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TEACHERS' DIFFICULTIES IN ARITHMETIC AND THEIR CORRECTIVES

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TABLE OF CONTENTS

PREFACE

Observation of teachers at work and conferences with them indicate that some are highly effective in handling certain phases of their work which others find difficult. Although all teachers may not be equally efficient in the application of a given method and teaching device, the pooling of successful practice should provide a store of procedures to which teachers could turn for assistance in handling difficulties which they encounter. This bulletin reports the results of an attempt to collect successful teaching procedures in the field of arithmetic. The list is doubtless far from complete but the bulletin is published with the hope that teachers in arithmetic will find it helpful.

The technique employed in carrying on this study has not been used extensively and the experience of Miss Streitz should be of interest to one contemplating an investigation of this type. One obstacle encountered is the fact that few teachers have analyzed their teaching so that they know what their difficulties are. This condition is significant both from the standpoint of educational research and with respect to efficient teaching.

This report represents the cooperation of a large number of teachers. To all who have contributed, the Bureau of Educational Research gratefully acknowledges its indebtedness.

Walter S. Monroe, *Director.*

February 13, 1924.

TEACHERS' DIFFICULTIES IN ARITHMETIC AND THEIR CORRECTIVES

CHAPTER I

INTRODUCTION

Need for specific correctives. A vast amount of literature on the curriculum and general principles of teaching is now available. Our attention has been called to the large individual differences which exist in most classes and much emphasis has been given to adapting instruction to the various levels of intelligence. General principles of teaching are, however, inadequate for attaining the highest degree of efficiency. Even when teachers are skillful in applying general methods they not infrequently obtain unsatisfactory results with some or all members of a certain class. Hence, they feel a need for correctives to deal with specific difficulties. Within recent years considerable attention has been given to the identification of specific difficulties and the correctives required.[1]

Teacher difficulties versus pupil difficulties. The teacher's task is to stimulate and direct the pupil in his learning. In assisting the pupil to overcome a particular difficulty which, as a learner, he may have encountered, the teacher herself may or may not meet with difficulties also. Her difficulty, however, although related to the pupil's, is not identical with it. In addition, the teacher because of the general conduct of her class may have to contend with obstacles which have no counterpart in the experiences of the pupils.

Need that difficulties be specific. The teacher may be aware that results are unsatisfactory. For example, Johnny can not work his long division examples readily, or is unable to solve written problems and Henry does not like arithmetic. When expressed in this way the difficulty is general rather than specific, and in order to be dealt with effectively it must be defined. Johnny's general difficulty with long division may be due to a lack of proficiency in subtraction or to the determination of the figures in the quotient or to particular cases

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[1]Gray, William Scott. "Remedial cases in reading: their diagnosis and treatment." Supplementary Educational Monograph No. 22. Chicago: University of Chicago, 1922.

Terry, Paul Washington. "How numerals are read." Supplementary Educational Monograph No. 18. Chicago: University of Chicago, 1922.

such as the placing of a zero in the quotient. Edith may be unable to solve problems because she does not know the meaning of certain technical words used in the statement, or she may not be sufficiently acquainted with the practical situation from which the problem is taken. Henry's dislike for arithmetic may be due to any one or a combination of a number of causes. Such causes must be analyzed into specific elements before they can be dealt with efficiently. This constitutes the teacher's difficulty and should be considered an important phase of her work. "Every teacher should resort to the method of analysis as it is only by this means that the teacher is able to apply proper instruction so that the pupil may be helped to improve."[2]

In many instances teachers have not determined the cause of the trouble and yet pupils have been able to advance in spite of the errors they have made. Any degree of success attained in such cases is due probably to ways devised by the pupils themselves for meeting their needs and not to conscious help on the part of the teacher. Such instances are far more numerous than we have been led to think. One of the most striking examples of such a procedure was reported by Uhl.[3] The use of standardized tests showed a boy weak in subtraction but unusually strong in multiplication. By inquiring into the method which he used it was found that when asked to subtract he turned the example into one of multiplication. If he had to subtract 9 from 46, he took 46, set aside 1 so as to secure a number that would be an exact multiple of 9. He then disintegrated his 45 into five 9's and dropped one of the 9's, thus performing the required subtraction. The reason for the boy's failure is obvious. The teacher, so long as the correct answer could be given, had not taken pains to find out exactly how the work was accomplished.

Purposes of investigation. The first purpose of the investigation reported in this bulletin was to compile a list of specific difficulties which teachers are actually encountering in the field of arithmetic. The second purpose was to formulate for each difficulty one or more proven correctives. These, for the most part, have been restricted to correctives which teachers are using successfully.

Plan of investigation. During the school year of 1921-22 a request was addressed to the superintendents of city school systems in

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[2]Judd, Charles H. "Analysis of learning processes and specific teaching," Elementary School Journal, 21:655-64, May, 1921.

[3]Uhl, W. L. "The use of standardized materials in arithmetic for diagnosing pupils' methods of work," Elementary School Journal, 18:215-18, November, 1917.

Illinois, asking them to invite their teachers to report specific difficulties which they were encountering in the teaching of arithmetic. The responses to this invitation were used to compile a tentative list of specific difficulties. The second step in the investigation was a visit to a number of school systems for the purpose of observing teaching in arithmetic, and of interviewing teachers in regard to the specific difficulties which they were experiencing. Correctives for various specific difficulties were formulated also during this visitation. In most instances the investigator asked the superintendent to direct her to some of his most successful teachers of arithmetic. Often these teachers were kind enough to demonstrate their teaching of a particular topic which was not a part of the regular work. In order to become familiar with the general principles governing the teaching of the subject and to secure additional devices for correcting specific difficulties, books and articles dealing with the teaching of arithmetic were also consulted.

Limitations of the method of investigation. The outstanding limitation of the investigation was the inability of teachers to define their difficulties. Many teachers even asserted that they experienced no difficulties in the teaching of arithmetic; others were able to mention only general items, some of which related to classroom management rather than to actual teaching; still others reported such matters as "how to secure interest" and "how to hold attention." A number of primary teachers stated difficulties relating to a particular textbook, complaining that the children, because of the complexity of the subject matter, were not able to advance beyond a certain point. Altogether, the total number of specific difficulties collected is much smaller than anticipated. In the writer's opinion, this is due largely to the fact that teachers were not able to analyze and define their difficulties.

The determination of the correctives is empirical rather than scientific. No attempt has been made to bring together a complete list of correctives for each difficulty. Neither have the correctives listed been tested experimentally, but they are believed to be in agreement with generally accepted educational principles.

Plan of report. In the following chapters specific difficulties are treated separately. A definite statement of the difficulty itself is first given, followed usually by a brief explanation. In succeeding paragraphs, under the head of "correctives" one or more explicit methods of dealing with this difficulty are described.

CHAPTER II

DIFFICULTIES RELATING TO GENERAL PHASES OF ARITHMETIC

Difficulty 1. How to create in young children a desire to learn the number facts which they need.

Although this difficulty was mentioned by a number of teachers the exact meaning is not clear. The number facts which a child actually needs in his own activities are very different from those which the teacher thinks he will need at some future time. The meaning of the word "young" is also indefinite but in the following discussion will be interpreted as referring to children in the primary grades.

Correctives: 1. Delaying teaching of number work. It is customary in many schools to begin formal instruction in arithmetic in the first grade. This is due in part to tradition but doubtless also to the fact that it is relatively easy to set exercises in arithmetic. As a result, children are frequently asked to learn many number facts before they have encountered any need for them in their activities. The simplest corrective would be to delay such teaching altogether until children experience the need for it. This suggestion is in agreement with the opinions held by many prominent educators.

2. Need for number work created in school situations. A primary teacher may very properly manipulate the school life of the child so that he will feel a need for number facts in his daily activities. Extremely artificial situations should be avoided as far as possible, but there are many devices which seem natural and which will probably prove interesting and enjoyable to the pupils.

a. Games as a means of creating need. Children like to play games which call for some knowledge of number facts. For example, counting and matching contests may be arranged with dominoes and loto; simple work in addition and multiplication is needed in bean-bag and ring-toss. Any game which requires that scores be kept creates a desire for some phase of number work.

b. Reproduction of adult activities as creating need. Many teachers find the reproduction of certain adult activities helpful in creating a need for various phases of number work. The use of these devices is not confined to the primary grades. For example, playing store is of great interest to children. It is just as much fun to be the purchaser as it is to be the storekeeper and both involve a certain amount of understanding of number facts. The making of change, adding of pennies, using pint and quart measures, and the counting of objects are all a part of the play. Some teachers have a "store" arranged in one corner of the room which serves as a basis for many types of activities. The "store" generally consists of several shelves, and a counter over which supplies are bought and sold. Upon the shelves may be found articles in their original wrappings, such as baking powder cans, breakfast foods of different kinds, coffee, tea, soap, etc. The appearance of these articles so arranged is not unlike a corner of a real grocery store. Perishable articles as fruit and vegetables are sometimes moulded in clay and painted with water colors. The money used should resemble real money, if possible, and when given in exchange for the article purchased, the pupil should know the denomination of the coin and its purchasing power. An understanding of the meaning of numbers in such a situation takes place quite naturally and the child's interest is much greater than it could possibly be if the material were presented in a remote or artificial environment. The teacher should not lose sight of the fact that a device of this kind is only a means to an end, and should be used only when there is need for securing a stronger motive or for interesting pupils in certain phases of number work.

c. **School activities as a means of creating need.** Some teachers have found it helpful to utilize certain school activities as a means of causing children to feel a need for arithmetic. For example, when health campaigns or health crusades were started in various school systems, the serving of milk followed in order that children might be helped in attaining the correct weight for their height. Milk was delivered each morning at the different schools and each child who could pay a certain amount was served. This necessitated bringing money from home to purchase the week's supply. In Urbana, one teacher looked upon the serving of milk as an opportunity for incidental teaching of number work. The children know how many pennies to bring in order to secure milk for the entire week. If a holiday occurs they know that no milk will be served and the price of one day's milk will be deducted.. Children like to handle money and like the feeling of responsibility which accompanies the making of purchases.

A banking system maintained in the schools may be an outgrowth of a thrift campaign. Quincy has a "banking day" when the children bring their pennies to be deposited in one of the banks. As each child comes up to the teacher's desk with his bank book and pennies the teacher asks how much money he has, how many pennies in a nickle, how many nickles in a dime, etc. In this way the child gains an idea of certain number facts. Although these serve to acquaint children with number facts the amount of time required to credit each child's account is questioned. Is there enough real educational material presented by this method to justify such an expenditure of time? Much would depend upon the system of banking as well as the plan which the teacher follows.

Difficulty 2. How to teach children to write numbers correctly.

This difficulty is found most frequently in the primary grades although in some cases children in the intermediate and even in the grammar grades have not learned how to write certain numbers correctly. It is not uncommon for some children to write figures backwards; figure 3 is sometimes written Ɛ, 6 is written ∂, and 9 is written P. There is also a tendency to reverse figures and when attempting to write 12, children will write 21, etc. Such errors, however, are not common to all children; with some there is little confusion regarding the form or arrangement of figures.

Corrective. Errors in writing numbers are due to the absence of certain specific habits. In most cases there has been a lack of sufficient practice. All learning is a matter of growth and children must have time in which to learn even those things which seem very simple. There are several devices which may be resorted to in helping the child overcome this difficulty. One teacher of the lower grades had several children who seemed unable to write 3 correctly. She placed a row of 3's on the board and beside each 3 was a "straight line" or tooth-pick drawing of a man facing in the same direction as the 3's. The teacher suggested that the 3's and the little men were soldiers who always faced in the right direction. The children compared their figures with the illustration and could easily recognize the correct and the incorrect forms.

Another teacher called attention to the difference between 6 and 9 and suggested that in writing the 9 they think always of the way the balloon man holds his balloons when offering them for sale. The top of the 9 is like a balloon on the end of a string. Later on the children are able to make 6 correctly by remembering to invert the balloon. Nearly everyone develops an individual system for remembering things which seems to aid in making correct responses to certain situations. Any device which is not too remote in its resemblance to the difficulty in question is justifiable if it facilitates in any way the overcoming of that difficulty.

Difficulty 3. How to secure accuracy in copying numbers.

Children are required to copy numbers in working the examples and problems given in the textbook. They also have to copy numbers in making calculations. The fact that children are surprisingly inaccurate in copying is probably due to the relatively little attention which is given to this phase of arithmetic work. Many teachers appear to assume that the process is so simple that it does not need to be taught.

Corrective. Teachers should give explicit training in copying figures. It is customary for them to give attention to the writing of figures from dictation, but this will not suffice. There should be definite exercises in which pupils are asked to copy figures from the blackboard or textbook. It is advisable for teachers to prepare simple tests in which both the rate and accuracy of copying will be measured.

Difficulty 4. How to teach the meaning of figures and other symbols.

Some children have trouble in associating appropriate meanings with numbers and other symbols which are used in arithmetic. This difficulty is confined for the most part to the primary grades but occasionally it persists even after the pupil has passed beyond the third grade.

Corrective. The attention of the child should be directed to the "number families" of 20, 30, 40, etc. A large calendar also serves to acquaint the children with numbers in a very useful way. This might be referred to daily until the children have learned to recognize the figures wherever they may be found. A "digit tree" drawn on the board or made in the form of a poster is a delight to young children; the numbers up to 10 are placed on the various branches and because zero is nothing it has no place in the branches but must remain on the ground. The teacher should write in both the figures and the words, the two being used together so that the children will see their relationship. For example, "22" and "twenty-two" should appear together until the child has learned that one is just another way of writing the other. There is no need, however, to write out figures of large denominations, for before using numbers of such magnitude, as 350,623 or even 1,725, the child will have advanced sufficiently in his school work to understand their meaning in words.

Difficulty 5. How to teach children to "see numbers" correctly.

This relates to the difficulty which some children experience in the perception of a figure. They are unable to pick out distinguishing characteristics so that they are able to recognize the figures easily.

Corrective. Children generally develop a system of their own to aid in the recognition of the number but the teacher can be of assistance in calling attention to its formation. She could point to the turn in the figure "8" and mention that it is like the beginning of the letter "s." In one room visited the children were writing 4's while the teacher kept saying *down, over, down,* with a certain marked rhythm. This no doubt directed the attention of the children to the formation of the number and aided in a later recognition.

CHAPTER III

DIFFICULTIES ENCOUNTERED IN THE OPERATIONS OF ARITHMETIC

Difficulty 6. How to teach addition.

This difficulty, as well as the corresponding ones for subtraction, multiplication, division, and fractions, is not specific. Study of arithmetical abilities has shown that there are a number of specific abilities in each of these divisions of arithmetic. Thus the general ability implied in the question "How to teach addition" should be broken up into specific difficulties, which will in general probably correspond to the various types of addition examples.[1]

Corrective. The first step then in securing fluency in addition is to break up the general difficulty into specific ones corresponding to the various types of addition examples. The second step is to make definite and adequate provision for teaching each type of example. Drill in addition must be appropriately distributed among the several types. As an illustration, the most effective procedure for securing proficiency in column addition is by drill upon column addition itself and not upon the combinations. Similarly in teaching addition which involves carrying, drill upon such examples must be given. Even in the teaching of the combinations it is not sufficient to drill pupils only upon $\begin{array}{r}4\\7\\\hline 11\end{array}$, but also upon $\begin{array}{r}7\\4\\\hline 11\end{array}$. In general, the combinations involving the larger digits are more difficult and require more practice. Usually this will mean that the teacher will need to exert special effort because it has been shown that in general, textbooks present a much larger number of exercises on the simpler combinations. The simpler combinations also are taught earlier when there is a longer period of practice.

There are many effective devices for giving drill upon number combinations and other types of examples. Some teachers keep a list of simple examples upon the blackboard and devote a few minutes each day to work upon them. In Jacksonville, a fourth-grade

[1]For a list of the types of examples in addition see
Monroe, Walter S. Measuring the Results of Teaching. Boston: Houghton Mifflin Company, 1918, p. 111-12.

teacher was observed to spend about two minutes each day with Thompson's Minimum Essential Practice Material. For this purpose, Studebaker Arithmetic Exercises and the Courtis Standard Practice Tests are also suitable. In practice upon the combinations, after the first stages of learning, there is no fixed order. Each combination should be mastered so thoroughly that the sight of it calls forth the correct answer. The arrangement of the addition facts in table form is merely a device which is useful in the beginning.

In the lower grades teachers were observed to make much use of concrete examples in teaching the fundamental combinations and in giving practice upon other types of examples. In connection with this corrective, it should be pointed out that a problem is not made concrete by giving numbers such labels as apples, dollars, pounds, etc.; it must be identified by the child with his own experience. Thus problems which are given a concrete interpretation in this way are frequently useful for teaching the concept of an operation. They also furnish practice upon the operation but there will be need probably always for some formal drill exercises. One teacher reported that concrete problems were inefficient for drill purposes because the children had difficulty in deciding just which operations were called for.

Difficulty 7. How to secure fluency in the fundamental operations.

This difficulty might be thought of as being included in the preceding one but a number of teachers mentioned it separately. In expressing this difficulty accuracy was usually given more emphasis than rapid work but both concepts were expressed.

Corrective. In securing fluency there should be emphasis upon rapid work. Practice should be timed. This can be done most effectively when the pupils are provided with printed or mimeographed lists of exercises. Scores in terms of both the number of examples attempted and the number of examples correct should be kept. If it is desired to give special emphasis to accuracy the penalty for errors may be increased.

In case mimeographed or printed practice exercises are not available appropriate examples may be placed on the blackboard. If space permits, the list may be duplicated so that two or more pupils can be working at the same time. In one school two pupils were sent to the board. They stood with their backs to the examples until the signal to begin was given. They then began work and the one who finished first was declared the winner, and was allowed to compete with another pupil. The contest feature of this drill creates a keen interest. In the case of addition the answers may be erased and the same exercises used by other pupils. The rules can be adjusted to give appropriate emphasis to rate of work and to accuracy.

Difficulty 8. How to teach the concept of subtraction.

A distinction is made here between the concept of subtraction and skill in working subtraction examples. Although an understanding of the meaning of subtraction is usually a prerequisite for skill in the operation, it will not insure the possession of such skill.

Corrective. There are two methods which have been widely used in explaining the process of subtraction: (1) the additive method, and (2) the take-away method. Pupils are familiar with addition when they reach the topic of subtraction. It would therefore seem logical to connect the two processes as closely as possible. Our observations and the testimony of several teachers have been to the effect that when the additive method is used in the lower grades the pupils discard it for the take-away method in the upper grades. A number of teachers expressed the conviction that because of this the take-away method should be used from the beginning. Beatty[2] states that pupils using the additive method are more accurate in subtraction but work more slowly. He concludes that although the additive method seems to be the more logical procedure its use is open to question. He states also that the exclusive use of either method is not justified.

It seems likely that the additive method is useful in explaining the process of subtraction to children who are having difficulty with the take-away method but our observations in this investigation justify the conclusion that subtraction should be taught by the take-away method. In any case, one method should be adopted by the school and followed by all teachers who give instruction in subtraction. "Nothing insures confusion more certainly than the indiscriminate use of different methods in different grades."[3]

In teaching subtraction by the take-away method the use of objects which may be counted is helpful. Blocks, splints, pencils,

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[2]Beatty, Willard W. "The additive versus the borrowing method in subtraction," Elementary School Journal, 21:198-200, November, 1920.

[3]Lennes, N. J. The Teaching of Arithmetic. New York: Macmillan Company, 1923, p. 227.

books, etc. may be used. If the child is given five objects and told to take away two, he can count the remainder.

Difficulty 9. How to teach the borrowing process in subtraction.

The process of borrowing in subtraction is one of the first serious difficulties which pupils encounter in learning arithmetic. The present tendency to teach addition and subtraction together so as to make clear to the pupil that the process of subtraction is the reverse of addition will probably tend to lessen the seriousness of this difficulty.

Corrective. One teacher suggested that this difficulty was due to confusion concerning the value of zero. One of her pupils stated that zero was more than nine, and in support of the assertion turned to his arithmetic and pointed out that the digits were printed in the following order: 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. Since 0 was given after 9 the natural inference was made that it was greater than 9. This confusion could easily be avoided by having "0" precede 1 whenever the digits are presented in their natural sequence.

In teaching the borrowing process, the small figures resulting from borrowing may be written above the minuend as follows:

$$\begin{array}{r} {\scriptstyle 7\ 13\ 16} \\ 846 \\ \underline{289} \end{array}$$

This device, however, is merely a "crutch" which should be used only in the earlier stages of learning. It is obvious that a pupil can not work subtraction examples rapidly by this procedure. In some instances where teachers have permitted the continued use of this "crutch," the individual has been unable, even in adult life, to subtract quickly and accurately.

A teacher in East St. Louis reported an ingenious device for explaining borrowing which she had found useful. In illustrating this device she used the example 6000 — 3876. Units, tens, hundreds and so forth were spoken of as families. Each one borrowed only from the family immediately to the left. The particular article borrowed was biscuits. In this example units had no biscuits for dinner; they tried to borrow from tens and tens then had to borrow from hundreds and hundreds from thousands. The peculiar thing about this borrowing was that thousands would not lend less than one pan of biscuits. When hundreds received this pan and attempted to put it into their pans they had enough for ten pans. They loaned one of these pans to tens, leaving them nine, and the process was repeated for the other places. As a result there were finally ten biscuits in units place, nine pans in tens, nine pans in hundreds, and five pans in thousands. This explanation was presented as an attractive narrative and pupils were required to answer questions, relative to the number of remaining biscuits, at various stages of the explanation.

Because of failure to understand the process of borrowing, some children do not realize that they can not take a larger number from a smaller. For example, such pupils will not hesitate to subtract 846 from 673. The fact that they are able to borrow in units place and perform the subtraction doubtless is responsible for their attempt to do likewise in hundreds place. This particular difficulty can be overcome by the use of concrete problems. Most pupils will readily understand that they can not take $800 from $600 when the example is expressed in this form.

Difficulty 10. How to secure proficiency in subtraction in the intermediate and upper grades.

Although corresponding difficulties exist for the other three fundamental operations it appears that the case of subtraction is peculiar. In some of the schools visited, pupils in the intermediate and grammar grades were observed to do satisfactory work in all the fundamental operations except subtraction. In some cases they were especially defective in this one operation.

Corrective. In correcting this difficulty it should be recognized that instruction in the operations of arithmetic can not be completed in the primary grades. In fact, it can not be entirely completed by the end of the intermediate grades. The teacher in each grade should recognize that she has a distinct responsibility, as it is only human for the pupils to forget and to lose skill. Even when the teaching in the lower grades has been excellent, additional training will probably be required each successive year in order to maintain a high degree of fluency. It is highly important that the pupils be given the right start and that the instruction in the lower grades be efficient, but this difficulty can not be corrected entirely by these means.

Difficulty 11. How to teach the multiplication tables.

This is one of the traditional difficulties in the field of arithmetic. Some of the harder tables have proven a serious stumbling block to many pupils. For some reason the multiplication tables constitute a more serious difficulty than the corresponding tables in the other operations.

Corrective. The writer believes that one of the reasons why the multiplication tables so frequently constitute a difficulty is that the teaching of them is spread out over too long a period of time. It seems reasonable that when the learning of the tables is spread out over several months or even over more than one year, as is true in some cases, the pupils fail to recognize it as a task to be undertaken and completed. If this thesis is true, one corrective would be to focus the attention of the pupils on the multiplication tables and to compress the teaching of them into two or three weeks.

Some teachers made the comment that pupils were not interested in learning the multiplication tables. This is probably due largely to the fact that they are bored with the slow pace expected of them in many schools. Children usually like to be given a definite task and to concentrate upon it. They are less capable of sustained interest than adults. In most cases, they will take an interest in something which appeals to them as constituting a real challenge to their abilities.

Because of the importance which teachers appear to attach to this difficulty, suggestions are given below for the teaching of the various tables. However, it should be borne in mind that after the first stages of learning the pupil should receive practice upon the combinations in miscellaneous order.

1. **The number 2 in multiplication.** Counting by 2's is useful in building up the table. The pupils seated in one row of seats may be asked to hold up both hands, then other pupils walk down the aisle counting by 2's. Others may be arranged two abreast in a march and counted by 2's. Such imaginary games as the mailing of letters where the cost of stamps must be computed, or the buying of certain articles as pencils, plums, etc. at two cents each, might prove helpful. One teacher suggested the use of pint and quart measures in teaching multiplication by the number 2. There are two pints in one quart and pupils may be asked how many pints in two quarts of milk, three quarts of milk, etc.

2. **The number 3 in multiplication.** Counting by 3's is likewise used in teaching the table of 3's. A foot rule and a yardstick may be used in much the same way as the pint and quart for the table of 2's. Simple concrete problems can be easily formulated.

3. **The number 4 in multiplication.** The suggestions already made for counting and for the use of concrete examples apply to the table of 4's. One teacher suggested the use of the fact that a horse

needs four shoes. Pupils may be asked to tell how many shoes are needed for two horses, three horses, etc. The use of the denominate number relation between quarts and gallons is also recommended.

4. **The number 5 in multiplication.** The table of 5's is perhaps the easiest and generally no special devices are needed for teaching it. Concrete problems involving the purchase of articles at five cents each will be found useful. Problems involving street-car fares at five cents each have also been suggested.

5. **The number 6 in multiplication.** The table of 6's may be illustrated by the purchase of articles by the half dozen. The use of six working days in a week was suggested. Problems such as "How many days will a man work in three weeks, five weeks, etc?" may be used.

6. **The number 7 in multiplication.** The table of 7's was mentioned frequently as the one causing the greatest difficulty. One teacher made this illuminating comment: "Children are so tired of the multiplication tables by the time the table of 7's is reached that it is very hard to get them interested enough to learn the facts." This comment serves to reinforce the thesis presented on page 20. If the facts in the multiplication tables were taught as one large piece of work to be accomplishd in a few days and not strung out over several weeks or months this tendency of the children to lose interest would be avoided. Furthermore, it may be pointed out that when the tables involving the higher numbers are delayed the pupils receive less drill upon the combinations because there is less time. The greater amount of the practice period has been given to the easier combinations. If the idea of multiplication has been rationalized in the teaching of the preceding tables, there will be little need for the use of concrete problems in connection with the teaching of the table of 7's and of the two following. The derivation of the multiplication combinations in the case of higher tables will be largely a waste of time.

7. **The number 8 in multiplication.** The comments just made with reference to the table of 7's also apply to the table of 8's. Furthermore, it should be noted that after the tables up to 8 have been learned, the table of 8's introduces only two new combinations, 8×8 and 8×9, provided the pupils have been taught both forms of a given combination, e. g., $7 \times 4 = 28$ and $4 \times 7 = 28$.

8. **The number 9 in multiplication.** The table of 9's should not constitute a real difficulty. The pupil should know all of the facts except 9×9. (If the tables beyond the 9's are to be taught this statement would need to be modified accordingly.) Practice should now be given involving all of the combinations in miscellaneous order. Various games and devices can be used. The pupil by this time has reached the place where the emphasis should be given to securing fluency in the use of multiplication combinations.

Difficulty 12. How to teach the multiplication tables so that pupils will be skillful in using combinations in other than the customary sequence.

This difficulty is really included in the preceding one since the tables should always be taught so that pupils will be skillful in using the combinations in other than the ordinary sequence. No teaching of this topic can be considered satisfactory which does not engender this ability in the pupils.

Corrective. After the first stages in the learning of the multiplication tables, drill upon the combinations should be in miscellaneous order rather than in the customary sequence. Skill will come only with practice. So far as possible, this practice should be given by doing exercises similar to those in which the pupils will use the combinations. Drill upon the combinations alone will not make pupils proficient in doing more complex types of multiplication examples.

A number of teachers were observed using effective devices for giving drill on the combinations. Among the best noted are the following.[4]

1. **Number races.** One type of a number race is to select two children to go to the board, each equipped with a pointer. A list of multiplication combinations are written on the board. Their classmates take turns in naming products of these combinations and the two at the board answer by pointing to the combination which belongs with the product. Each child tries to be the first to touch the right number with the pointer. The game can be varied by having the products written on the board and the pupils at their seats name the combinations.

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[4]For other games and similar devices see,
Smith, David Eugene and others. "Number games and number rhymes," Teachers College Record, 8:1-3, November, 1912.

A further variation of this game may be secured by dividing the class into two teams. In addition one pupil should be chosen as a score keeper. One pupil from each team is sent to the board and each pupil in his seat then asks for one number. After all have participated two other children are sent to the board and the process is repeated. A score of one point is given for each time a child at the board is the first to point to the correct answer, and the team receiving the greater number of points wins the game.

2. **Bean bags.** (For use in the primary grades.) Draw a circle on the floor like the one in the picture. Choose sides with a leader for each side. The game consists in throwing the bean bags at the circle (Fig. 1) and in striking the largest numbers possible. The leader has the first turn, then each child comes forward for his turn. The score equals the number struck multiplied by itself. When a line is struck the score is zero.

3. **Multiplication game.** This game is similar to the bean bag game except that you multiply the number in the circle by those around the rim (Fig. 2). One child can be selected to point to the numbers while the children in the different rows perform the multiplication.

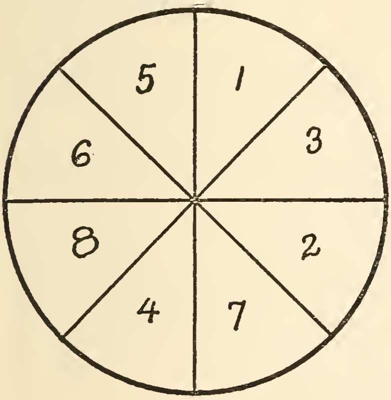

Fig. 1

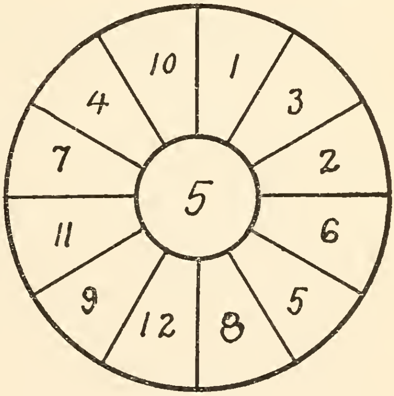

Fig. 2

Difficulty 13. How to get pupils to understand the close relationship which exists between multiplication and division.

The occurrence of this difficulty is probably due to the fact that multiplication and division are treated as separate topics.

Corrective. As in other instances, pupils will be unlikely to grasp the relationship between multiplication and division unless it is definitely taught. Thus to overcome the difficulty one should make specific provision for teaching this relationship. One suggestion is to teach multiplication and division simultaneously. Such a procedure would probably serve to engender an understanding of the relationship but it is not unlikely that it would interfere with the effective teaching of the multiplication tables. After the pupils have grasped the idea of multiplication and have become familiar with at least some of the combinations it is not inappropriate to introduce the idea of division, particularly in dealing with concrete problems. When the pupils have solved a problem in which multiplication is required they may be given an inverse problem calling for division.

Difficulty 14. How to teach pupils to divide an uneven number by 2.

This difficulty was mentioned frequently by primary teachers but probably does not apply to intermediate and grammar grades.

Corrective. One teacher stated that she met this difficulty by calling the one left over a remainder. To express it as a fraction appears to be confusing to pupils and in the primary grades there is little or nothing to be gained by insisting upon the expression of the complete quotient.

Difficulty 15. How to avoid confusion due to the variety of forms by which division is indicated.

There are at least four ways in which division is indicated:

$$42 \div 7 = \quad , \qquad 8)128(, \qquad 87\,\overline{\big|\,3654,}\quad \frac{56}{8}\ .$$

Teachers report that pupils tend to be confused when these methods of indicating division are presented together.

Corrective. The obvious corrective is to use only those forms of expressing division that are necessary. The second and fourth forms can easily be avoided. Several teachers mentioned that pupils were confused because different teachers used different forms in long division. This suggests that it is highly desirable for teachers in the successive grades to agree upon the same form.

Difficulty 16. How to secure accuracy in the determination of quotient figures in long division.

Pupils make many errors in the determination of quotient figures. One teacher mentioned that many of her pupils set down a quotient figure which was too small and as a result secured a remainder larger than the divisor, and that they failed to recognize this as an indication of a mistake in their work. Another type of error is the omission of zeros in the quotient. Occasionally pupils make errors due to bringing down two figures of the dividend at one time. Teachers generally appear to agree that long division constitutes one of the most serious difficulties in the field of arithmetic.

Corrective. One corrective which was suggested is to train pupils in estimating quotients and to insist that this is done before the actual work of division is started. If pupils are skillful in estimating quotients, gross errors can be easily detected.

There is not just one difficulty in long division; there are several. Investigation has indicated that there are several types of examples, each of which presents a distinctive difficulty.[5] For example, an exercise which results in such a quotient as 508 involves a difficulty due to the presence of a zero in the quotient. Another difficulty is encountered when there is a remainder. Still other difficulties grow out of the relation of the trial divisor to the dividend. Explicit training should be given on each type of example. It is doubtless true that many pupils experience difficulty in long division because in the training which they have received one or more types of examples have been neglected.

Difficulty 17. How to teach reduction of common fractions.

The topic of common fractions has suffered in somewhat the same way as that of the multiplication tables. Fractions are commonly introduced very early in the child's educational career and each year he encounters examples which are slightly more difficult. There are certain advantages in this plan but there is this weakness also; the pupil fails to approach the study of fractions with the attitude that it constitutes a real task and demands the concentration of his attention. Some of the difficulties in handling fractions could doubtless be overcome by singling them out as specific tasks and by concentrating upon them.

Corrective. In connection with the reduction of fractions pupils should be made familiar with the principle that "fractions may be changed in form, without altering their values, by multiplying or dividing both terms by the same number." If the meaning of this principle is made clear through appropriate illustrations there should be frequent reference to it in the reduction of fractions.

The reduction of fractions to the lowest common denominator

[5]Monroe, Walter S. Measuring the Results of Teaching. Boston: Houghton Mifflin Company, 1918, p. 112-13.

was specifically mentioned. As a partial corrective for this difficulty it may be pointed out that it is not imperative that the common denominator be the lowest one. Any denominator which is a multiple of each of the denominators will be satisfactory; the lowest is desirable only as a means for reducing the necessary calculations to a minimum. In some cases the reduction of fractions to a common denominator has been made artificially difficult by insisting upon the *lowest* common denominator.

Difficulty 18. How to teach multiplication of common fractions.

Most of the difficulties in common fractions occur when one of the factors is an integer or a mixed number. The multiplication of a fraction by a fraction does not usually constitute a serious difficulty but the multiplication of a mixed number by a mixed number was mentioned as being especially difficult.

Corrective. A fundamental principle which should be recognized in overcoming this difficulty is to recognize each type of example and to single it out for specific instruction. Training upon the multiplication of one fraction by another fraction will help pupils very little in multiplying one mixed number by another. The presentation of more than one procedure for doing examples will probably tend to confuse most pupils. Some teachers recommend that both integers and mixed numbers should be reduced to improper fractions and then the multiplication may be performed in exactly the same way as when the product of two fractions is required.

Difficulty 19. How to teach division of common fractions.

Most of the difficulties encountered in the division of common fractions center around the inversion of the divisor. The relative magnitude of the divisor and dividend is also a source of trouble. Many pupils are confused when the divisor is greater than the dividend.

Corrective. One means of correcting this difficulty which pupils encounter is to rationalize the process of division, i. e., train pupils to think through the whole process step by step. Although this suggestion has many advocates, it is not inappropriate to raise the question, "Is it desirable to rationalize the topic of division of fractions?" Suzzallo has stated that "The rationalization of a process which should be performed mechanically is merely to stir up unnecessary trouble, trouble unprompted by a demand of actual efficiency."[6] This state-

[6]Suzzallo. Henry. The Teaching of Primary Arithmetic. Boston: Houghton Mifflin Co., 1912, p. 64-65.

ment appears to be particularly applicable to the topic of division of fractions. The process should be performed mechanically and there is little or no advantage in understanding the reason for each step. It seems likely that the most effective way of controlling this difficulty is by training the pupils to invert the divisor and then multiply without attempting any explanations of the rule.

A number of teachers have found it helpful to analyze the written work of pupils for the purpose of determining the types of errors which they most frequently make. In such studies it has been found that certain types occur much more often than others. When a teacher knows the errors which her class tends to make she is able to focus her instruction upon their correction.

Difficulty 20. How to secure accuracy in placing the decimal point in products and quotients.

It may be pointed out that aside from the reading and writing of decimal fractions, this is the only serious difficulty which this topic involves. Otherwise decimal fractions may be treated much as integers.

Corrective. Investigation[7] has indicated that pupils tend to place decimal points by means of specific rules rather than by a single general rule. If this is the case there should be explicit training in each type of example. Some drill of course is provided when an example involving decimal fractions is worked but it is advisable to single out the activity of placing the decimal point and drill upon that. This can be done by using examples with products and quotients given correctly except for the decimal point. The pupils may then be required to insert the decimal point in the correct place. The following are types of exercises which have been found useful:

Multiplication

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657.2          67.50
   .7            .03
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46004          20250
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Division

.47 ⟌ 2758.9   Answer 587

8.2 ⟌ 38.54   Answer 47

In the case of multiplication the pupil merely inserts the decimal

[7] Monroe, Walter S. "The ability to place the decimal point in division," Elementary School Journal, 18:287-93, December, 1917.

point. In the division exercises he is required to write the quotient in the appropriate place and to insert the decimal point.

Another corrective which is used by some teachers is to require pupils to estimate the product or quotient. This procedure will tend to prevent gross errors. Checking the work is also a means of detecting errors but in general will require considerable time.

CHAPTER IV

DIFFICULTIES RELATING TO DENOMINATE NUMBERS AND PROBLEM SOLVING

Difficulty 21. How to present the study of denominate numbers so that pupils will be interested in the topic and will remember the facts.

As stated this is a general difficulty. In practice, it is likely that pupils may be interested in some denominate numbers and not in others. Thus the following discussion of correctives should be thought of as applying to those denominate numbers which given groups of pupils do not find interesting.

Corrective. In the earlier grades, interest in tables of denominate numbers can be secured by utilizing practical problems which arise in the regular work of the school and by playing such games as store, bank, etc. The teacher may appropriately manipulate the activities of the children so that a need for denominate numbers will be created.

In the intermediate grades where denominate numbers are studied more systematically the use of concrete problems will serve to engender interest in many pupils. It is advisable to show measuring instruments such as a foot rule, yardstick, pint measure, gallon measure, bushel measure, etc. Some teachers have pupils measure a liquid in order to determine the relation between pint and quart, quart and gallon, etc. If the pupil is given a pint measure and a quart measure he will find that two pints of water will fill the quart measure. Information can be derived in the same way from the quart measure and the gallon measure.

Another means of overcoming this difficulty is by eliminating from the course of study all obsolete or obsolescent measures. It appears that frequently interest is lessened because pupils are asked to learn tables of denominate numbers which are no longer used.

Difficulty 22. How to make pupils understand that the units of square measure are used to measure areas.

Pupils have difficulty in identifying the table of square measure as a means of describing areas. They are also confused over the products of the dimensions of a rectangle being expressed in terms of a square unit.

Corrective. Doubtless one cause for this difficulty is the insistence that the multiplier is always an abstract number and that the product must be of the same denomination as the multiplicand. Although there is logical justification for this principle, for practical purposes we may justify the conflicting principle that feet multiplied by feet gives square feet and that the area expressed in terms of square units is the product of the two dimensions.

As in the case of many other difficulties, the fundamental rule to be observed is to single out the difficulty and give specific attention to overcoming it. In the field of arithmetic where the outcome of instruction consists largely of habits, repetition must be provided for.

Difficulty 23. How to teach cubic measure.

The difficulty in the case of cubic measure is similar to that mentioned above for square measure.

Corrective. It is desirable to have a number of blocks whose dimensions are one inch. With a sufficient number of these blocks a cubic foot can be built up and parallelepipeds of various dimensions can be constructed. Pupils, who have difficulty in understanding the relation between cubic content and the unit of measurement, may then count the number of cubic inches in a cubic foot or in the particular parallelepiped under construction. They will probably grasp the idea that a convenient way of counting these blocks is to ascertain the number in the top layer and then to multiply this by the number of layers. In this way the pupil should easily grasp the rule that volume is equal to the product of length by width by depth or height.

Difficulty 24. How to overcome the pupil's lack of self-reliance in solving problems.

A lack of self-reliance is frequently indicated by the pupils when he erases all of his work and begins anew on the solution. Some pupils "give up." Still others are content to accept any answer which they obtain.

This difficulty is not easy to overcome, because the trait exhibited functions in other activities. In many cases it is characteristic of the pupil. The teacher faces the problem of causing the pupil to acquire self-reliance not only in problem solving but in other phases of his school work and even in the things which he does outside of school.

Corrective. In our emphasis upon the need for training pupils to study, it has been observed that teachers should not give too much assistance. If pupils do not have an opportunity to face their difficulties and to overcome them they will never become self-reliant. It is sometimes advisable to allow pupils to make mistakes because they need the experience of determining the corrections. They should be given time to think. The solving of problems is something which can not be done in a mechanical way. Timid pupils are frequently discouraged because they are not given a chance to complete their problems. The teacher's task is to stimulate and direct the learning activities of his pupils; and whenever he does any of the work for them he thereby deprives them of an opportunity to learn. It is likely that he also tends to destroy their self-reliance.

Difficulty 25. How to teach pupils to solve problems.

Many pupils fail to grasp the idea of problem solving. Some of them think of it as a mechanical application of certain rules or the duplication of a procedure illustrated in a problem worked out in the text. Still others think of it as a trial and error process in which they perform certain operations upon certain numbers in the hope that they will secure an answer which will be accepted as correct.

Corrective. A fundamental principle which should be recognized in dealing with this difficulty is that pupils must be trained to *solve* problems rather than to secure correct answers. The emphasis should constantly be placed upon learning how to solve problems rather than upon the answers obtained. A pupil may learn a great deal about the solving of problems even when he is failing to secure a correct answer. Furthermore, by doing a very small number of exercises, he may learn much about the solving of problems.

In training pupils to solve problems the teacher should have clearly in mind the steps of the process.

(1) The first step is to read the statement of the problem. In this statement there are two kinds of words, first, those which describe the setting of the problem or the particular environment in which it occurs, and second, those which define quantities or quantitative relationships. The second class of words may be called "technical" and precise meanings must be associated with them. The reading of a problem is a complex process and generally a higher

degree of comprehension is required than in our reading of ordinary printed material.

(2) The principles applicable to the problem must be recalled. For example, in the problem "A man invests $893 in property. He sells the property for $1050. What is his rate of profit?", it is necessary for the pupil to recall the principle that the rate of profit is calculated upon the amount invested, and not upon the selling price.

(3) The meaning of the technical words and the principles recalled are used in formulating a plan of procedure for solving the problem. In doing this one must not overlook words which may appear inconspicuous in the statement of the problem but which, nevertheless, are important in specifying the relationships between the quantities.

(4) The plan of solution must be verified. In some cases this verification may be made before calculations are begun. Sometimes the pupil will be unable to verify his plan until he has worked the problem through.

The performance of the operations specified in the plan of solution is not a part of the reasoning process. The reflective thinking has been completed when the plan of solution has been formulated.

Difficulty 26. How to train pupils to make accurate statements.

Many of the errors in arithmetic are due to permitting inaccurate statements of problems in the classroom. One authority has said that, "It is the loose manner of writing out solutions, tolerated by many teachers that gives rise to half the mistakes in reasoning which vitiate the pupils' work and teachers are coming to realize that inaccuracies of statement tend to beget inaccuracy of thought and so should not be tolerated in the classroom."[1]

Corrective. It is likely that careless reading of problems and of explanations is one cause for inaccurate statements by pupils. Insistence upon correct reading will do much to overcome this difficulty. The teacher should also insist that the pupils attach precise meanings to the technical terms used in the statements of problems and in the explanation of the procedures employed in their solution. Investigation has shown that pupils use many words without being sufficiently acquainted with their meaning.

The correction of the tendency to make inaccurate statements is to be accomplished in much the same way as the correction of other

[1]Brown, J. C., and Coffman, L. D. How to Teach Arithmetic. Chicago: Row, Peterson and Co., 1914, p. 44-45.

errors. There must be painstaking attention to details on the part of the teacher and no errors should be permitted to pass unnoticed.

Difficulty 27. How to train pupils to read problems.

It has frequently been said that pupils would have much less difficulty in their study of arithmetic if they knew how to read problems. Investigations have shown that children are very careless in the reading of ordinary prose. Usually they are not able to read critically and carefully when the material is very simple. However, arithmetic problems represent a special type of reading matter and a peculiar type of comprehension is required.

Corrective. The teacher of arithmetic must assume some of the responsibility for training pupils to read the problems. It is frequently said that children do not know the meanings of such technical words as "factor," "product," "value," "per pound," "are obtained," etc. These words define relationships which exist between the quantities and are cues for formulating the plan of solution. The teacher must take into consideration the fact that the vocabulary is different from the one used in ordinary conversation. The child may have heard of such words as "average," "cost," and "profit" but when asked *to find the average, to find the cost,* etc., he is confronted with a problem that can not be solved from his general knowledge. *To find the average* a certain definite procedure is required. The vocabulary in arithmetic should receive the same treatment as a vocabulary in a foreign language, i. e., it should be taught.

One teacher stated that she tried to get her pupils to read the problems orally with as much expression as they would use in reading a story. This device may prove helpful in many cases but, in general, problems will be read silently rather than orally. One teacher frequently reminded her pupils that a "problem always tells you directly or indirectly what you are to do if you understand the statement of it."

Difficulty 28. How to teach pupils to give a good oral explanation of problems.

Although the explanation of problems has occupied a very prominent place in arithmetic teaching, it is not inappropriate to ask the question, "Is it necessary to have problems explained orally?" The explanation of a problem is justified only if it assists the pupil giving the explanation or his classmates in learning how to solve the problem. Whenever the explanation becomes merely an end in itself and is mechanical it serves no useful purpose. The writer has observed many teachers who required an explanation of the process of working an example in multiplication or division. In one instance all the pupils went to the board and an exercise in multiplication was given. All performed the necessary calculation and then one child was called upon "to explain the problem." The "problem" (exercise) was to multiply 4569 by 43. The child proceeded—"3 times 9 are 27, put down the 7 and carry the 2, 3 times 6 are 18 and 2 are 20, put down the 0 and carry the 2, 3 times 5 are 15 and 2 are 17, put down the 7 and carry the 1," etc. It is doubtful if such a procedure is beneficial either to the one who gives the explanation or to those who listen.

Corrective. Instead of a formal explanation the teacher and other members of the class may ask the pupil why he did this or that. In the case of a problem which has not been solved by other members of the class, the pupil should feel that he has the responsibility of making his classmates understand how he solved the problem and the reasons for his procedure. Occasionally formal explanations may be justified but not as a usual rule. When a formal explanation is given the pupils should be required to present it effectively and to use good English.