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Progressivity of the Income Tax
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ABSTRACT

In this paper, the before and after tax distribution of family utility is used to measure the "true" progressivity of the personal income tax. We suggest that the distribution of money income is a less useful standard for assessing vertical tax equity than is the distribution of utility. An index of household utility is derived indirectly from household labor supply behavior and before- and after-tax Lorenz curves are calculated. A measure of the effective progression of the income tax is then used to compare progressivity under the utility standard with progressivity under a money income standard. We find that the income tax is less progressive under the utility standard than under the income standard.

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## PROGRESSIVITY OF THE INCOME TAX

Judging the vertical equity of a tax system demands a measure of economic welfare. Traditionally, taxable income has provided that standard. However, as many authors have noted, taxable income is an inadequate measure of economic welfare since it does not account for the nonmarket activities of the household. ${ }^{1}$ Activities such as cooking, car raintenance, volunteer work, and leisure improve economic well-being but do not contribute to taxable income. Consequently, measures of tax progressivity based on taxable income are distorted. Becker (1965) and Musgrave (1976) have suggested that instead of taxable money income, full income be used to measure economic wellbeing. Full income is the total income of the household when all family members work the full time available. It can be calculated by multiplying the time available to each family member by the respective wage and adding the unearned income of the family. Since full income depends only on time available and not on the allocation of time between market and nonmarket activities, it provides a more comprehensive measure of economic well-being than does money income. The disadvantage of full income as a measure of economic welfare is that it does not discriminate between increases in full income due to better wage opportunities and those due to increases in unearned income even though the latter contribute more to increasing well-being than do the former. This last point is illustrated in Figure 1 showing the choice between income and leisure. The original budget constraint is $A B C$, and full income is $O A$. An increase in full income to $O A^{\prime}$ can come about in

$$
-2-
$$

Figure 1 Income-Leisure Choice

Income

two ways: by an improvement in wage opportunities (shifting the budget constraint to $A^{\prime} B C$ ) or by an increase in nonwork income (shifting the budget constraint to $A^{\prime} D C$ ). As seen in the figure, the consumer is made better off (i.e., is moved to a higher indifference curve) by the increase in nonwork income than by the increase in wage opportunities.

We submit that a better measure of economic welfare than either taxable income or full income is the utility level of the household. Since utility is a function of taxable income and the quantity of leisure consumed, it reflects both market and nonmarket uses of time. It also has an advantage over full income in that the utility level is sensitive to increases in income due to improved wage opportunities and those due to increased nonwork income. In Figure 1, the utility level is clearly higher after the increase in nonwork income than after the improvement in wage opportunities even though the level of full income is identical under the two circumstances.

In this study, economic welfare is approximated by a measure of household utility derived indirectly from household labor supply behavior. First, work offer curves for the family are estimated to obtain estimates of the parameters of the household utility function. The parameters of the utility function are then used to calculate a welfare index for each household. Households are next ranked according to their utility levels before and after tax so that before- and after-tax Gini coefficients can be calculated. The Gini coefficients are used to compare tax progressivity using the utility standard with tax progressivity using the taxable income standard. According to our measure,
the income tax is progressive by both standards, but less progressive under the utility standard than under the incore standard.

In Section I, our methodology for estimating household utility levels is explained and in Section II, the data used in the study are described. Section III contains the results of estimating household utility levels, while section $I V$ presents the results on tax progressivity. Section $V$ is the conclusion.

## I. Methodology

It is assumed that household utility is Cobb-Douglas in after-tax incore, leisure of the husband, and leisure of the wife:

$$
\begin{equation*}
U=a_{1} \ln Y_{d}+a_{2} \operatorname{lnL} 1+a_{3} \ln L_{2} \tag{1}
\end{equation*}
$$

where $Y_{d}$ is household disposable income, $L_{1}$ and $L_{2}$ are the leisure hours of the husband and wife, and the a's are positive constants that sum to one. In its maximization of utility, the household is constrained by an income and a time constraint. According to the income constraint, family income must be equal to the sum of the husband's and wife's earnings and the nonwork income of the family minus the family tax liability:

$$
\begin{equation*}
Y_{d}=Y-T(Y-e)=W_{1} H_{1}+W_{2} H_{2}+I-T(Y-e) \tag{2}
\end{equation*}
$$

where $Y$ is family income before tax, $T(Y-e)$ is the family income tax liability, $e$ is total exemptions and deductions, $w$ is the hourly wage, $H$ is hours of work, I is nonwork income, and the subscripts denote the husband and wife, respectively. The family time constraint limits hours of work and hours of leisure to the total time available:
(3)

$$
K=H_{j}+L_{j} \quad j=1,2
$$

where $K$ is the fixed amount of time available.
The income tax is a bracket tax on family taxable income, Y-e. For a family in the ith tax bracket, the tax function is given by:
(4)

$$
T=B_{i}+t_{i}\left(Y-e-b_{i}\right)
$$

where $t_{i}$ is the ith bracket tax rate, $b_{i}$ is the minimum income in that tax bracket, and $B_{i}$ is the tax payable on that minimum income. For example, under current law, a taxpaying family in the 40 percent bracket filing jointly pays tax equal to $\$ 10,334$ (B) plus . 40 (t) times taxable income in excess of $\$ 45,800(b)$.

Maximization of household utility (1) subject to the income and time constraints yields the following work offer curves for the husband and wife:

$$
\begin{equation*}
H_{1}=\left(1-a_{2}\right) K-a_{2}\left[\frac{I(1-t)+t(e+b)-B}{w_{1}(1-t)}\right]-a_{2} K \frac{w_{2}}{w_{1}} \tag{5a}
\end{equation*}
$$

$$
\begin{equation*}
\mathrm{H}_{2}=\left(1-a_{3}\right) K-a_{3}\left[\frac{I(1-t)+t(e+b)-B}{w_{2}(1-t)}\right]-a_{3} K \frac{w_{2}}{w_{2}} \tag{5b}
\end{equation*}
$$

which relate hours of work to the own wage, the spouse's wage, nonwork income, and the parameters of the tax system.

Since $H_{j}=K-L_{j}$, we can then substitute (5a) and (5b) back into the utility function (1). Simplifying yields:
(6)

$$
\begin{aligned}
V= & \ln \left[w_{1}(1-t) K+w_{2}(1-t) K+I(1-t)+t e+t b-B\right] \\
& -a_{2} \operatorname{lnw_{1}}(1-t)-a_{3} \operatorname{lnw_{2}}(1-t)+a_{1} \ln a_{1}+a_{2} \ln a_{2}+a_{3} \ln a_{3}
\end{aligned}
$$

which is the indirect utility function of the household. ${ }^{2}$ Note that the first argument of $V$ is full income but that $V$ is also a function of the wage rates of the husband and wife after tax and the parameters of the utility function. Estimates of $a_{1}, a_{2}$, and $a_{3}$ obtained by estimating (5a) and (5b) using restricted least squares allow us to calculate $V$ for each household in our sample. A description of the sample data follows in the next section.

## II. Data

The sample of husband-wife families used in this study was drawn from the 1980 Michigan Survey of Income Dynamics. The data set contains information on income, wage, hours of work, and family demographic characteristics as well as tax information on total tax liability and marginal tax rate. A subset of 1,972 husband-wife families formed the basis for our study. Families on welfare, families in which the husband was less than 18 or more than 65 years of age, and families with negative nonwork income were excluded from the sample because we felt that behavior of these families would not be well described by the work-leisure choice model of our study. Since, in the case of some families the wife was not employed and therefore had no observed wage, and since the wage is measured with error for wives who were employed, we used the Heckman (1980) method to obtain a consistent estimate of the potential wages of wives in our sample. Using the Heckman method, we first estimated (using probit analysis) the probability that a wife worked outside the home. Second, we estimated a wage equation through regression analysis, using the
parameters of the probit estimation to adjust for bias in the estimation of the wage. The wage equation was then used to impute a wage to all wives in the sample.

The data contained information on the family's marginal tax rate, $t$, and cotal income tax liability, $T$. This information was used to infer the other tax parameters by solving for the unknown tax parameters in equation (4). This gave us:

$$
\begin{equation*}
t(e+b)-B=t Y-T \tag{7}
\end{equation*}
$$

which we calculated for each household in the sample. In estimating our model it was assumed that the tax brackets are sufficiently wide that small changes in income do not cause changes in tax brackets. This allowed us to disregard nonlinearities in the tax function and use ordinary least squares to estimate our model. ${ }^{3}$

Measurement of the other variables in the study was straightforward. Hours of work were measured annually, the husband's wage was computed by dividing earnings by hours worked, and nonwork income was computed by subtracting the earned income of the husband and wife from total family income. Estimates of the wage equations for black and nonblack wives appear in Table 1. The columns headed LFP give the results of the probit estimation of the probability that the wife participates in the labor force. The wage equation shows that city size has a positive influence on the wife's wage while the wage increases with years worked but at a decreasing rate. The education level of the wife was entered as a series of dummy variables with 17 or more years the omitted category. The negative coefficients indicate that those with

Table 1
Estimates of the Wage Equations for Black and Nonblack Wives (t ratios in parentheses)

| Explanatory Variables | Nonblack |  | Black |  |
| :---: | :---: | :---: | :---: | :---: |
|  | LFP | Wage | LFP | Wage |
| Constant | $\begin{gathered} 2.821 \\ (10.221) \end{gathered}$ | $\begin{gathered} 8.585 \\ (13.540) \end{gathered}$ | $\begin{gathered} 3.684 \\ (5.218) \end{gathered}$ | $\begin{gathered} 7.159 \\ (6.465) \end{gathered}$ |
| Nonwage income | $\begin{gathered} -.484 \mathrm{E}-5 \\ (-1.254) \end{gathered}$ | -- | $\begin{aligned} & -.978 \mathrm{E}-5 \\ & (-.674) \end{aligned}$ | -- |
| City size | $\begin{array}{r} -.0416 \\ (-.559) \end{array}$ | $\begin{gathered} 1.049 \\ (5.085) \end{gathered}$ | $\begin{gathered} -.121 \\ (-.761) \end{gathered}$ | $\begin{gathered} 1.150 \\ (3.934) \end{gathered}$ |
| Education |  |  |  |  |
| 0-5 years | -. 747 | -8.049 | . 718 | -5.884 |
| 6-9 years | -. 748 | -6.156 | -. 509 | -5.360 |
| 10 years | -. 703 | -5.186 | -. 625 | -4.932 |
| 11 years | -. 276 | -4.843 | -. 727 | -6.521 |
| 12-13 years | -. 474 | -4.935 | -. 542 | -4.321 |
| 14 years | -. 165 | -3.887 | -. 356 | -3.627 |
| 15 years | -. 284 | -3.285 | -1.914 | -5.551 |
| 16 years | -. 118 | -2.762 | . 492 | -2.573 |
| Years worked | $\begin{gathered} .896 \\ (5.149) \end{gathered}$ | $\begin{gathered} .133 \\ (3.043) \end{gathered}$ | $\begin{gathered} .124 \\ (3.875) \end{gathered}$ | $\begin{gathered} .065 \\ (1.061) \end{gathered}$ |
| Years worked squared | $\begin{aligned} & -.000983 \\ & (-1.649) \end{aligned}$ | $\begin{gathered} -.00215 \\ (-1.448) \end{gathered}$ | $\begin{gathered} -.00298 \\ (-1.983) \end{gathered}$ | $\begin{gathered} -.00173 \\ (-0.852) \end{gathered}$ |
| Husband's wage | $\begin{array}{r} -.0182 \\ (-3.520) \end{array}$ | -- | $\begin{aligned} & -.00582 \\ & (-.589) \end{aligned}$ | -- |
| Children |  |  |  |  |
| 1-2 years | -. 705 | -- | -. 482 | -- |
| 3-5 years | -. 499 | -- | -. 203 | - |
| 6-13 years | -. 140 | -- | -. 109 | -- |
| Age | $\begin{array}{r} -.0577 \\ (11.448) \end{array}$ | -- | $\begin{array}{r} -.0858 \\ (-6.550) \end{array}$ | -- |
| Home ownership | $\begin{gathered} .178 \\ (1.785) \end{gathered}$ |  | $\begin{gathered} .259 \\ (1.608) \end{gathered}$ |  |
| Probit Lambda | -- | $\begin{gathered} -.191 \\ (-.730) \\ \hline \end{gathered}$ | -- | $\begin{gathered} .459 \\ (1.198) \\ \hline \end{gathered}$ |
| $R^{2}$ | -- | . 205 | -- | . 208 |
| -2 times log |  |  |  |  |
| Sample size | 1507 | 1014 | 465 | 353 |

less than 17 years of education earn a lower wage and that generally, the lower the education level, the lower the wage.

## III. Estimation Results

After the wage function was used to impute a wage to each woman in the sample, the work offer curves were estimated using restricted least squares. The results of the estimation are in Table 2 . The estimates of the utility function parameters are all between zero and one as required by theory.

Table 3 shows the utility parameters implied by the regression coefficients for the four population subgroups. The parameter $a_{1}$ reflects the family's utility weight on income and the estimate shows it to be higher for those with more than 12 years of school. This is an expected result if families view education as an investment in higher future income. The parameters $a_{2}$ and $a_{3}$ are the family utility weights on the husband's and the wife's leisure, respectively. For nonblacks, our estimates show little difference in these weights between husbands and wives although black families place a higher weight on the husband's than on the wife's leisure according to our estimates. The utility level was calculated on the basis of the estimated weights for each of the four subgroups and is shown in the table. Since the utility level has only a relative significance, it is shown as a ratio to the population mean utility level. The group with the highest utility level is nonblacks with more than 12 years of education while the group with the lowest utility level is blacks with less than 12 years of education.

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$$

Table 2
Estimates of the Work Offer Curves

|  | Nonblack |  | Black |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Slope | $\bar{R}^{2}$ | Slope | $\bar{R}^{2}$ |
| Education $\leqslant 12$ |  |  |  |  |
| Husbands | $\begin{array}{r} -.249 \\ (-40.71) \end{array}$ | . 669 | $\begin{array}{r} -.387 \\ (-64.04) \end{array}$ | . 918 |
| Wives | $\begin{array}{r} -.241 \\ (-58.48) \end{array}$ | . 806 | $\begin{array}{r} -.184 \\ (-25.64) \end{array}$ | . 644 |
| Education > 12 |  |  |  |  |
| Husbands | $\begin{array}{r} -.224 \\ (-31.27) \end{array}$ | . 588 | $\begin{array}{r} -.384 \\ (-34.55) \end{array}$ | . 923 |
| Wives | $\begin{array}{r} -.224 \\ (-51.26) \end{array}$ | . 793 | $\begin{array}{r} -.141 \\ (-10.71) \end{array}$ | . 537 |
| All |  |  |  |  |
| Husbands | $\begin{array}{r} -.237 \\ (-50.68) \end{array}$ | . 630 | $\begin{array}{r} -.387 \\ (-72.80) \end{array}$ | . 919 |
| Wives | $\begin{array}{r} -.233 \\ (-77.49) \end{array}$ | . 799 | $\begin{array}{r} -.172 \\ (-27.16) \end{array}$ | . 614 |

Notes: $\bar{R}^{2}$ is adjusted $R^{2}$. Numbers in parentheses are $t$-ratios.

## Table 3

Utility Parameters by Type of Family

| Group | $a_{1}$ | $a_{2}$ | $a_{3}$ | Utility <br> level | Sample <br> size |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Nonblack, Education $\leqslant 12$ | .510 | .249 | .241 | .971 | 822 |
| Nonblack, Education $>12$ | .522 | .224 | .224 | 1.199 | 685 |
| Black, Education $\leqslant 12$ | .429 | .387 | .184 | .727 | 365 |
| Black, Education $>12$ | .474 | .385 | .141 | .866 | 100 |
| All |  |  |  |  | 1.000 |

IV. Measuring Tax Progressivity

A measure of tax progressivity developed by Musgrave and Thin (M-T) (1948) is used to compare the progressivity of the income tax under a utility standard with its progressivity under a taxable income standard. The $M-T$ measure is based on the coefficient of equality defined as one minus the Gini coefficient. If $E_{a}$ is the after-tax coefficient of equality and $E_{b}$ the before-tax coefficient of equality, then $M-T$ measures the effective progression of $a$ tax by the ratio $E_{a} / E_{b}$. If the ratio is equal to one, the tax is proportional; if it is greater than one, the tax is progressive; and if it is less than one, the tax is regressive. Hence, the effective progression of the income tax depends on the extent to which it shifts the distribution of income toward equality.

The Musgrave-Thin measure of effective progression is illustrated in Figure 2. The coefficient of equality before tax is the ratio of the area beneath the before-tax Lorenz curve and the area under the line of equality. The after-tax coefficient of equality is measured similarly with respect to the after-tax Lorenz curve. If the after tax Lorenz curve lies wholly to the left of the before-tax Lorenz curve (as it does in the diagram), the tax is progressive. Conversely, the tax is regressive if the after-tax Lorenz curve lies wholly to the right of the before-tax Lorenz curve. Using 1948 data, Musgrave and Thin found the coefficient of equality for the distribution of income before tax was .853 , the coefficient of equality after tax was .872 , implying a coefficient or progression of $1.022 .^{4}$

Figure 2

## Before- and After-Tax Lorenz Curves

Cumulative


In order to compute the M-T measure of progressivity, the households in our sample were ranked by their income and utility levels. The population was divided into deciles and the proportion of income and utility attributable to each decile was computed. These calculations were made before and after tax. The results are shown in Table 4 .

The Gini coefficient and the coefficient of equality are shown in the last two lines of the table. The Gini coefficient was calculated using the trapezoidal method which assumes that the Lorenz curve can be approximated by a series of linear relationships. The coefficient of equality is simply one minus the Gini coefficient. The larger the coefficient of equality and the smaller the Gini coefficient, the more equal is the distribution. As seen in the table, utility is more equally distributed than is income and taxes tend to equalize both the distribution of income and the distribution of utility. ${ }^{5}$

In order to measure the progression of the income tax with respect to income and utility, we use the Musgrave-Thin measure of effective progression, $E_{a} / E_{b}$. With respect to income, effective progression of the income tax is 1.041 while with respect to utility, effective progression is 1.033. Since effective progression is greater than one in both cases, the income tax is progressive by this measure; however, the effective progression of the tax is greater with respect to income than with respect to utility. Compared with the Musgrave-Thin measure of effective progression in 1948 of 1.022 , both our measures show the income tax to have greater progressivity 32 years later (1980).

Table 4
Distribution of Pre- and Post-Tax Income and Utility for Families, 1980

| $\begin{aligned} & \text { Income (Utility) } \\ & \text { Decile } \end{aligned}$ | \% Share of Total Income |  | \% Share of Total Utility |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Before Tax | After Tax | Before Tax | After Tax |
| Bottom decile | 3.4 | 3.8 | 6.0 | 6.5 |
| 2nd decile | 5.3 | 5.7 | 6.9 | 7.4 |
| 3rd decile | 6.4 | 6.9 | 7.6 | 8.0 |
| 4th decile | 7.4 | 7.8 | 8.3 | 8.6 |
| 5 th decile | 8.4 | 8.7 | 9.0 | 9.2 |
| 6th decile | 9.4 | 9.6 | 9.7 | 9.8 |
| 7 th decile | 10.6 | 10.6 | 10.4 | 10.4 |
| 8th decile | 12.1 | 12.0 | 11.2 | 11.1 |
| 9 th decile | 14.3 | 14.1 | 12.4 | 12.1 |
| Top decile | 22.7 | 20.7 | 18.2 | 16.8 |
| Gini coefficient | . 276 | . 246 | . 174 | . 147 |
| Coefficient of equality | . 724 | . 754 | . 826 | . 853 |

V. Conclusion

In this paper, the before- and after-tax distributions of family utility are used to measure the "true" progressivity of the personal income tax. We find that while utility is more equally distributed than taxable income, the personal income tax is actually less progressive by the utility standard than by the taxable income standard.

From a practical point of view, basing the personal income tax on household utility levels is infeasible. The estimate of the utility level depends on the choice of utility function. In this study, we selected the Cobb-Douglas utility function because of its conveniently linear offer curves. An unfortunate feature of this utility function, however, is that it restricts the elasticity of substitution between income and leisure to one. Relaxing this assumption requires more complicated estimation techniques. The gain in terms of getting a better measure of utility must be weighed against the loss in estimation efficiency.

Admitting that our measure of household utility is imperfect, we nonetheless maintain that it provides a better picture of income tax progressivity than does taxable income. While there can be no scientific resolution to the debate on how progressive should be the income tax, rational political decision-making depends on a realistic measure of tax progressivity. ${ }^{6}$ The distribution of household utility provides that measure and provides further evidence that the income tax may be less progressive than has been previously believed.

## Footnotes

$1_{\text {Boadway }}$ and Wildasin (1984), pp. 265-266.
${ }^{2}$ See Varian (1978, p. 94) for the derivation of the indirect utility function in the one-person Cobb-Douglas case.
${ }^{3}$ This approach was followed successfully by Wales and Woodland (1977).
${ }^{4}$ Musgrave and Thin (1948), p. 511.
${ }^{5}$ Since the before- and after-tax Lorenz curves do not cross, the Gini coefficient is an appropriate means for comparing income inequality. Atkinson (1970) showed that the Gini coefficient can be misleading if Lorenz curves intersect.
${ }^{6}$ See Blum and Kalven (1953) for an interesting discussion of the pros and cons of progressive taxation.

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