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After Three Centuries and a Half, What Have We Learned?

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Abstract

Distinguish between quantity and price and between macroeconomics and microeconomics, let the two distinctions intersect, and see the core of economic theory as the four resulting elements: (1) seventeenth-century unemployment theory, (2) early eighteenth-century allocation theory, (3) mid-eighteenth-century inflation theory, and (4) late-eighteenth and early-nineteenth-century theory of relative price.

In our early centuries insights were gained and lost, and elements remained irreconciled. The paper sees one reason for our slow progress in our obsession with value judgments, another in our barriers to communication, i.e., the mathematics barrier and the language barrier. The paper concludes by characterizing the reconciliation attempted in our own century.

AFTER THREE CENTURIES AND A HALF, WHAT HAVE WE LEARNED?

By HANS BREMS

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I. THE CORE OF ECONOMICS

For three centuries and a half economists have concentrated their efforts on a compact core of well-defined problems[1]. Two distinctions will bring out our core. The first is the distinction between quantity and price as they manifest themselves in competitive markets. The second is the distinction between macroeconomics and microeconomics. The two distinctions will give us the simple two-by-two matrix shown in our table. We have written its elements in the order in which they were discovered. Let us take a brief look at the four elements.

OUR CORE

	Macro	Micro
Quantity	Unemployment Theory	Allocation Theory
Price	Inflation Theory	Theory of Relative Prices

II. EARLIER CENTURIES

1. Unemployment Theory

Assuming the economy to produce a single good, macroeconomic unemployment theory determines the physical output of that good. Seventeenth-century mercantilist unemployment theory saw physical output as its equilibrating variable. Physical output was bounded by demand, and there was always excess capacity. Supply was no problem: in the seventeenth century we were confident that demand would always create its own supply.

What was our theory of interest? We were sure that the rate of interest was determined by the supply of and the demand for money rather than by the supply of saving and the demand for investment. Because our rate of interest had everything to do with the money supply, we could discuss the effects of the money supply upon it: an expanding money supply would lower the rate of interest, stimulate investment, and expand physical output.

But how exactly could the money supply be expanded? As long as money was metal and the country possessed no mineral deposits of that metal, an export surplus was the only available source of money. Logically the exporter, the merchant, became the hero of the mercantilist piece and was duly glorified, for example by Mun [1964 (1949: 88)]: "... Forraign Trade ... is ... The Noble profession of the Merchant, The School of our Arts, the supply of our wants, The employment of our poor, The improvement of our Lands, The Nurcery of our Mariners, The walls of the Kingdoms, The means of our Treasure, The Sinnews of our wars, The terror of our Enemies."

In England money was metal, but Yarranton [1677 (1854: 38)] called attention to the practice of Dutch banks of extending credit with mortgages as collateral: "Observe all you that read this, and tell to your children this strange thing, that paper in Holland is equal with moneys in England ..." and believed that following the Dutch example would lower the rate of interest from six to four percent.

In the seventeenth century, in other words, we thought of money as nonneutral: it would always affect real but never affect nominal variables, and we may not have been entirely wrong. The economy was not yet fully monetized. Further monetization might still be a prerequisite for more division of labor, hence higher labor productivity. In that sense the economy still had excess capacity.

So much for monetary policy. As for fiscal policy, we had nothing but kind words for taxes and public works. A late mercantilist, Steuart

[1767 (1805: 271-272)] thought "that taxes promote industry; not in consequence of their being raised upon individuals, but in consequence of their being expended by the state; that is, by increasing demand and circulation ... It is no objection to this representation of the matter, that the persons from whom the money is taken, would have spent it as well as the state. The answer is, that it might be so, or not: whereas when the state gets it, it will be spent undoubtedly."

Public works, too, were a good thing. In Petty's [1662 (1899: 29-31)] words, such works may be useful: "... making all High-wayes so broad, firm, and eaven, as whereby the charge and tedium of travelling and Carriages may be greatly lessened. The cutting and scowring of Rivers into Navigable; the planting of usefull Trees for timber, delight, and fruit in convenient places." But public works would still be a good thing even if they were useless: "... 'tis no matter if it be employed to build a useless Pyramid upon Salisbury Plain, bring the Stones at Stonehenge to Tower-Hill, or the like; for at worst this would keep their mindes to discipline and obedience, and their bodies to a patience of more profitable labours when need shall require it."

In the seventeenth century did we worry about the crowding-out effect of taxes, public works, fiscal deficits? We did not. Our rate of interest had nothing to do with the supply of saving and the demand for investment but was fully controlled by the money supply!

The seventeenth century, then, was a time at which a modern Keynesian would have felt quite at home. He would have found serious doubt that, left to itself, capitalism was at all capable of utilizing its own resources. Government action was believed to be the remedy.

2. Allocation Theory

Assuming the economy to produce more than one good, early eighteenth-century microeconomic theory came to grips with allocation in accordance with preferences. Writing around 1730, Cantillon [1755 (1931)] used preferences to determine sustainable employment. The employment an available physical stock of land could support depended on the land absorption by the necessities needed to feed labor as well as on the land absorption by the luxuries demanded by landlords. Such absorption differed among crops, and there was a choice among crops. The choice expressed preferences, i.e., [1755 (1931: 70-71)] labor's "Manner of Living," "maniere de vivre," and [1755 (1931: 80-81)] the "Taste, Humours and Manner of Living of the Proprietors of Land," "des volontés, du goût & de la facon de vivre des Propriétaires de terres." As a result, Cantillon [1755 (1931: 84-85)] could ask "whether it is better to have a great multitude of Inhabitants, poor and badly provided, than a smaller number, much more at their ease: a million who consume the produce of 6 acres per head or 4 million who live on the produce of an Acre and a half." But Cantillon could also distinguish analysis from value judgment and dismiss the question as "outside of my subject."

But alas! For another century and a third Cantillon's insights were ignored by mainstream economics.

3. Inflation Theory

Assuming the economy to produce a single good, macroeconomic inflation theory determines the price of that good--and how rapidly that price is rising. Hume's mid-eighteenth-century inflation theory saw price as its equilibrating variable. Physical output was bounded by supply, and there was never any excess capacity. Demand was no problem: in the mid-eighteenth century we were confident that supply would always create its own demand.

What was our theory of interest? We were sure that the rate of interest was determined by the supply of saving and the demand for investment rather than by the supply of and demand for money. Hume

[1752 (1875: 322)] put it this way: "Low interest ... proceeds from ... three ... circumstances: A small demand for borrowing; great riches to supply that demand; and small profits arising from commerce: And these circumstances are all connected together, and proceed from the encrease of industry and commerce, not of gold and silver." Because our rate of interest had everything to do with saving and investment, we could discuss the crowding-out effects of taxes and fiscal deficits upon it. In Smith's [1776 (1805: 89)] words, "[Kings and ministers] ... are themselves always, and without any exception, the greatest spendthrifts in the society. Let them look well after their own expence, and they may safely trust private people with theirs."

Because our rate of interest had nothing at all to do with supply of and demand for money, we saw no direct effect of the money supply upon the rate of interest. The direct effect of the money supply was upon price: a mercantilist export surplus would simply generate inflation or, in Hume's [1752 (1875: 333)] own words: "Must not all labour and nations could afford to buy from us; while their commodities, on the other hand, became comparatively so cheap, that, in spite of all the laws which could be formed, they would be run in upon us, and our money flow out." In other words, the mercantilist export surplus would eliminate itself.

If the money supply had any effect upon the rate of interest at all, it would be the opposite of what the mercantilists had believed: two-thirds of the way through the eighteenth century Turgot [1769-1770 (1922: 76)] saw an indirect effect of an increasing money supply upon the rate of interest: "it may on the contrary happen that the very cause which increases the money in the market, and which increases the prices of other commodities by lowering the price of money, is precisely that which increases the hire of money or the rate of interest." Here is the first glimpse of a distinction between a nominal and a real rate of interest: a nominal rate will exceed a real rate by the rate of inflation.

[illegible] eighteenth century, in other words, we thought of money as

commodities rise to such an exorbitant height, that no neighbouring

preindustrial technology had, perhaps, been exploited. In that sense the economy had no more excess capacity.

The eighteenth century was a time at which a modern monetarist would have felt quite at home. He would have found no doubt whatever that, left to itself, capitalism was fully capable of utilizing its own resources. Government action, however well-meant, was the real problem.

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In the eighteenth century, in other words, we thought of money as neutral: it would never affect real but always affect nominal variables, and we may not have been entirely wrong. The economy was by now fully monetized. Further monetization could generate no additional division of labor, hence no higher labor productivity. The full potential of a preindustrial technology had, perhaps, been exploited. In that sense the economy had no more excess capacity.

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4. Theory of Relative Price

Assuming the economy to produce more than one good, microeconomic theory may come to grips with the relative price of those goods. Late eighteenth and early nineteenth century English classical theory tried to do so but excluded preferences, referred to as "value in use." Preferences were banished because of the value paradox: ever since Aristotle everybody had observed that goods having the highest value in use often had the lowest value in exchange and vice versa. Everybody had duly concluded that value in use could not explain value in exchange.

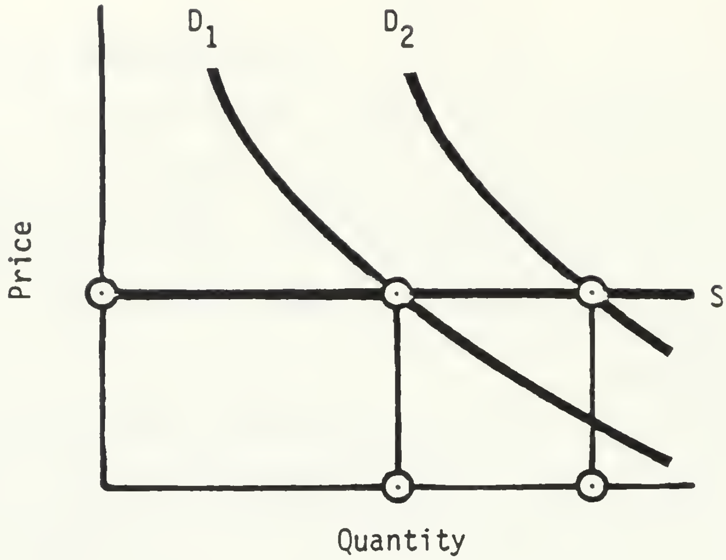
The value paradox is easily resolved once the derivative of value in use with respect to available quantity is taken, and a member of the Basle lineage of brilliant mathematicians took that derivative: Bernoulli [1738 (1896: 27)] took the derivative of "advantage" with respect to "possession" and suggested that "any arbitrarily small gain will produce an advantage which will be in inverse proportion to the already existing possession." Now everything should have fallen into place: because available quantity is so large, the last gallon of water has a low value in use, hence a low value in exchange. Because available quantity is so small, the last stone of diamonds has a high value in use, hence a high value in exchange. On the margin, then, value in exchange does follow value in use.

But everything did not fall into place. Bernoulli's results were noticed and remembered by probability theorists like Laplace and Poisson and by experimental psychologists like Fechner. But for another century and a third economists remained convinced that value in use could not explain value in exchange.

We thought we had better ways of explaining it anyway, for example, as Smith did, by relative costs of input required--all input: land, labor, and capital. Or perhaps, as Ricardo [1817 (1951)] did, by the relative requirement of a single input, i.e., labor. Ricardo was skating on thin ice. For one thing, capital could be ignored only if in all industries the capital-labor ratio were the same and the durability of capital goods were the same. Of this Ricardo was aware. What he was not aware of was that in a multi-crop economy the rent an acre could earn in one crop would be an opportunity cost to any other crop, hence could not be ignored.

Retreating from such thin Ricardian ice, could we reach safety by going back to Smith and use his relative costs of all input, not just labor input? Mill tried to do so but distinguished between two subclasses of reproducible commodities of which Smith had thought of only the first. Let us draw a diagram for each subclass.

In Mill's first subclass, commodities were reproducible at constant cost per unit, so we draw the horizontal supply curve S. Here, may we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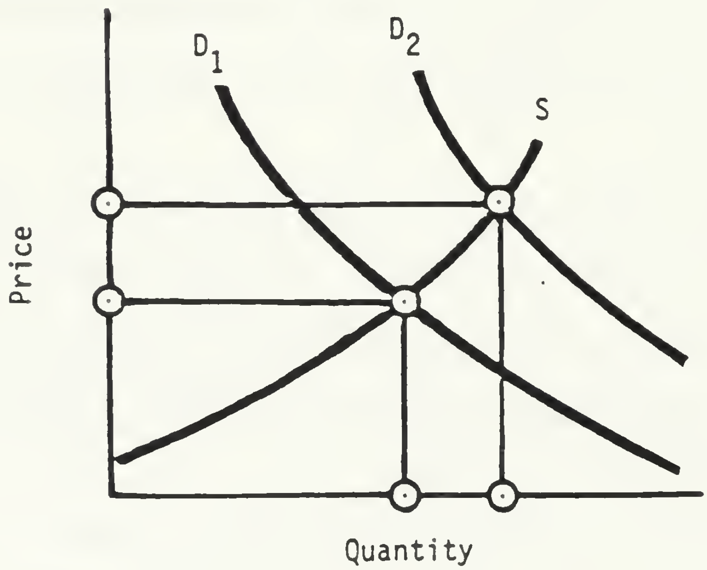
D1
D2
S
Price
Quantity

safely ignore demand? Let a demand curve D_1 intersect our horizontal supply curve. Clearly the ordinate (price) of the intersection point but not the abscissa (quantity) will be the same for D_1 and a new demand curve D_2. Demand, then, matters for allocation but not for price.

In Mill's second subclass, commodities were only reproductible at rising cost per unit, so we draw a rising supply curve S. Here, may we safely ignore demand? Now neither the ordinate (price) nor the abscissa (quantity) of our intersection point will be the same for D_1 and a new demand curve D_2. Demand, then, matters for allocation as well as for price.

Mill [1848 (1923: 456)] never grasped this. "Demand and supply," he said, "only determine the perturbations of value... They themselves obey a superior force, which makes value gravitate towards Cost of Production." Indeed Mill saw nothing left for any future writer to clear up. For a century and a third after Cantillon and Bernoulli, our allocation theory was nonexistent, and our price theory was a <u>non sequitur</u>.

III. WHY DID WE PROGRESS SO SLOWLY?

1. Value Judgment versus Analysis

One reason for the slowness of our progress was our obsession with value judgments. Again and again we preferred easy value judgments to hard analysis. We wasted the entire millennium between Aristotle and the Renaissance by asking if it was right to charge interest instead of asking why, always and everywhere, time has been an economic good and has had a positive price called the rate of interest.

Another reason was our lack of training in the use of appropriate tools. An untutored mathematical mind may handle arithmetic safely and go quite far with it: a mathematical restatement[2] of, say, Cantillon's or Böhm-Bawerk's arithmetic is straightforward and will impeccably derive their conclusions. But tutored mathematical minds may go farther. Of such minds we had few, and with the exception of Fisher [1892 (1925)] they used non-English languages: Bernoulli [1738 (1954)] Latin, Cournot [1838 (1897)] French, von Thünen [1850 (1960)], Wicksell [1893 (1954)], Wald [1935, 1936 (1968)], and von Neumann [1937 (1968)]

German, and Kantorovich [1939 (1960)] Russian. That brings us to our two barriers to communication.

2. The Mathematics Barrier

The small core of problems to which economists have for so long confined themselves, involves quantity and price or their product, quantity times price: output times its price and summed over all output demanded by a household is its budget. Output times its price and summed over all output supplied by a firm is its revenue. Input times its price and summed over all input demanded by a firm is its cost. Input times its price and summed over all input supplied by a household is its income. Our core, then, is always quantitative, and for any number of variables larger than a handful, an important class of economic problems will exist which can be solved mathematically but hardly verbally. It took us our first two centuries and a half merely to formulate such problems and our first three centuries to actually solve them.

The mathematics barrier was a one-way barrier: the mathematicians were fully literate. They would be at no disadvantage should an important class of economic problems exist which could be solved verbally but not mathematically.

3. The Language Barrier

Smith spent the years 1764-1766 in France and may have learned much from the physiocrats; we don't know. Academic etiquette of his day demanded no acknowledgments, and he offered none.

The language barrier was magnified by an attitude barrier: who cared, really? Ricardo, who believed that supply would always create its own demand, spent a few days in Geneva in 1822 discussing the matter with Sismondi, who believed that demand would always create its own supply. Neither convinced the other. Sismondi [1824 (1827: 411)] was not at all surprised by the encounter with Ricardo: "Il apporta à son examen l'urbanité, la bonne foi, l'amour de la vérité qui le distinguaient, et une clarté à laquelle ses disciples eux-mêmes ne se seraient pas attendus ... ." Poise, suavity, good faith, love of truth, lucidity! Ricardo [1822 (1962: 243)], on his side, was visibly surprised by Sismondi: "I am a great admirer of his talents, and I was favorably impressed by his manners--I did not expect from what I had seen of his controversial writings to find him so candid and agreeable. M. Sismondi takes enlarged views... ."

Victorian England cared even less than did Smith and Ricardo. Gårdlund (1958: 341) reprints Marshall's remarkable letter of August 26, 1904, to Wicksell: "I will be frank. I have decided not to answer, probably not even to read Professor Böhm-Bawerk's criticisms on myself."

Like the mathematics barrier, the language barrier was a one-way barrier: the Continentals did read English. But behind their one-way language barrier English economists enjoyed a protection enhancing their reputation among themselves and, until recently, in the United States. Paying full tribute to Continentals like Bernoulli, Cournot, Dupuit, and Gossen, Jevons [1879 (1931: xlii and xlv) may have been the first Englishman to characterize English economics in terms as strong as "insular narrowness" and "a fool's paradise."

In our own century the barriers are vanishing: everybody is now being trained in mathematics, and the Continentals and the Japanese not only read English, as they always did, but also publish in it. Let us turn to that happy century.

IV. OUR OWN CENTURY

1. Macroeconomics

The macroeconomics of the second third of the twentieth century was dominated by unemployment theory in its Keynesian form. But already in the fifties Hicks (1956: 150) had "a feeling that the world of the fifties ... may be Keynesian in its policies, but it is not Keynesian in its working." What Hicks had suspected became fully apparent with the oil shocks of 1974 and 1979.

For five reasons a Keynesian model could not accommodate the oil shocks. First, the price of oil was something on the supply side, but to Keynes supply was never a problem: demand would always create its own supply. Second, the price of oil was something on the price side, but to Keynes the decisive equilibrating variable was physical output, never price. Third, the price of oil was something microeconomic, a relative price, but Keynes's model was macroeconomic with no room for the relative price of two or more goods. Fourth, the price of oil was something international, but Keynes's model visualized a closed economy with no room for international transactions. Fifth, the oil shocks started inflationary spirals, but Keynes's model was static with no room for derivatives with respect to time such as the rate of inflation.

No wonder that the instinctive reaction of central bankers and secretaries of treasuries was to fling aside the Keynesian model and look for something else, anything else. A monetarist model would repair the first two deficiences of a Keynesian one: it was a supply-side model whose decisive equilibrating variable was price, never physical output. But vague as the Friedman (1968), (1970) model was, it was as macroeconomic, as closed, and as static as the Keynesian one had been.

In our own century, then, the two halves of macroeconomics have been alternating in favor, at least among central bankers and secretaries of treasuries. A beginning reconciliation became visible only in the seventies, both in theory [Turnovsky (1977)] and in the second and third generation large-scale macroeconometric models.

2. Macroeconometrics

Like the high-speed electronic digital computer itself that was to become its base, macroeconometrics has outgrown its academic birthplace and become an industry.

First-generation macroeconometric models by Klein (1950) and Klein-Goldberger (1955) were Keynesian, used the large aggregates of the new national income accounts, and were severely constrained by computational facilities. Klein's original model had 12 equations! Soon computational facilities made much larger models possible, and macroeconometrics rose to the challenge.

The second-generation models of the sixties were less Keynesian: income elasticities and multipliers were shrinking, and sensitivities to inflation and the rate of interest were expanding. As for size, the original Federal Reserve-M.I.T. model had 66 equations, the Brookings model 150, and the Data Resources model about 300.

The third-generation models of the seventies were even less Keynesian: had even weaker fiscal multipliers, displayed even more crowding-out, were even more sensitive to inflation and the rate of interest, and were more cyclical. As for size, the Data Resources model is now approaching a thousand equations, ever more of them simultaneous rather than recursive.

Thus in the third third of our own century the computer is freeing us from the straitjacket of heavy aggregation. Macroeconomics is beginning to look much like microeconomics. Keynesian and monetarist models alike are beginning to look parochial.

3. Microeconomics

Already in the late nineteenth century Walras (1874-1877) had consolidated the two halves of microeconomics and formulated, but not solved, the problem of a general economic equilibrium: Walras merely counted his equations and his unknowns and found their numbers equal.

4. The Saddle Point

To his friend, Georges Renard, Walras wrote: "If one wants to harvest quickly, one must plant carrots and salads; if one has the ambition to plant oaks, one must have the sense to tell oneself: my grandchildren will owe me this shade," [Antonelli (1939: 8)].

Walras's work was completed by grandchildren like Wald [1935 (1968)], [1936 (1968)], and [1936 (1951)], von Neumann [1937 (1968)], Koopmans (1951), (1957), Arrow-Debreu (1954), Arrow-Hurvicz (1958), Dorfman-Samuelson-Solow (1958), McKenzie (1959), and Debreu (1959).

But first a new mathematical tool had to be discovered, and it took one of our century's foremost mathematicians to discover it. Replacing calculus by finite mathematics von Neumann maximized a primal constrained by one set of inequalities and minimized a dual constrained by another set. For two important economic problems he proved the existence of a saddle point at which the primal reached its maximum and the dual its minimum. His first [1928 (1944)] problem was mixed-strategy games, his second [1937 (1968)] problem was a competitive growing economy.

Von Neumann's work was pure theory. Soon after, urgent practical work uncovered more saddle points. Kantorovich (1939), a Leningrad mathematician, worked on the allocation of available machine-tool time among components in Soviet industrial planning. He saw a saddle point whose dual was a set of accounting prices representing opportunity cost. Koopmans [1942 (1970)], a Dutch physicist, worked on the allocation of available tonnage among global shipping routes in the Second World War. He, too, saw a saddle point whose dual represented opportunity cost.

5. Our Heartland

Our heartland is an economy in which industry demands inputs and supplies outputs; households demand outputs and supply inputs. Both inputs and outputs are transacted in competitive markets and have prices. Some inputs may remain free goods and have zero prices. Some outputs may not cover cost and remain unproduced. How does a saddle point apply to such an economy? Any competitive economy must meet two elementary constraints.

First we can always make industry outputs high enough to generate positive excess demand for at least one input. But how high can we make them without doing that? Our primal problem is to maximize the value of all output subject to the constraint that excess demand for any input must be nonpositive. When industry outputs reach their highest possible value, the equilibrium value, excess demand has become zero for at least one input. All other inputs will have negative excess demand, hence be free goods. So our equilibrium solution will tell us which inputs will be economic goods and which will be free goods.

Second, we can always make input prices low enough to generate positive profits in at least one industry. But how low can we make them without doing that? Our dual problem is to minimize the value of all input subject to the constraint that in any industry under freedom of entry and exit, profits must be nonpositive. When input prices reach their lowest possible value, the equilibrium value, profits have become zero in at least one industry. All other industries will have negative profits, hence be nonexistent. So our equilibrium solution will tell us which outputs will be produced and which will not.

Taking their primal and their dual together Dorfman-Samuelson-Solow (1958) proved the existence of a saddle point at which the maximized value of all output equaled the minimized value of all input.

The saddle point is a powerful tool and a practical tool. But it also has a beauty of its own, certainly mathematical beauty, perhaps even poetic beauty:

At nonpositive excess demand
required by all feasible practice
and nonpositive industry profits
required by competitive vigor
the maximum value of output
will equal in saddle-point rigor
the minimum value of input

As result, both for price and for quantity
solutions exist and will offer
a key to the door of reality

FOOTNOTES

[1] On recent wider horizons, see Hirschleifer (1985).

[2] as attempted, for example, in Brems (1986).

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