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Adsplit: An Advertising Budget
Allocation Model

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Budget Allocation Model

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ADSPLIT is a computer-based, interactive, marketing model which allocates
a specified corporate promotional budget among individual brands competing
for the limited resources available. The model first asks the manager of
each brand to input his/her judgments on what the sales should be for
different values of price and advertising expenditure for the brand. It
is assumed that sales are an S-shaped function of advertising expenditure
and a constant elasticity (Cobb-Douglas) function of the price. The
parameters of these functions are estimated using the judgmental data
provided by the managers. The model uses the estimated functions to
compute the optimal budget allocation for the brands, given an upper
bound for the total promotional expenditure as well as an upper and a
lower bound for the budget for every brand considered. Optimal prices
for the brands are also computed.

1. INTRODUCTION

This paper presents a computer-based, interactive, marketing model which allocates a specified corporate promotional budget among individual brands (competing for limited corporate promotional resources) in a manner which utilizes judgmental response data inputs (by brand) to maximize total corporate Contribution.

This model – which we call ADSPLIT – has been developed because of our observation that while the "real world" advertising (or other promotion) budgeting process is a combination of "bottom up" (product-market need) analysis iterated with "top down" resource constraint specification, advertising budgeting models developed to date are **single product** maximizing models which ignore the reality of corporate (or profit center) funds constraints which typically force the total of all-brand spend levels to be less than the sum of their individually "optimal" levels.

In other words, if the product-by-product "optimal" levels (as determined by existing single-product ad budgeting models) were to be aggregated, they would (typically) exceed the funds available to the corporation for total advertising. When this happens, the corporate allocating entity is faced with the problem of making cuts in individual product-budgets such that the revised aggregated budgets meet the overall funds-available constraint. This "cutting" process inevitably involves making **across-product** comparisons. Such comparisons are by definition impossible in current single-product models, but form the distinguishing feature of ADSPLIT.

It follows that current single-product models would obviously suffice when the corporate funds available exceed the sum of individual "optimal" levels. Even here, however, the model presented in this paper has the following design advantages: while it uses judgmentally-derived response input data (in the style of ADBUDG (Little , 1970) and other decision-calculus models), it uses these response-functions to derive a mathematical "near-

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optimum" (by a heuristic, not a programming technique) spend level for each brand such that corporate **Contributions** will be "maximized." Unlike ADBUDG and similar models, therefore, the model uses an optimizing heuristic, and not numerical analysis; and unlike ADBUDG again, it operates on a (sales and cost data derived) "contribution response" function, not a sales response one. In deriving this "contribution function," sales are made a function not only of advertising but also of price, using the multiplicative form. We have deliberately ignored distribution and product quality on the grounds that distribution strength and product innovation are longer-term variables, changes in which are not really controllable for any one year, which forms the planning horizon for ADSPLIT. Users are therefore required to provide ADSPLIT with response judgment data not just for the advertising-sales but also for price-sales response relationships.

Furthermore, ADSPLIT has been designed to also meet certain other "real world" needs:

a. It allows the specification of not only a "grand" corporate funds constraint but also the specification of "lower bound" and "upper bound" budget constraints for **each** brand such that spend levels may be "locally" optimized within a historically- or competitively-acceptable range, by brand.

b. By not constraining individual brands in this fashion (as an option), however, it also permits the derivation of "zero base", "globally optimal" solutions.

The program does, of course, allow sensitivity analysis to be carried out, to answer "what if" questions for various possible scenarios, in view of the fact that the specification of funds available for promotion is itself somewhat flexible. This is especially important for cases where estimates of funds available change over the year; through ADSPLIT, cuts or increases can be made for the most appropriate product rather than across-the-board.

Certain limitations of ADSPLIT must also, however, be pointed out:

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a. It uses a planning horizon of one year and assumes no carryover effects. While this may seem unreasonable, it must be pointed out that current thinking on the "duration interval" of advertising's effect on sales for mature packaged goods is that 90% of the effect occurs within 3 to 9 months of the advertising (Clarke, 1977, p.60). Furthermore, decisions on the amounts to be budgeted and spent for individual products for smaller periods within a year are more tactical (within product) than strategic (across products) and can be tackled adequately by existing ADBUDG-like models. We do, however, plan to extend ADSPLIT to multi-period cases to take care of this problem.

b. While it uses a non-linear (S-shaped) sales response function, it ignores inter-dependencies between products which may exist in actuality, such as interdependencies in demand, the joint utilization of production or salesforce capacities, or media discounts specific to certain products. (To the extent that media discounts are a function of total corporate spend levels, which they usually are, these should not be a problem. They could be troublesome if media discounts are specific to seasons and buys rather than to a company over a year.)

c. Marketing mix variables other than advertising and price are ignored, as are competitive spend levels, though both are implicitly taken into account in the judgmental derivation of the sales response functions which are specific to a **given** scenario of competitive and other marketing mix elements.

d. It assigns equal weights to dollar contributions from each product. This may be inappropriate if (i) long term strategy considerations imply the need for differential weighting (ii) large changes in the product-mix ensue from the recommended budget-mix such that fixed cost estimates go awry because of write-offs of raw and packaging material inventory, non-utilization (or over-utilization) of certain production or sales force capacities, etc. While the former (strategic) weighting will be a part of our planned multi-period

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ADSPLIT extension, the latter (fixed cost) weighting can be managed to some extent through the appropriate modification of the variable cost inputs to the program.

2. FORMAT

The rest of this paper is structured as follows: after a literature review, we present the problem specification and program logic. We then describe the response functions and optimization heuristics used. We conclude with brief reports on program tests to date, and future extensions planned.

3. LITERATURE REVIEW

There is enough support in the literature for our assertion that advertising budgeting within a corporation is an iterative combination of product/market level need analysis and total all-brand resource constraint specification. Evidence for this can be found in Hurwood (1968) who reports the results of a Conference Board survey of 267 executives on advertising budgeting methods. The survey showed that allocations across products ("setting priorities") almost always had to be made and that this was done on the basis of either present brand profit contribution (usually for mature brands) and potential profit contribution (for new brands). The total corporate budget was specified either on an "earnings required" basis, or in relation to "alternative uses," or on a historical A/S ceiling basis. San Augustine and Foley have shown in their more recent survey (1975) that Hurwood's conclusions are still largely valid. Little (1975) made explicit mention of this allocation problem and process in his introduction to BRANDAID.

Like the other models in the ad budgeting area, however, BRANDAID – and its predecessor, ADBUDG – are both models which model the ad budgeting process for individual products only, and take no account of the across-products allocation problem. This is the typical situation in the modeling literature (see, for example, Horsky, 1977 and Picconi and Olsen, 1978) and it should be apparent that the optimal levels for individual

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brands derived from such models lose meaning if arbitrary cuts are made to fit all-brand totals within corporate constraints. The need for a model like ADSPLIT is thus evident.

It would thus seem that the development of ADSPLIT would have had to start from scratch, but for the following:

a. literature on the requirements for marketing models generally and "decision calculus" models specifically (Little, 1970; Montgomery and Weinberg, 1973).

b. literature on the requirements for advertising-sales response functions (Little, 1979)

c. Two models specifically:

(i) DETAILER (Montgomery, Silk and Zarazoga, 1971), which allocates salesforce promotion time across pharmaceutical products

(ii) ADBUDG (Little, 1970), which provides the pattern for the judgmental response function specification

d. literature on the **geographical** allocation of promotion effort for one brand (Rao and Miller, 1975) and on multi-brand models (Urban, 1969).

4. PROBLEM SPECIFICATION

The optimization problem addressed by ADSPLIT is as follows:

$$\text{Max} \sum_{i=1}^{n} CAA_i \quad (1)$$

subject to

$$\sum_{i=1}^{n} \quad \leq \quad G \quad \text{(total advertising budget)} \quad (2)$$

and

$$BU_i \quad \geq \quad \text{Advtg.}_i \quad \geq \quad BL_i, \quad i = 1, \ldots, n \quad (3)$$

$$\text{cost}_i \quad \leq \quad \text{price}_i \quad \leq \quad PU_i \quad (4)$$

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$$\text{price}_i \ \geq \ 0 \quad i = 1, \ldots n \quad (5)$$

where BU_i and BL_i are upper and lower bounds of the advertising budget, and PU_i the upper bound for price for brand i, respectively. The user specifies BU_i, BL_i and PU_i for the different brands.

The other terms used are described below.

$$CAA_i \ = \ (CBA_i \ - \ \text{Advtg}_i)$$

$$= \ ((\text{Sales}_i\# \ . \ \text{GM}_i) \ - \ \text{Advtg.}_i)$$

$$= \ [(\text{Sales}_i\# \ . \ (\text{Price}_i/\# \ - \ \text{Cost}_i/\#)) \ - \ \text{Advtg}_i]$$

$$n \ = \ \text{number of brands (maximum} = 20).$$

$$CAA_i \ = \ \text{Contribution after advertising for } i$$

$$CBA_i \ = \ \text{Contribution before advertising for } i$$

$$\text{GM}_i \ = \ \text{Gross Margin for } i$$

$$\text{Advtg}_i \ = \ \$ \ \text{level of Advertising for } i$$

$$\text{Sales}_i\# \ = \ f(\text{price}_i) \ . \ g(\text{Advtg}_i)$$

where $f(\text{price})$ = Cobb-Douglas function,

and $g(\text{Advtg})$ = S-shaped function.

The parameters of f and g are determined from user supplied judgments.

5. PROGRAM LOGIC

Figure 1 gives the flow chart of the program.

[Figure 1 About Here.]

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6. RESPONSE FUNCTIONS

a. Sales Response Functions

ADSPLIT takes the sales **of each brand** to be the multiplicative product of units sold times the price per unit, with the units sold being a function of both the price and the advertising level for the brand, with reference to base conditions. It is assumed that the sales response function for **advertising** will be S-shaped and that the sales response function for **price** will be of the Cobb-Douglas, constant elasticity type.

Throughout the model, the following relation is assumed to exist between sales S, advertising A and price p.

$$S = L + (\frac{p}{p_0})^{-\beta}(U - L)\frac{A^\alpha}{A^\alpha + B} \quad (6)$$

where

$$p_0 = \text{a reference price,}$$

$$L = \text{sales at zero advertising,}$$

$$U = \text{sales at infinite advertising,}$$

$$B = \text{a constant,}$$

$$\alpha = \text{the advertising response coefficient,}$$

and

$$-\beta = \text{price elasticity of sales (negative).}$$

The expression (6) assumes that the sales are multiplicatively separable in price and advertising. Figure 2 graphically illustrates the response functions relating advertising-sales and price-sales.

[Figure 2 About Here.]

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Estimating the Advertising-Sales Response Function

While the user directly inputs the values of L and U, the values of B and α are obtained by a regression procedure using user-input judgments of what sales will be at levels of advertising $X\%$ below/above current levels ($X\%$ ranges from 50% to 150%, at 10% increments).

Let price be fixed at the reference level p_0. Using eqn. (6),

$$S \quad = \quad L \quad + \quad (U - L)\frac{A^\alpha}{A^\alpha + B}$$

$$\Rightarrow \quad \frac{(S - L)}{(U - L)} \quad = \quad \frac{A^\alpha}{A^\alpha + B}$$

$$\Rightarrow \quad \frac{(U - L)}{(S - L)} \quad = \quad 1 \quad + \quad BA^{-\alpha}$$

$$\Rightarrow \quad \frac{(U - L)}{(S - L)} \quad = \quad BA^{-\alpha}.$$

Therefore,

$$\ln\left(\frac{U - L}{S - L}\right) \quad = \quad \ln B \quad - \quad \alpha \ln A \quad (7)$$

which is used as the estimating equation to derive values for B and α.

Estimating the Price-Sales Response Function.

Let us assume that advertising is fixed at some level A. Let S and S_0 be sales corresponding to prices p and p_0, respectively. Then, using eqn. (6),

$$\frac{S}{S_0} \quad = \quad \left(\frac{p}{p_0}\right)^{-\beta}$$

$$\Rightarrow \quad \ln\left(\frac{S}{S_0}\right) \quad = \quad -\beta \ln\left(\frac{p}{p_0}\right) \quad (8)$$

This relation is used to estimate the price elasticity $-\beta$.

b. Contribution-Response Function.

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The contribution for any one brand is defined as

$$CAA = CBA - A \quad (9)$$

where   CAA   = Contribution after advertising,

CBA   =   Contribution before advertising,

A =   Advertising spend level,

and CBA =   Sales units . Contribution/#

=   Sales units . (Price/# − Variable Costs/#).

Let c_i represent the variable costs per unit for brand i. Then using eqns. (6) and (9), our contribution function for brand i is given by

$$\Pi_i = (p_i - c_i)(\frac{p_i}{p_{0,i}})^{-\beta_i}[L_i + (U_i - L_i)\frac{A_i^{\alpha_i}}{B_i + A_i^{\alpha_i}}] - A_i \quad (10).$$

The maximization procedure (described in the next section) is carried out over this contribution function for all brands (functions) simultaneously.

7.   OPTIMIZATION HEURISTICS

We have three alternatives for the optimization algorithm: the easier but less reliable marginal analysis technique, which considers the response functions only one at a time and proceeds sequentially; the reverse marginal analysis technique, which finds the profit maximizing point ignoring the overall budget constraint and then works its way backward to meet it, minimizing the profit lost in the process; and the more difficult but more efficient gradient search technique. Within each, results are presented for (a) the case when no price changes are allowed and (b) when the price also, in addition to the ad allocation, is allowed

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to vary, but only such that it always exceeds cost and stays below a user supplied upper bound.

We have used all three, as alternatives. Flowcharts of the three are given in the pages following the verbal description which follows now. Our observations on the relative merits of the three alternatives are given in a later section.

DESCRIPTION OF OPTIMIZING ROUTINES

The objective of the optimization is to maximize profit by choosing appropriate levels of price p_i and advertising expenditure A_i for each brand i.

Using eqn. (10), the total profit can be expressed as

$$\Pi \;=\; \sum_{i=1}^{n} \; \sum_{i=1}^{n} f_i(p_i) g_i(A_i) \;-\; \sum_{i=1}^{n} A_i, \quad (11)$$

where

$$f_i(p_i) \;=\; (\frac{p_i}{p_{0,i}})^{-\beta_i}(p_i - c_i),$$

$$g_i(A_i) \;=\; L_i + (U_i - L_i)(\frac{A_i^{\alpha_i}}{B_i + A_i^{\alpha_i}}).$$

Clearly, for any level of A_i, since $g_i(A_i)$ is a positive multiplier independent of p_i, the same value of p_i will maximize Π_i, and that is the p_i that maximizes $f_i(p_i)$.

Therefore, the optimization procedure has the following two phases:

Phase I: Price optimization.

For each brand i, p_i is chosen within its allowed bounds to maximize $f_i(p_i)$.

Phase II: Advertising level optimization.

Each p_i is held fixed at the value derived in stage I. Three algorithms are then used to find the advertising expenditures (A's) which maximize Π.

a. Marginal Analysis. The procedure starts at the user-specified level of advertising spending for each brand i, denoted by A_i. Using the formulation (11), we compute Π_i and

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$\frac{\partial \Pi_i}{\partial A_i}$, the partial derivative of profit Π_i with respect to A_i, for each i. Comparing the partial derivatives for the different i's, we find the brand j which has the largest partial derivative, i.e. the largest marginal profit from an additional advertising dollar, i.e.,

$$\frac{\partial \Pi}{\partial A_j} \geq \frac{\partial \Pi}{\partial A_i} \quad \text{for} \quad i = 1, \ldots, n.$$

Next, we increment A_j to $A_j + \Delta A_j$, where ΔA_j is a prespecified small positive quantity. This is carried on until either the total budget is exceeded or the increasing brand budget A_j attains upper bound U_j. In the latter case, the brand in question becomes ineligible for further increases. An upper bound of 5000 is used for the number of iterations. The flowchart of the Marginal Analysis algorithm in presented in Figure 3.

[Figure 3 About Here.]

b. Reverse Marginal Analysis.

The procedure has two stages:

Stage 1: Total Budget constraint ignored.

The total budget constraint is ignored. For each brand i, the advertising A_i is varied in small steps of ΔA_i, where ΔA_i is a prespecified small positive quantity (usually $\frac{U_i - L_i}{100}$) between L_i and U_i, to maximize Π_i. Let AMAX_i be the value of A_i which maximizes Π_i.

Stage 2: Total Budget constraint satisfied.

If $\sum_i AMAX_i \leq B$ (total advertising budget), we stop.

If $\sum_i AMAX_i$ exceeds B, the algorithm finds the brand j which has the minimum marginal profit for an additional advertising dollar:

$$\frac{\partial \Pi_j}{\partial A_j} \big|_{A_j = AMAX_j} \leq \frac{\partial \Pi_i}{\partial A_i} \big|_{A_i = AMAX_i}, \quad \text{for} \quad i = 1, \ldots, n.$$

The algorithm then decreases A_j to $A_j - \Delta A_j$ where ΔA_j is a prespecified small positive quantity. Again the algorithm checks whether the total budget constraint is satisfied. If

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not, the partial derivatives $\frac{\partial \Pi_i}{\partial A_i}$'s are again computed for all i's and as before, the advertising budget is reduced for the brand which will register the minimum loss in profit due to it. This is carried on until the total budget constraint is satisfied. Figure 4 presents a flowchart of the Reverse Marginal Analysis algorithm.

[Figure 4 About Here.]

c. The Gradient Search Procedure.

The algorithm starts with a user specified advertising budget vector. Let A_i denote the budget for brand i, and Π_i the contribution to profit due to brand i. The partial derivative $\frac{\partial \Pi_i}{\partial A_i}$ is computed for each brand i. Next, the algorithm finds for each brand i,

$$y_i \;\; = \;\; \frac{\frac{\partial \Pi_i}{\partial A_i}}{[\sum_j (\frac{\partial \Pi}{\partial A_j})^2]^{.5}} \quad for \quad i = 1, \ldots, n$$

and sets

$$A_i \;\; \leftarrow \;\; A_i \;\; + \;\; y_i \cdot \Delta A \, ,$$

where ΔA is a prespecified small positive number. If any of the individual budgets goes above or below the upper or lower bounds, respectively, that budget is set at the boundary value.

The above procedure is repeated until one of the following happens:

(i) The total budget $\sum_i A_i$ exceeds B.

(ii) There is no appreciable improvement in profit from one iteration to the next.

(iii) The number of iterations exceeds 5000.

Figure 5 gives the flowchart for the Gradient Search algorithm.

[Figure 5 About Here.]

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8. PROGRAM TESTING

8.1. Numerical Analysis

The program as presented here was tested using a set of data which (as best as could be recollected) was used by one of the authors while a Brand Manager over 3 years ago. Some of the major results are presented in Table 1.

[Table 1 About Here.]

As Table 1 shows, the current allocation could be modified to yield a higher profit, with the allocation changed altogether in a zero-base situation. Plan 2 would indicate that in the event of greater funds availability more money should go to Brand 2. The figures presented are for allocations with no price changes; even higher profits could be realized if prices are flexible.

8.2. Differences in Heuristics.

When the same data was run using the three different heuristics, it was found that (1) the reverse marginal technique always gave the best solutions, and (2) the values given by the marginal and gradient techniques are sensitive to their starting points.

For this reason, the program is written to generate three different starting points, for each of the gradient search and marginal analysis techniques, on its own; it then finds the best solutions for each of these trials, plus the reverse marginal solution; stores all these in a file; and then sorts the results by profit value from this file, so that the highest profit solution (regardless of the algorithm used to locate it) can be identified and printed out.

8.3. Other Criteria

The other decision calculus model criteria – simple to use, easy to control, uses managerial judgment and is adaptive, is interactive, is (relatively) complete (uses price) etc. – are met. Further, Little's (1979) criteria for validity of the response function model are also met (zero advertising has intercept sales, user inputs take creative element, and competition,

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into account) except that since this is a single-period model no carryover and decay rates have been specified.

For a description of our planned extensions to ADSPLIT, see that section below.

9. FUTURE EXTENSIONS

It is our intention, sometime in the near future, to extend the present ADSPLIT program so as to increase its utility and robustness/reliability, as well as validity, in the following ways:

a. to model synergy, goodwill interdependencies, and cannibalization effects, across the brands in a company's product line.

b. to include more than one period in the planning horizon, and thus to introduce the carryover effect and other strategic weights and, if necessary, to include discounted profits. Strategic weights are necessary because investing today by advertising in a mature, low growth market obviously has lesser value than investing in a higher growth market. The strategic value of brand loyalty may be included in the computation of the carryover rate, as a weighting factor.

c. to do further work in the evaluation of the three heuristics so that we can choose one for ultimate implementation.

d. to increase the interactive character and user-ease of the program presented by including more user instructions within the program itself.

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Table 1

Numerical Analysis Without Price Changes
for Two Brands

(All figures in thousands of dollars)

Description of Plan	Total Budget	Specified Minima		Allocated Budgets		Total Contribution
		1	2	1	2	
CURRENT	1500			500	1000	51620
Plan One	1500	200	500	200	1300	54726
Plan One (zero base)	1500	0	0	0	1500	56302
Plan Two	2000	200	500	200	1800	58645
Plan Three	2500	200	500	500	2000	60755

Figure 1. Program Logic

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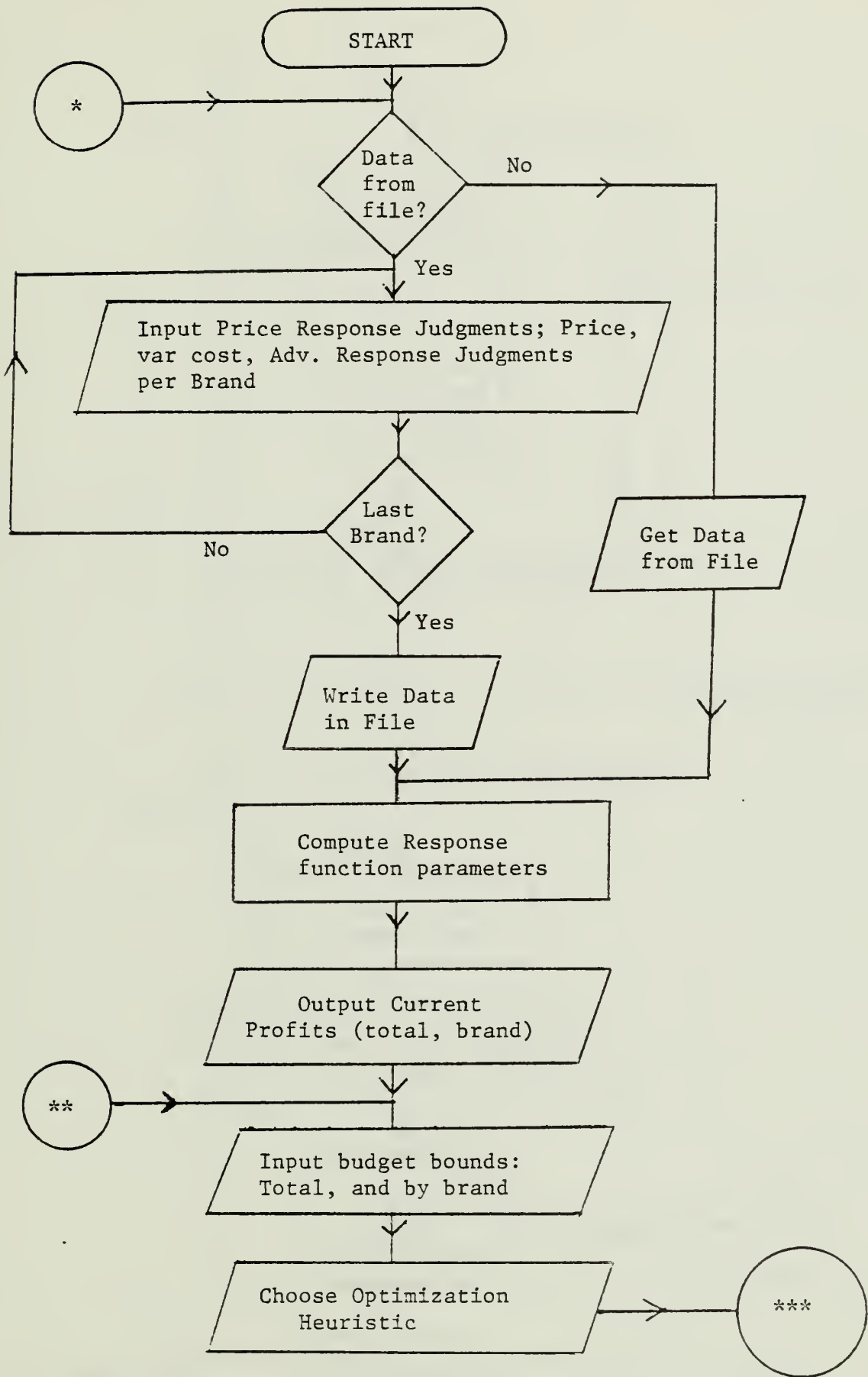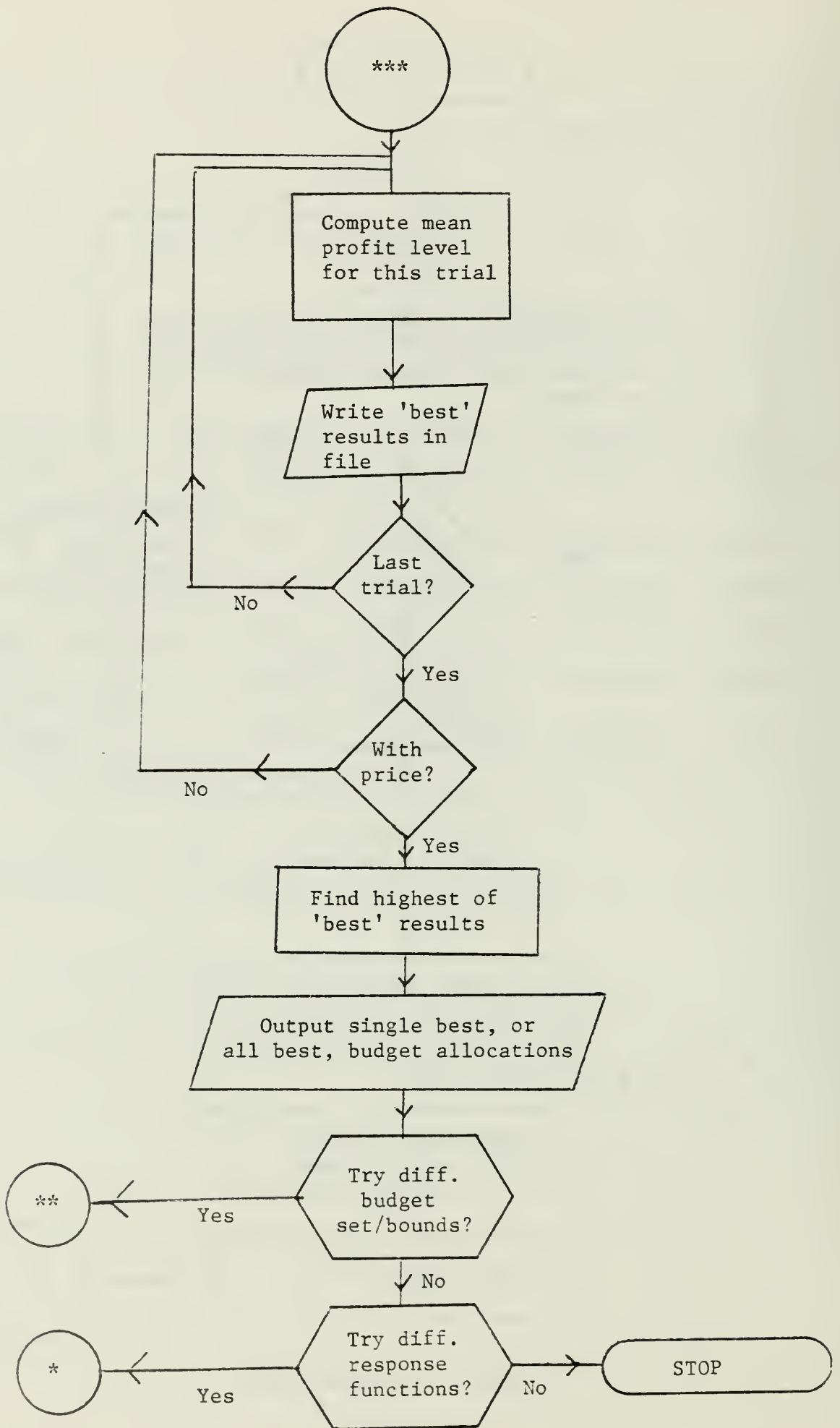
Figure 1. Program Logic (continued)
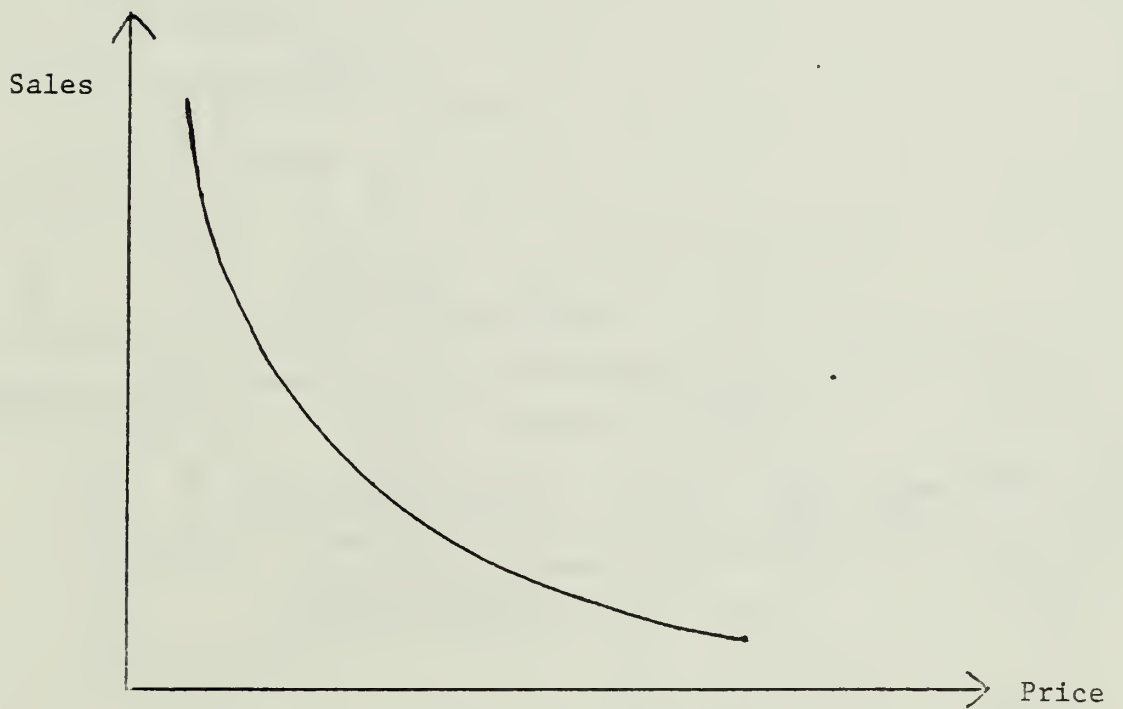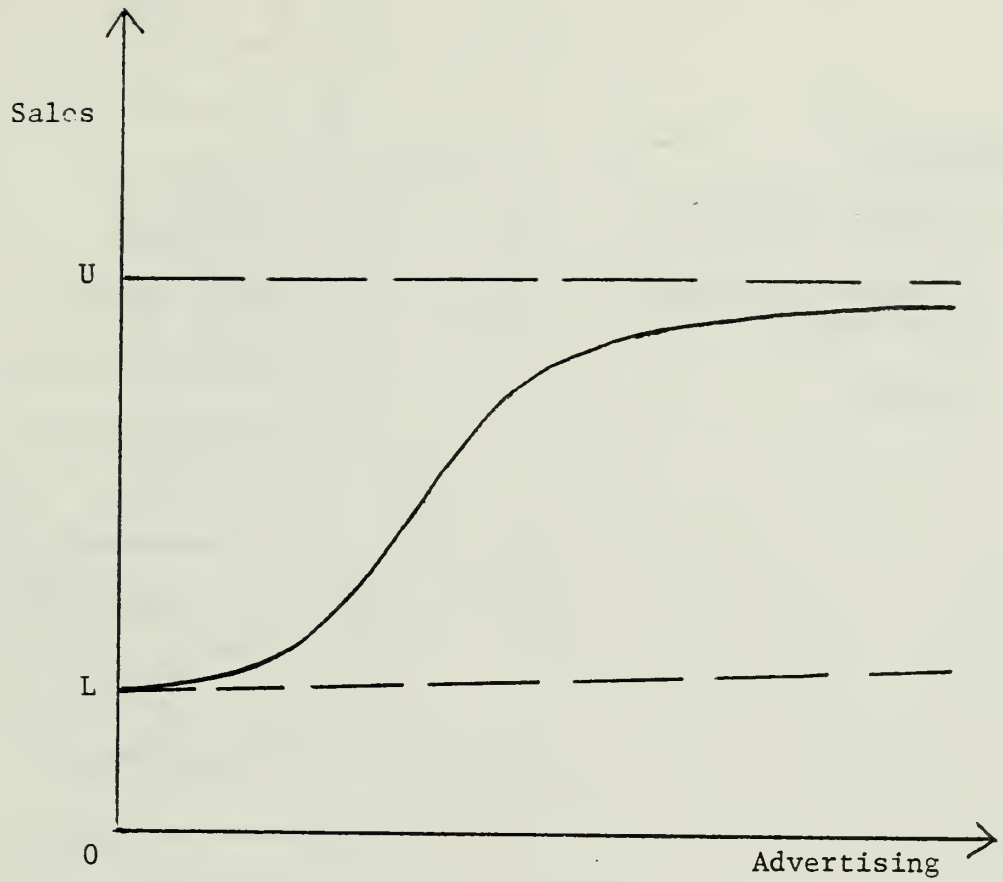
Figure 2.   Response functions relating
           advertising-sales and price-sales

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Figure 3. Marginal Analysis

Notation:

N = number of brands

A_i = advertising budget for brand i
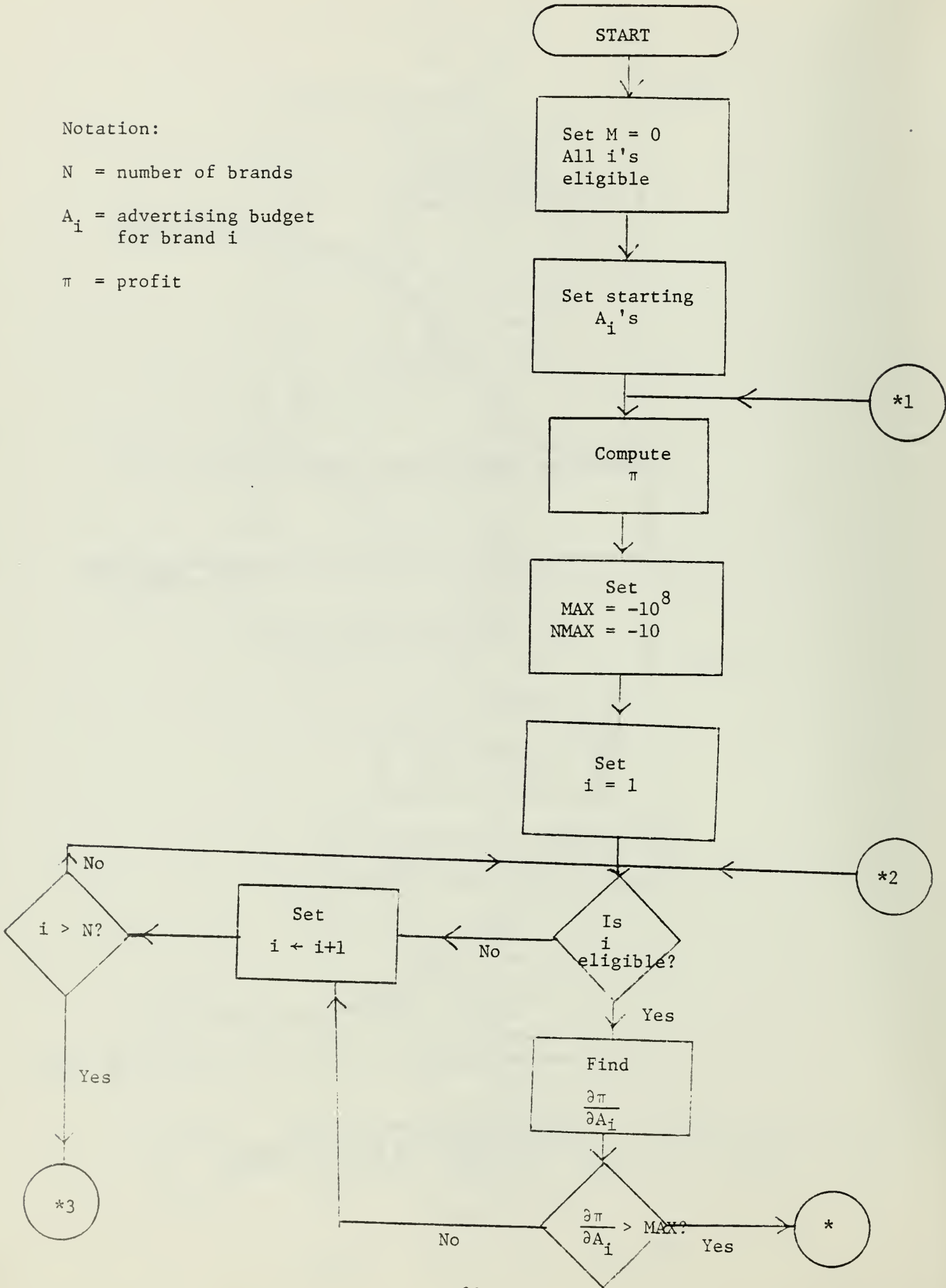
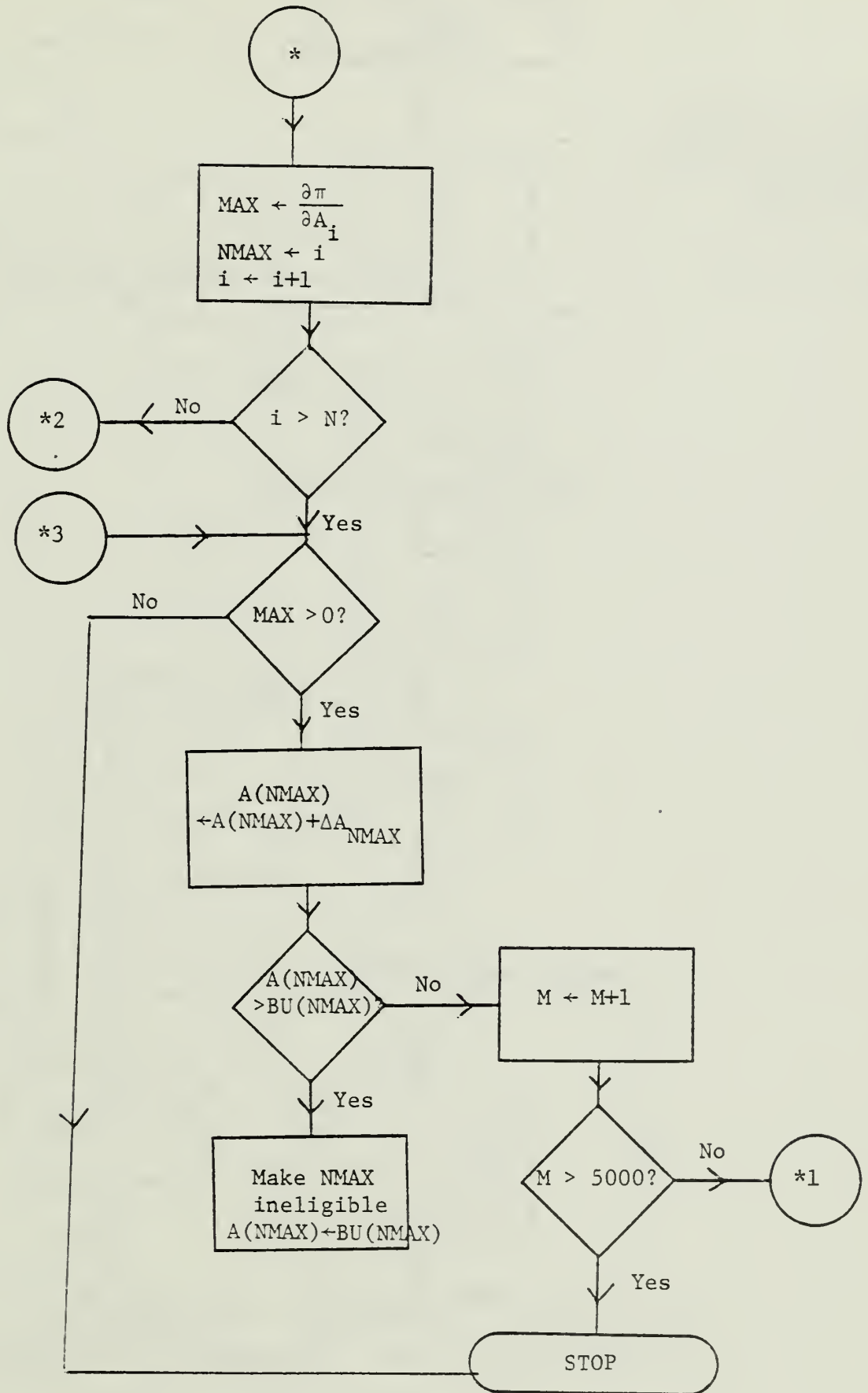π = profit

Figure 3. Marginal Analysis (continued)

Figure 4. Reverse Marginal Analysis

Stage 1. To find optimal advertising expenditures without a total budget constraint.
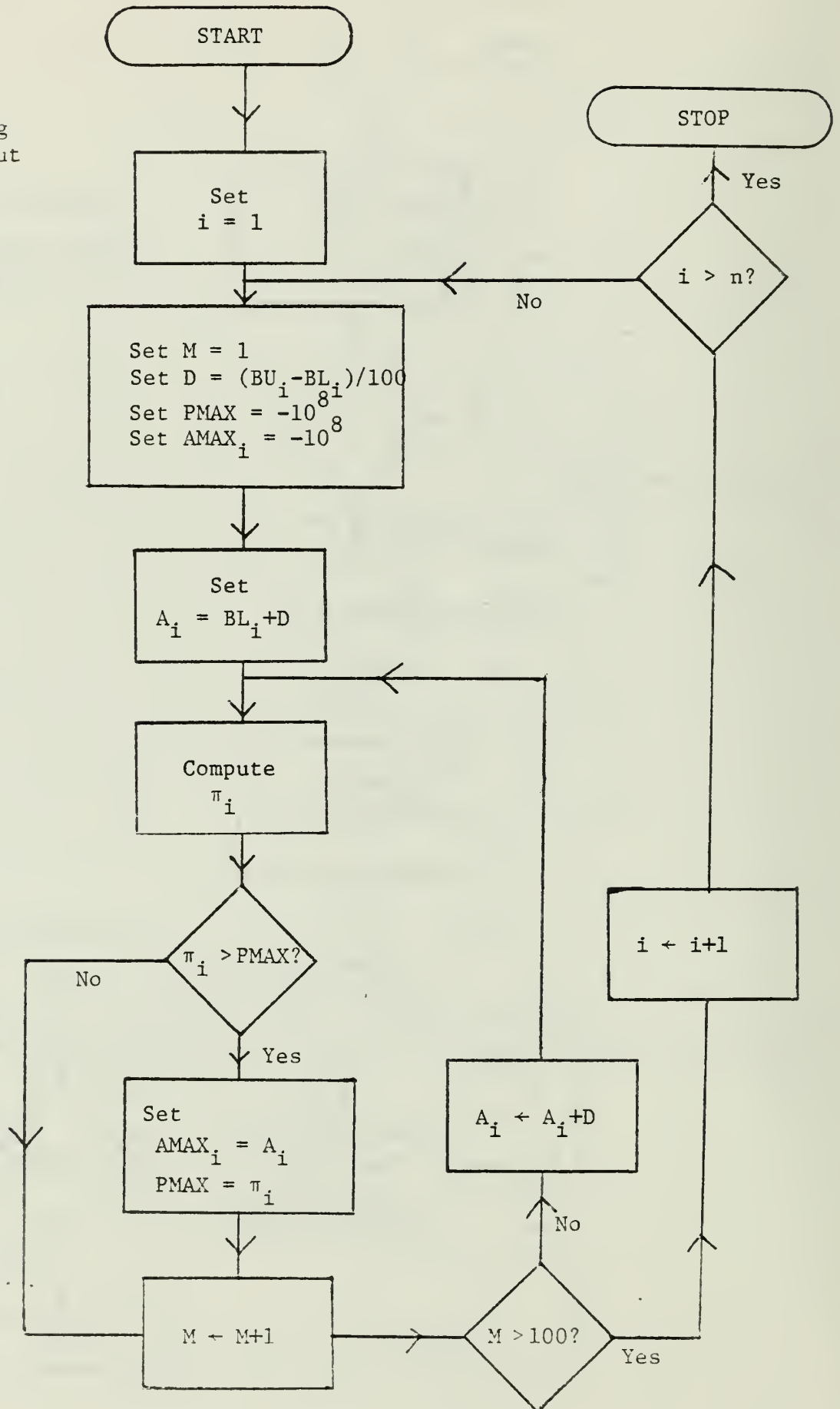
π_i = profit for brand i

START

STOP

Set
i = 1

Set M = 1
Set D = $(BU_i - BL_i)/100$
Set PMAX = -10^8
Set $AMAX_i = -10^8$

Set
$A_i = BL_i + D$

Compute
π_i

$\pi_i > PMAX?$

No

Yes

i > n?

Yes

No

i ← i+1

Set
$AMAX_i = A_i$
$PMAX = \pi_i$

$A_i \leftarrow A_i + D$

M ← M+1

M > 100?

No

Yes

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Figure 4. Reverse Marginal Analysis

Stage 2. To satisfy the total budget constraint.
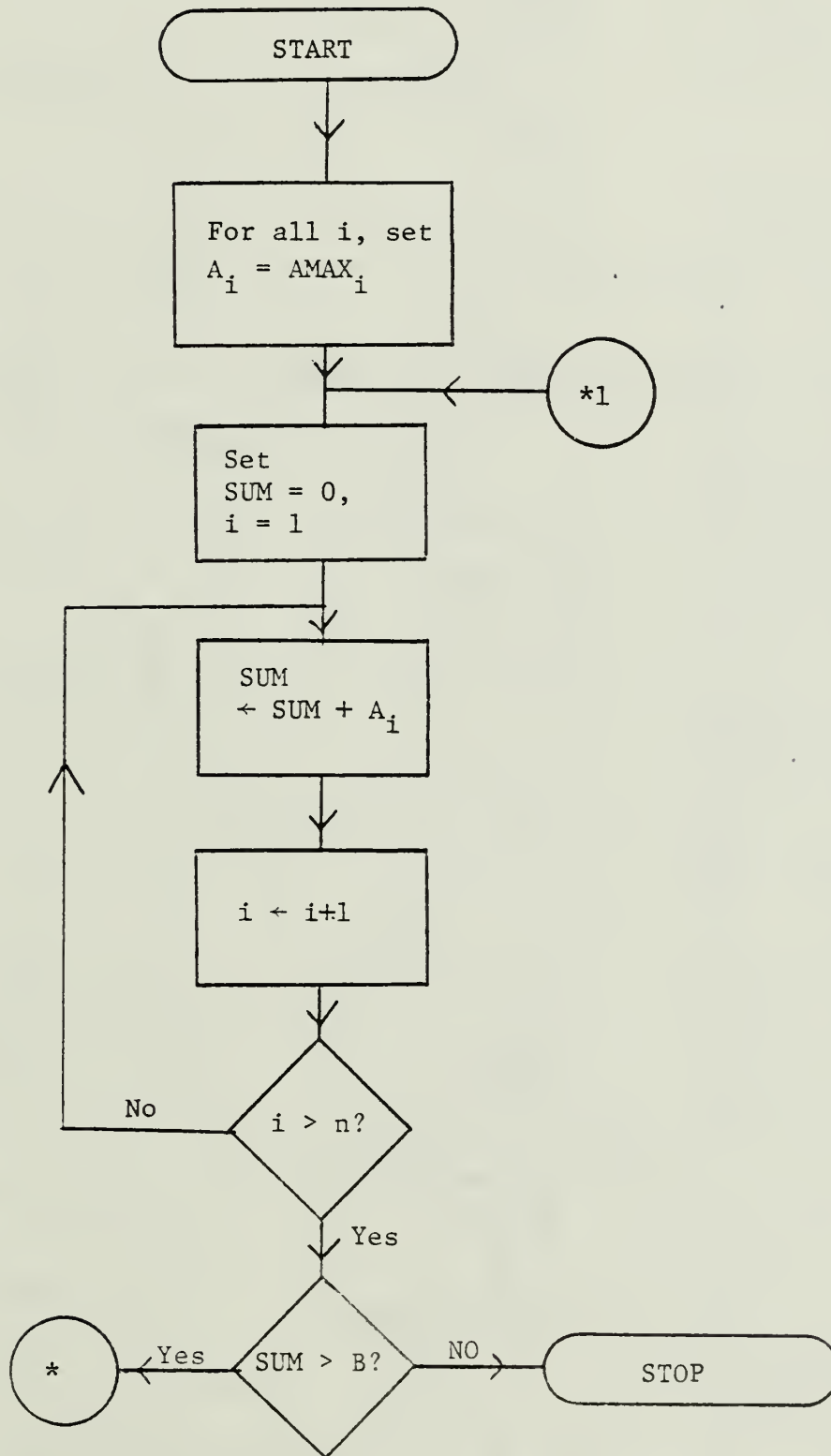
Figure 4. Reverse Marginal Analysis, Stage 2 (continued)

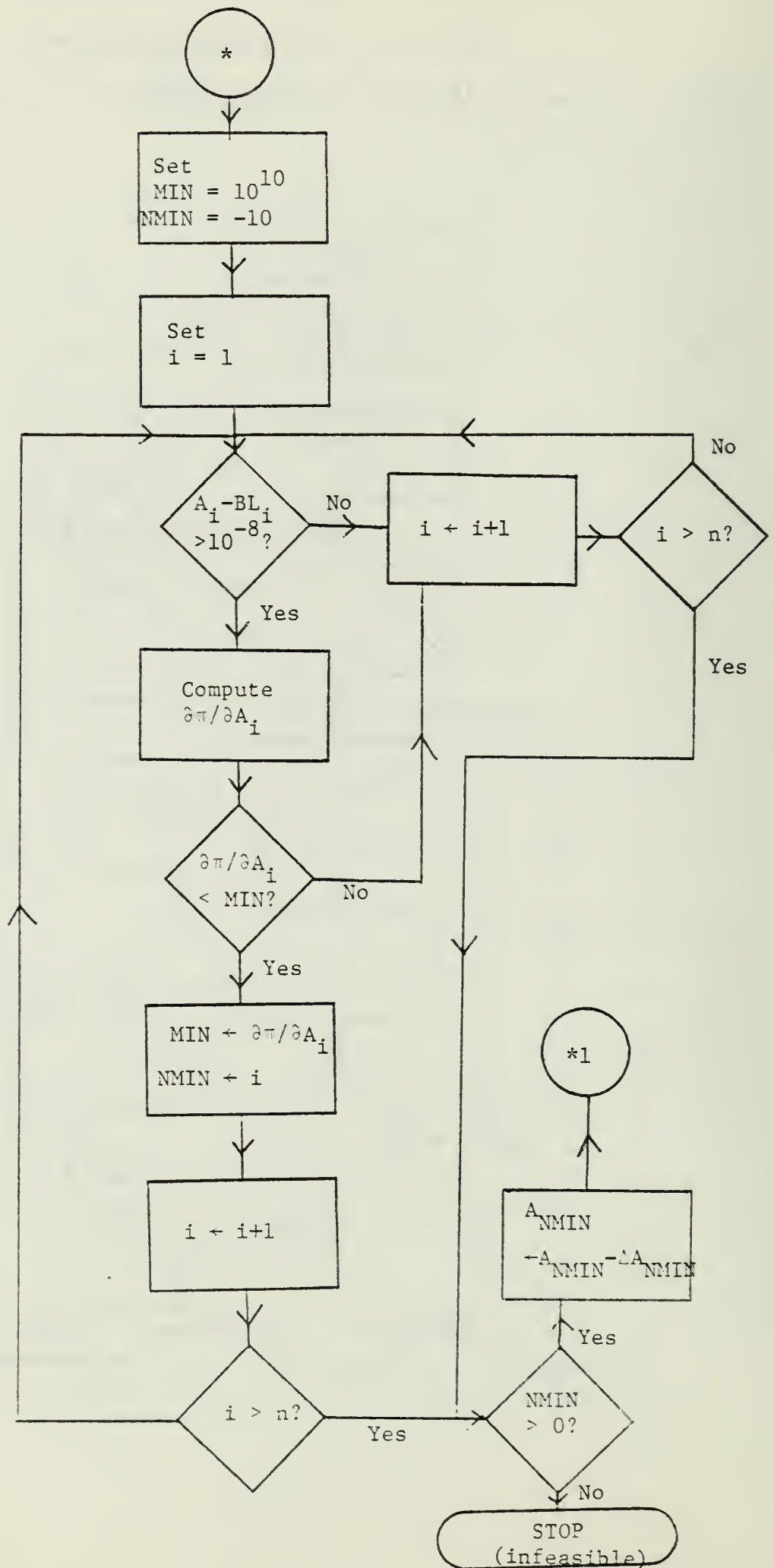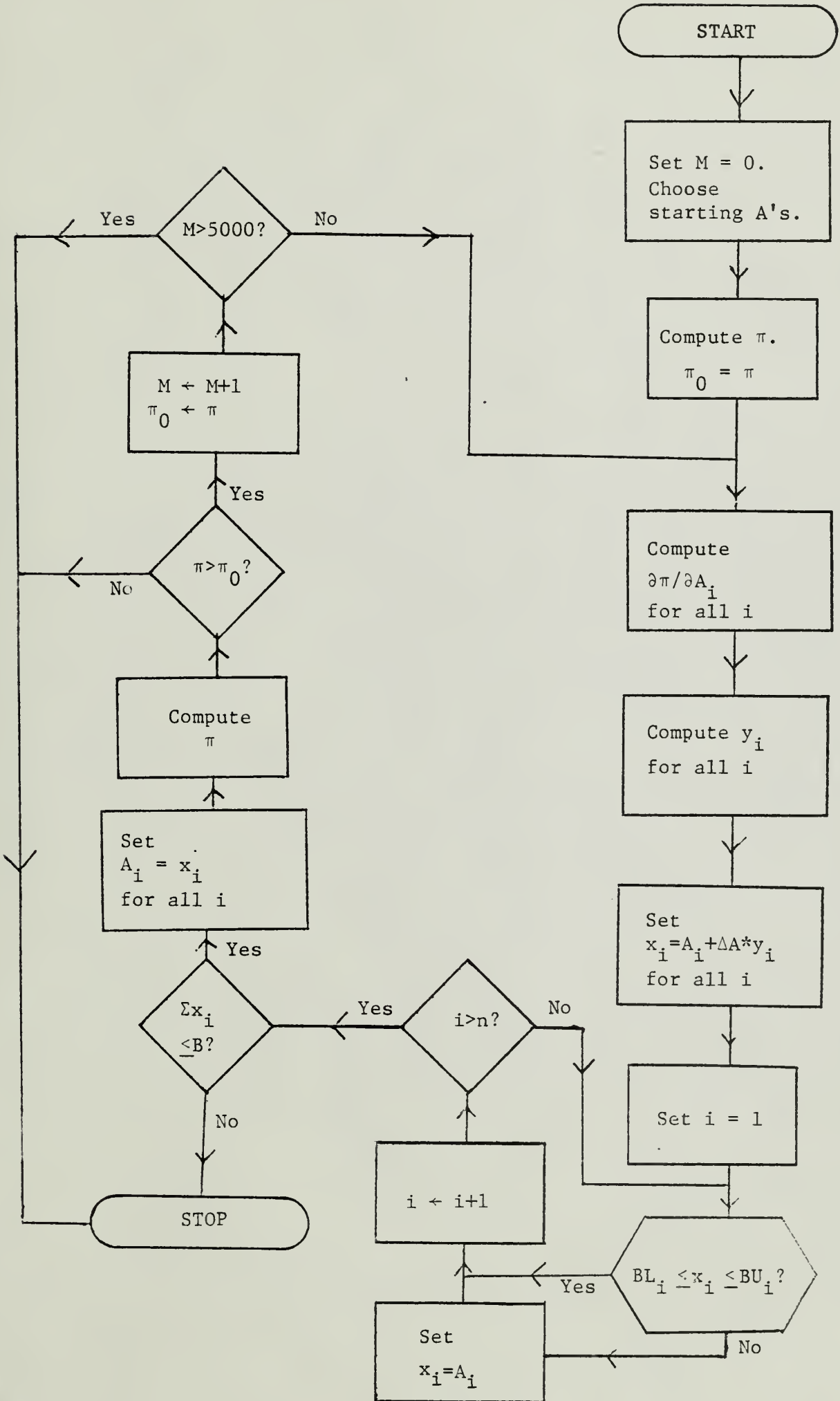24

Figure 5.   Gradient Search

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