)	10-40
Stone column diameter (m)	0.5-1.0
Stone column length (m)	3.5-3.2
Width of footing (m)	4.2, 7 and 9.8

4.2. Influence of the Area Replacement Ratio on the Critical-Length Ratio

The analysis of the influence of the column length ratio (L/B) on the bearing-capacity ratio (q_u/q_o) of reinforced-soil, give the possibility to determine directly the optimum or critical-length ratio (L_c/B) for which the maximum value of the ultimate bearing-capacity is reached in the reinforced-soil.

In order to determine the critical-length ratio, obtained results are presented in Figure 4 in dimensionless form as the bearing-capacity ratio (q_u/q_o) versus the column length ratio (L/B), for different values of area-replacement ratio (A_s) ranging from (10 to 40%), with a value of the undrained-cohesion of ($C_u=25, 35\text{kPa}$). The curves in Figure 4 show a similar shape for all the considered values of the area replacement ratio (A_s) as well as for the undrained-cohesion. These curves clearly illustrate, that the bearing-capacity ratio (q_u/q_o) increases linearly with the increase of the column length ratio (L/B) and reaches its maximum value at the critical-length ratio (L_c/B), beyond this critical value no significant increase in bearing-capacity ratio (q_u/q_o) is noted. This observed behavior can be explained by the distribution of stresses in the soil-stone columns system which decomposes in two cases: for short columns ($L \leq L_c$), the stresses caused by the vertical displacement of the footing will be transferred from the top to the bottom through the full length of the stone column, while for the case of long columns ($L > L_c$), only a part of the total length of the column (critical-length) contribute to support the load, and the stresses reach just the critical depth. In addition, for stone columns shorter than the critical-length, the plastic shear zone takes place and extends below the limit of the reinforced soil, and inversely, if the stone columns are longer than the critical-length, the limit depth of stone columns is always under the plastic zone [17], consequently, no beneficial improvement will be observed for additional column length.

Besides, it can be seen from Figure 5 that by increasing the area replacement ratio from 10% to 40%, the critical-length ratio (L_c/B) increases from ($L_c/B=1.1$) to ($L_c/B =3.1$) and from ($L_c/B=1.1$) to ($L_c/B =2.6$) for ($C_u=25, 35\text{kPa}$), respectively. It should also be noted that the critical-length ratio (L_c/B) increases around 180% and 140% for the considered values of the undrained-cohesion, reflecting the great influence of the area replacement ratio on the value of the critical-length. This is probably due to the fact that when the area replacement ratio increases, the circular contact area between the column and the footing becomes larger, thus, providing more ability to the stone column to receive and transfer more stresses toward the lower zone at great depths [2].

Castro (2014) [27] reported that groups of stone columns with the same area replacement ratio but with different number, spacing and configuration of stone columns, will lead approximately to the same load-settlement behavior. For this reason, only the influence of area replacement ratio (A_s) is taken for the analysis of the critical-length ratio, then; it is considered as a representative parameter, without taking into account the influence of the rest of the parameters (the number (N), the spacing (S) and the diameter (D) of columns).

If we consider from Figure 4 the bearing-capacity ratio (q_u/q_o) of 2 curves associated with 2 areas replacement ratio in which the difference does not exceed 10%, the variation is seen to be approximately similar and has the same values of (q_u/q_o) this until the curve associated with the lowest A_{s1} ratio reaches its maximum at the corresponding critical-length ratio (L_c/B_1). After this inflection point the difference of variation between the 2 curves becomes much more marked in the sense that the bearing-capacity ratio curve associated with (A_{s1}) takes an asymptotic form, while the curve related to (A_{s2}) continues to increase as a function of (L/B) until reaching its maximum value at the critical-length ratio (L_c/B_2), and keeps constant value for length ratios greater than this critical value. As example, for the case of ($A_{s1} = 32\%$) and ($A_{s2} = 40\%$) the corresponding curves show quite identical values of the bearing-capacity ratio (q_u/q_o) until reaching the inflection point (critical-length ratio) ($A_{s1} = 32.5\%$) for ($L_c/B_1 = 2.6, q_u/q_o_{\max1}=1.85$). Beyond the inflection point, the value of the bearing-capacity ratio (q_u/q_o)_{max1} linked to (A_{s1}) remains constant, while the curve corresponding to ($A_{s2} = 40\%$) continues to increase until it achieves its maximum value at the corresponding critical value ($L_c/B_2 = 3.2, q_u/q_o_{\max2}=2.5$). as reflected in Figure 4, it can be seen that this kind of variation remains valid for all the curves associated to areas replacement values ranged from 10 to 40%. It can be concluded from this observation that the bearing-capacity is controlled by the length of the column before reaching the critical-length. Exceeding the later makes the area replacement ratio the predominant parameter for each case.

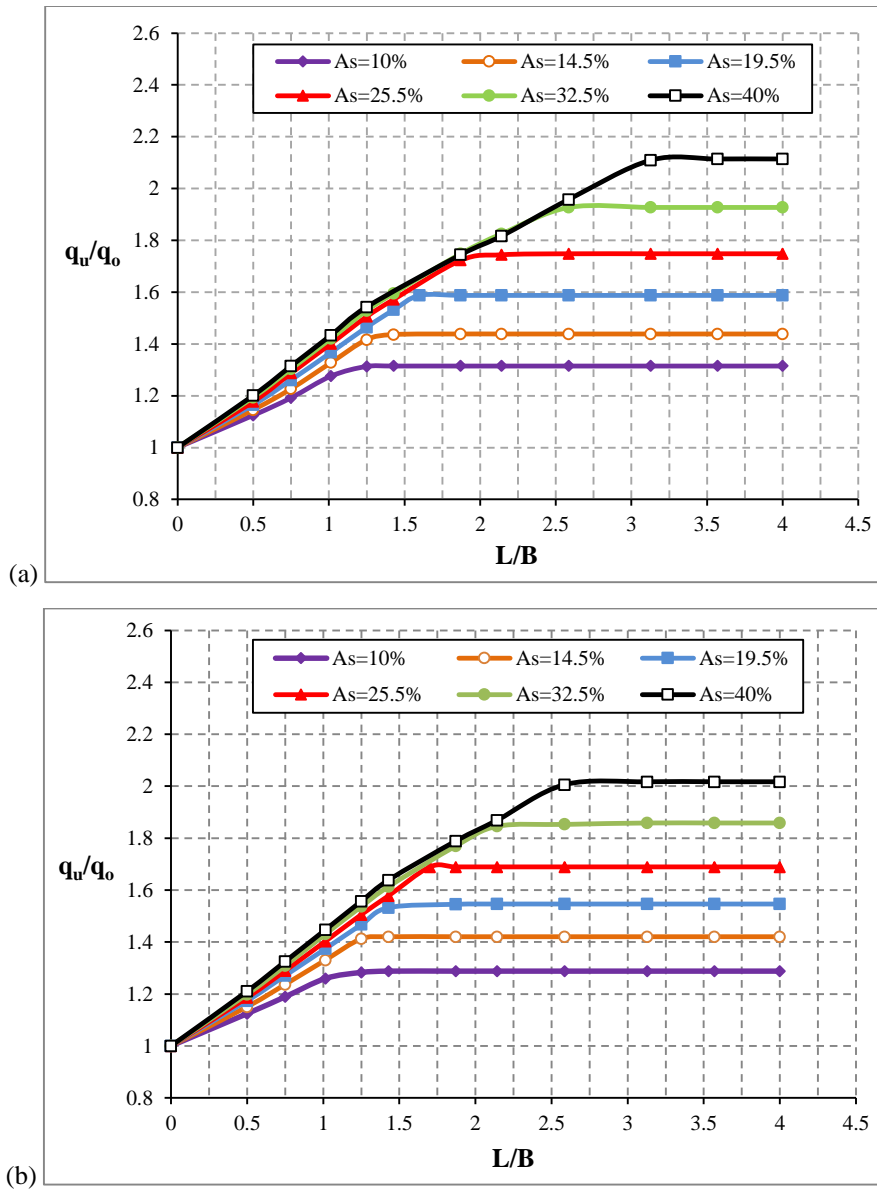


Figure 4. Variation of the bearing-capacity ratio (q_u/q_o) with the length of column to footing width (L/B): (a) $C_u=25\text{kPa}$, (b) $C_u=35\text{kPa}$

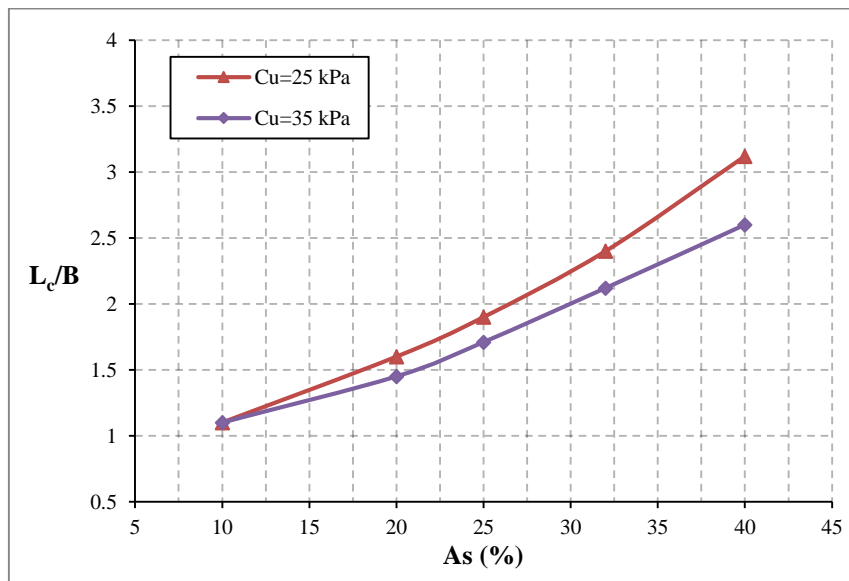


Figure 5. Variation of the critical length ratio (L_c/B) with the area replacement (A_s)

Furthermore, the given curves show that for low values of length ratio ($L/B < 1$), the influence of the area-replacement ratio (A_s) on the bearing-capacity ratio is practically not significant in the range considered (10 - 40%). However, this influence starts to be remarkable for values of length ratios beyond ($L/B > 1$) and becomes greater for high values of (L/B) and (A_s). This means that the favorable effect of area replacement ratio on the bearing-capacity of reinforced-soil is more effective for depths longer than the width of the foundation. Consequently, the value of the length of the column must always to be bigger than the width of the foundation ($L/B > 1$), to ensure efficient reinforcement and better improvement of reinforced-soils.

4.3. Influence of the Undrained-Cohesion on the Critical-Length Ratio

In order to characterize the effects of the undrained-cohesion (C_u) on the bearing-capacity as well as on the critical-length ratio of the column, the results presented in Figure 4 for ($C_u=25, 35$ kPa) are extended to a series of the undrained-cohesion values ($C_u = 15, 20, 30$ and 35 kPa), using the same procedure. The analysis of the obtained results reflect that for the considered values of (C_u) the shape of variation of the corresponding curves remains similar to those presented in Figure 4, except for the change observed in values of the bearing-capacity ratio (q_u/q_o) and the critical-length ratio of the column (L_c/B). The above curves have been only used to derive the maximum values of the bearing-capacity ratios (q_u/q_o_{max}) and the critical-length ratios (L_c/B), respectively, which are necessary for the analysis, and there is no need to present a series of curves similar to those presented in Figure 4, corresponding to the different values of (C_u) and (A_s). To analyze the influence of the undrained-cohesion (C_u), the presented values of the critical-length ratios (L_c/B) are plotted in Figure 6 versus the undrained-cohesion (C_u) for different values of area replacement ratio (A_s). Based on the variation of these curves it can be noticed that the critical-length ratio (L_c/B) decreases as the undrained-cohesion (C_u) increases. However, for low values of area replacement ratio ($A_s = 10\%$), the influence of variation of (C_u) is insignificant. Nevertheless, this influence is noticeable for moderate values of area replacement ratio ($A_s \geq 20\%$), and becomes much more sensitive for the highest values. In other words, the decreasing rate of the critical ratio becomes greater by increasing the area replacement ratio (A_s), as it can be seen from Figure 6, if we consider $A_s=40\%$ and by varying the undrained-cohesion (C_u) from 15 to 35 kPa, the effect on the critical length ratio (L_c/B) will be a decrease of 30%.

Therefore, for a given value of (A_s), the lowest value of the critical-length is observed at the highest value of (C_u), and conversely its maximum value is obtained for the lowest value of (C_u). In addition, for the ranges of values adopted for A_s and C_u in this study, the maximum value of the critical-length ratio ($(L_c/B)_{max}$) corresponding to the case ($A_s = 40\%$, $C_u = 15$ kPa) while the minimum value ($(L_c/B)_{min}$) is obtained for the case ($A_s = 10\%$, $C_u = 35$ kPa). The rise in the critical-length observed in the present study for lower undrained-cohesion (C_u) is mainly due to the stresses transferred by the horizontal section of stone column to the lower zone which is characterized by the stress concentration ratio (n) defined as the stresses of the stone column over the stresses on the surrounding soil. Similar to the critical-length, the stress concentration ratio (n) rises as the undrained-cohesion of the soft-soil decreases [29], resulting in more stresses reported to the column, which in turn transfers them to great depths in the soil. In other words, stress concentration on the surface of the column increases with the increase on the stiffness ratio (E_c/E_s) [33], where E_c and E_s are the Young's modulus of the stone column and the soft soil, and by taking in consideration the proportional relation between the undrained-cohesion (C_u) and young's modulus (E_s) of the surrounding soil [32], it can be concluded that the increase of the undrained-cohesion lead to decrease in the stress concentration, consequently, the critical length decrease.

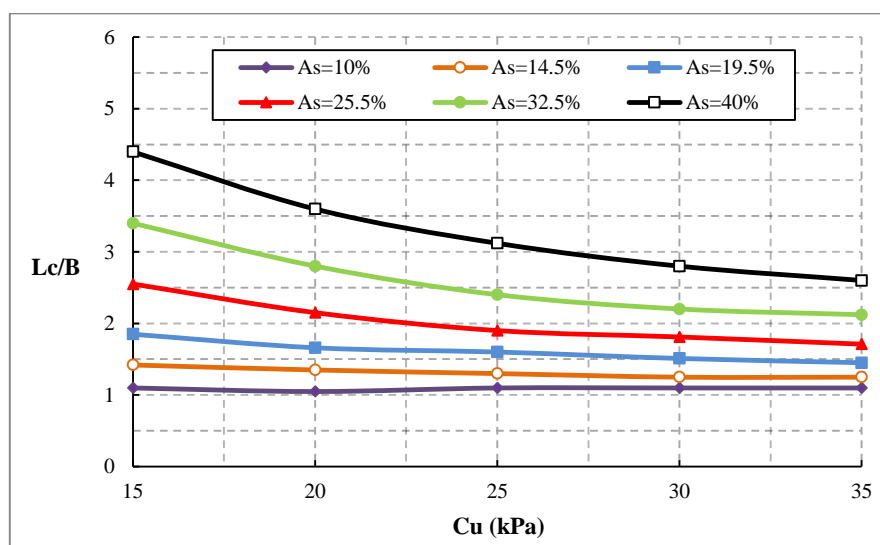


Figure 6. Variation of the critical-length to footing width (L_c/B) with the undrained-cohesion (C_u) for different values of area replacement ratio (A_s)

4.4. Optimal Values of the Bearing-Capacity Ratio

The maximum values of the bearing-capacity ratios (q_u/q_0) developed by the reinforced-soil for the critical-lengths of stone columns ($L = L_c$), are determined as shown in Figure 7 for different conditions of (C_u) and (A_s). As it can be seen from this figure that for a fixed value of (A_s), the two curves associated with the two ratios (critical-length ratio and bearing-capacity ratio presented in Figures 6 and 7 respectively) exhibit the same type of variation. Therefore, the concluded remarks concerning the influence on the critical-length ratio as well for (C_u) as for (A_s) remain also valid for the case of the bearing-capacity ratio (q_u/q_0).

It can also be noted that Figure 7 give in practice the upper and lower limits of the bearing-capacity ratios providing an efficient reinforcement of a soft-soil having a value of (C_u) included in the interval ($C_u = 15, 20, 25, 30$ and 35 kPa). The latest is reinforced by a group of floating stone columns characterized by an area replacement ratio (A_s) adopted within the interval of values (10-40%). Similar to the case of the critical-length ratio, these limits can also be set from Figure 7. As it is shown, the lower limit of bearing-capacity ratio (q_u/q_0)_{inf} = 1.28 is obtained for the largest value of ($C_u=35$ kPa) and for the smallest value of ($A_s = 10\%$) corresponding to ($L_c/B=1.1$). Whereas; the upper limit (q_u/q_0)_{sup} = 2.32 is obtained for the case of lowest value of ($C_u = 15$ kPa) and the greatest value of ($A_s = 40\%$) corresponding to ($L_c/B=4.4$). For the intermediate values of the (A_s) and (C_u), the values of the bearing-capacity ratio (q_u/q_0)_{max} are always within the interval defined by the values of the lower and upper limits of the ratio as derived from Figure 7: $1.28 < (q_u/q_0)_{max} < 2.32$.

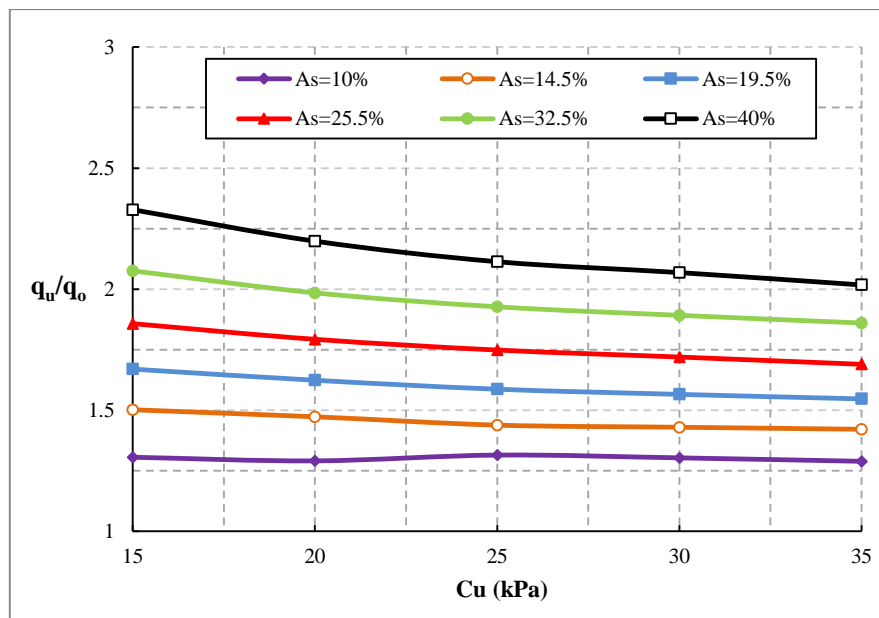


Figure 7. Variation of the bearing-capacity ratio (q_u/q_0) with the undrained-cohesion (C_u) for different values of (A_s)

Zhou et al. (2017) [17] calculated the ultimate bearing-capacity of soft-soils reinforced by a group of floating stone columns using a homogenous approach. The later method has been proposed in previous studies [11, 34]. Equivalent parameters have been considered for the soft-soil and the stone columns, and were expressed as follows:

$$C_{eq} = C_c \cdot A_s + (1 - A_s) \cdot C_s \quad (2)$$

$$\varphi_{eq} = \tan^{-1}[A_s \cdot \mu_c \cdot \tan \varphi_c + (1 - A_s) \cdot \mu_s \cdot \tan \varphi_s] \quad (3)$$

$$\mu_c = n/[1 + (n - 1) \cdot A_s] \quad (4)$$

$$\mu_s = 1/[1 + (n - 1) \cdot A_s] \quad (5)$$

where C_{eq} , C_s and C_c are the equivalent cohesion, the cohesion of the soft-soil and the cohesion of the stone column, respectively; and φ_{eq} , φ_s , φ_c are the equivalent friction angle, the friction angle of the soft-soil and the the friction angle of the stone column, respectively, A_s and n are the area replacement ratio and the stress concentration, respectively.

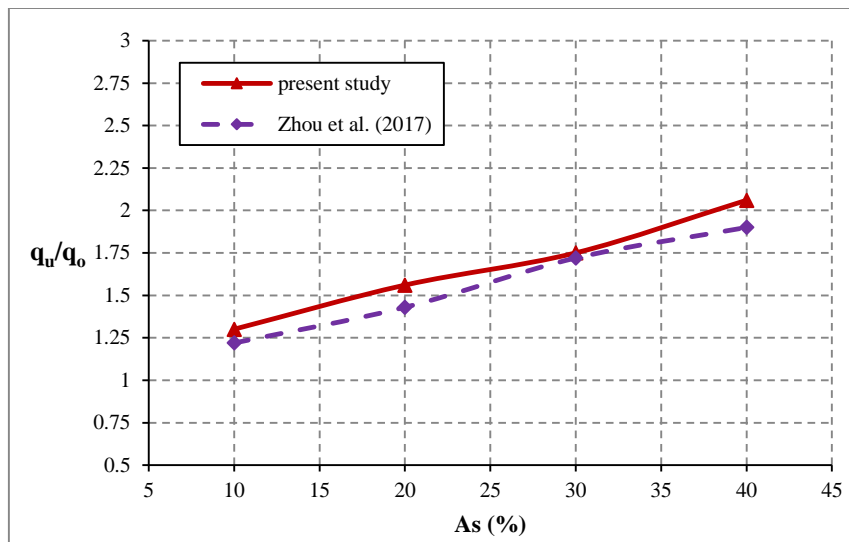


Figure 8. Comparison of the bearing-capacity ratio (q_u/q_o) of the present study with that of Zhou et al.(2017) for $C_u=30kPa$

The ultimate bearing-capacity has been estimated for different area replacement ratio ($A_s=10-40\%$) for a given undrained-cohesion ($C_u=30kPa$). In addition, the length ratio (L/B) and the stress concentration (n) were taken equal to 4 and 1.2, respectively; this later was chosen according to the results of Zhang et al. (2014) [35]. It should be noted that the length ratio (L/B) used in the present comparison is higher than the critical-length ratio (L_c/B), which will results in a maximum ultimate bearing-capacity. Figure 8 shows that there is a good agreement between the present study and Zhou’s method.

4.5. Influence of Footing Width on the Critical-Length Ratio

Figure 9 shows the variation of the bearing-capacity (q_u/q_o) ratio versus (L/B) using three different number of columns ($N=3, 5$ and 7) and taking an undrained-cohesion ($C_u = 30$ kPa). For each configuration two areas replacement ratios (A_s) are chosen (10% and 19.5%) for this analysis.

It can be observed that the curves exhibit the same shape of variation and they vary linearly with the length ratio increasing (L/B), until reaching its critical value (L_c/B). Beyond this value, the curves keep a constant form. These curves also illustrate that the critical-length ratio and the bearing-capacity ratio are practically not affected by the variation of the number (N) of columns, and both the ratios are only affected by the variation in the value of the area replacement ratio (A_s). Since an increase in the number of columns results in an increase in the foundation width (B), then it can be said that the critical-length ratio (L_c/B) is not affected by the variation of the foundation width (B). In fact it can be seen from Figure 9, that for the two values of (A_s), considered in the analysis, the corresponding values of the critical-length ratios remain the same, despite that the number of columns is varied: ($L_c/B = 1.15$ for $A_s = 10\%$) and ($L_c/B = 1.55$ for $A_s = 19.5\%$).

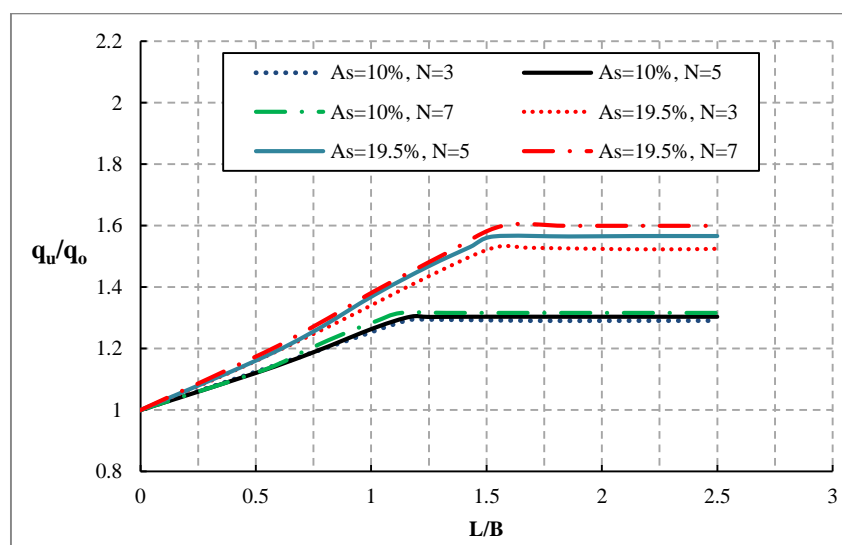


Figure 9. Variation of the bearing-capacity ratio (q_u/q_o) with the length of column to footing width (L/B) for different value of footing width ($B= 4.2, 7$ and $9.8m$)

Finally it can be mentioned from the results above that the increase in the critical-length, as result of an increase of footing width is due mainly to the mode of distribution of the pressure bulb within the soil beneath the foundation. It is recognized that this distribution depends on the foundation width and can reach deeper depths as the footing width increases [7], leading then to greater values of critical-length of the columns.

4.6. Analytical Form Proposed for Estimation of the Critical-Length

4.6.1. Expression of the Analytic Form

The different values of the critical-length (L_c) of the column, derived from the results of Figure 9 have been reported in Figure 10 as a function of the width (B) of the foundation, for the two considered values of the area replacement ratio ($A_s = 10\%$ and 19.5%). Figure 10 shows that in both cases, the representative curves are straight lines where the column length ratio (L_c) increases by increasing the footing width (B). Thus; the critical-length (L_c) may be related to the width (B) with a good level of accuracy, through a simple linear equation such as:

$$L_c = k \cdot B \tag{6}$$

where, the k factor represents the critical-length ratio presented in Figure 6.

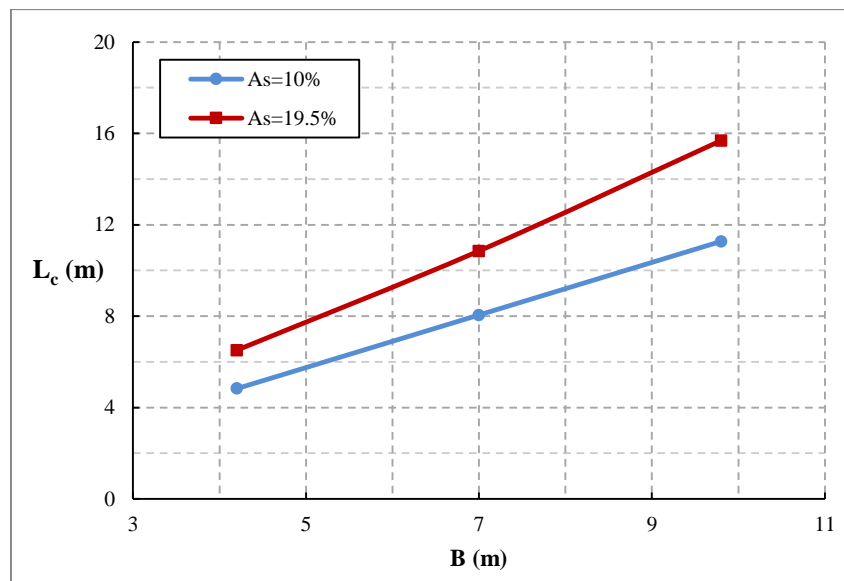


Figure 10. Variation of the critical-length (L_c) with the footing width (B) for two area replacement ratio ($A_s = 10\%$ and 19.5%)

The analysis and interpretation of the obtained results concerning the critical-length ratio $k = (L_c/B)$ showed that this parameter depends only on the area replacement ratio (A_s) and the undrained-cohesion of the soft-soil (C_u). Then Equation 2 can be rewritten in more explicit form as:

$$L_c = f(A_s, C_u) \cdot B \tag{7}$$

On the basis of a series of computational and verification tests which are carried out on the numerical results obtained in the present study and by using Microsoft Excel Solver function, a simple analytical form is built up for the function $f(A_s, C_u)$ (Equation 4) to express the later with a high degree of reliability the relation between the (L_c) and (B), as follows:

$$L_c/B = f(A_s, C_u) = \alpha \cdot \log(C_u/C_0) + \beta \tag{8}$$

$$L_c = (\alpha \cdot \log(C_u/C_0) + \beta) \cdot B \tag{9}$$

In this function, \log is the decimal logarithm and the ratio (C_u/C_0) represents the undrained-cohesion (C_u) normalized by the minimum value of undrained-cohesion of the soft-soils ($C_0 = 15$ kPa) selected for the study. While, the terms α , and β called critical-length factors are two linear dimensionless functions of parameter (A_s) varying within the range (10-40%) and expressed as:

$$\alpha = -17 A_s + 1.95 \tag{9-a}$$

$$\beta = 10.78 A_s - 0.14 \tag{9-b}$$

These factors are represented in Figures 11 and 12.

To summarize, it should be noted that the use of the present proposed analytical approach is to determine the optimal conditions for an efficient reinforcement through the critical-length, and it remains valid only for soft-soils having an undrained-cohesion between (15 and 35kPa), and reinforced by a group of floating stone columns characterized by an area replacement ratio, chosen in the ranged from (10-40%).

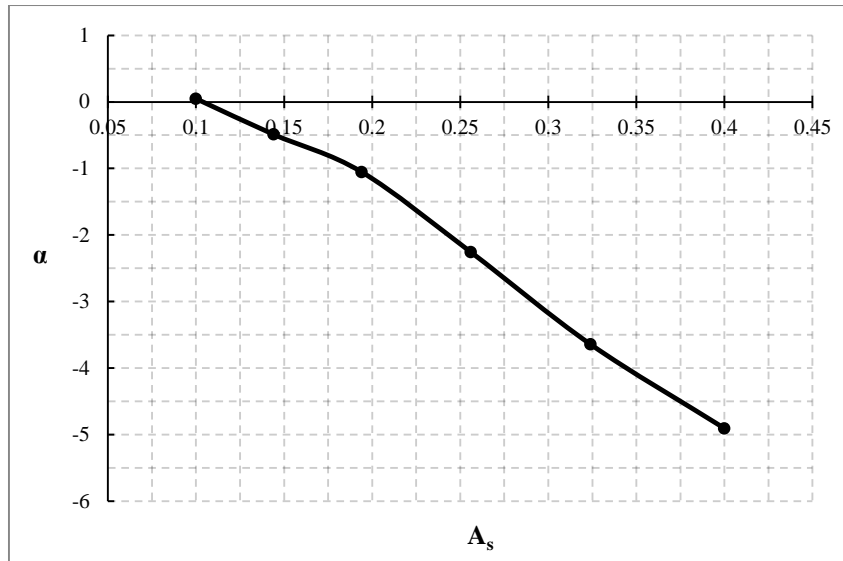


Figure 11. Variation of the critical-length factor α with the area replacement ratio (A_s)

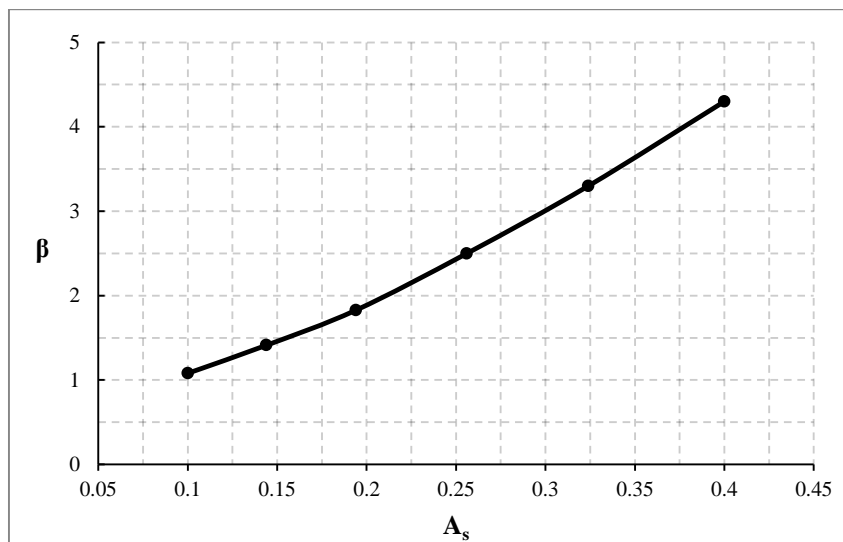


Figure 12. Variation of the critical-length factor β with the area replacement ratio (A_s)

4.6.2. Validation of the Proposed Analytic Form

For more efficiency and accuracy, the analytical expression proposed to estimate the critical-length of floating stone columns is verified and validated with results available in the literature such as experimental works of McKelvey et al. (2004) [24] and numerical study of Zhou et al. (2017) [17]. The results of validation are presented in Table 3. The critical-length factors (α , β) are determined for each case of validation. It should be noted that the critical ratio ($L/d = 6$) reported by McKelvey et al. (2004) [24] is equivalent to ($L_c/B = 1.5$) according to the same study. Notably, the values predicted using the new approach are in good agreement with those obtained from the previous studies.

Table 3. Values of material properties and results used in the validation of the new approach

Case	c_u (kPa)	A_s (%)	α	β	Previous Study (L_c/B)	Present study (L_c/B)	Difference (%)
Zhou et al. 2017	30	10	0.25	0.938	1.1	1.01	8.1
	30	30	-3.15	3.094	2.3	2.15	3.15
McKelvey et al. 2004	32	24	-2.13	2.447	1.5	1.74	16

5. Conclusions

A numerical approach was used to investigate the optimal conditions for soft soils reinforced by groups of floating stone columns, beneath a strip footing. A numerical simulation was validated by comparing it with data available in the literature.

A series of parametric studies were conducted on different type of soft soils with various values of undrained-cohesion ($C_u = 15$ to 35 kPa). The stone columns are of variable length (L) with area replacement ratios (A_s) selected in the range (10-40%) and by considering different footing width (B). Analysis of the obtained results showed that the bearing capacity reaches its maximum value within the reinforced soil at the critical column length ratio (L_c/B), illustrating the important influence of this parameter on the performance of reinforcement. It should be noted that the maximum ultimate bearing capacity is always obtained for ($L/B=L_c/B$ and $A_s=40\%$).

Concluding remarks indicated that the critical-length ratio increase with the increase of the area replacement ratio (A_s), and reaches its maximum value for the less cohesive soils corresponding to the greatest value of ($A_s=40\%$). Inversely, the minimum value of the critical ratio was obtained for the most cohesive soils corresponding to the lowest value of ($A_s=10\%$).

After the numerical calculations, a reliable and a useful analytical form for estimating the critical length (L_c) has been proposed by the authors to determine the optimal conditions for an efficient reinforcement, which remain valid only for the values of (C_u) and (A_s) considered in the present study. The proposed analytical equation was compared with the results available in the literature and showed good agreement.

6. Declarations

6.1. Author Contributions

The basic theme and the methodology of the research were discussed and decided by all authors. The manuscript was written by N.B., M.M. and A.M. while the numerical analysis work was carried out by N.B. and M.L., the results, discussions, interpretation and conclusion sections were completed by all authors. All authors have read and agreed to the published version of the manuscript.

6.2. Data Availability Statement

The data presented in this study are available in article.

6.3. Funding

The authors received no financial support for the research, authorship, and/or publication of this article.

6.4. Conflicts of Interest

The authors declare no conflict of interest.

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