Prospects for Analogue Confirmation

Abstract: In analogical reasoning, observations about one or more source domains provide support for a conjecture about a target domain. Analogical support can range from plausibility to strong confirmation. In modeling this type of reasoning, two recent discussions are relevant. The first is Norton's challenge to formal models of analogical inference (Norton 2021). The second, a debate about whether analogue experiments can confirm theories about an inaccessible target domain, provides impetus to develop just such formal models (Dardashti et al. 2019). This paper argues that we can navigate these discussions with *quasi-formal* models of analogical reasoning. Such models are broadly compatible with Norton's position. They help to clarify the structure and strength of different forms of analogical inference, and to identify basic requirements for a good analogical inference, even when the target domain is inaccessible.

1. Introduction

In analogical reasoning, observations about one or more source domains provide inductive support for a hypothesis about a target domain. This support can come in different strengths. Work by Froude in the 1800s showed that tests conducted on model ships (the source) can provide highly reliable information about the stability of full-size vessels (the target) (Froude 1874; Sterrett 2017a). By contrast, many analogical arguments are offered merely to show that a hypothesis is plausible, i.e., a serious possibility. An ethnographic analogy appeals to our knowledge of an object produced by a familiar culture to motivate a potential explanation for a similar artefact from a vanished culture. Finally, there are analogical arguments of intermediate strength. Neuroscientists interested in the genetic mechanisms that lead to neurodegenerative disorders in humans employ animal models, typically mice, to support or refine hypotheses about how these diseases may be caused or treated in humans (Ahmad-Annuar et al. 2003; Fisher et al. 2019).

These examples correspond to a familiar distinction among three grades of inductive support. In Bayesian terms, where E represents a new observation and H represents a hypothesis, *strong confirmation* corresponds to $Pr(H / E) > r$ for some threshold r; *incremental confirmation* corresponds to $Pr(H / E) > Pr(H)$; and plausibility may be interpreted as non-negligible prior probability $Pr(H)$. The examples suggest a corresponding distinction for analogies: *strong* and *intermediate analogue confirmation*, and *analogical plausibility*.

The purpose of this article is to develop a general framework for evaluating whether an analogical argument provides strong, weak or intermediate inductive support. In modeling analogical reasoning, two current discussions are relevant. The first is Norton's challenge to formal models of analogical reasoning. The second, a debate about analogical inferences with inaccessible target domains, provides impetus for developing just such formal models.

For Norton (2021), a formal model of analogical reasoning is an abstract, universal schema that sets normative standards. The main thesis of his material theory of induction is that inductive inferences are warranted entirely by “local” material facts. Inductive schemas, to the extent that they are valid, derive their legitimacy from these local facts (2021, 270). In the case of analogical reasoning, Norton rejects formal schemas altogether. We should focus not on abstract rules but rather on empirical investigation of the source and target domains. In a similar spirit, Currie writes that a formal approach can “obscure the local warrants” of analogical inferences and “misses where the action is” (2018, 197). I agree with Norton and Currie that local facts do the heavy lifting in assessing analogical arguments. I argue, however, that an orientation towards local warrant leaves room for what I shall call *quasi-formal* models, and that they are broadly compatible with Norton’s position.

Now consider the problem of analogies whose target domains are, in crucial respects, inaccessible to observation. Some physicists and philosophers believe that experiments on black hole analogues can confirm the existence of Hawking radiation in real black holes (Dardashti et al. 2017, 2019). The target domain is inaccessible because actual black holes, in relevant respects, are *astronomically remote*. Hawking radiation cannot be detected from earth. A second prominent group of examples comes from historical sciences, such as archaeology and evolutionary biology, where scientists use analogies to make inferences about the distant past. The target domains are inaccessible because they are *historically remote*.

The important question, when the target is inaccessible, is whether an analogical inference can provide *intermediate* (incremental) confirmation. There is no consensus on whether the analogue gravity experiments confirm the existence of Hawking radiation.[1] There are both optimists and pessimists on analogue confirmation in archaeology.[2] The material theory of analogy seems to have little to offer in resolving such debates. If the target domain is inaccessible, then empirical investigation, almost by definition, cannot settle disagreements about whether an analogical argument provides confirmation or just plausibility. This motivates us to reconsider formal approaches to analogical inference, and indeed (Dardashti et al. 2019) develop a Bayesian approach.

I shall steer a middle course through these debates by arguing for the value of quasi-formal models of analogical reasoning. The starting point is the thesis that good analogical arguments are related to background generalizations or uniformities, but in different ways.[3] Strong analogical arguments are "powered" by an underlying generalization that establishes a reliable correlation between features of the source and target domains. Weak analogical arguments proceed in the opposite direction: they aim at a potential generalization. Finally, intermediate analogical arguments rely on the refinement of partially articulated generalizations that generate inter-domain correlations of intermediate strength. In all three cases, the "action" is local, but quasi-formal models let us distinguish between the three types

[1] Crowther et al (2019), in particular, reject claims of confirmation.
[2] (Chapman and Wylie 2016) review decades-long debates about analogies in archaeology.
[3] This idea, which builds on (Bartha 2010), is challenged by Fraser (this symposium).

by spelling out the role played by background generalizations. The models incorporate standards that are helpful in assessing the prospects for analogue confirmation, even for inaccessible (or partially inaccessible) target domains.

Section 2 of the paper discusses Norton's material theory. Section 3 provides examples of quasi-formal models for strong and weak analogical arguments. Section 4 argues for the value of similar models for intermediate analogue confirmation. Section 5 explores the Bayesian framework in Dardashti et al. (2019), and assesses its potential application to inaccessible target domains.

2. Norton's material theory of analogy

Norton argues negatively that formal analyses of analogical reasoning are pointless, and positively for an analysis in which analogical inferences are warranted entirely on the basis of local facts.[4] The essence of the negative argument is that any abstract formal schema for analogy will "at best fit a range of cases imperfectly" (Norton 2021, 105). Even elaborate schema "will still never be adequate to all the cases. Gaps will remain." There is "no universal schema" which tells us when properties of the source can legitimately be passed to the target.

On Norton's positive account, analogies are "factual matters to be explored empirically" (2021, 96). A good analogical inference is "powered" by some material fact that embraces the two systems, which Norton calls the *fact of analogy*. In a successful analogical argument, the fact of analogy, together with additional observations about one or both

[4] This section draws on the assessment of Norton's theory of analogy in (Bartha 2020).

systems, warrants the conclusion. Norton argues that this analysis matches scientific practice. Analogical reasoning in science is oriented towards "empirical investigations" of the facts, rather than useless formal principles (2021, 106).

As an illustration, Norton considers Galileo's use of analogy to infer the existence of mountains on the moon. In *Siderius Nuncius* (1610), Galileo records his observation of the advancing edge of sunlight on the moon. Bright points of light appear ahead of the boundary, and eventually join up with the illuminated area. Galileo compares this to what happens at sunrise on earth, when the "highest peaks [are] illuminated by the sun's rays while the plains below remain in shadow" (1610, 33). He infers that the points of light mark the location of mountains on the moon. On Norton's reading, Galileo's inference is not warranted by conformity to some formal rule. Rather, the warrant is this: "the mode of creation of shadows on the earth and of the moving dark patterns on the moon is the same: they are shadows formed by straight rays of sunlight" (2021, 108). Norton adds: "the inference is not driven as much by analogy as by subsumption of the moon into a larger class of illuminated bodies" (2021, 109).

In this example, the "fact of analogy" is best understood as a background generalization sufficiently broad that it applies to both domains.[5] This generalization, together with additional facts about the two domains, provides both formal and material warrant for the

[5] This reading of Norton is defended in (Bartha 2020).

analogical inference, and eliminates the need for any universal schema. We can, however, characterize Galileo's argument in formal terms, using the following pattern:

S (Earth)		T (Moon)
	$(x)(P(x) \rightarrow Q(x)) \xrightarrow{1}$	
P(S): points of light/shadows		$P(T) \rightarrow Q(T)$
Q(S): mountains		$P(T)$
		$\downarrow_2$
		$Q(T)$

Fig. 1: Mountains on the moon

The local generalization is that patterns of light and shadow on illuminated bodies are always produced by rays of sunlight striking regions of different elevation. This generalization derives support both from terrestrial observations and from general theory (optics). The two arrows represent deductive inferences from the generalization, together with the observation P(T) of the points of light on the moon. We could invoke a broader generalization (the laws of optics), but the overall reasoning pattern would be similar.[6]

Quasi-formal models are compatible with Norton's rejection of universal schemas. They are templates that indicate how an analogical inference relates to particular facts and to

[6]Norton notes that there is an element of inductive risk in the conclusion. We could represent the argument form as abductive (see section 3.2). In short, different quasi-formal models might be applied, each specifying a different "formal basis" for the argument.

the background generalization (fact of analogy). Norton insists that the fact of analogy always "powers" the argument. This position limits the material theory to models of strong analogue confirmation (section 3.1). Other types of analogical inference require different models.

3. Strong and weak analogical arguments

Predictive analogical arguments use analogies to predict specific properties of the target domain. *Explanatory* analogical arguments use analogies to support an explanatory hypothesis about the target domain, one that would explain the observed features. I argue in this section that this distinction about objectives aligns with a distinction in logical structure between strong and weak analogical arguments. In strong analogical arguments, a well-supported background uniformity powers the inference. In weak arguments, the background uniformity may not be articulated and the direction of inference is reversed. The structural distinction leads to different models, and different standards, for the two categories.

3.1 Strong analogue confirmation and predictive analogies

Experiments and observations on a source domain sometimes lead to highly reliable predictions about the target domain. This type of analogical reasoning, important in practical settings, draws upon empirical observation and theoretical understanding of both domains.

Sterrett (2017a, 2017b) provides historical and philosophical examination of the *method of physically similar systems*. One of the examples that she discusses is the work of William Froude, a 19th century English engineer, on model ships. Prior to Froude, model ships were used in the design of full-size vessels, but predictions were unreliable. Froude determined that the key concept was a dimensionless characteristic now known as the *Froude number*,

$$F = \frac{v}{\sqrt{lg}} \ ,$$

where *v* represents ship speed, *l* is the characteristic length of the hull and *g* is the gravitational constant. A model ship with the same Froude number as a full-sized vessel could be used to predict, for the full-sized ship, the residual resistance of the water due to created waves and eddies. *Froude's Law of Comparisons* states that if S and T are ships with the same Froude number ($F_S = F_T$), then their residual resistance is in the ratio of the cubes of their lengths:

$$\frac{R_T}{R_S} = \frac{l_T^3}{l_S^3} \ .$$

Froude's results allowed for "the estimation, with reasonable accuracy, of the resistance and horse-power of full-sized ships from experiments with small and inexpensive models" (Taylor 1907, 418).[7]

Efforts to analyze the reasoning in this and similar examples culminated in two papers by Edgar Buckingham (1914a, 1914b). Buckingham begins with a standard characterization of two physically similar systems S and S′:

[7]Taylor mentions a number of constraints for such inferences.

> If the relation in S′ is of the *same form* as the relation in S and is describable by the same equation, then the two systems are *physically similar* as regards this relation. (1914b, 353)

In many cases, we don't need to know the underlying physical laws. It is enough to know that certain dimensionless quantities determine the feature of interest, and that these dimensionless quantities are identical in the two systems. (The Froude number is one such dimensionless quantity.) Buckingham writes $\Pi_1, \ldots, \Pi_p$ for the dimensionless parameters and $\Psi(\Pi_1, \ldots, \Pi_p) = 0$ for the *reduced equation* that indicates the dependence relations. He states:

> If the values of the dimensionless parameters… are the same for S and S′, then we can determine the values of any [physical variable] Q_i in S′ given the others, and given values of Q_i in S. (1914a)

This gives us a quasi-formal model for analogical reasoning, described in (Sterrett 2017b):

$$\begin{tikzcd}
\textbf{S (source)} & & \textbf{T (target)} \\
& \Psi(\Pi_1, \ldots, \Pi_p) = 0 \arrow[dl] \arrow[dr] & \\
\Pi_1 = \pi & & \Pi_1 = \pi_1 \\
Q(S) & & Q(T)
\end{tikzcd}$$

Fig. 2: Common reduced equation

The template in Fig. 2 is quasi-formal because it has the kind of "gaps" that Norton notes. Correct application requires expertise about the range of applicability for the reduced equation. The template is nevertheless useful because it illustrates the basic structure and requirements of strong analogue confirmation:

- A well-confirmed local generalization that lets us move reliably between features of the source and target domains;
- Typically, a *predictive* analogical inference: the inferred conclusion, Q(T), is a particular property of the target.

3.2 Weak analogical arguments

Analogical reasoning is often used to show that a conjecture is plausible, i.e., worthy of serious consideration. Bartha (2010) proposes that such arguments are successful if they establish the potential truth of a generalization that covers both source and target domains. Bartha's articulation model is based on a two-step evaluation. The first step is to articulate the *prior association*, a causal or logical relationship among the properties of the source domain. The second step is to assess the *potential for generalization* by verifying that no crucial element of the prior association lacks an analogue in the target domain.

As an illustration, consider the *acoustical analogy*, employed by some 19th century physicists seeking to explain the discrete lines in the visible spectrum of Hydrogen.[8] Around

[8] This example expands on the discussion in (Bartha 2010). For a historical account, see (Maier 1981).

1870, Stokes suggested that the lines might be explained using a model analogous to a vibrating string or tuning fork. If such a model were correct, then we could identify the frequencies with some type of oscillation. We should expect to find that frequencies f_n of the spectral lines are integral multiples of a fundamental frequency f_1, and therefore that the frequencies should be related by simple whole-number ratios. Although Stoney (in 1871) found that some of the frequencies could be related in this way, there were many missing spectral lines. Furthermore, the whole-number ratios that he discovered involved the numbers 20, 27 and 32 – hardly simple ratios. As Bartha (2010) suggests, the acoustical analogy has an initial measure of plausibility but, on close scrutiny, fails to satisfy the criterion of potential for generalization.

Consider the structural difference between the acoustical analogy and analogical inference using model ships. In the latter case, the background generalization, *Froude's Law*, is well understood in advance and drives the analogical reasoning. In the acoustical analogy, the analogical inference is powered in the reverse direction: *from* observed features of the two domains (the discrete frequencies) *towards* a possible generalization that is not fully articulated. This is an *explanatory* analogical inference: its purpose is to suggest the kind of hypothesis that might explain the spectral lines of hydrogen. We can represent the inference with the following diagram:

S (vibrating string)		**T (hydrogen spectral lines)**	
	[General harmonic oscillator model]		
Q:	$f_n = \mathrm{n}\, f_1$ for n = 1,2,…	*Q**:	$f_n = \mathrm{n}\, f_1$ for n = 1,2,…
E:	$f_n / f_m = n/m$	E*:	? $f_n / f_m = n/m$

Fig. 3: Acoustical analogy

Q represents an *explanatory* feature of the source whose analogue is projected to hold for the target. The positive analogy, E and E*, is the observed evidence of discrete frequencies in whole number ratios. The dashed arrows point towards a tentative background generalization. The argument fails for lack of evidence that spectral line frequencies f_n occur in the right ratios. The logic is captured in the quasi-formal model of Fig. 3.

Weak analogical arguments derive their cogency from the possibility of a background generalization. Quasi-formal models indicate the structure of this relationship, which supplies the material and formal foundation for the argument.

4. Intermediate analogical arguments

A positive result for medical treatment tested on an animal model may count as evidence (incremental confirmation) for its effectiveness in humans. Currie (2018) seems to acknowledge a similar function for analogies in the historical sciences. He identifies a role for background uniformities in such arguments:

> One does not move from analogue-features to the target having features without mediation. The mediation in historical science is often via some process type that is taken to have been active in both analogue and target… (2018, 197)

This diverges from the models in section 3. We must have prior knowledge of a background uniformity over the two domains (in contrast to plausibility arguments), but this may be a broad uniformity (in contrast to strong analogical arguments). Particular facts about the domains are then used to refine the uniformity. Intermediate analogue confirmation, therefore, is "powered" in both directions. This section outlines two models.

4.1 Analogical reasoning used to refine a broad regularity

Donnan (1971) uses ethnographic analogy to explain the significance of odd markings on the necks of Moche clay pots found in the Peruvian Andes (Donnan 1971). Donnan learned that contemporary Peruvian potters in the region employ similar markings, known as *signáles*, to indicate ownership when multiple potters fire their pots in a common kiln. The analogical reasoning appears to confirm that the marks served the same purpose for the Moche. The conclusion is strengthened by direct historical analogy: the present-day population traces a continuous link to the Moche culture.

The pattern of inference may be represented as follows:

$$\begin{tikzcd} Q \arrow[d] \arrow[r, leftrightarrow, "X"] & Q^* \arrow[d] \\ P \arrow[r, leftrightarrow, "X'"] & P^* \end{tikzcd}$$

Fig. 4: Refinement of a broad regularity

P and P* denote the positive analogy: analogous effects (*signáles*). Q denotes the known explanation for current production practices, and Q* denotes the analogous explanation for the Mochica. X represents a broad background uniformity: *production processes operate in the same way* in both domains. X is refined to a more specific uniformity, X′: "ceramic technologies… are maintained over long periods of time" (Donnan 1971, 466). The refined uniformity supports the analogical conclusion, Q*.

It might seem that this explanatory analogy provides incremental confirmation. Interestingly, Donnan makes no such claim, contenting himself with a modest assertion of plausibility:

> The ethnographic analogy does offer a possible explanation for the marks… and provides an interesting hypothesis which could be tested when more data are made available (1971, 466).

Whether this sort of analogical argument counts as confirmation appears to depend upon our ability to exclude rival explanatory hypotheses. This difficulty is specific to explanatory analogies, which reinforces the idea that they are naturally classified as plausibility arguments.

4.2 Engineering the right source domain

Researchers studying neurodegenerative diseases rely on animal models, typically mice or rats, to understand how the diseases work in humans. Consider SMA, Spinal Muscular Atrophy, a disease caused by defective motor neurons. Humans have one copy of the SMN1 (survival motor neuron 1) gene and up to four copies of SMN2, a "backup" gene that imperfectly duplicates the protein-producing function of SMN1. Mutations in SMN1 result in SMA, a disease in which motor neurons in the brain stem and spinal cord gradually die. The death rate is inversely related to the amount of functional SMN2.

Mice are used to study SMA, but the genetic mechanism in mice is different. Mice have a single SMN gene (Fig. 5, from (Fisher and Bannerman 2019)). If one or both alleles are normal, the mouse is viable and does not develop SMA; if both alleles are mutated, the mouse dies in embryo. Consequently, SMA never occurs in naturally born mice.

Mouse **Human**

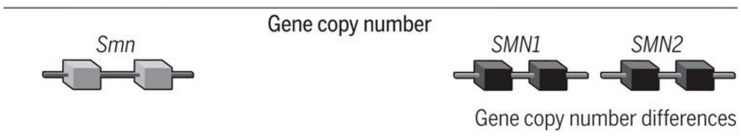

Fig. 5: SMN genes in mice and humans

For this reason, engineered mouse models are used. One SMN allele is deleted and the other is modified to resemble various mutations of human SMN2. The mouse is viable and develops SMA. Researchers can study the rate of neuron loss in relation to the amount of functional SMN2, and they can test gene therapies. This research has had considerable success in developing treatments for humans.

Do the experiments on mice provide incremental confirmation for hypotheses about neurodegenerative diseases in humans? Arguably, yes. The argument begins with a broad background uniformity, in this case our common genetic inheritance:

> 99% of human genes have a mouse homolog and more than 90% of the genes that have been implicated in human disease are present in the mouse genome" (Ahmad-Annuar et al 2003, 451).

This provides a basis for using the animal model in plausibility arguments, aimed at exploring possible genetic mechanisms for a disease. In the case of SMA, where the causal gene is known, researchers adopt a more precise approach in which the gene is mutated to create the mouse model. Researchers can then rely on a specific uniformity to explore causal mechanisms and develop treatments. These predictive analogical arguments are good candidates for incremental confirmation.

In short: intermediate analogue confirmation seems to require an independently established background generalization which is refined using particular facts about the two domains. The prospects for incremental analogue confirmation are better for predictive analogies than for explanatory analogies.

5. Bayesian analysis of analogue confirmation; application to inaccessible domains

Dardashti et al. (2019), henceforth (DHTW 2019), propose a Bayesian analysis of intermediate analogue confirmation. This section briefly explains their Bayesian model (in a simplified fashion) and explores its applicability to black holes and other inaccessible target domains.

In an earlier paper (Dardashti et al. 2017), the authors argue that analogue confirmation of a hypothesis about an inaccessible target domain rests on an assumption of *universality*. Two systems belong to the same universality class if variation in physical type is irrelevant to the physical properties of interest. The assumption that source and target belong to the same universality class is appropriate for physically similar systems (section 3.1). More generally, the requirement of universality is similar to the requirement, in section 4, of an independently established background generalization for incremental confirmation. In the examples of section 4, however, justification comes from observation of both source and target domains. If the target is inaccessible, the authors suggest, "model-external and empirically grounded arguments", or MEEGA, might still be given for universality (2017, 73).

In the black hole example, the challenge is to justify the assumption, call it X, that laboratory analogues of black holes belong to the same universality class as actual black holes. Given X, the observation of phenomena analogous to Hawking radiation in the analogue experiments can provide incremental confirmation for Hawking radiation in actual black holes. But it is unclear exactly how X is to be justified.

The Bayesian analysis of (DHTW 2019) offers two improvements. First, it provides a general model for incremental analogue confirmation. Second, it replaces the assumption of universality (X) with the seemingly weaker assumption $0 < Pr(X) < 1$: universality has non-zero prior probability. Let $\mathcal{A}$ represent the source domain and $\mathcal{M}$ the target. The Bayesian model introduces four binary variables:

X: Universality assumptions hold.

M: The model of the target is adequate.

A: The model of the source is adequate.

E: Empirical evidence is observed for the model $\mathcal{A}$.

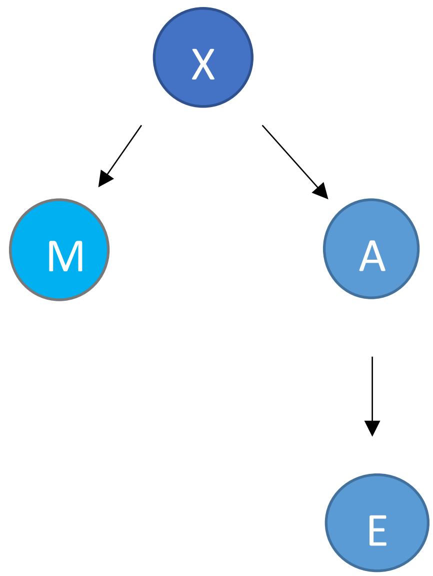

Fig. 6: Bayesian representation of analogue confirmation

The argument for confirmation rests on the following assumptions:

(1) The relationship between X, M, A, E is appropriately modeled with the Bayesian network in Fig. 6.

(2) $1 > \Pr(X) > 0$

(3) $\Pr(M / X) > \Pr(M / \neg X)$: Universality supports M.

(4) $\Pr(A / X) > \Pr(A / \neg X)$: Universality supports A.

(5) $\Pr(E / A) > \Pr(E / \neg A)$: E is supported by A.

From these assumptions, one can prove

$$\Pr(M / E) > \Pr(M).$$

After arguing that conditions (1) – (5) are satisfied for the black hole example, the authors conclude that observation of phenomena analogous to Hawking radiation in the laboratory analogues would incrementally confirm the reality of Hawking radiation.

The Bayesian analysis provides a useful model for incremental analogue confirmation. There is an important concern, however, about whether the argument gains in generality by substituting the assumption $0 < \Pr(X) < 1$ in place of X.

Consider the following simple experiment. A coin of unknown bias is tossed. If *Heads* comes up, a coin biased in favour of *Heads* is placed in each of two boxes, labelled $\mathcal{M}$ and $\mathcal{A}$. If *Tails* comes up, coins biased in favour of *Tails* are placed in the two boxes. Box $\mathcal{M}$ is removed from the room; it is now unobservable. Box $\mathcal{A}$ is opened, the coin inside is tossed and the result is *Heads*. This evidence provides incremental confirmation that the coin in box M is biased for *Heads*. To see that this experiment conforms to assumptions (1) – (5), let X signify *Heads* on the original toss, let M and A signify the placement of a coin biased for *Heads* in the two boxes, and let E stand for a result of *Heads* when the coin in box $\mathcal{A}$ is tossed. Because (1)– (5) are satisfied, we have analogue confirmation: $\Pr(M / E) > \Pr(M)$. But the assumptions *guarantee* the existence of a correlation between the variables M and A: $\Pr(M / A) > \Pr(M)$ and $\Pr(M / \neg A) < \Pr(M)$ regardless of the value of $\Pr(X)$, so long as $0 < \Pr(X) < 1$. The relevant universality assumption here is not X but an experimental set-up that guarantees that both coins have the same bias.

Similarly, in the black hole example, the universality assumption responsible for incremental confirmation is not X, but the Bayesian network set-up that guarantees a correlation between M and A. The universality assumption is no weaker than in the (2017) paper. We face the same difficulty: to understand how MEEGA can justify a universality assumption for an entirely inaccessible target domain. Perhaps this concern is similar to the one expressed by Crowther et al (2019), who insist that analogue confirmation depends on prior confirmation that source and target belong to a common universality class.

6. Conclusion

I close with two optimistic comments about analogue confirmation. First: there are ways of establishing the universality assumption in cases of partially inaccessible targets (as we saw in the SMA example), and perhaps we can do the same for black holes, based on general theoretical considerations or accessible knowledge of black holes. Second: the distinction between confirmation and plausibility arguments is not always critical. Plausibility arguments count towards overall probability and, in a Bayesian framework, they are part of the logic of confirmation. Furthermore, as Reiss (2019) suggests, there is much to be said for broadening our understanding of model-based reasoning beyond a narrow focus on confirmation.

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