

Learning Trajectory of Dilation and Reflection in Transformation Geometry through the Motifs of Bamboo Woven

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Abstract. Cultural diversity around students can be integrated into learning mathematics in the classroom. Yet, Indonesian students still have difficulty applying mathematical concepts to solve problems related to everyday life, such as dilation and reflection in transformation geometry. Therefore, this study aims to design the dilation and reflection in transformation geometry learning trajectory using the motifs of bamboo woven. This context was used as the starting point in the learning process by applying the Pendidikan Matematika Realistik Indonesia approach. The research method used is design research consisted of three stages: preliminary design, teaching experiment, and retrospective analysis. This study describes how the motifs of bamboo woven made a real contribution for ninth-grade students to understand the concept of dilation and reflection in transformation geometry. The design experiments showed that this context can stimulate students to understand their knowledge of the idea of dilation and reflection in transformation geometry. All of the strategies and models that students find illustrate and discuss show how the construction or contribution of students can be used to help their initial understanding of the reflection and dilation.

Keywords: design research, ethnomathematics, learning trajectory, the motifs of bamboo woven, dilation, reflection

Introduction

The results of the PISA (Programme for International Student Assessment) show that Indonesian students still have difficulty applying mathematical concepts in solving problems related to daily life (Hadi, Retnawati, Munadi, Apino, & Wulandari, 2018). Several factors are caused by learning that have not connected mathematics with culture or students' daily activities (Arisetyawan, Suryadi, Herman, & Rahmat, 2014; Naidoo, 2012). On the other hand, most people consider mathematics as a complicated and abstract subject (Artemenko, Soltanlou, Dresler, Ehliis, & Nuerk, 2018; Tsami & Kitsou, 2017). Therefore, the learning process is needed to connect culture and students' daily lives in mathematics learning so that mathematics learning becomes more meaningful.

Education in the 21st Century has a fundamental principle that learning must be student-centered, collaborative, contextual, and integrated with society (Adarlo & Jackson, 2017). Furthermore, Komara (2018) explains the learning model in the 21st Century consists of four items. Firstly, learning is directed to encourage students to find out from various sources. Secondly, learning is directed to formulate problems or ask questions, not just solve problems or

answer. Thirdly, learning is directed to practice analytical thinking, as in the decision-making process, not mechanical and routine review. Lastly, learning emphasizes the importance of collaboration in solving problems. It means that the teacher must facilitate or allow students to solve a problem in life, and students must be able to link the material they are learning with real life.

Transformation geometry is a mathematical concept that is widely applied in everyday life, such as making batik motifs, woven ornaments, making pottery, and others (Clements & Sarama, 2009; Ditasona, 2018; Purniati, Turmudi, Juandi & Suhaedi, 2021; Risdiyanti & Prahmana, 2017). It is fundamental to learn in school because it underlies other concepts such as function and symmetry (Hollebrands, 2003). In conveying the idea of transformation must utilize student experience and active student involvement (Brown & Heywood, 2011). For example, in games and daily activities, students often explore mathematical ideas as they search, classify, compare, and pay attention to shapes and patterns (Naidoo, 2012). Thus, students must understand this concept with their experience and active involvement in the learning process.

One of the solutions is to design innovative and meaningful learning for students so that they can place the teacher as a facilitator in learning and involve students actively. This learning is in line with the Pendidikan Matematika Realistik Indonesia (PMRI) approach, which is an adaptation of Realistic Mathematics Education (RME) and has been developed by the context, cultural values, and local wisdom in Indonesia (Hadi, 2017; Prahmana, 2012). The first characteristics of PMRI is to use contexts that are close to students' lives as a starting point in learning mathematics, such as the culture that surrounds students' lives. Cultural diversity around students can be integrated into learning mathematics in the classroom (Chahine, 2011; Ogunkunle & George, 2015; Rosa, D'Ambrosio, Orey, Shirley, Alangui, Palhares, & Gavarrete, 2016). Also, in the cultural value, there is a mathematical concept that can be used as a starting point in learning and can also be a solution in introducing culture as well as learning mathematics (Maryati & Prahmana, 2018; Maryati & Prahmana, 2019; Risdiyanti, Prahmana, & Shahrill, 2019).

Haris and Putri (2011) explained that woven is suitable for the transformation geometry of mathematical concepts. This is in line with the results of several studies that demonstrated the motifs of woven can be used as a starting point in learning transformation (Fauzi & Setiawan, 2020; Khaerunnisa, Setiani, & Rafianti, 2018). On the other hand, the motifs of Bamboo woven have been successfully implemented in improving students' understanding of the concept of translation and rotation in transformation geometry (Maryati & Prahmana, 2020a; 2020b). The researcher used the woven bamboo motif to help students understand the two others transformation geometry, which is dilation and reflection topic. This is different from the previous research on learning transformation

geometry because it uses the cultural context, namely the motifs of bamboo woven, as a starting point in the learning process. This context was chosen because it is close to students and easily found in students' daily life. Therefore, the research question of this study is finding to what extent the role of a bamboo woven motif in learning dilation and reflection topic.

Method

The research method used is design research consisted of three stages, namely preliminary design, teaching experiment, and retrospective analysis (Plomp, 2013; Prahmana, 2017). This research aims to improve the quality of learning practices in the classroom. The essence of design research is shaped by classroom teaching experience developing teaching sequences and Local Instruction Theory (LIT) that supports it (Gravemeijer, 2004; Prahmana, 2017). LIT is learning process theory that describes the trajectory of learning on a particular topic with a set of supporting activities (Gravemeijer & Van Eerde, 2009; Prahmana, 2017). It is called a local theory because the theory only addresses specific domains, which are particular topics in particular learning.

Preliminary Design

At this stage, the researcher implements the initial idea of using the context of the motifs of Bamboo woven in learning dilation and reflection in transformation geometry by studying the literature. Furthermore, the researcher did observations at one private junior high school in Yogyakarta, Indonesia, to see the students' initial abilities used as the basis for designing the prototype of Hypothetical Learning Trajectory (HLT).

HLT development in each learning activity is an essential part of designing student learning activities. The design of learning activities is inseparable from the learning trajectory, which contains an itinerary of learning material and visual representations of information that students experiences during the learning process. Furthermore, the learning trajectory used in dilation and reflection in the transformation geometry learning process is developed in two cycles so that it is feasible to become a LIT.

A set of activities for dilation and reflection in the transformation geometry learning process has been designed based on students' hypothesized learning trajectories and thought processes consisting of several conjectures that the researcher hypothesis what students will do in the learning process. This set of instructional activities has been divided into four activities completed in 4 meetings with 100 minutes for each session. The relationship between student learning paths, learning activities, and the basic concepts of dilation and reflection in transformation geometry can be seen in Figure 1.

Teaching Experiment

Researchers tested the learning activities in this phase that have been designed at the preliminary design stage in two cycles. The first cycle is a teaching experiment that aims to evaluate and improve the learning path that has been created. Furthermore, based on the results of the teaching experiment (cycle 1), the researcher developed HLT to be implemented in the pilot experiment (cycle 2). The second cycle is a teaching experiment that aims to apply the learning trajectory. This purpose of this activity is to evaluate and then revise the HLT in a teaching experiment.

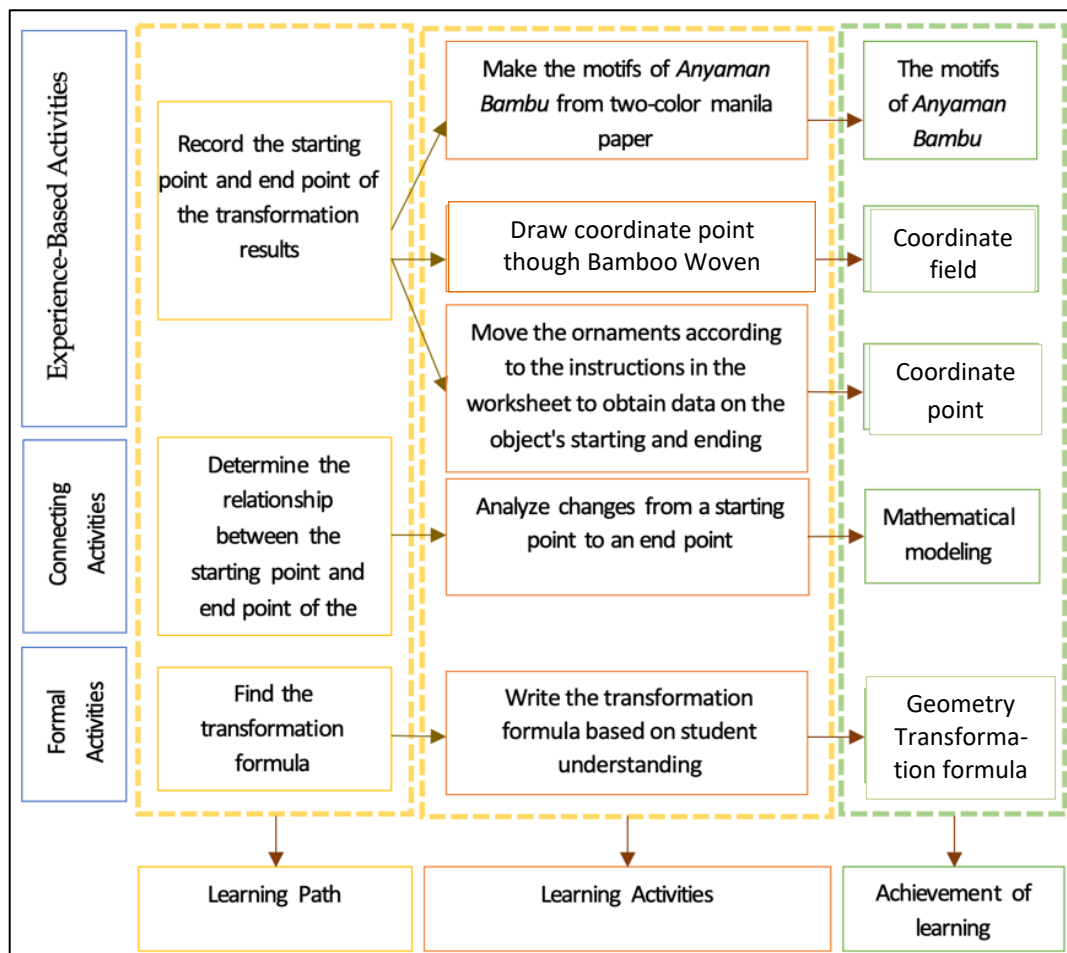


Figure 1. The design of HLT in transformation geometry through bamboo woven

Retrospective Analysis

Data obtained from learning activities in class are analyzed after the teaching experiment has been applied. Furthermore, the results are used to design activities or to develop designs for subsequent learning activities. The purpose of retrospective analysis, in general, is to establish local instruction theory. At this stage, the hypothetical learning trajectory is compared with actual student learning, so the results of this analysis can describe the learning trajectory of dilation and reflection in transformation geometry through the motifs of bamboo woven.

Results and Discussion

This study indicates the implementation of dilation and reflection in transformation geometry learning trajectory using the motifs of Bamboo woven as a starting point in the learning process. The learning trajectory consists of four activities, as explained in Table 1.

Table 1. Reflection and dilation in transformation geometry learning activities using the motifs of bamboo woven

Transformation Geometry Learning Concepts	Description of Learning
Learning about reflection concepts	Preparing Woven Arrange the wicker to form an ornament Reflect the ornament and record the starting point and endpoint according to the instructions in student worksheet namely "Activity 1." Recording the starting and ending points in a table Analyzing point changes after reflection Reinvention the concept of reflection Determining the reflection formula
Learning about dilation concepts	Preparing Woven Arrange the wicker to form an ornament Dilation the ornament and record the starting point and endpoint according to the instructions in student worksheet namely "Activity 2." Recording the starting and ending points in a table Analyzing point changes after dilation Reinvention the concept of dilation Determining the dilation formula

Understanding the Activities of Reflection Concepts

This activity aims to bring up the students' language (mother tongue) or understanding of the concept of reflection. Hadi (2017) explains that the use of contexts which closes to the students, which is one of the characteristics of PMRI, can explore the students' language that is useful for improving students' understanding of a specific mathematical concept. Furthermore, students are given assignments in groups (4-5 people per group) to work on Activity 1 in the student worksheet.

The activities in this learning process begin with the teacher instructing all students to gather with their respective groups as in the previous meeting. Furthermore, the teacher gives assignments to each group to work on Activity 1 in the student worksheet about the concept of reflection. Activities in the student worksheet start from preparing the woven and ornaments.

After students complete their woven and ornaments, the teacher gives some instructions for each group to record the initial and the final coordinates after they are reflected according to the instructions in the student worksheet. In this situation, several students were able to develop some strategies for solving the given problem. This is one of the characteristics of PMRI, which uses models generated by students to reinvention students' understanding of a particular mathematical concept (Hadi, 2017; Prahmana, 2012). Furthermore, Figure 2 shows that students are seen rotating the ornaments according to the instructions in the student worksheet, namely

Activity 1, to get a starting point and endpoint. All groups can write the coordinate points correctly.



Figure 2. Students reflect the ornament and record the coordinates

After all data obtained, students write it in a table, as shown in Figure 3.

Pemodelan

Apakah hubungan antara posisi awal titik, letak cermin, dan posisi akhir titik setelah dicerminkan? Apakah terdapat pengaruh antara letak cermin dengan posisi akhir titik setelah dicerminkan? Untuk memudahkan menjawab soal ini, silah tabel berikut dengan mencermati soal nomor 1 dan 7 pada bagian "Interaktivitas dan Keterkaitan".

Soal Nomor	Nama Titik	Posisi Awal Titik	Letak Cermin	Posisi Akhir Titik
1	A	(1, 2)	Sumbu x	(1, -2)
	B	(2, 2)		(2, -2)
	C	(2, 3)		(2, -3)
	D	(1, 3)		(1, -3)
2	A	(2, 2)	Sumbu y	(-2, 2)
	B	(3, 2)		(-3, 2)
	C	(3, 3)		(-3, 3)
	D	(2, 3)		(-2, 3)
3	A	(1, 1)	(0, 0)	(-1, 1)
	B	(2, 1)		(-2, 1)
	C	(2, 2)		(-2, 2)
	D	(1, 2)		(-1, 2)
4	A	(1, 4)	$x = y$	(4, 1)
	B	(2, 4)		(4, 2)
	C	(2, 5)		(5, 2)
	D	(1, 5)		(5, 1)
5	A	(2, 4)	$-x = y$	(-4, 2)
	B	(-1, 4)		(-4, -1)
	C	(-1, 5)		(-5, -1)
	D	(2, 5)		(-5, 2)
6	A	(1, 4)	$y = a$	(1, a)
	B	(2, 4)		(2, a)
	C	(2, 5)		(2, a)
	D	(1, 5)		(1, a)
7	A	(4, 4)	$x = b$	(b, 4)
	B	(5, 4)		(b, 4)
	C	(5, 5)		(b, 5)
	D	(4, 5)		(b, 5)

Figure 3. The results of student work in activities to record the starting point and endpoint of the reflection results in a table

Next, students analyze the change from the starting point to an endpoint and make interpretations related to the concept of reflection with their language as shown in Figure 4.

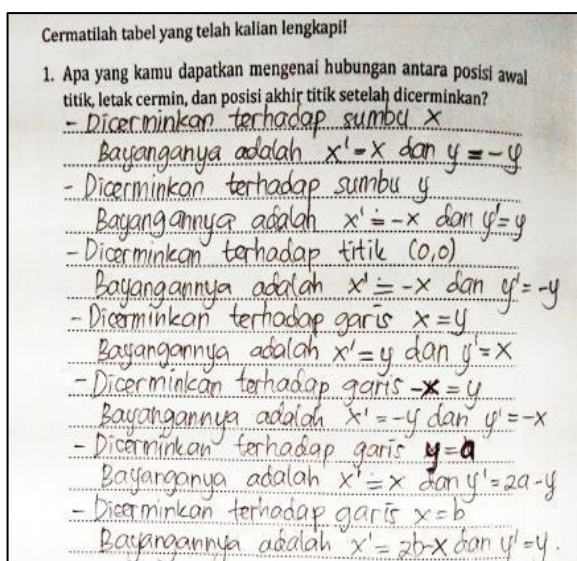


Figure 4. Students write the results of their reflection interpretations in their worksheet

Based on the results of their interpretation, students make mathematical modeling in the form of a reflection formula according to their understanding. The results of students' mathematical modeling can be seen in Figure 5.

2. Dapatkah kalian menentukan rumus bayangan, jika diketahui koordinat titik awal dan letak cermin untuk suatu titik tertentu?

Letak cermin	Bayangan
Sumbu x	$(x, -y)$
Sumbu y	$(-x, y)$
Titik $(0,0)$	$(-x, -y)$
garis $x=y$	(y, x)
garis $-x=y$	$(-y, -x)$
garis $y=a$	$(x, 2a-y)$
garis $x=b$	$(2b-x, y)$

Figure 5. The results of students' mathematical modeling related to the reflection formula

To classify the results of student answers listed in the worksheet, class discussions are needed. Therefore, the teacher invites each group to present their work. This is one of the characteristics of PMRI, namely interactivity (Hadi, 2017; Prahmana, Zulkardi, & Hartono, 2012). Furthermore, Figure 6 shows that students are seen performing the concept of reflection.



Figure 6. Students present reflection concepts in front of class

During the discussion process, it seemed that the participants were very enthusiastic about expressing their opinions and ideas on each group's worksheet. This is caused by the position of the ornament starting point that is different from each discussion group, so that the location of the ornament endpoint will also be different. This problem is called open-ended problems which have many correct answers in solving one problem. Furthermore, the teacher guides students to have a common perception of the concept of reflection, namely (1) if the starting point is reflected against the x-axis, then the end point will be $(x, -y)$, (2) if the starting point is reflected against the y-axis, then the end point will be $(-x, y)$, (3) if the starting point is reflected against the origin $(0,0)$, then the end point will be $(-x, -y)$, (4) if the starting point is reflected against the line $y = x$, then the end point will be (y, x) , (5) if the starting point is reflected against the line $y = -x$, then the end point will be $(-y, -x)$, (6) if the starting point is reflected against the line $y = h$, then the end point will be $(x, 2h-y)$, and (7) if the starting point is reflected against the line $x = h$, then the end point will be $(2h - x, y)$.

The activity in the third meeting was ended by evaluating the questions in the form of formal questions, such as Question 1: "Build ABCD is a rectangle formed from 4 coordinates, namely A (1,1), B (3,1), C (3,3), and D (1,3). The rectangle will be reflected against the y axis. How to position the coordinates of the shadow now?"; and Question 2: "Build PQR is a triangle formed from 3 coordinates, namely A (-4, -1), B (0, -1), and C (-2, -2). The triangle will be reflected against $x = 1$. How to position the coordinates of the shadow now?". The results of the answers of one student are shown in Figure 7.

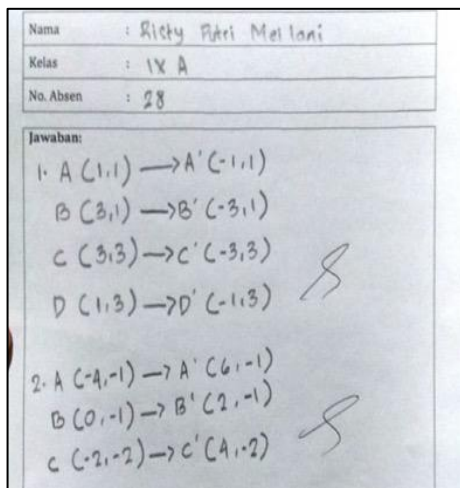


Figure 7. The results of student answers

Based on the analysis of student's answers, several students were able to apply the basic concept of reflection to solve the given problem. So, it can be concluded that after students do reflection learning using the motifs of bamboo woven, students get a good understanding of the concept of reflection. These results indicate that mathematics learning associated with local culture (in this case, the culture of bamboo woven), namely ethnomathematics, can be used as a starting point in learning mathematics to develop students' mathematics understanding, as documented by several previous researchers (Ditasona, 2018; Fauzi & Setiawan, 2020; Risdiyanti & Prahmana, 2018; Utami, Sayuti, & Jailani, 2019).

Understanding the Activities of Dilation Concepts

This activity aims to bring up the students' language (mother tongue) or understanding of the concept of dilation. Students are given assignments in groups (4-5 people per group) to work on Activity 2 in the student worksheet. The activities in this learning process start with the teacher gives instruction to all students to gather in their respective groups as in the previous meeting. Furthermore, the teacher gives assignments to each group to work on Activity 2 in the student worksheet about the concept of dilation. Activities in the student worksheet start from preparing the woven and ornaments.

After all students finished their woven and ornaments, the teacher instructs each group to record the initial coordinates and the final coordinates after they are reflected according to the instructions in the worksheet. Figure 8 shows that students are seen dilating the ornaments according to Activity 2 in the student worksheet to get a starting point and endpoint. All groups can write the coordinate points correctly. After all data is obtained, students write it in a table, as shown in Figure 9.

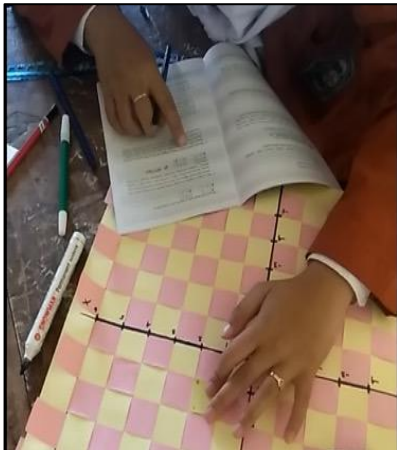


Figure 8. Students dilate the ornament and record the coordinates

Soal Nomor	Nama Titik	Posisi Awal Titik	Perbesaran/Pengcilan		Posisi Akhir Titik
			Faktor Skala	Titik Pusat	
	C	(2,2)		(0,0)	(4,4)
	D	(1,2)		(0,0)	(2,4)
3	A	(1,1)	3 kali	(0,0)	(3,3)
	B	(2,1)		(0,0)	(6,3)
	C	(2,2)		(0,0)	(6,6)
	D	(1,2)		(0,0)	(3,6)
4	A	(1,1)	2 kali	(1,1)	(1,1)
	B	(2,1)		(1,1)	(3,1)
	C	(2,2)		(1,1)	(3,3)
	D	(1,2)		(1,1)	(1,2)
5	A	(1,1)	3 kali	(1,1)	(1,1)
	B	(2,1)		(1,1)	(4,1)
	C	(2,2)		(1,1)	(4,4)
	D	(1,2)		(1,1)	(4,4)

Figure 9. The results of student work in activities to record the starting point and endpoint of the dilation results in a table

Furthermore, students analyze the transformation from the starting point to the endpoint and make interpretations related to the concept of dilation with their language, as shown in Figure 10. Based on the interpretation results, students make mathematical modeling in the form of a dilation formula according to their understanding. The results of students' mathematical modeling can be seen in Figure 11.

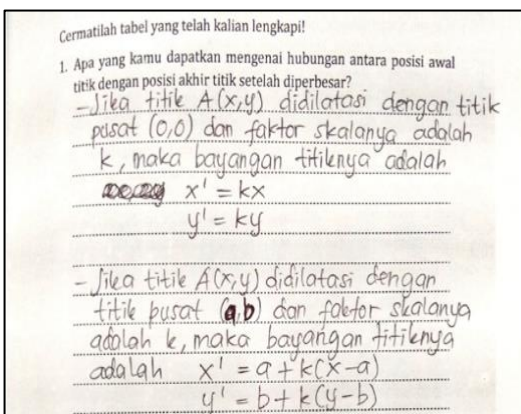


Figure 10. Students write the results of related interpretations the concept of dilation in the worksheet

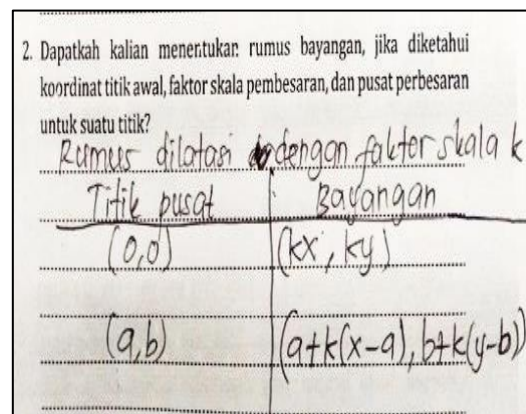


Figure 11. The results of students' mathematical modeling related to the dilation formula

To clarify the results of student answers listed in the student worksheet, class discussions are needed. Therefore, the teacher invites each group to present their work. Students are seen giving the concept of dilation, as shown in Figure 12.

During the discussion process, it seemed that the participants in the discussion were very enthusiastic about expressing their opinions and ideas on each group's worksheet. It is caused by the position of the ornament starting point that is different from each discussion group so that it will also alter the location of the ornament endpoint.

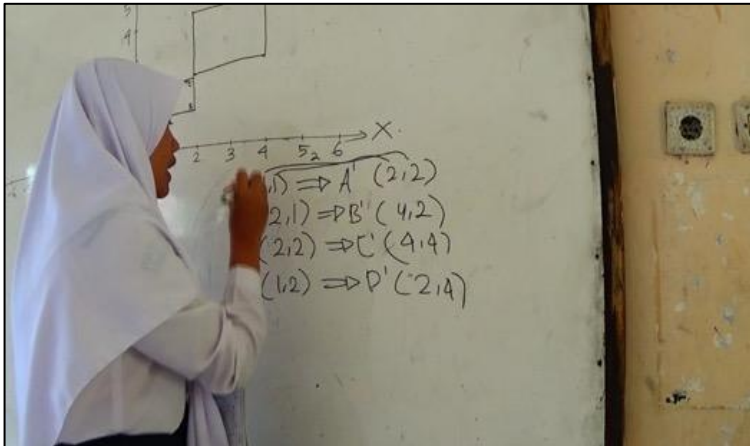


Figure 12. Students present the dilation concepts

Furthermore, the teacher guides students to have a common perception of the concept of dilation, namely, if an object is enlarged, then the position of the shadow point is the result of multiplication between the initial coordinate point with the scale factor. In contrast, if the object is enlarged against the center point (a, b) , then the shadow coordinate point is $x' = a + k(x - a)$ and $y' = b + k(y - b)$.

The activity at the fourth meeting was ended by evaluating the questions in the form of formal issues, such as Question 1: "Write KLM point is a triangle from 3 coordinates, namely $K(12,4)$, $L(4,8)$, and $M(8,-8)$. After twice sequent dilation on the center point $(0, 0)$ with the same scale factor, the image coordinates become $K''(3,1)$, $L''(1,2)$, and $M''(2,-2)$. Determine the scale factor k used to dilate $\triangle KLM$ to $\triangle K''L''M''$ "; and Question 2: "Build ABC is a triangle formed from 3 coordinates, namely $A(6,12)$, $B(-9,3)$, and $C(6,-6)$. If the triangle is dilated using $1/3$ scale factor with the center point $(0,0)$, draw the starting coordinates point and their image!". The results of the answers from one student are shown in Figure 13.

Students could apply the basic concept of dilation to solve the given problem in terms of the analysis of students' answers. So, it can be concluded that after students do dilation learning by using the motifs of bamboo woven, students get a good understanding of the concept of dilation.

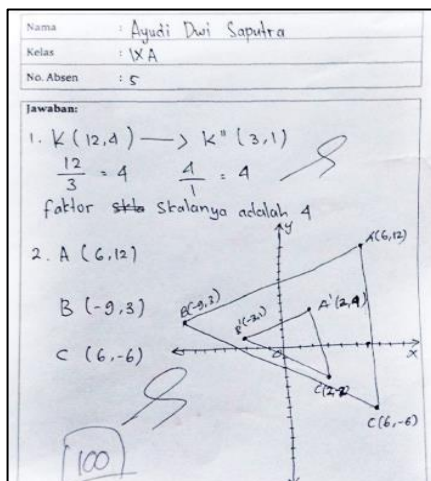


Figure 13. Results of student answers

Several activities that students have passed to enhance students' understanding of the concepts of reflection and dilation have several characteristics. The first characteristic is using the context of woven bamboo activities that are close to students in exploration activities. Furthermore, students use some strategies in solving problems given in the form of mathematical models. Third, the teacher appreciates and uses all students' creations and contributions in constructing students' understanding of the concepts of reflection and dilation. Furthermore, discussions are interactive and intertwining with other mathematics materials during the learning process. Finally, the context used in this study is a cultural activity that exists in Indonesia. All of these activities are in line with the characteristics of PMRI, as explained by Sembiring, Hoogland, and Dolk (2010).

Furthermore, the use of woven bamboo activities in this study is part of the results of exploring cultural activities close to students who have mathematical concepts. The movement of integrating culture in mathematics learning is one of the characteristics of ethnomathematics learning (Chahine, 2011; Ogunkunle & George, 2015; Rosa et al., 2016). This is also part of preserving cultural activities for students, which students in the last decade have abandoned (Risdiyanti & Prahmana, 2020; Risdiyanti, Prahmana, & Shahrill, 2019). Therefore, besides enhancing students' understanding of the concepts of dilation and reflection, the results of this implementation are also able to introduce and preserve the culture of woven bamboo for students by integrating ethnomathematics learning with the PMRI approach.

Finally, the results of the design experiments show that this context stimulate students to understand their knowledge of the concept of transformation. All the strategies and models that students find and discuss, demonstrate how the construction or contribution of students can be used to help their initial understanding of the concept of dilation and reflection in transformation geometry. These results support some of the effects of previous studies, which state that the

cultural context can be used as a starting point in learning mathematics (Hendriana & Fitriani, 2019; Hilaliyah, Sudiana, & Pamungkas, 2019; Maryati & Prahmana, 2018; Utami, Sayuti, & Jailani, 2019; Risdiyanti, Prahmana, & Shahrill, 2019). Besides, some researchers have also made mathematics learning designs using PMRI approaches and cultural contexts, such as learning number patterns using “Barathayudha” war stories (Risdiyanti & Prahmana, 2020), designing the transformation learning using Sidoarjo written batik motifs (Lestariningsih & Mulyono, 2017), and learning number operation using traditional Indonesian game “Tepuk Bergambar” (Prahmana, Zulkardi, & Hartono, 2012). Therefore, the contribution of this study is to enrich the study of mathematics learning design, that is, dilation and reflection in transformation geometry learning design using the context of the woven bamboo motif.

Conclusion

A learning trajectory can support students' understanding of dilation and reflection in transformation geometry from informal to formal mathematical forms. This trajectory consists of several activities, including recording the starting points and endpoints in the table, analyzing and interpreting the transformation of the starting point into an ending point using students' language, and lastly, writing the dilation and reflection formula in transformation geometry.

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